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A Two Dimensional Jet Flowing into a Semi-Infinite Flow Field with an Ambient Velocity

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A TWO DIMENSIONAL JET FLOWING INTO A SEMI-INFINITE FLOW FIELD WITH AN AMBIENT VELOCITY

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ABSTRACT

The circulation of the Chesapeake effluent into the Atlantic coastal waters was simulated with a steady state model which described the flow pattern averaged over several tidal cycles. Considering the surface circulation only, the flow field was simulated by a two dimensional jet flowing into a semi-infinite flow field with an ambient velocity. The Navier-Stokes equations were transformed to a coupled set consisting of a dynamic vorticity equation and a stream function/vorticity equation, and the coupled equations were non dimensionalized. The non-dimensionalized equations were written in finite difference form and solved numerically for the appropriate boundary conditions. In particular, the circulation patterns for the steady state were developed for various computational parameters. These parameters were divided into two categories: one, the parameters that characterize the flow; and two, the computational parameters that determine the stability and efficiency of the model. Two computational techniques were investigated, the boundary value scheme and the explicit time dependent scheme. The results were presented for the boundary value scheme and the circulation patterns were discussed.

A TWO DIMENSIONAL JET FLOWING INTO A SEMI-INFINITE FLOW FIELD WITH AN AMBIENT VELOCITY

I INTRODUCTION

Early studies of the Atlantic coastal currents have emphasized the general circulation from Florida to Nantucket, followed by studies of the currents of sections of the Atlantic coast. One of the sections is the region from Cape Henlopen to Cape Hatteras. A major component of the surface circulation is the effluent of the Chesapeake Bay. The circulation of Chesapeake Bay water into the Atlantic Ocean influences the distribution and abundance of the fisheries, the flushing rate of the Chesapeake Bay, and the erosion and deposition of beach material of the Atlantic coast.

Miller (1952) used drift bottles and reported the formation of gyres in the coastal waters off Cape Charles and off Cape Henry. Bumpus and Lazarier (1965) computed surface currents from drift bottle recoveries in a study of the Atlantic from Nantucket to Cape Hatteras. The range of the speeds measured by Bumpus and Lazarier (1965) was 1 to 10 nautical miles per day and generally to the south to southwest in direction.

Norcross and Stanley (1967) proposed a general circulation pattern for each month from drift bottle recovery data. The area covered seven zones paralleling the coast, with the velocity inferred from the frequency and location of drift bottles recovered. They noted a strong surface wind dependence in the summer for the surface currents, with a weak reversal of the currents in June, July and August.

Bumpus, Lynde and Shaw (1973) published an atlas of oceanographic data of the Atlantic Coastal Zone from Nantucket to Cape Hatteras, including a summary of surface current measurements. For each month, the average surface velocities ranged from 1 to 15 nautical miles per day, to the south and southwest, with an average speed of 10 nautical miles per day. The currents measured at the fixed locations of the lightships at the entrance of Chesapeake Bay and at Diamond Shoals agree with both the speed and direction computed by drift bottle methods.

Bue (1970) computed the average flux over a tidal cycle through the mouth of the Chesapeake Bay from the fresh water runoff in the entire drainage basin. He determined the flux to be 75,000 c.f.s.

Thus far the surface circulation has been measured on a scale that does not illustrate the detailed circulation patterns. The interest in the detailed circulation has motivated a study of the dynamics of the circulation. This study investigated the steady state circulation of the Atlantic Coastal region contiguous to the mouth of the Chesapeake Bay. Particular attention was paid to the conditions of the existence or absence of the recirculating gyres, that were speculated by the previous investigators. The region was simulated as a two dimensional jet entering a flow field with an ambient velocity. The main features of the circulation were determined by the ambient velocity of the oceanic drift and the velocity of the Chesapeake effluent through the Bay mouth. For the idealized jet the two dimensional Navier-Stokes equations govern the surface description when neglecting all variation with depth. The Navier-Stokes equations were transformed to a coupled set consisting of a dynamic vorticity equation and a stream function/vorticity equation, and the coupled equations were non dimensionalized. The non-dimensionalized equations were written in finite difference form and solved numerically for the appropriate boundary conditions. Figure 1 defines the limits of the computational region.

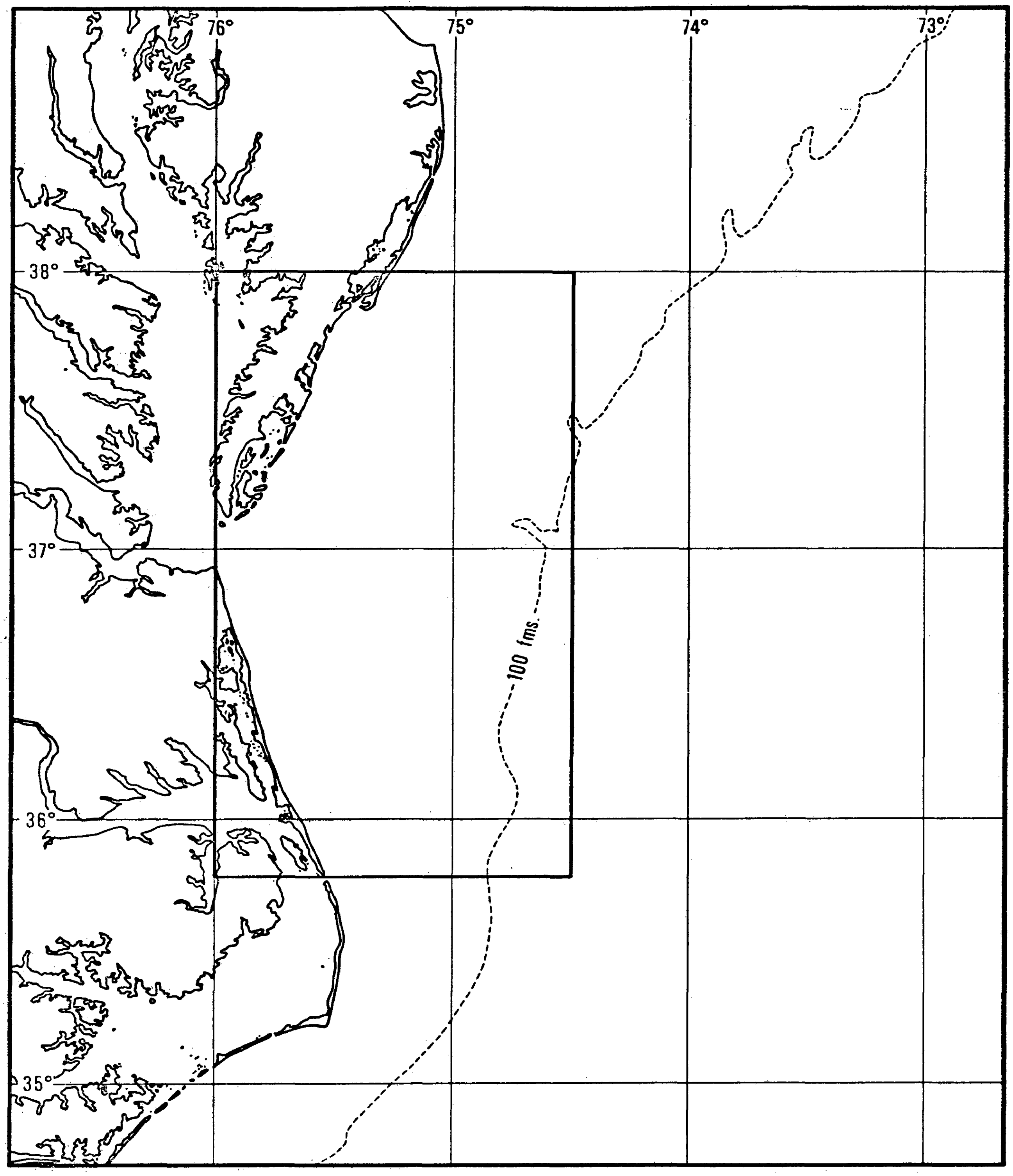


Figure 1. Computation region at the mouth of the Chesapeake Bay

II EQUATIONS

The dynamic equations for incompressible flow include the frictional forces, the inertial terms, and the affect of the rotation of the earth. The two dimensional form is given by the two equations:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - 2\Omega v\sin\theta = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right], \quad (1)$$

and

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + 2\Omega u\sin\theta = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right], \quad (2)$$

where u is the velocity in the x direction (east-west), v the velocity in the y direction (north-south), t the time, ρ the density, p the pressure, Ω the angular velocity of the earth, θ the latitude, and ν the kinematic viscosity, or eddy viscosity in the case of turbulent flow.

The continuity equation for incompressible flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

Because equation (3) holds everywhere in the flow field, the velocity components can be written in terms of a stream function such that

$$u = \frac{\partial \psi}{\partial y} \quad (4a)$$

and

$$v = -\frac{\partial\psi}{\partial x} \tag{4b}$$

The asymmetry of the flow determines the local rate of rotation and this local rotation is defined as the local vorticity, as

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{5}$$

or using equations (4a) and (4b),

$$\omega = -\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2} \tag{6}$$

To simplify the notation, let $2\Omega\sin\theta = f$ and note that f is a function of y only.

To eliminate the pressure term, take the partial derivative of y for equation (1), the partial derivative of x for equation (2), and subtract the first equation from the second. Noting that the operations of differentiation are interchangeable and rewriting the equation, the two dynamic equations reduce to the single equation

$$\frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial x} + v\frac{\partial\omega}{\partial y} = -v\frac{df}{dy} + \nu\left(\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2}\right) \tag{7}$$

where ω is the vorticity. The term $v\frac{df}{dy}$ can be approximated by the term βv where β, beta plane approximation, equals the quantity $\frac{2\Omega\cos\theta}{R}$ where R is the radius of the earth. The dynamic equation becomes

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = -\beta v + \nu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) , \tag{8}$$

or

$$\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} = \beta\frac{\partial \psi}{\partial x} + \nu\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) \tag{9}$$

To insure generality, the equations are non-dimensionalized.

The scaling factors are L_o, the width of the mouth of the Chesapeake Bay and V_o, the ambient velocity of the Atlantic Ocean. All quantities can be non-dimensionalized through these scales as following:

$$t' = \frac{t}{L_o/V_o} \qquad v' = v/V_o$$

$$\psi' = \psi/L_oV_o$$

$$x' = x/L_o$$

$$\beta' = \frac{\beta}{V_o/L_o^{\,2}}$$

$$y' = y/L_o$$

$$u' = u/V_o \qquad 1/Re = \nu/L_oV_o$$

Substituting these nondimensionalized quantities into equation (9) yields,

$$\frac{\partial \omega'}{\partial t'} + \frac{\partial \psi'}{\partial y'}\frac{\partial \omega'}{\partial x'} - \frac{\partial \psi'}{\partial x'}\frac{\partial \omega'}{\partial y'} = \beta'\frac{\partial \psi'}{\partial x'} + \frac{1}{Re}\left(\frac{\partial^2 \omega'}{\partial x'^2} + \frac{\partial^2 \omega'}{\partial y'^2}\right) \tag{10}$$

and substituting into equation (6) yields

$$\omega' = -\frac{\partial^2\psi'}{\partial x'^2} - \frac{\partial^2\psi'}{\partial y'^2} \tag{11}$$

All succeeding equations will be derived from these non-dimensional forms and the primes will be dropped for convenience. The finite difference form of equations (10) and (11) is defined for a grid with constant spacing. A typical section is given in Figure 2.

