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## Original Article

# Estimating fishing and natural mortality rates, and catchability coefficient, from a series of observations on mean length and fishing effort 

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Gedamke and Hoenig (2006) (Transactions of the American Fisheries Society, 135: 476-487) developed a non-equilibrium version of the Beverton and Holt estimator of total mortality rate, $Z$, based on mean length and thereby increased the usefulness of length-based methods. In this study, we extend their model by replacing period-specific $Z$ parameters with the year-specific parameterization $Z_{y}=q f_{y}+M$ where $q$ is the catchability coefficient, $f_{y}$ is the fishing effort in year $y, F(=q)$ is the fishing mortality rate, and $M$ is the natural mortality rate. Thus, the problem reduces to estimating just three parameters: $q, M$ and residual variance. We used Monte Carlo simulation to study the model behaviour. Estimates of $q$ and $M$ are highly negatively correlated and may or may not be reliable; however, the estimates of corresponding $Z$ 's are more precise than estimates of $F$ and are generally reliable, even when uncertainty about the mean lengths is high. This length-based method appears to work best for stocks with rapid growth rate. Contrast in effort data may not be necessary for reliable estimates of $Z$ 's. This approach forms a bridge between data-limited models and more complex models. We apply the method to the Norway lobster Nephrops norvegicus stock in Portugal as an example.

Keywords: catchability coefficient, data limited, data-poor, fishing effort, mean length, natural mortality rate, Nephrops, non-equilibrium conditions.

## Introduction

Reliable and representative age information remains scarce for assessing the status of many fished stocks. Size information, on the other hand, is widely collected and much more readily available for data-poor stocks. Utilizing the latter information to obtain reasonably useful estimates of stock parameters, such as total mortality rate, $Z$, is highly desirable especially when collection of age data is highly resource-intensive or accurate aging is not possible. Hence continual research to improve and extend the utility of existing length-based methods is warranted and urgently needed for many unassessed stocks.
A number of length-based estimators of $Z$ require restrictive assumptions which are generally untenable for real fisheries
settings. One such estimator was developed by Beverton and Holt (1956) to estimate $Z$ from sample mean length and von Bertalanffy growth parameters. The Beverton and Holt estimator assumes equilibrium length composition such that the mean length reflects the current $Z$ rate experienced by the stock.

Gedamke and Hoenig (2006) modified the Beverton and Holt estimator by relaxing the strict assumption of equilibrium. This was done by modelling the transition of mean length from one equilibrium period to the next, following step-wise changes in $Z$. Using a time series of mean length observations, the GedamkeHoenig estimator yields period-specific estimates of $Z$ and the corresponding years of change in mortality (Gedamke and Hoenig, 2006). The number of parameters depends on the
number of change points in $Z$. Using this methodology, Cardinale et al. (2010) were able to detect a total of seven changes in $Z$ in the plaice stock of the Kattegat-Skagerrak area from a century-long time series of mean length data.

The objective of this work is to increase the generality and reliability of non-equilibrium length-based estimators. Specifically, our article extends the work of Gedamke and Hoenig (2006) by utilizing additional information from a time-series of fishing effort data, $f$, to estimate $Z$ rates. The data required to apply the model are length measurements coupled with a time-series of standardized fishing effort or times series of catch and catch rates. Our model estimates three parameters-the catchability coefficient, $q$, post-recruitment natural mortality rate, $M$, and residual variance.

Gedamke and Hoenig (2006) showed that for a given change in the mortality rate experienced in the population from $Z_{1}$ to $Z_{2}$, the mean length requires a longer time to reach equilibrium (i.e. to reflect $Z_{2}$ ) when the von Bertalanffy parameter $K$ is low than when it is high. This suggests that the non-equilibrium, length-based method may be more useful for stocks with relatively higher $K$ rates. Hence, we utilize a simulation framework with realistic biological parameterizations and plausible scenarios to assess the accuracy and precision of the extended model. Generalizations and recommendations for application of the extended method are provided. We then estimate mortality rates for a stock of Norway lobster Nephrops norvegicus in South and Southwest Portuguese waters (ICES areas FU28 and FU29) using mean lengths and effort as an example application.