For the steady state solution, the finite difference form of equation (10) is

$$-\left(\frac{\psi(i,j+1) - \psi(i,j-1)}{2h}\right)\cdot\left(\frac{\omega(i+1,j) - \omega(i-1,j)}{2h}\right)$$
$$+\left(\frac{\psi(i+1,j) - \psi(i-1,j)}{2h}\right)\cdot\left(\frac{\omega(i,j+1) - \omega(i,j-1)}{2h}\right)$$
$$= \beta\left(\frac{\psi(i+1,j) - \psi(i-1,j)}{2h}\right) +$$
$$\frac{1}{Re}\left(\frac{\omega(i+1,j) + \omega(i-1,j) + \omega(i,j-1) + \omega(i,j+1) - 4\omega(i,j)}{h^2}\right) \tag{12}$$

For equation (11) the form is

$$\omega(i,j) = -\frac{\psi(i+1,j)+\psi(i-1,j)+\psi(i,j+1)+\psi(i,j-1)-4\psi(i,j)}{h^2} \tag{13}$$

Equation (12) is regrouped as

$$\frac{4}{Re}\,\omega(i,j) = \left(\frac{1}{Re} + \frac{\psi(i,j+1) - \psi(i,j-1)}{4}\right)\cdot\omega(i+1,j)$$
$$+\left(\frac{1}{Re} - \frac{\psi(i,j+1) - \psi(i,j-1)}{4}\right)\cdot\omega(i-1,j)$$

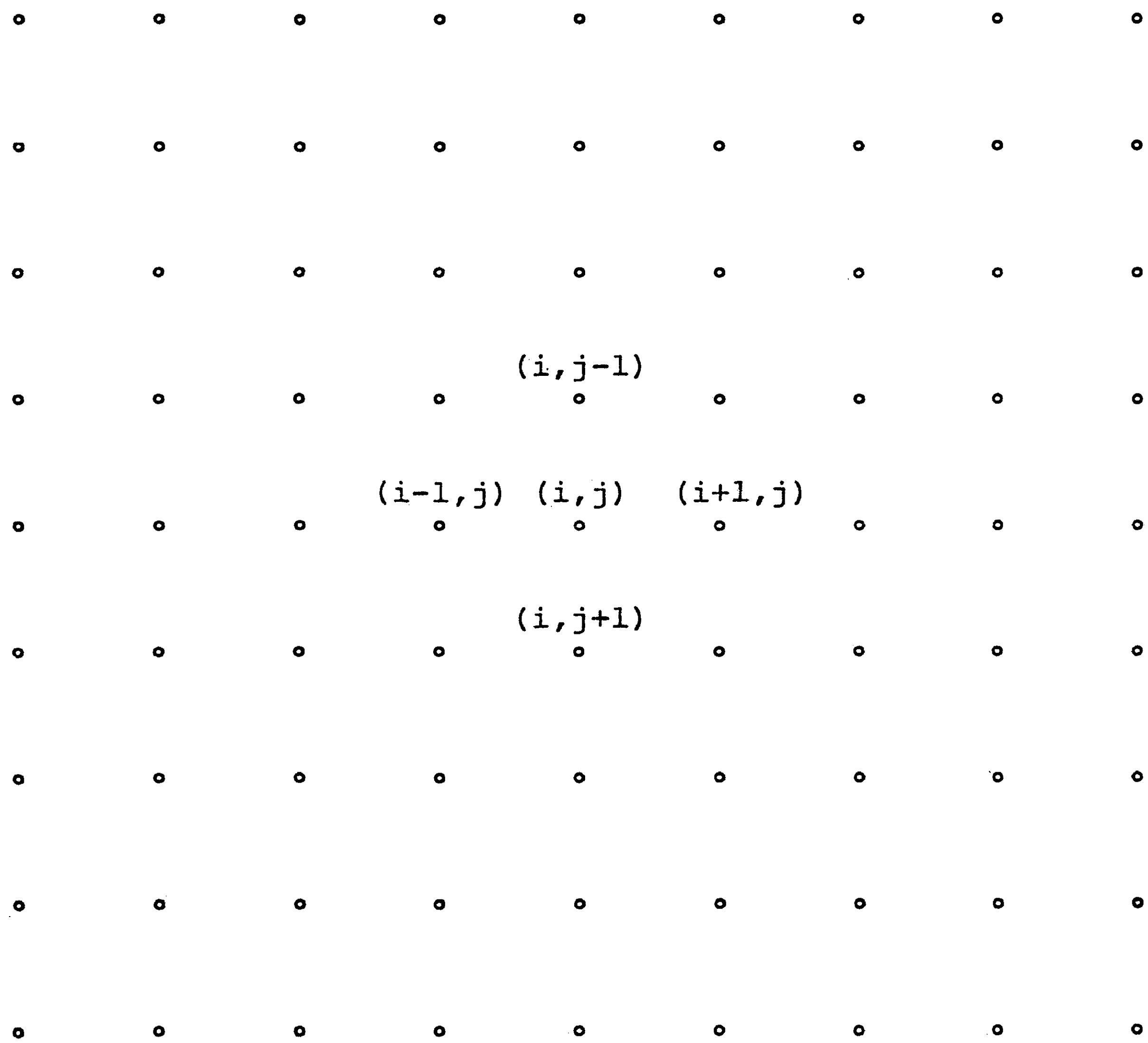


Figure 2. Indices for the computation grid.

$$+\left(\frac{1}{Re} - \frac{\psi(i+1,j) - \psi(i-1,j)}{4}\right) \cdot \omega(i,j+1)$$
$$+\left(\frac{1}{Re} + \frac{\psi(i+1,j) - \psi(i-1,j)}{4}\right) \cdot \omega(i,j-1)$$
$$+ \beta h\left(\frac{\psi(i+1,j) - \psi(i-1,j)}{2}\right) \qquad (14)$$

The equation (13) is regrouped as

$$\psi(i,j) = \frac{\psi(i+1,j)+\psi(i-1,j)+\psi(i,j+1)+\psi(i,j-1)+h^2\omega(i,j)}{4} \qquad (15)$$

These are the two equations to compute values for the vorticity, equation (14), and the stream function, equation (15), in the interior. The values on the boundaries must be treated separately, incorporating the boundary conditions with the particular geometry.

For the explicit time dependent scheme the term $\frac{\partial \omega}{\partial t}$ will be included in the dynamic equation and the finite difference form is $\frac{\omega_n(i,j) - \omega_o(i,j)}{2\tau}$. Transforming to the finite difference form for central time differences, the equation becomes, for τ the non dimensional time step,

$$\frac{\omega_n(i,j) - \omega_o(i,j)}{2\tau}$$
$$-\left(\frac{\psi(i,j+1) - \psi(i,j-1)}{2h}\right) \cdot \left(\frac{\omega_p(i+1,j) - \omega_p(i-1,j)}{2h}\right)$$
$$+\left(\frac{\psi(i+1,j) - \psi(i-1,j)}{2h}\right) \cdot \left(\frac{\omega_p(i,j+1) - \omega_p(i,j-1)}{2h}\right)$$

$$= \beta \left[\frac{\psi(i+1,j) - \psi(i-1,j)}{2h} \right]$$

$$+ \frac{1}{Re} \left[\frac{\omega_p(i+1,j) + \omega_p(i-1,j) + \omega_p(i,j+1)}{h^2} \right.$$

$$\left. + \frac{\omega_p(i,j-1) - 2\omega_n(i,j) - 2\omega_o(i,j)}{h^2} \right] \quad (16)$$

where ω_o refers to vorticity for the previous time step, ω_p refers to vorticity for the present time step and ω_n refers to vorticity for the new time step, with all other terms having the same definition as in the boundary value scheme.

The equation (16) is rewritten in the computational form of the equation as

$$\left(\frac{2}{Reh^2} + \frac{1}{2\tau} \right) \omega_n(i,j) = \left(\frac{1}{2\tau} - \frac{2}{Reh^2} \right) \omega_o(i+1,j)$$

$$+ \left(\frac{1}{Reh^2} + \frac{\psi(i,j+1) - \psi(i,j-1)}{4h^2} \right) \cdot \omega_p(i+1,j)$$

$$+ \left(\frac{1}{Reh^2} - \frac{\psi(i,j+1) - \psi(i,j-1)}{4h^2} \right) \cdot \omega_p(i-1,j)$$

$$+ \left(\frac{1}{Reh^2} - \frac{\psi(i+1,j) - \psi(i-1,j)}{4h^2} \right) \cdot \omega_p(i,j+1)$$

$$+ \left(\frac{1}{Reh^2} + \frac{\psi(i+1,j) - \psi(i-1,j)}{4h^2} \right) \cdot \omega_p(i,j-1)$$

$$+ \beta \left(\frac{\psi(i+1,j) - \psi(i-1,j)}{2h} \right) \quad (17)$$

The stream function that corresponds to this format is given as

$$\psi(i,j) = \frac{\psi(i+1,j)+\psi(i-1,j)+\psi(i,j+1)+\psi(i,j-1)+h^2\omega_p(i,j)}{4}. \tag{18}$$

III COMPUTATION TECHNIQUE

Fromm (1963) and Fromm and Harlow (1963) described a method to solve the central time difference equations (17) and (18) explicitly for a two-dimensional grid with constant spacing. Using this technique as a method for solving the equations, the problem, defined as a jet flowing into a flow field with ambient velocity, can be solved numerically. The geometry of this problem is given in Figure 3 for the computation region CDEF, with the jet entering through the opening AB. The constant grid spacing h is defined as the distance AB divided by the number of intervals between AB. The distance from C to D and from F to E is Nh and the distance from C to F and from D to E is Mh where N is the number of intervals in the x direction to the right boundary and M is the number of intervals from the top to the bottom boundary. The boundaries from C to A and from B to F correspond to the coast lines to the north and south of the bay mouth.

To check solutions computed with the explicit time scheme, steady state solutions are computed and compared to results from computed solutions considering the problem as a boundary value problem. The solutions of boundary value problems can be computed from equations (14) and (15) by applying the boundary conditions and computing the solutions until they converge.

Computing the steady-state solutions by boundary value problems has several advantages: one, by considering the

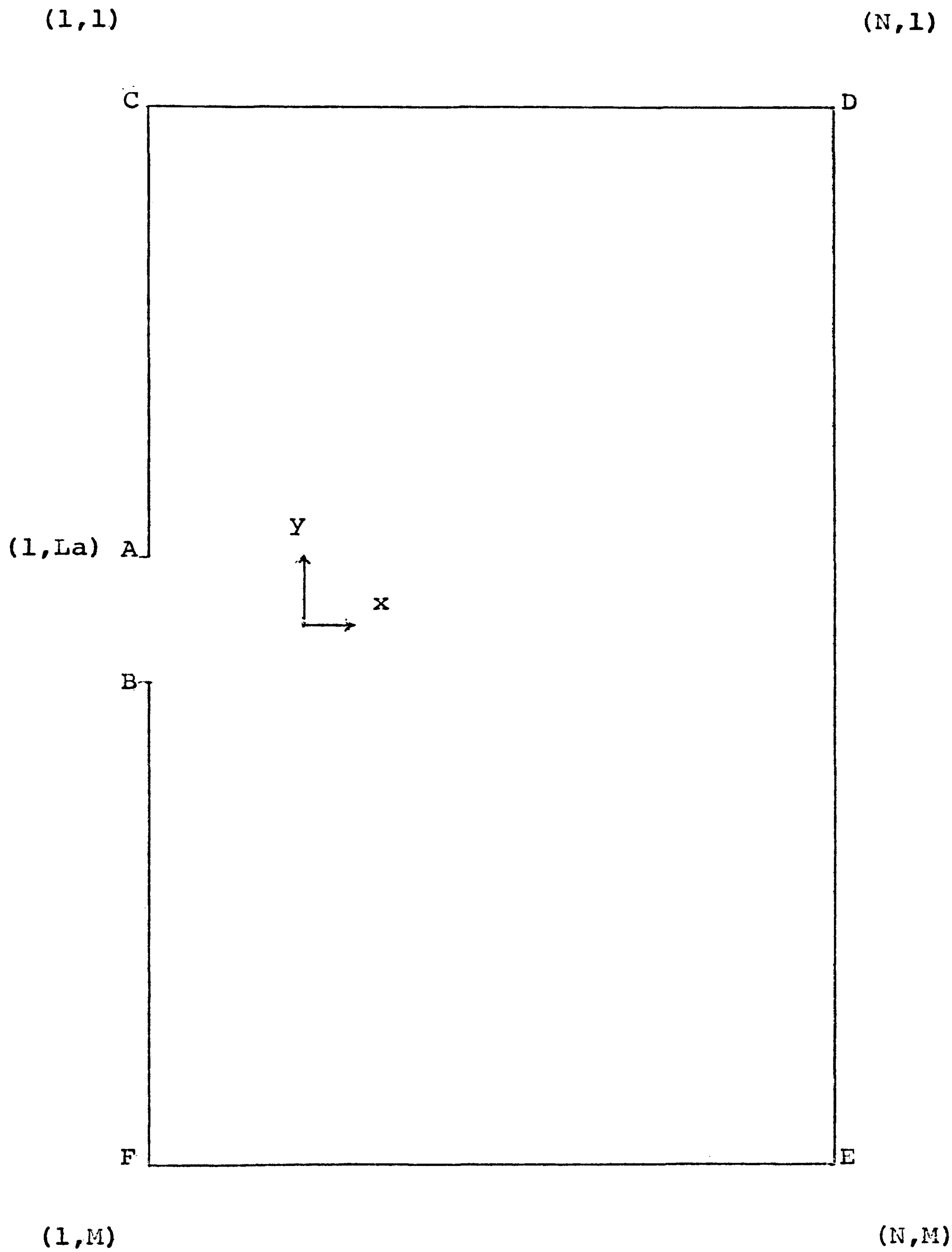


Figure 3. Computation region.

problem with one less variable, the solutions are less complicated; two, by examining the computational parameters, the optimum value for each one can be determined for the time dependent equation; and three, the efficiency and speed can be estimated by the boundary-value scheme.

The computation region given in Figure 3 holds for a two-dimensional jet flowing into a semi-infinite flow field with an ambient velocity. The boundary value problem for this region treats the flow as uniform along DE, with a prescribed velocity distribution along CD and with a prescribed velocity distribution in the jet AB. For the boundary CA and BF, the no slip velocity condition holds, and along FE, there is no advection in the x direction. The minimum domain to satisfy the boundary conditions and the grid spacing h define the number of grid points for the computation region.

In this scheme the equations (14) and (15) are solved until the values ψ and ω converge. To begin the calculation, both variables ψ and ω are assigned initial values at each grid point corresponding to a flow field with a parabolic layer from the wall and uniform flow extending from the boundary layer to the far right hand boundary.

The technique is outlined as follows:

(A) Compute the values for the vorticity at the wall.

(B) Compute the values for the vorticity in the interior using equation (14).

(C) Solve the stream function by iterating equation (15) to convergence.

(D) Compute the values for the vorticity at the wall.

(E) Compare the new wall vorticities with the previous values.

(F) If the values do not converge repeat steps B through E.

(G) Plot flow field if the values converge.

The flow diagram for the computer program is given in Appendix A.

To solve for the vorticity at the wall, one must apply the boundary conditions; i.e. $\frac{\partial\psi}{\partial x} = 0$ and $\frac{\partial\psi}{\partial y} = 0$. Because equation (14) does not apply at the boundary, a finite difference form must be found for the vorticity at the wall. The vorticity can be expanded in a Taylor series from the wall to the vorticity in the interior. Macagno and Hung (1970) described a particular expansion, including second order terms in the vorticity, with the following equation:

$$\omega(i,j) = \frac{3}{h^2}\left[\psi(i,j) - \psi(2,j)\right] - \frac{1}{2}\omega(2,j) + \frac{h^2}{8}\nabla^2\omega\Big|_{1,j} \qquad (19)$$

where $\nabla^2\omega$ is evaluated at the wall. The operator $\nabla^2\omega\Big|_{1,j} = 0$ for the steady flow.

Therefore, the vorticity at the wall is computed by the equation

$$\omega(1,j) = \frac{3}{h^2}\left[\psi(1,j) - \psi(2,j)\right] - \frac{1}{2}\omega(2,j) \qquad (20)$$

$$\omega(1,j) = -\left.\frac{\partial^2\psi}{\partial y^2}\right|_{1,j}$$ where the operator is evaluated from the top of the jet to the bottom of the jet for a prescribed function of ψ. The vorticity along the top is given as

$$\omega(i,1) = -\left.\frac{\partial^2\psi}{\partial x^2}\right|_{i,1}$$ for i varying from 1 to Lb where the operator is evaluated at the top, where the stream function ψ has a boundary layer with thickness Lb from the boundary. The flow is assumed to be uniform at the right boundary and the vorticity for uniform flow is zero. The bottom boundary is assumed to have no advection in the x direction, and expressed as a linear combination of values in the y direction near the bottom boundary.

Macagno and Hung (1970) described a method for predicting the values at the bottom boundary from the interior values. The vorticity values and the stream function values are given by

$$\omega(i,M) = \omega(i,M-4) - 2\omega(i,M-3) + 2\omega(i,M-1) \tag{21}$$

and

$$\psi(i,M) = \psi(i,M-4) - 2\psi(i,M-3) + 2\psi(i,M-1) \tag{22}$$

where the index i covers the entire interval from F to E.

At the beginning of each computation the boundary values are computed first and the interior values next. The vorticity is computed first, then the stream function is computed. The values at the boundary for the stream function are prescribed except at the bottom boundary which is given by equation (22).

The vorticity at the boundary is computed by equation (20) along the wall and equation (21) at the bottom and prescribing in the jet, the top and the far right boundary.

For the interior region equation (14) computes the vorticity values for that particular boundary value of the vorticity and the previous values of the stream function. This constitutes the next iteration for the vorticity.

The stream function is computed by iterating equation (15) over the interior and applying equation (22) at the bottom. To aid convergence, the following equation computes the stream function with over relaxation parameter ℓ,

$$\psi(i,j) = \psi_O(i,j) + \ell\left(\psi_T(i,j) - \psi_O(i,j)\right) \tag{23}$$

where $\psi_T(i,j)$ is the temporary value, computed by equation (15) and ψ_O is the stream function value from the previous iteration. The stream function value ψ_O is updated to the over-relaxed value ψ, i.e. $\psi_O(i,j) = \psi(i,j)$ and equation (15) is used to compute $\psi_T(i,j)$. This process is repeated until the stream function converges for the given boundary values of the stream function and the vorticity values at the interior. The test for convergence ε_1 is an error limit of the normalized difference of the stream function values between successive iterations, i.e. $\frac{\psi_T - \psi_O}{\psi_O}$ is less than ε_1.

This completes the iteration over both variables, ψ and ω. To begin the next iteration for the vorticity, first compute a new vorticity at the wall from equation (20). Checking the convergence of the method, the error limit ε_2 is compared to the normalized difference between the vorticity values at the boundary for successive iterations. When the values of the vorticity at the boundary have converged, the flow field features are characterized by the plots of the stream function. The steady-state solutions will form a basis for evaluating the time dependent solution.

The computational technique for the time dependent scheme is similar to the boundary value scheme. The computation region for the time dependent scheme is given by Figure 3. The flow is uniform at the right hand boundary, prescribed along the top and in the jet, and the boundary conditions at the wall are $\frac{\partial\psi}{\partial y} = 0$ and $\frac{\partial\psi}{\partial x} = 0$. The initial values for the stream function and the vorticity are determined by the boundary layer thickness from the wall. The flow diagram for the time dependent program is given in Appendix B.

For the time dependent case, the initial values are assigned to each variable and the jet applied impulsively, as in the boundary value technique. From this point, the first time step is computed for the vorticity by equation (17) and for the stream function by equation (18). Again, the convergence of the method is tested for each time step until the steady state solution is achieved. The computation begins with the values at the boundary positions, and then the interior is computed using equation (17) for the vorticity and equation (18) for the stream function.

For the time dependent scheme, the vorticity at the wall is solved using the method outlined by Macagno and Hung (1969) where the operator, $\nabla^2\omega\Big|_{(i,j)}$ evaluated at the boundary, is not necessarily zero. Because this operator is not zero, the vorticity values at the wall must be estimated, then iterated to convergence. The following finite difference equation estimates the vorticity at the wall

$$\omega_N(1,j) = \frac{3}{h^2}\Big(\psi(1,j) - \psi(2,j)\Big) - \frac{1}{2}\,\omega_N(2,j) \quad . \tag{24}$$

The following equation is iterated to convergence

$$\omega_N(1,j) = \frac{3}{h^2}\Big(\psi(1,j) - \psi(2,j)\Big) - \frac{1}{2}\,\omega_N(2,j)$$

$$+ \frac{1}{8}\Big(\omega_N(1,j+1)+\omega_N(1,j-1)-5\omega_N(2,j)+4\omega_N(3,j)-\omega_N(4,j)\Big) \tag{25}$$

where ω_N is the vorticity at the next time step. The test for convergence of the vorticity at the wall is ε_3.

After the boundary values are determined for the vorticity for the new time step, the interior values are determined by computing equation (17). The vorticity at the bottom boundary is given by

$$\omega_N(i,M) = \omega_N(i,M-4) - 2\omega_N(i,M-3) + 2\omega_N(i,M-1) . \qquad (26)$$

Now the stream function is computed using equation (18) and the boundary values of the stream function. This computation is similar to the boundary value scheme, with the vorticity value at the new time step used in equation (18). The stream function at the bottom boundary is given by equation (22) and the interior values are over relaxed by equation (23). The stream function converges when the normalized difference is less than ε_1.

The convergence of the method is a test of the error tolerance ε_2 with the normalized difference between the vorticities at the boundary for the present time step and the new time step. The procedure repeats for the vorticity and stream function until the vorticity converges. The convergent solutions will be plotted and compared to the boundary value scheme.

IV RESULTS

Preliminary runs establish the size of the computation region given in Figure 3. To satisfy the boundary conditions of zero velocity at the wall, a parabolic boundary layer is assumed initially along the wall. The first velocity profile of the jet is parabolic. The grid spacing h, determined by the number of intervals to describe the width of the jet, is a constant. The values ε_1 and ε_2 are chosen for a particular error tolerance for the solutions.

The computation time depends on the total number of grid points and the number of iterations to achieve convergence for the stream function at each step. To minimize the computation time, the domain is minimized and the convergence criteria is chosen to minimize both the error and the computation time. The domain was determined by the potential flow solution of a jet with a uniform velocity entering a flow field with a uniform cross stream. The distance in the x-direction was 10.0 from the wall, and the distance in the y-direction was 9.5 above and below the axis of the jet. The jet velocity u_o was 1.0 and the cross stream velocity v_o was -1.0. The minimum domain was the minimum distance in each direction that satisfied the boundary conditions for the problem. For the minimum domain the distance in the x-direction was 4.0 and the distance in the y-direction was 6.5 above and below the axis of the jet. Figure 4 outlines the potential flow for the minimum domain.