## Methods

## Estimation procedure, assumptions, and data requirements

We used an age-structured geometric decline model as the underlying population model in contrast to Gedamke and Hoenig (2006) who used a continuous (exponential) model. For this model formulation, we assume constant fishery recruitment such that relative abundance at age of full recruitment $N_{t_{c}}=1$. In practice, this assumption can be addressed when applying the model to a stock with an available time-series of an index of recruitment (see Gedamke et al. (2008) for an example with the barndoor skate, Dipturus laevis). We also assume knife-edge selection of lengths (flat topped selectivity curve) by the fishery gear, such that all lengths $\geq L_{\mathrm{c}}$ corresponding to ages $\geq t_{\mathrm{c}}$ are fully vulnerable to the gear. Other model assumptions include:
(i) Mean length at age is known and constant over time.
(ii) There is no individual variability in growth.
(iii) Natural mortality $M$ is independent of stock size and constant with age and over time.
(iv) Catchability $q$ is constant over time and over age for all ages $\geq t_{c}$.

Abundance (in numbers) at age $t$ for a given year $y, N_{t, y}$ (expressed as a fraction of the recruitment, assumed to be constant) is modelled as:

$$
N_{t, y}= \begin{cases}1, & t=t_{c} \text { for all } y  \tag{1}\\ N_{t-1, y-1} e^{-Z_{y-1}}, & t=t_{c}+1, \ldots, \infty ; y=1,2, \ldots, \mathrm{n}\end{cases}
$$

with

$$
\begin{equation*}
Z_{y}=q f_{y}+M \tag{2}
\end{equation*}
$$

where $t_{c}$ is the age at first capture; $M$ is the instantaneous natural mortality rate; $q$ is the catchability coefficient; $f_{y}$ is the standardized fishing effort in year $y ; Z_{y}$ is the instantaneous total mortality rate for year $y$, assumed to be linearly related to $f$.

Estimates of the mean length at each discrete age $t$ are needed for ages $\geq t_{c}$. In this article, we used mean length-at-age $\bar{L}_{t}$ from the von Bertalanffy growth function (3):

$$
\begin{equation*}
\bar{L}_{t}=L_{\infty}\left(1-e^{\left(-K\left(t-t_{0}\right)\right)}\right) \tag{3}
\end{equation*}
$$

Where $K$ is the von Bertalanffy growth coefficient $\left(\mathrm{yr}^{-1}\right) ; L_{\infty}$ is the von Bertalanffy asymptotic length (scaling parameter); $t_{0}$ is the age-axis intercept (year); The growth parameters are assumed to be constant over time. In practice, the mean length at age information can be obtained via other methods apart from the von Bertalanffy growth function. The predicted mean length in year $y$ is modelled as

$$
\begin{equation*}
\bar{L}_{\mathrm{pred}, y}=\frac{\sum_{t=t_{c}}^{\infty}\left(\bar{L}_{t, y} \cdot \hat{N}_{t, y}\right)}{\sum_{t=t_{c}}^{\infty} \hat{N}_{t, y}} \tag{4}
\end{equation*}
$$

where $\hat{N}_{t, y}$ is obtained from Equation (1) using estimates of $Z_{y}$.
In theory, the age summations in Equation (4) extend to infinity but, for computational purposes, the infinite summation can be approximated to any desired degree of precision by choice of an upper limit of summation, e.g. a specified maximum age. For a long lived species with many age groups, this creates a theoretical problem of requiring a long time series of observations. As a practical matter, the older age groups that are no longer abundant become less important since less weight is given to these age groups in computing the mean length.