After limits are established for the computation region,

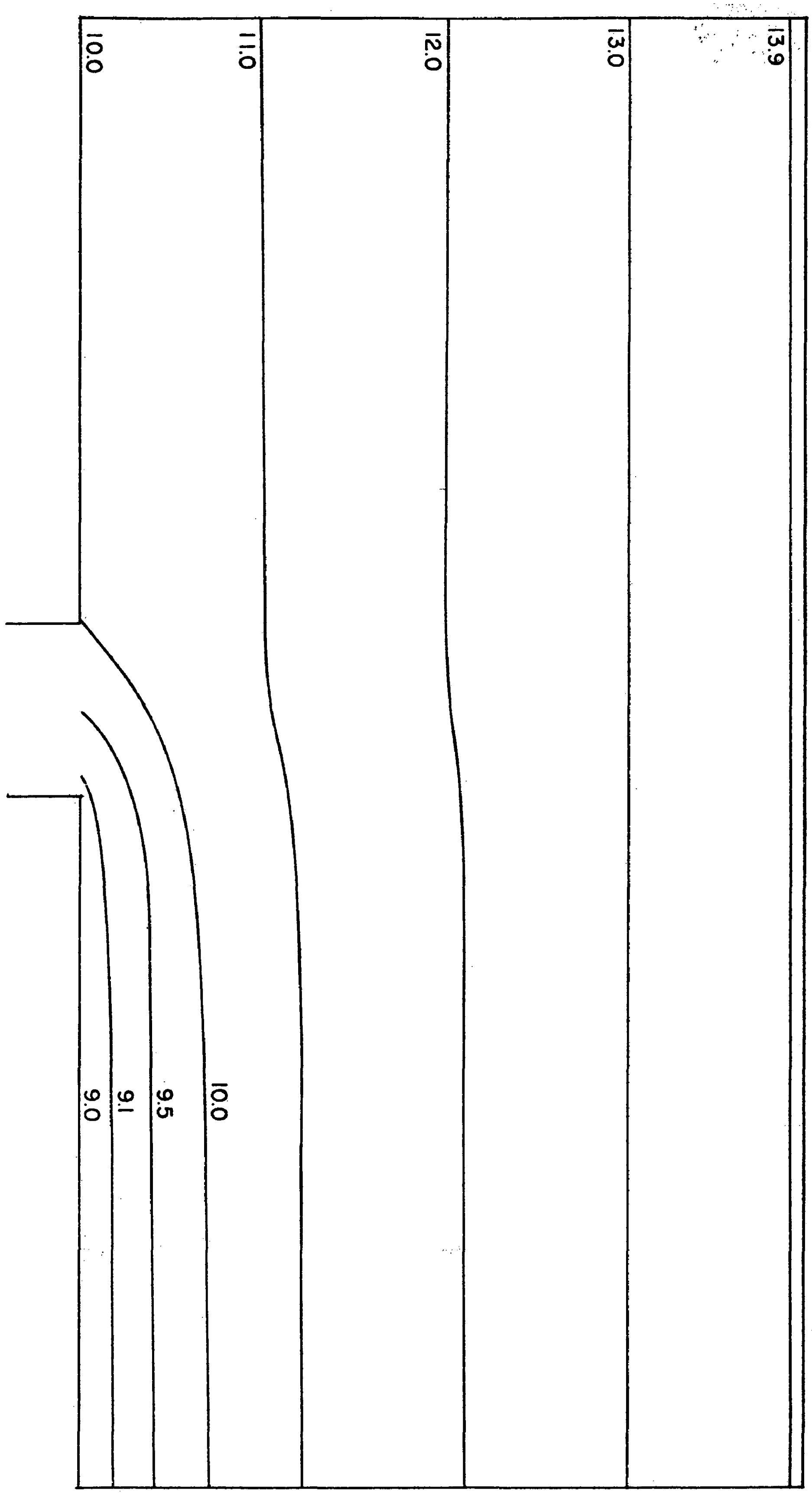


Figure 4. Stream function graph of the potential flow solution.

typical values for the Chesapeake Bay are assigned to the parameters, and the prescribed flow field at the top boundary and at the jet are chosen. The parabolic velocity profile of the top boundary depends on the boundary layer thickness, Lb. The parabolic velocity profile with center velocity u' is weighted so that the total discharge of the jet approximates the flux through the cross section of the Chesapeake Bay mouth determined by Bue (1970). Table I summarizes the non-dimensionalized parameters characteristic of the Chesapeake Bay region.

The boundary-value scheme does not converge for typical values of the region. The vorticity values at the wall near the jet oscillate during the computation from one iteration of the vorticity to the next iteration. To reduce the gradient of the vorticity along the wall and in the jet, a fourth order velocity profile for the jet is used in the computation. The boundary scheme for this jet velocity profile does not converge either, because the values of the vorticity at the wall do not converge.

The next step is to define the range of each parameter where the boundary value technique will be stable and converge to a solution. The problem is reduced by considering a non rotating coordinate system and finding the range of the other parameters for a non rotating coordinate system, i.e. $\beta = 0$. The dynamic parameters are assigned minimal values, but the computational parameters remain the same. The computational parameters are the size of the computation region, the location

TABLE 1

The values of the non-dimensional parameters for the Chesapeake Bay

Non-dimensionalized parameter	Value	Reference
u'	0.05	Bue (1970)
v'	-1.0	Bumpus et al (1973)
β'	3×10^{-3}	
Re	300	Bowden (1962)
Lb	.5	
h	.1	
$\varepsilon_1, \varepsilon_2$	10^{-4}	
ℓ	1.6	
τ'	1.0*	Harlow and Amsden (1971)

* for time dependent scheme only

of the jet axis, the convergence criteria, ε_1 and ε_2, the grid size h, and the over relaxation parameter ℓ. The dynamic parameters are the center velocity of the jet u_o, the velocity profile of the jet, the boundary layer width Lb, and the viscosity related by the Reynolds number Re. For all cases, the ambient velocity of the computation region is V_o which is equal to -1.0, the scaling factor in the velocity domain in the -y direction. Table II summarizes the range of values for a non rotating coordinate system and lists the figure number for the plot of the stream function corresponding to those values for each parameter.

From Figures 5 through 8 the stream lines define the velocity and the features of the circulation of several cases of the dynamic parameters. To illustrate the asymmetry of the flow field and advection of the vorticity, Figure 9 is the vorticity regime with the same dynamic parameters as those in Figure 5.

The large, negative vorticity values at the bottom of the jet at the wall mark a strong shear indicated by the stream lines in Figure 5 and by the vorticity contours contained in Figure 9.

In Figure 10 the vorticity contours correspond to the flow field in Figure 8. The solutions would not converge for Reynolds number greater than 50 for a fourth order jet with a center velocity magnitude 0.05. For smaller Reynolds number the method converged for jets with magnitude 0.5 and 1.0. For larger Reynolds numbers, the method does not converge for any

Table II. Non dimensional computational parameters

Figure number	5	6	7	8
u'	0.5	0.5	0.05	0.05
v'	-1.0	-1.0	-1.0	-1.0
β'	0	0	0	0
Re	1.0	1.0	20.0	50.0
Lb	0.5	3.8	3.8	3.8
h	0.1	0.1	0.1	0.1
ε_1	10^{-4}	10^{-4}	10^{-4}	10^{-4}
ε_2	10^{-2}	10^{-2}	10^{-2}	10^{-2}
ℓ	1.6	1.6	1.6	1.6
Computation size *	4.0X13.0	4.0X13.0	4.0X13.0	4.0X13.0

* jet centered on left wall

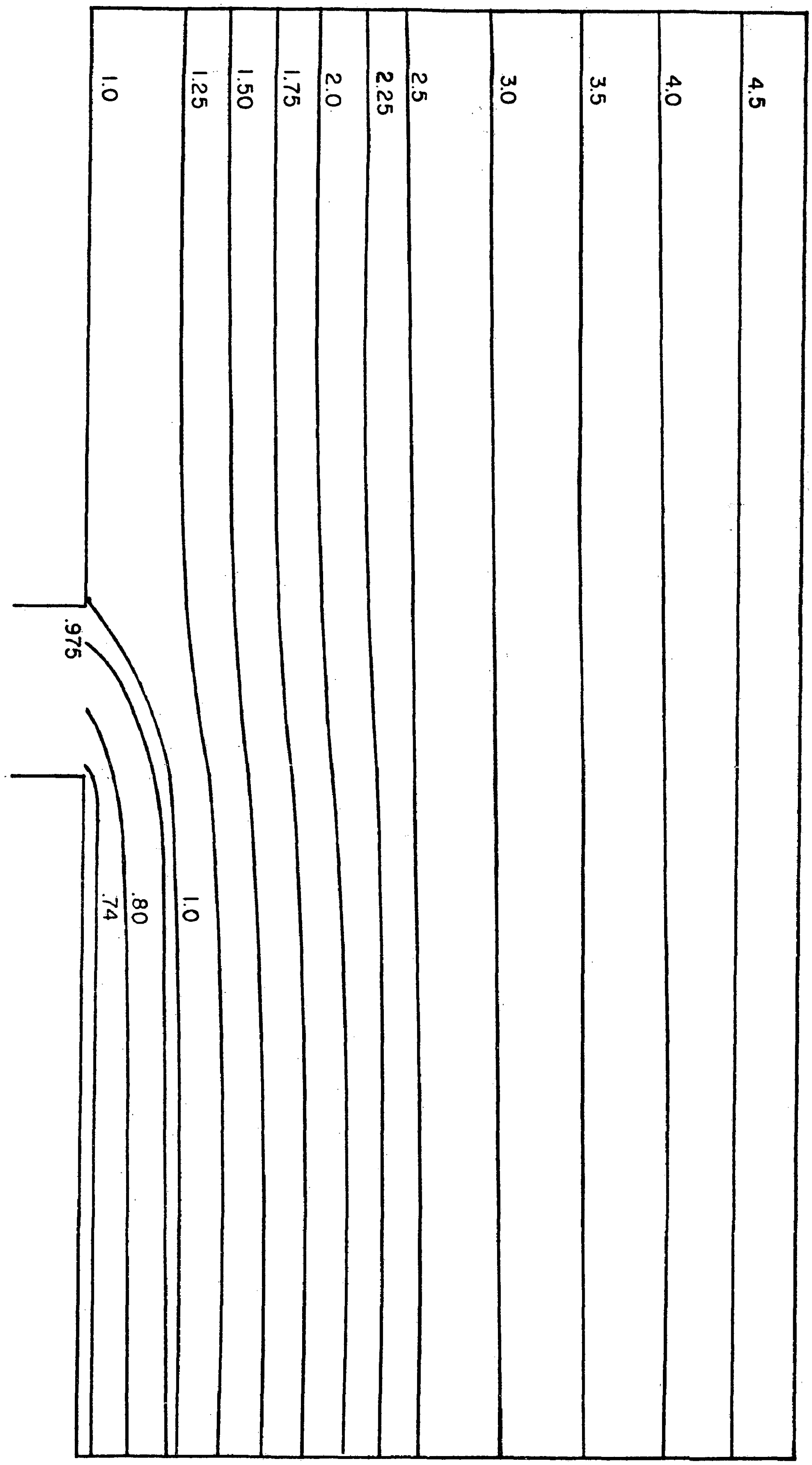


Figure 5. Stream lines from boundary value scheme.