To compute predicted mean length in year $y$, we need the fishing efforts going back in time for an infinite number of years if we assume an infinite number of cohorts are present [see Equation (4)]. This creates a minor difficulty in specifying the predicted mean lengths for the initial years of the time series. In the absence of fishing effort information for years prior to the first year of mean length observation, one approach is to assume zero fishing effort. Hence the mortality rate depends solely on the parameter $M$, which is estimable. Another approach is to assume equilibrium conditions at the start of the time series and use the (average) fishing effort(s) in the first year(s) of the time series as the effort prior to the start of the time series. A third approach is to divide the time series into two parts; the fishing efforts in the first part of the time series are used to compute predicted mean lengths in the second part. Essentially, the observed lengths in the first part of the time series are disregarded. The third possibility may not be a viable option if the time series is too short or the analyst does not wish to 'waste' data. In practice, one could explore all possible options and see if the results are very different from each other. We note that as the time series grows longer, this problem diminishes.

By the Central Limit Theorem, sample means will tend toward a normal distribution with increasing sample size. Hence, the sample mean length is modelled as being normally distributed, i.e., $\bar{L} \sim N\left(\mu, \frac{\sigma^{2}}{m}\right)$, with the associated probability density
function $f\left(\bar{L} ; \mu, \sigma^{2}\right)=\frac{\sqrt{m}}{\sqrt{2 \pi \sigma^{2}}} \cdot e^{-\left(\frac{m}{2 \sigma^{2}}\right) \cdot(\bar{L}-\mu)^{2}}$, where $\mu$ is the predicted mean length $\bar{L}_{\mathrm{pred}, y} y$ from Equation (4) and $m$ is the sample size of observed lengths $>L_{c}$. The product likelihood function for $n$ years of observed mean lengths with sample size $m_{y}$ is given as:

$$
\begin{equation*}
\Lambda=\prod_{y=1}^{n} \frac{\sqrt{m_{y}}}{\sqrt{2 \pi \sigma^{2}}} e^{-\left(\frac{m_{y}}{2 \sigma^{2}}\right)\left(\bar{L}_{y}-\mu_{y}\right)^{2}} \tag{5}
\end{equation*}
$$

and $\mu_{y}$ is a function of $q$ and $M$. It should be noted that while standard deviation of mean lengths is assumed as constant (yearly sample sizes are used to weight the observations), it is also possible to make the standard deviation year-specific. Maximum likelihood estimation is employed to estimate the parameters $q$ and $M$ from the kernel of the $\log$ likelihood $\log _{e}(\Lambda)$ which is proportional to

$$
\begin{equation*}
-\mathrm{n} \cdot\left(\log _{e} \sigma\right)-\frac{1}{2 \sigma^{2}} \sum_{y=1}^{n} m_{y} \cdot\left(\bar{L}_{y}-\mu_{y}\right)^{2} . \tag{6}
\end{equation*}
$$

Year-specific $Z$ is then obtained using Equation (2).

## Diagnostic procedures

Two principal assumptions underlying the method in this article are that the selectivity curve is flat topped above the length $L_{\mathrm{c}}$ and that recruitment is constant. These assumptions can be examined through diagnostic procedures outlined below.

To estimate mortality using the estimation procedure described earlier, the data analyst first sets the value of $L_{\mathrm{c}}$ and computes mean length of those animals above $L_{\mathrm{c}}$. Suppose that the selectivity curve is indeed flat topped (constant for all lengths $>L_{\mathrm{c}}$ ). Then if estimates of $Z$ are computed for a series of increasing values of $L_{\mathrm{c}}$ (to the right of the modal length) there should be no trend in the estimates of $Z$ for any particular year. This is because the assumptions of the model are met for each of the values of $L_{\mathrm{c}}$. However, if $L_{\mathrm{c}}$ is set to be very large, a considerable amount of length data is discarded so the estimates will become unstable.