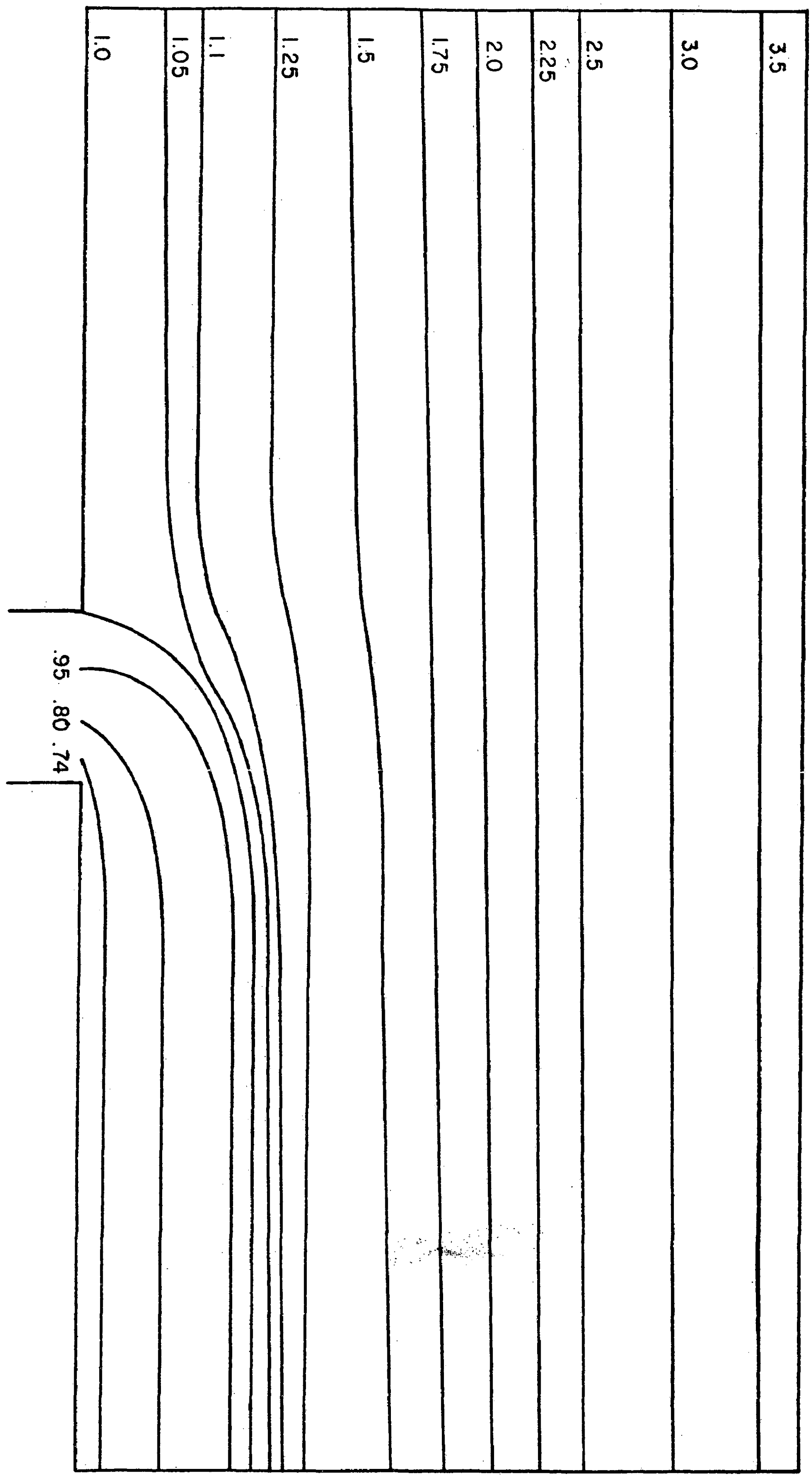


Figure 6. Stream lines from boundary value scheme.

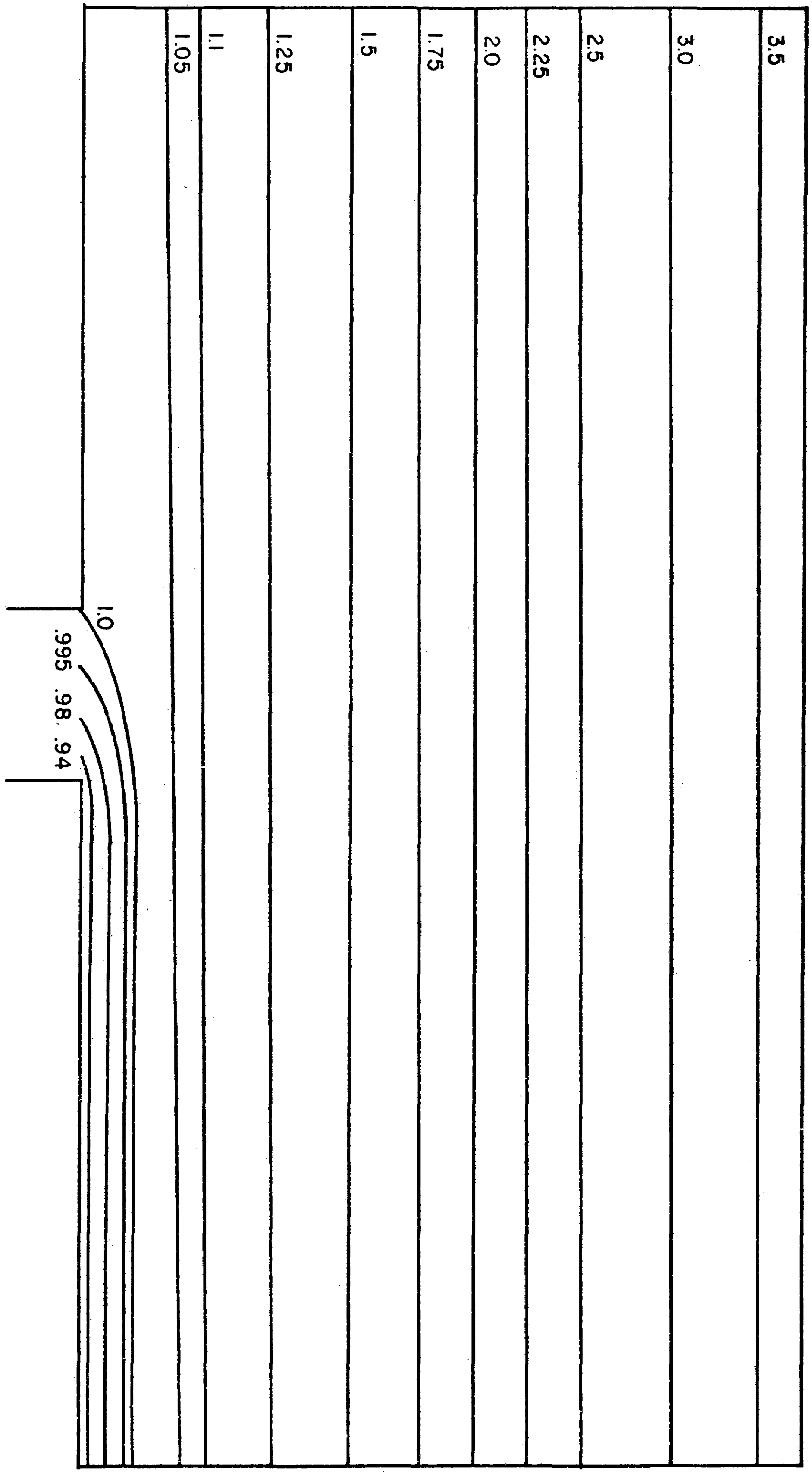


Figure 7. Stream lines from boundary value scheme.

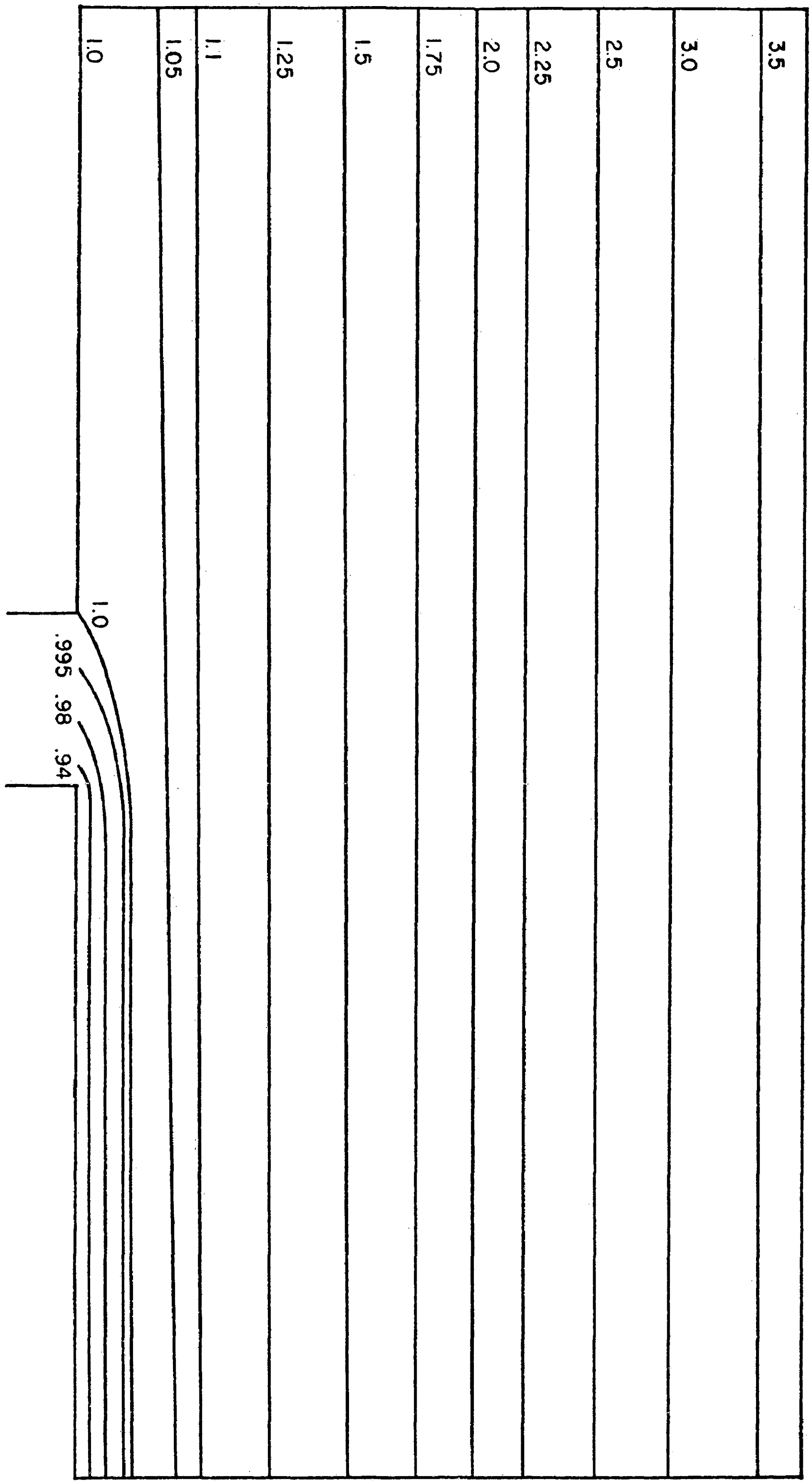


Figure 8. Stream lines from boundary value scheme.