Now assume that the estimates of $Z$ (for any particular year) show an increasing trend with increasing values of $L_{c}$. This would be consistent with a dome shaped selectivity curve; lower selectivity on the right accounts for the fish disappearing faster with length than if the selectivity were constant; as length data on the left ( $<$ specified $L_{\mathrm{c}}$ ) are discarded, the effect of declining selectivity on the right would be attributed to increasing mortality. This diagnostic procedure is not specific to errors in the assumption about selectivity. If $L_{\infty}$ is misspecified in the model, one can also get trends in $Z$ (for any given year) with changes in $L_{c}$. Thus, trends in the estimates with change in $L_{\mathrm{c}}$ indicate a problem but it is not clear what the problem may be. The absence of a trend can be considered a quality control check.

An extreme year class moving through the annual length frequency distributions over time can potentially cause bias in the Z estimates, although this would be less of a problem in a sufficiently long time series. One way to check for a moving year class is to compare the modal value in the length frequency distributions from year to year. If the modal length is stable or varies randomly about a mean there is no suggestion of a problem. However, if the mode migrates successively to the right over a


Figure 1. Time series of length frequency distributions for the northern stock of hake (Merluccius merluccius) from the northeast Atlantic (ICES, 2015). The modal length frequency appears stable from 2004 to 2006 but a pulse of strong recruitment can be seen to migrate to the right in successive years after 2006.
period of several years this suggests that the effect of a dominant year class may be confounded with the effects of mean length. Both situations-stable modal length (in the first part of the time series) and migrating modal length (in the second part) can be seen in the length frequency distributions for hake (Merluccius merluccius) in Figure 1 (ICES, 2015).

## Simulation procedure and model evaluation

Various authors have found that estimates of the parameters $q$ and $M$ are highly, negatively correlated when estimated within a common model framework (e.g. Megrey, 1988; Wang, 1999; Fu and Quinn II, 2000). These parameters are also difficult to estimate reliably when there is little contrast in the catch or effort data. Hence we used simulations to evaluate the performance of our estimator under varying scenarios and to determine if the estimates of $q, M$ and the corresponding $Z$ s (denoted as $\hat{q}, \hat{M}$, and $\hat{Z})$ are useful and reliable. Stock with parameters given in Table 1 was generated using Equations (1-4).

In addition to reflecting the underlying mortality rates, mean length data are also subject to other sources of variability, i.e. sampling and non-sampling errors, which we will refer to as pooled errors. The former can be reduced by increasing sample size $m$ of measured lengths, but the latter, such as recruitment variability, may not be accounted for in a like manner. To explore
the effect of pooled errors in mean length on the reliability of the model estimates, we added random errors $\epsilon \sim N\left(0, \sigma^{2} / m\right)$ to generated mean length data with $\sigma / \sqrt{m}$ (denoted henceforth as $\sigma^{\prime}$ ) varying from 1 to 5 in magnitude. A single realization of mean length data generated with three levels of $\sigma$ ' is shown in Figure 2 (top row). When $\sigma^{\prime}=1$, the trend in mean length is clear; when $\sigma^{\prime}=5$, one can still see a decline in mean length although the details of when and how much are obscured.

Table 1. Parameters of total mortality rates $Z\left(\mathrm{yr}^{-1}\right)$ and von Bertalanffy growth coefficient $K\left(\mathrm{yr}^{-1}\right)$ used in the simulations.

|  |  |  |  | Changes in <br> $\mathbf{Z}\left(\mathbf{Z}_{2} / \mathbf{Z}_{\mathbf{1}}\right.$ | von <br> Bertalanffy | No. of mean <br> length <br> observations, |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Scenario | $\mathbf{Z}_{\mathbf{1}}$ | $\mathbf{Z}_{\mathbf{2}}$ | ratio $)$ | $\boldsymbol{K}$ | $\boldsymbol{n}$ |  |
| (a) Base Case | 0.6 | 1.0 | $+0.4(1.67)$ | 0.4 | 15 |  |
| (b) Low $K$ | 0.6 | 1.0 | $+0.4(1.67)$ | 0.1 | 15 |  |
| (c) Decreased $Z$ | 1.0 | 0.6 | $-0.4(0.6)$ | 0.4 | 15 |  |
| (d) Less contrast | 0.6 | 0.8 | $+0.2(1.33)$ | 0.4 | 15 |  |
| $\quad$ in $Z$ |  |  |  |  |  |  |
| (e) Low $K$ | 0.6 | 0.8 | $+0.2(1.33)$ | 0.1 | 15 |  |
| (f) Low $K, n=20$ | 0.6 | 1.0 | $+0.4(1.67)$ | 0.1 | 20 |  |

Other parameter constants used in the simulation are natural mortality $M=$ 0.4 , catchability coefficient $q=0.002$ and the change in $Z$ rates occurs in the sixth year.