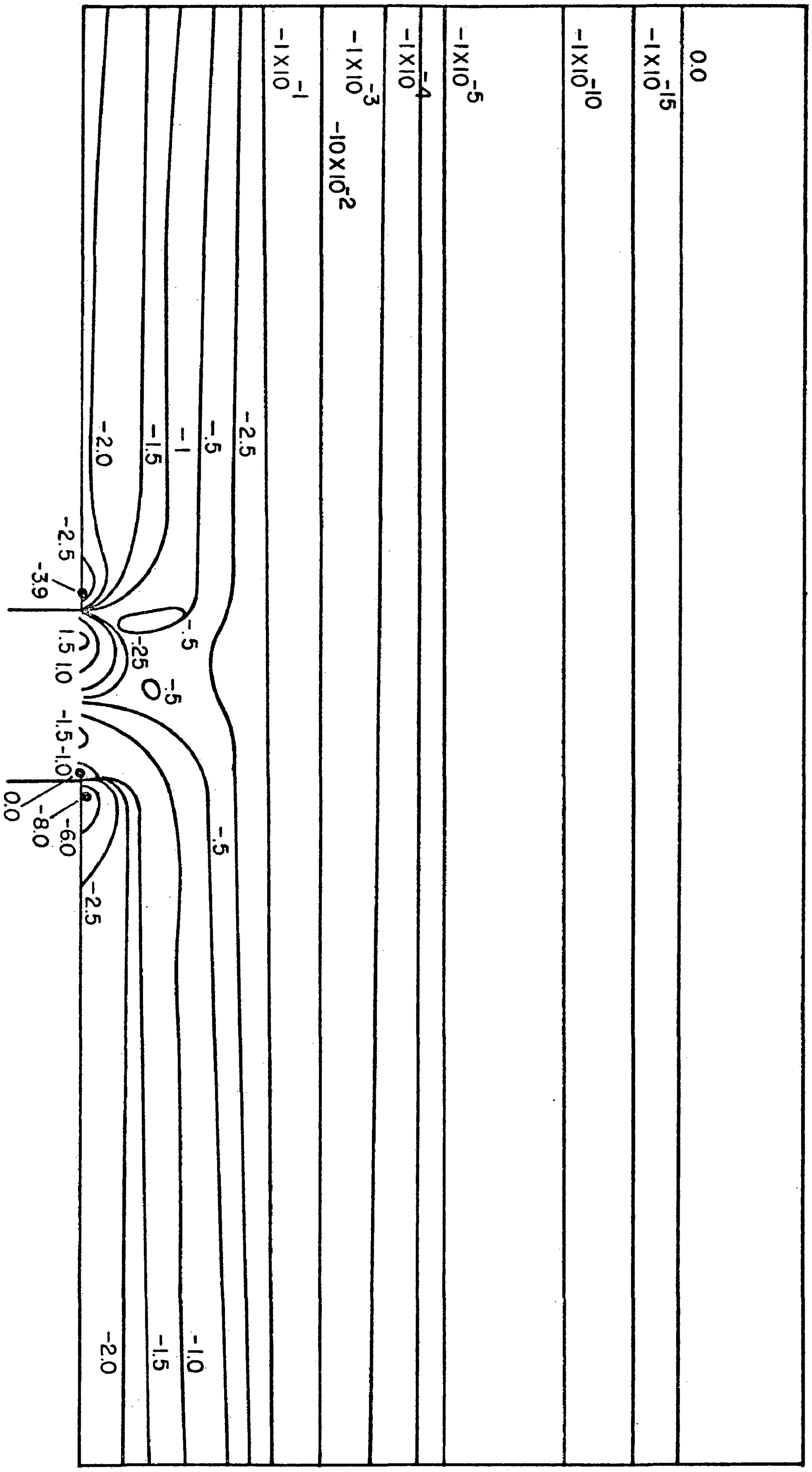


Figure 9. Vorticity contours of the flow field corresponding to Figure 5.

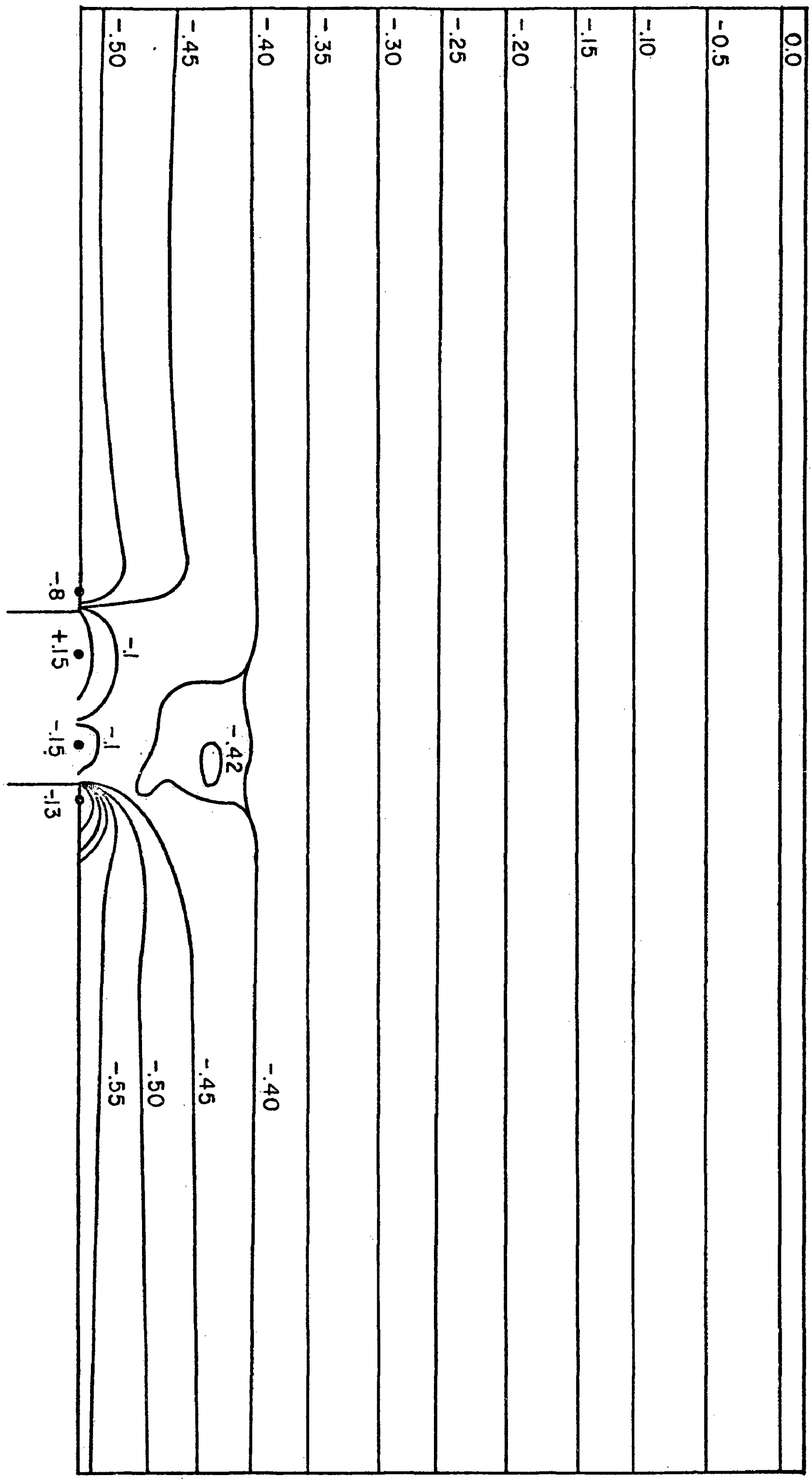


Figure 10. Vorticity contours of the flow field corresponding to Figure 8.

value of the jet center velocity.

In the time dependent technique the values in Table I are run for a fourth order jet. These do not converge; the vorticity at the wall near the jet oscillates. Repeating the values of Table II, all cases do not converge. The time step for the time dependent scheme is 0.02 for stability requirements.

For both time dependent scheme and the boundary value scheme, the methods do not converge for a rotating coordinate system; i.e. β not equal to zero.

For the cases of the time dependent scheme that do not converge, the vorticity at the wall near the jet does not converge to a solution, because these positions have large oscillations in the vorticity values.

V CONCLUSION

The finite difference form of the Navier-Stokes equations can be solved numerically for a two dimensional jet entering a flow field with an ambient velocity. The steady state solutions are computed by a boundary value scheme applying equation (14) and (15) with the appropriate boundary conditions. The boundary value method converges for a range of jet velocity values on the order of those assumed for the steady state at the mouth of the Chesapeake Bay. The boundary value scheme does not converge for a Reynolds number larger than 50, which is smaller than typical values= for the coastal sea around the Chesapeake Bay mouth.

The time dependent scheme does not converge for any value of the parameters. The non-converging cases have large oscillations in the values of the vorticity on the wall near the jet. Reducing the gradient of the vorticity in the jet does not reduce these oscillations.

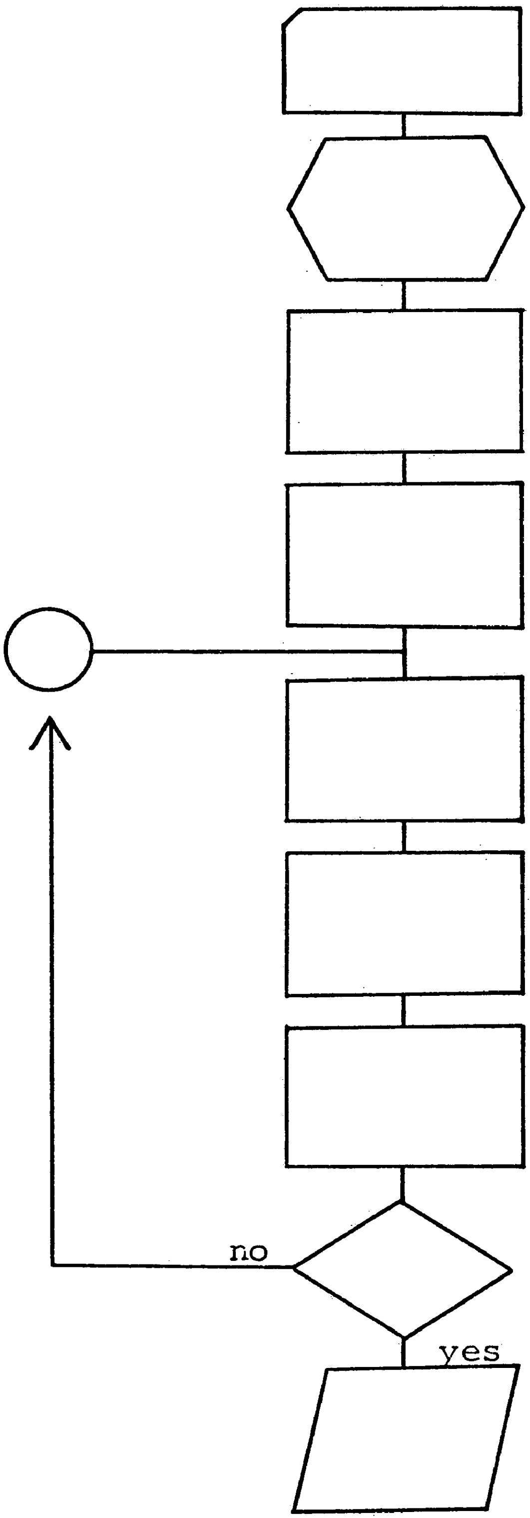
The critical part of the calculation is the determination of the vorticity at the boundary. For the boundary value scheme and the explicit time dependent scheme, the boundary vorticity does not depend on the vorticity and stream function values in the interior near the boundary at the current iteration, but uses the values from the previous iteration as estimates for the current iteration. This severely limits the explicit time scheme for stability requirements.

The stream lines in Figures 5, 6, 7 and 8 do not form closed loops that would indicate the existence of a gyre. It may be concluded by this study that a gyre is not a stable feature of the circulation averaged over several tidal cycles, if the gyre does exist. The gyre might be generated and dissipated in response to the tidal current fluctuation and a model that has an intra-tidal time scale may be more appropriate for investigating the existence of absence of a gyre.