For clarity, we present scenarios with a single change in $Z$ over the simulated time-series. Specifically, we simulated two levels of fishing effort corresponding to two time periods with periodspecific constant $F$ and $Z$. Our base case scenario is modelled after a stock with high $K$ subjected to increased fishing effort such that the $Z$ rate of that stock increased by $0.4 \mathrm{yr}^{-1}$ from $Z_{1}=0.6$ to $Z_{2}=1.0 \mathrm{yr}^{-1}(\Delta Z=+0.4)$. We present additional scenarios with a single parameter modification to systematically examine the effect on the model performance (see Table 1 for details). It should be noted that mean length takes a longer time to reach equilibrium for a decrease in $Z$, e.g. $\Delta Z=-0.4 \mathrm{yr}^{-1}$, than an increase in the same magnitude $\left(\Delta Z=+0.4 \mathrm{yr}^{-1}\right)$ (see Figure 3).

In our simulation, the age summations in Equation (4) were arbitrarily approximated to an upper age limit $t_{\max }$ that corresponded to $0.2 \%$ of the original population size when subjected to a constant decline rate $Z\left(=\min \left\{Z_{1}, Z_{2}\right\}\right)$. The underlying constants for all scenarios were: $L_{\infty}=100 \mathrm{~cm}, t_{0}=0$ year, $M=0.2 \mathrm{yr}^{-1}, q=0.002$ per unit effort, $t_{\mathrm{c}}=1$ year, $t_{\max }=10$ years (i.e. exploitable life span of 9 years), 15 years of mean length observations, and change in fishing effort occurred at the start of the sixth year. The population was in equilibrium (i.e. experiencing $Z_{1}$ ) prior to the start of the mean length data collection. These parameters and conditions were used to generate mean length datasets using the population and growth model outlined earlier. Predicted values of mean length at the start of the time series were calculated consistent with the


Figure 2. Simulated data and parameter estimates for the base case scenario (total mortality rates $Z_{1}=0.6, Z_{2}=1$, and von Bertalanffy $K=0.4$ ). Top row: Generated mean length data with added variability ( $\sigma^{\prime}$ ); a single realization is shown here for $\sigma^{\prime}=$ (a) 1 , (b) 3 , and (c) 5 . Scatterplots of 5000 estimates of natural mortality rate $M$ versus estimated catchability $q$ (middle row) and corresponding $Z_{2}$ versus $Z_{1}$ (bottom row). Dashed lines denote the true parameter values simulated and the dotted lines denote the 2.5 th and 97.5 th percentiles of the estimates. Insets of histogram (bottom row) show the ratio of $Z_{2} / Z_{1}$ estimates, with dashed lines denoting the ratio of 1 and the solid triangles denoting the true ratio of $Z_{2} / Z_{1}$ estimates $(=1.67)$.


Figure 3. The contrasting responses in mean length to approach equilibrium when subjected to conditions of (a) a single increase in total mortality rate, $Z$, from 0.6 to $1.0 \mathrm{yr}^{-1}\left(\Delta Z=+0.4 \mathrm{yr}^{-1}\right)$ versus (b) a single decrease in $Z$ from 1.0 to $0.6 \mathrm{yr}^{-1}\left(\Delta Z=-0.4 \mathrm{yr}^{-1}\right)$. The stock shown here is simulated with the von Bertalanffy parameter $K=0.4$ and $L_{\infty}=100 \mathrm{~cm}$.
data generation process, i.e. on the basis that the population was in equilibrium to start. The mean length at the start of the time series in the base case computed from nine age groups is 52.04 cm while that computed from 50 age groups is 52.15 cm . This highlights the negligible effect of truncating the age distribution in the summation procedure earlier.