In addition to changing the time scales, the method can be improved by considering the stresses at the surface and the bottom. A vertically integrated model would include the stresses at the surface and bottom in a two dimensional form. To improve the stability of the computation, a more stable scheme is to treat values at the boundary and in the interior at the same time step, i.e., an implicit time scheme.

By neglecting these stresses, the application of the solutions of a two dimensional model to the coastal sea around the Chesapeake Bay mouth would be difficult to relate drift data.

Appendix A. Flow Diagram and Program Listing of Boundary Value Scheme.



Read all computation parameters

Initialize the variables at each grid point for a boundary layer flow

Apply the jet impulsively and solve for the stream function iterating equations 15, 22 and 23 to convergence

Compute the vorticity at the wall with equation (20)

Compute the vorticity in the interior with equation (14)

Iterate the stream function with Equation 15, 22 and 23 to convergence

Compute the vorticity at the wall with Equation (20)

Has the vorticity at the wall converged?

Print the final values of the variables

```
FORTRAN IV G LEVEL  21                BAYMD6            DATE = 73362          20/40/36              PAGE 0001
              C     THIS PROGRAM WILL CALCULATE THE STREAM FUNCTION VALUES AND VORTICITY
              C     FOR A JET FLOWING INTO A FLUID MOVING PERPENDIULAR TO THE AXIS OF THE JET
              C     THE PARAMETERS USED IN THE CALCULATION WILL BE READ AND THE INITIAL VALUES
              C     WILL BE GIVEN FOR EACH CALCULATION
              C     ALL INPUT VALUES WILL BE PRINTED AND THE CONVERGENCE WILL BE CHECKED BY
              C     PRINTING EVERY MTH ITERATION FOR SELECTED TEST POINTS WITHIN THE GRID
              C
              C
              C
              C
              C
 0001               COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
                   1LN,LP,H,UO,VO,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
                   2,RO,LT,KZZ
              C
              C
              C*****THIS SECTION READS ALL THE INPUT DATA
              C
              C
              C
              C
              C     THE FIRST READ WILL BE THE GRID SIZE AND DIMENSION
              C
              C     THE VALUE N IS THE NUMBER OF POINTS IN THE X DIRECTION, THE NUMBER M IS
              C     THE NUMBER OF POINTS IN THE Y DIRECTION
              C
              C
 0002            10 READ (5,11) N,M
 0003            11 FORMAT (I3,7X,I3)
              C
              C
              C     THE VARIABLE B IS THE BETA PLANE COEFFECIENT, RE IS THE REYNOLDS NUMBER
              C     UO IS THE VELOCITY OF THE JET, VO IS THE VELOCITY OF THE FLUID, LA IS THE
              C     BOTTOM IN THE JET, L IS THE FULL WIDTH OF THE JET
              C
 0004            16 READ (5,17) B,RE,UO,L,LA
 0005            17 FORMAT (F6.5,4X,F7.1,3X,F4.2,16X,I2,2X,I2)
              C
 0006            18 READ (5,19) TESTA,TESTB,TESTC
 0007            19 FORMAT (3(F6.5,4X))
              C
 0008            20 READ (5,21,END=9) KD,A,LB,RO
 0009            21 FORMAT (10X,I3,2X,F2.1,2X,I2,2X,F6.2)
              C
 0010               READ (5,2000) PS(1,LA)
 0011          2000 FORMAT (F6.2)
              C
              C*****THIS SECTION CALCULATES THE STREAM FUNCTION VALUES FROM THE VELOCITY
              C     SCALE AND GRID DIMENSIONS
              C
 0012               CALL PSIA
              C
 0013               CALL VORIN
```

```
0014              READ (5,1000) UO
0015         1000 FORMAT (F4.2)
            C*****THIS SECTION WRITES THE CALCULATION PARAMETERS
            C
            C
            C
            C
0016              WRITE (6,22)
0017           22 FORMAT (1H1//////1X,'       N       M              H             B
               1          RE           UO                VO       L     LA          A'
               2///)
0018              WRITE (6,23) N,M,H,B,RE,UO,VO,L,LA,A
0019           23 FORMAT (5X,I3,4X,I3,4X,F11.4,4X,F11.4,4X,F12.4,4X,F9.4,4X,F11.4,4X
               1,I2,4X,I2,4X,F8.4////)
            C
0020              WRITE (6,30)
0021           30 FORMAT    (/////7X,'          TESTA         TESTB         TESTC
               1  KD          Q1        Q2        Q3         Q4       LB   '/)
            C
0022              WRITE (6,31) TESTA,TESTB,TESTC,KD,Q1,Q2,Q3,Q4,LB
0023           31 FORMAT (10X,3(F11.6,2X),5X,I3,5X,3(F6.1,3X),F8.2,5X,I2/)
            C
            C
            C*****THIS BEGINS THE CALCULATION OF THE VORTICITY AND THE STREAM FUNCTION
            C
            C
0024              KX=0
            C
0025              CALL BOUND
            C
0026           28 CONTINUE
0027              KW=0
0028           93 CONTINUE
            C
            C
0029              CALL VORCAL
0030              CALL UPDATE
            C
0031              CALL PSICAL
            C
            C
0032           57 KW=KW+1
0033              KZZ=KW+KX
            C
            C     THIS PART CHECKS FOR PRINT OUT OF TEST POINTS WITHIN THE GRID
            C
            C
0034              IF (KW.EQ.KD) GO TO 70
0035              GO TO 90
            C
            C
0036           70 KX=KX+KD
0037              WRITE (6,71)     KX
```

```
FORTRAN IV G LEVEL  21                BAYMD6           DATE = 73362        20/40/36              PAGE 0003

0038            71 FORMAT (1H1,  '        THIS IS THE',I4,'TH ITERATION STEP FOR THE
                  1VORTICITY'///)
             C
             C
             C
0039               WRITE (6,72) KX
0040            72 FORMAT (1H1,'        THIS IS THE',I4,'TH ITERATION STEP FOR THE
                  1STREAM FUNCTION'///)
             C
0041               CALL PRINT2
             C
0042               KW=0
             C
             C
0043            90 CONTINUE
             C
0044               CALL VORWAL
             C
0045               IF (ITEST.EQ.1) GO TO 93
             C
             C
             C*****THIS SECTION PRINTS OUT THE CALCULATED ANSWERS FOR ALL PARAMETERS
             C
             C
0046            95 WRITE (6,96)
0047            96 FORMAT (1H1,'     THIS IS THE PRINTOUT OF THE FINAL VALUES FOR THE
                  1VORTICITY'////)
0048               CALL PRINT1
             C
0049               WRITE (6,97)
0050            97 FORMAT (1H1,'      THIS IS THE PRINTOUT OF THE FINAL STREAM FUNCTI
                  1ON VALUES'/////)
0051               CALL PRINT2
             C
0052               GO TO 902
0053             9 WRITE (6,901)
0054           901 FORMAT (1H1,'    THERE IS AN ERROR IN THE READING OF DATA CARDS')
0055           902 CONTINUE
             C
             C     THIS SECTION WRITES OUT THE DATA ONTO TAPE OR DISK
             C
0056               CALL EXIT
0057               END
```

```
      SUBROUTINE PSIA
      COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
     1LN,LP,H,UO,VO,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
     2,RO,LT,KZZ
      H=1./L
      VO=-1.0
      LN=LA-1
      LC=LB+1
      LP=LA+L+1
      M1=M-1
      N1=N-1
      LBA=N1-LB
      LT=LA+L
      Q1=PS(1,LA)
      Q3=Q1+(2.*LB*H)/(3.0)
      Q2=Q3+(LBA*H)
      DO 15 J=1,M
      DO 14 I=1,LB
      PS(I+1,J)=Q1+(((I*H)*(I*H))/(LB*H))-(((I*H)*(I*H)*(I*H))/((3.0)*(L
     1B*H)*(LB*H)))
   14 CONTINUE
      PS(1,J)=Q1
      PS(N,J)=Q2
      DO 13 I=1,LBA
      PS(LC+I,J)=Q3+(I*H)
   13 CONTINUE
   15 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE VORIN
      COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
     1LN,LP,H,UO,VO,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
     2,RO,LT,KZZ
C
C     THIS LOOP INITIALIZES THE VORTICITY
C
      DO 12 J=1,M
      DO 12 I=1,N
      W(I,J)=0.
   12 CONTINUE
C
      IF (LB.EQ.0) GO TO 26
      LBC = LB - 1
      DO 25 J=1,M
      DO 25 I=1,LB
      W(I,J)= ((2.0*(1.0-(((I-1)*H)/(LB*H))))/(LB*H))*VO
   25 CONTINUE
      MOUT=UO
      IF (MOUT.EQ.0) GO TO 27
      W(1,LA)= 4.0*UO
      DO 16 J=1,L
      W(1,LA+J)  =  UO*(4.0 - 8.0*(J*H))
   16 CONTINUE
      LMF=LA-(L/2)
      W(1,LMF)=0.
   27 CONTINUE
   26 CONTINUE
      DO 1 J=1,M
      DO 1 I=1,N
      W1(I,J) = W(I,J)
    1 CONTINUE
      RETURN
      END
```