To evaluate the estimator's performance in relation to variability in mean length, other input parameters were fixed and assumed to be known correctly when applying the estimator to the generated dataset. To quantify the reliability of the resulting estimates of the parameters of interest, $\theta=\left\{q, M, Z_{1}, Z_{2}\right\}$, we used the measures of percent bias $\left(\%\right.$ Bias $\left.=\frac{100 \cdot \sum(\hat{\theta}-\theta)}{n} / \theta\right)$, percent root mean square error $\left(\left(\% \mathrm{RMSE}=\frac{100 \cdot \sqrt{\sum\left((\hat{\theta}-\theta)^{2}\right) / n}}{\theta}\right)\right.$ and the coefficient of variation ( $\mathrm{CV}=$ ratio of the $S D$ to the mean of $\hat{\theta}$ ) based on 5000 simulated datasets. The ratio of $\hat{Z}_{2} / \hat{Z}_{1}$ was also examined because an estimator may be better at capturing the trend of change in a parameter than it is in quantifying the absolute value of the parameter. Ratio $>1$ indicates an increase in $Z$ and vice versa.

All analyses and plotting were conducted using the R statistical programming language ( R Development Core Team, 2014). We used the nlminb minimization routine for the maximum likelihood estimation.

## Application to Norway lobster

We illustrate use of the method using data for the Norway lobster stock in South and Southwest Portugal [Functional Units (FUs) 28, 29]. The two FUs are currently assessed and managed as a single stock. Previous assessments used data-rich methods but, due to poor model performance, this stock is currently assessed using datalimited methods (ICES, 2014). Landings, CPUE, and length composition data from the commercial trawl fleet were obtained from the ICES Working Group for Data-Limited Stocks (WKLIFE V; ICES, 2015). Effective effort of the trawl fleet from 1998 to 2014 was calculated as the ratio of commercial landings and CPUE. Length composition data from 2000 to 2014 were sex-specific and mortality rates were consequently estimated separately for both female and male Norway lobster. The $L_{\mathrm{c}}$ was selected to be 32.5 mm carapace length which is the mode of the length frequency distributions of both sexes (Figure 4). However, the analysis was repeated for a series of values of $L_{\mathrm{c}}$ above 32.5 as a diagnostic check on constant


Figure 4. Observed length frequency distribution by sex of Norway lobster. Thick, vertical line indicates value of $L_{c}$ chosen.

Table 2. Mean lengths above the length at first capture, $L_{c}=$ 32.5 mm , and effort data for the Norway lobster.

|  |  | Mean length (mm) |  |
| :--- | :--- | :---: | :---: |
| Year | Effort | Females | Males |
| 1998 | 3.75 | - | - |
| 1999 | 3.49 | - | - |
| 2000 | 4.57 | 38.1 | 42.5 |
| 2001 | 7.37 | 37.7 | 42.2 |
| 2002 | 5.76 | 37.5 | 41.9 |
| 2003 | 4.07 | 38.6 | 41.5 |
| 2004 | 4.74 | 37.3 | 40.5 |
| 2005 | 3.94 | 36.6 | 39.0 |
| 2006 | 3.28 | 37.1 | 40.3 |
| 2007 | 3.50 | 36.5 | 39.5 |
| 2008 | 2.50 | 37.9 | 42.6 |
| 2009 | 2.41 | 37.9 | 41.5 |
| 2010 | 2.24 | 38.0 | 41.6 |
| 2011 | 2.21 | 39.8 | 45.0 |
| 2012 | 2.87 | 36.3 | 38.3 |
| 2013 | 2.66 | 36.6 | 40.8 |
| 2014 | 2.40 | 37.9 | 42.32 |

Unit of effort is $10^{4}$ trawl hours.
selectivity and correct specification of $L_{\infty}$. Annual length frequency distributions were examined for trends in the modal length over time to check on the constancy of recruitment. The annual mean lengths above $L_{\mathrm{c}}$ were then calculated (Table 2).