FORTRAN IV G LEVEL 21 BOUND DATE = 73362 20/40/36 PAGE 0001

```
      SUBROUTINE BOUND
      COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
     1LN,LP,H,UO,VO,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
     2,RC,LT,KZZ
      LZ=L/2
      IF ((L-2*(L/2)).GT.0) LZ=(L/2)+1
      CPS=  -4./15.*UO     +PS(1,LA)
      PS(1,LA+LZ) =  CPS
      DO 300 J=1,LZ
      PS(1,LA+5-J)  =   ((16./5.)*(J*H)*(J*H)*(J*H)*(J*H)*(J*H)
     1  -(8./3.)*(J*H)*(J*H)*(J*H)     +(J*H))*UO      +CPS
      PS(1,LA+5+J)  =  -((16./5.)*(J*H)*(J*H)*(J*H)*(J*H)*(J*H)
     1  -(8./3.)*(J*H)*(J*H)*(J*H)     +(J*H))*UO      +CPS
  300 CONTINUE
      NUO=UO
      DO 3 J=LP,M
      PS(1,J)= PS(1,LA+L)
    3 CONTINUE
      DO 4 I=1,5
      W(1,LA+I-1)  =  -16.0*((4.0*(5-I+1)*(5-I+1)*(5-I+1)*H*H*H) - (5-I+
     11)*H)*UO
      W(1,LA+L-I+1)  =  -16.0*((4.0*(I-1-5)*(I-1-5)*(I-1-5)*H*H*H) - (I-
     11-5)*H)*UO
    4 CONTINUE
       W(1,LA+LZ)=0.0
   26 CONTINUE
      DO 10 J=1,M
      W1(1,J)=  W(1,J)
   10 CONTINUE
      WRITE (6,1000) (PS(1,J),J=1,M)
 1000 FORMAT (7(10(E10.4,1X)/)/)
      WRITE (6,1000) (W(1,J),J=1,M)
      WRITE (6,1000) (W1(1,J),J=1,M)
      RETURN
      END
```

```
FORTRAN IV G LEVEL  21              PRINT1           DATE = 73362        20/40/36              PAGE 0001

0001                SUBROUTINE PRINT1
0002                COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
                   1LN,LP,H,UO,VO,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
                   2,RO,LT,KZZ
              C     THIS IS THE PRINT OUT OF THE VORTICITY VALUES
              C
0003                WRITE (6,24)
0004             24 FORMAT (1H1///30X,'        VORTICITY VALUES',///)
              C
0005                WRITE (6,25) ((W(I,J),I=1,N),J=1,M)
0006             25 FORMAT (90(4(7X,10E11.3/)/))
              C
0007                RETURN
0008                ENC
```

```
FORTRAN IV G LEVEL  21                PRINT2            DATE = 73362         20/40/36             PAGE 0001

0001                 SUBROUTINE PRINT2
0002                 COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
                    1LN,LP,H,U0,V0,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
                    2,RO,LT,KZZ
             C       THIS IS THE PRINTOUT OF THE STREAM FUNCTION VALUES
             C
0003                 WRITE (6,26)
0004              26 FORMAT (/////////30X,'     STREAM FUNCTION VALUES',///)
             C
0005                 WRITE (6,27) ((PS(I,J),I=1,N),J=1,M)
0006              27 FORMAT (90(4(7X,10E11.3/)/))
             C
             C
             C
0007                 RETURN
0008                 END
```

```
0001            SUBROUTINE VORCAL
0002            COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
               1LN,LP,H,UO,VO,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
               2,RC,LT,KZZ
          C
0003            KJ=0
0004            M2=M-2
0005            M3=M-3
0006            M4=M-4
          C
0007         35 CONTINUE
          C
0008            KJ=KJ+1
          C
0009            ITEST=0
0010            REG=1./RE
0011            C=4.*REG
0012            DO 40 J=2,M1
0013            DO 40 I=2,N1
0014            D1=REG+ (PS(I,J+1)-PS(I,J-1))/4.
0015            D2=REG- (PS(I,J+1)-PS(I,J-1))/4.
0016            D3=REG- (PS(I+1,J)-PS(I-1,J))/4.
0017            D4=REG+ (PS(I+1,J)-PS(I-1,J))/4.
0018            E =  B*((PS(I+1,J)-PS(I-1,J))*H/2.0)
0019            WTEMP = ( D1*W(I+1,J) + D2*W(I-1,J) + D3*W(I,J+1) + D4*W(I,J-1) +
               1 E) / C
0020            W1(I,J)=WTEMP
0021         40 CONTINUE
0022            DO 41 I=1,N1
0023            WTEMP  =  W1(I,M4) - (2.0*(W1(I,M3))) + (2.0*(W1(I,M1)))
0024            W1(I,M) = WTEMP
0025         41 CONTINUE
0026            WRITE (6,24)  KZZ
0027         24 FORMAT (/////50X,'     THIS IS ITERATION NUMBER ',I3//)
0028            RETURN
0029            END
```

```
FORTRAN IV G LEVEL  21                 PSICAL              DATE = 73362          20/40/36                PAGE 0001

0001            SUBROUTINE PSICAL
0002            COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
               1LN,LP,H,UO,VO,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
               2,RC,LT,KZZ
0003            KK=0
0004            M2 = M - 2
0005            M3= M-3
0006            M4 = M - 4
0007         38 CONTINUE
0008            KK=KK+1
          C
          C
          C
          C
          C     THIS SECTION CALCULATES THE INTERIOR STREAM FUNCTION VALUES
          C
0009            JTEST=0
0010            CO 50 J=2,M1
0011            CO 50 I=2,N1
0012            PSTEMP=(PS(I+1,J)+PS(I-1,J)+PS(I,J+1)+PS(I,J-1)+(H*H)*W(I,J))/4.
0013            IF  (ABS((PS(I,J)-PSTEMP)/PS(I,J)).GT.TESTC) GO TO 54
0014            GO TO 52
0015         54 JTEST=1
0016         52 PS(I,J)=PS(I,J)+(A*(PSTEMP-PS(I,J)))
0017         50 CONTINUE
          C
0018            CO 51 I=2,N1
0019            PS(I,M)=(PS(I,M4)-(2.*(PS(I,M3)))+(2.*(PS(I,M1))))
0020         51 CONTINUE
          C
0021            IF (KK.GT.500) GO TO 905
0022            IF (JTEST.EQ.1) GO TO 38
          C
0023            WRITE (6,72) KK
0024            RETURN
0025        905 WRITE (6,72) KK
0026         72 FORMAT (/////50X,I3,'  ITERATIONS FOR CONVERGENCE FOR THE STREAM F
               1UNCTION'///)
0027            CALL PRINT2
0028            CALL EXIT
0029            ENC
```

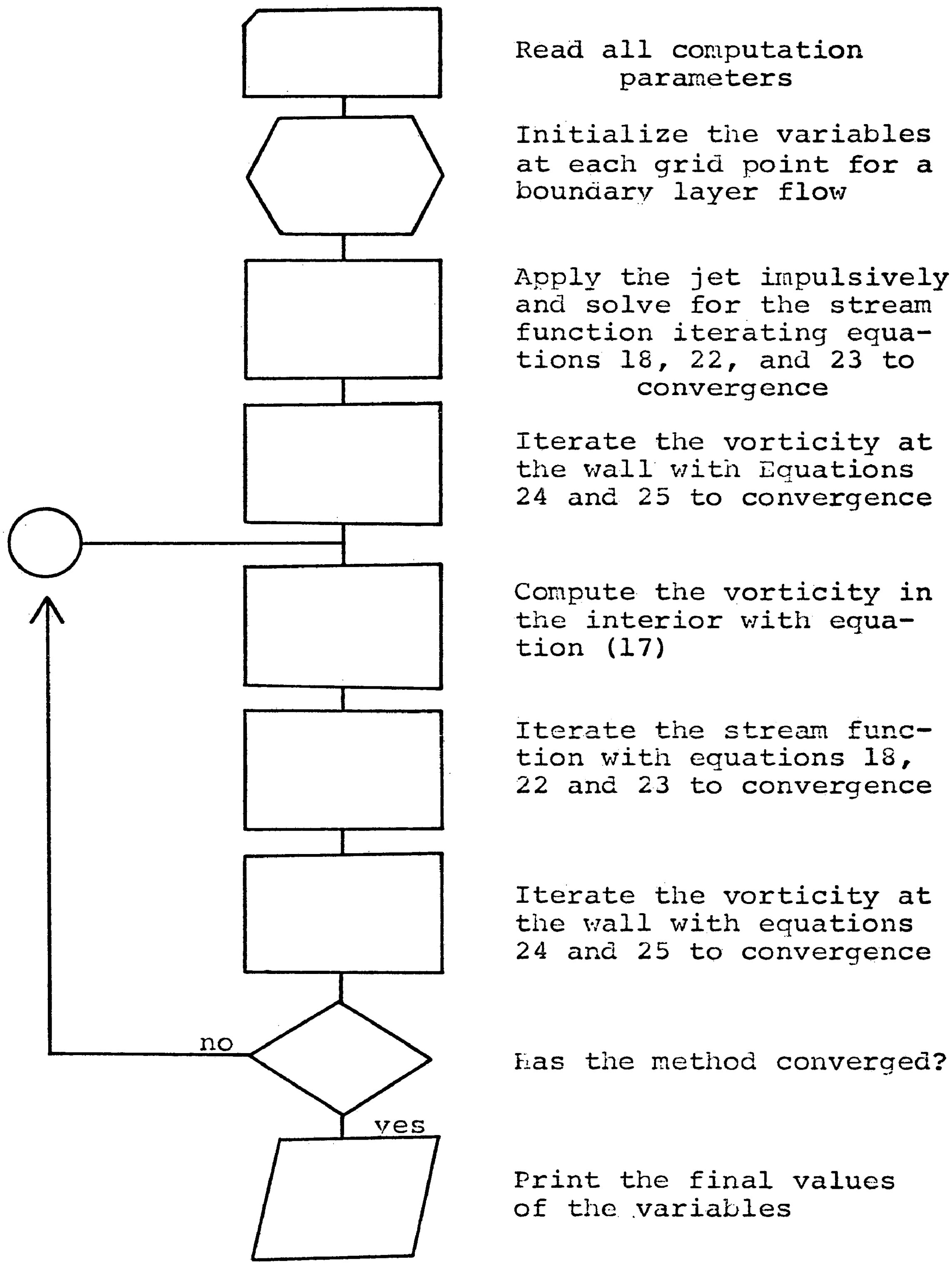
```
FORTRAN IV G LEVEL  21                UPDATE            DATE = 73362         20/40/36              PAGE 0001

 0001               SUBROUTINE UPDATE
 0002               COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
                   1LN,LP,H,UD,VD,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
                   2,RD,LT,KZZ
 0003               CO 1 J=2,M
 0004               CO 1 I=2,N1
 0005               W(I,J)=W1(I,J)
 0006             1 CONTINUE
 0007               RETURN
 0008               ENC
```

```
      SUBROUTINE VORWAL
      COMMON W(51,220),W1(51,220),PS(51,220),N,M,N1,M1,M2,M3,M4,L,LA,LB,
     1LN,LP,H,UO,VO,RE,A,B,C,ITEST,JTEST,TESTA,TESTB,TESTC,Q1,Q2,Q3,Q4
     2,RO,LT,KZZ
      DIMENSION WJ(100)
      WJ(LA+5)=0.0
      CO 250 I=1,5
      WJ(LA+I-1)=     -16.0*((4.0*(5-I+1)*(5-I+1)*(5-I+1)*H*H*H) - (5-I+
     11)*H)*UO
      WJ(LA+L-I+1)=     -16.0*((4.0*(I-1-5)*(I-1-5)*(I-1-5)*H*H*H) - (I-
     11-5)*H)*UO
  250 CONTINUE
      WJ(LA+5)  = 0.
C
      ITEST=0
      I=1
      CO 91 J=2,LN
        WTEMP=((3.0*(PS(1,J)-PS(2,J)))/(H*H))-(W(2,J)/2.0)
      IF (ABS((W(1,J)-WTEMP)/W(1,J)).GT.TESTA) GO TO 92
      GO TO 93
   92 ITEST=1
   93 W(1,J)=WTEMP
   91 CONTINUE
C
      CO 87 J=LP,M1
        WTEMP=((3.0*(PS(1,J)-PS(2,J)))/(H*H))-(W(2,J)/2.0)
      IF (ABS((W(1,J)-WTEMP)/W(1,J)).GT.TESTA) GO TO 86
      GO TO 88
   86 ITEST=1
   88 W(1,J)=WTEMP
   87 CONTINUE
C
C
      MOUT = UO * 1000.
      IF (MOUT.EQ.0) GO TO 97
      WRITE (6,41)
   41 FORMAT (//////40X,'        THESE ARE THE VALUES OF THE VORTICITY AT
     1 THE WALL'////)
      WRITE (6,25) (W(1,J),J=1,M)
   25 FORMAT (28(10(7X,10E11.3/)////))
C
      RETURN
   97 LX=L+1
      CO 96 J=1,LX
        WTEMP=((3.0*(PS(1,J)-PS(2,J)))/(H*H))-(W(2,J)/2.0)
      IF (ABS((W(1,LN+J)-WTEMP)/W(1,LN+J)).GT.TESTA) GO TO 95
      GO TO 98
   95 ITEST=1
   98 W(1,LN+J)=WTEMP
   96 CONTINUE
      WRITE (6,41)
      WRITE (6,25) (W(1,J),J=1,M)
      RETURN
```

```
0044                ENC
```

Appendix B. Flow Diagram of the Time Dependent Scheme.


Read all computation parameters
Initialize the variables at each grid point for a boundary layer flow
Apply the jet impulsively and solve for the stream function iterating equations 18, 22, and 23 to convergence
Iterate the vorticity at the wall with Equations 24 and 25 to convergence
Compute the vorticity in the interior with equation (17)
Iterate the stream function with equations 18, 22 and 23 to convergence
Iterate the vorticity at the wall with equations 24 and 25 to convergence
Has the method converged?
no
yes
Print the final values of the variables

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