For both sexes, models were fitted with the natural mortality rate estimated or fixed to the assumed value used in previous assessments (ICES, 2014). Natural mortality is generally assumed to be lower for females based on the burrowing behaviour and implied reduced predation risk. Sex-specific von Bertalanffy growth parameters were also used in the model (Table 3). Effort prior to 1998 was assumed to be equal to that in the first year of the model. Confidence intervals with accelerated bias correction (BCa; Efron, 1987) for estimated and derived parameters were obtained from 1,000 bootstraps.

## Results

## Simulations

Estimates of $q$ and $M$ were highly negatively correlated in all scenarios, with $r^{2} \approx 0.8$ (Figure 2, middle row). For the base case
scenario ( $Z_{1}=0.6, Z_{2}=1, Z_{2} / Z_{1}=1.67$ ), $\hat{q}$ and $\hat{M}$ were reliable when $\sigma^{\prime}=1(q: \%$ Bias $=0.06, \%$ RMSE $=7.4, \mathrm{CV}=0.07 ; M: \%$ Bias $=-0.5, \% \mathrm{RMSE}=6.2, \mathrm{CV}=0.06$ ) but the reliability of the estimates decreased with increasing $\sigma$ ' (see Figure 2, middle row). With $\sigma^{\prime}=3, \%$ RMSE $=21.8$ and 18.8 for $\hat{q}$ and $\hat{M}$, respectively and when $\sigma^{\prime}=5$, the corresponding $\%$ RMSE $=37.1$ and 31.2. The CV values increased from $0.22\left(\sigma^{\prime}=3\right)$ to $0.36\left(\sigma^{\prime}=5\right)$ for $\hat{q}$ and from 0.19 to 0.31 for $\hat{M}$.

The \% RMSE increased linearly with increasing $\sigma$ ' across all the scenarios simulated (Figure 5). Relative to the base case scenario (Figure 5a, top row), the $q$ and $M$ parameters appeared to be almost as well estimated when a reversed directional change in $Z$ of equal magnitude was simulated (Figure 5c, top row). With less contrast in the change in $Z$ (Figure 5d, top row), the $q$ and $M$ parameters were less well estimated than the base case, especially for $q$. The least favourable scenario for the performance of the estimator appeared to be the case of a low von Bertalanffy $K$ (Figure 5b, top row), with \% RMSE as high as $73 \%$ for $\hat{q}$ and $55 \%$ for $\hat{M}$ when $\sigma^{\prime}=5$. The model appeared to perform better

Table 3. Life history values used in the Norway lobster application.

| Parameter | Female | Male |
| :--- | :--- | :--- |
| Von Bertalanffy $L_{\infty}(\mathrm{mm})$ | 65 | 70 |
| Von Bertalanffy $K\left(\mathrm{yr}^{-1}\right)$ | 0.065 | 0.200 |
| Von Bertalanffy $t_{0}(\mathrm{yr})^{\text {Natural mortality } M\left(\mathrm{yr}^{-1}\right)}$ | 0 | 0 |

in the estimation of $M$ rather than $q$ across the simulation scenarios based on the \% RMSE.

However, $Z_{1}$ and $Z_{2}$ estimates were not correlated with each other and were reliable even at high levels of $\sigma^{\prime}$ (Figure 2, bottom row). For the base case scenario, \% RMSE ranged from 2.2 to $11.3 \%$ for $\hat{Z}_{1}$ and from 2.5 to $12.5 \%$ for $\hat{Z}_{2}$ across the levels of $\sigma$ '. The CV values were $0.02,0.07$, and 0.11 for $\hat{Z}_{1}$ and $0.02,0.07$, and 0.12 for $\hat{Z}_{2}$ when $\sigma^{\prime}=1,3$ and 5 respectively. Histograms of the ratio of $\hat{Z}_{2} / \hat{Z}_{1}$ indicated that the center of the distribution coincided with the true $Z_{2} / Z_{1}$ ratio and the model reliably predicted an increase in $Z$ over the time series (ratio> $1100 \%$ of the time) even with increasing $\sigma^{\prime}$ (Figure 2, bottom row).

Relative to the base case scenario (Figure 5a, middle row), the $Z_{1}$ and $Z_{2}$ parameters were almost as well estimated in the scenarios of decreasing $Z$ (Figure 5c, middle row) and less contrast in $Z$ (Figure 5d, middle row) with \% RMSE of no more than $15 \%$. However, $Z_{1}$ and $Z_{2}$ were less well estimated in the case of the low $K$ (Figure 5b, middle row), with $\%$ RMSE up to $27 \%$ when $\sigma^{\prime}=$ 5. Nevertheless $Z_{1}$ and $Z_{2}$ were reliably predicted across all the scenarios simulated (ratio was $>1$ for scenarios with an increase in $Z$ and $<1$ for the scenario of a decrease in $Z 100 \%$ of the time; see Figure 5, bottom row).

From the scatterplots of Figure 5 (bottom row), we glean additional information on the behaviour of the estimator. $Z_{1}$ was estimated slightly better than $Z_{2}$ when an increase in $Z$ was simulated (base case, see Figure 5a bottom row), but the opposite was observed when a decrease of the same magnitude $(\Delta Z=-0.4)$ was


Figure 5. \% RMSE as a function of variability in mean length ( $\sigma^{\prime}$ ), for the estimates of natural mortality rate $M$ and catchability $q$ (top row) and corresponding $Z_{2}$ and $Z_{1}$ (middle row) for varying scenarios: (a) Base case, (b) Low $K$, (c) Decreased $Z$, and (d) Less contrast in $Z$ (see Table 1 for actual parameter values). Bottom row: Scatterplots of 5000 estimates of $Z_{2}$ versus $Z_{1}$, given $\sigma^{\prime}=3$. Insets of histogram show the ratio of $Z_{2} / Z_{1}$ estimates, with dashed lines denoting the ratio of 1 and the solid triangles denoting the true ratio of $Z_{2} / Z_{1}$ for each scenario.


Figure 6. Model output time series for female (left column) and male (right column) Norway lobster. Top row: observed (grey dots and lines) and predicted (black lines) mean lengths when natural mortality is estimated (solid lines) or fixed (dashed lines). Bottom row: estimated instantaneous fishing mortality rate (grey lines) and total mortality rate (black lines) when natural mortality is estimated (solid lines) or fixed (dashed lines).
simulated (Figure 5c bottom row). In addition, less contrast in $Z$ ( $\Delta Z=+0.2$, Figure 5 d bottom row) did not appear to affect the reliability of the $Z$ estimates relative to the base case scenario.

Estimates of $q$ were almost always positively biased in our simulation while $\hat{M}$ may be biased negatively or positively depending on the scenario. The magnitude of the $\%$ Bias for $\hat{q}$ was greater than that of $\hat{M}$. However, bias appeared to be a negligible component in the RMSE as evidenced by a maximum of $10 \%$ bias in the $\hat{q}$ estimates and $5 \%$ in $\hat{Z}$ when $\sigma^{\prime}=5$ in the least favourable scenario of a low $K$. Although the \% RMSE of $\hat{q}$ and $\hat{M}$ can be very high when the $\sigma^{\prime}$ levels are high, the $\%$ RMSE of $\hat{Z}$ were often $<15 \%$ with the exception of the low $K$ scenario.
We examined additional scenarios specifically in relation to a low $K$ situation. When less contrast in $Z$ was simulated (Table 1e), the \% RMSE for $\hat{q}$ and $\hat{M}$ increased; however the \% RMSE for $\hat{Z}$ improved. When additional years of mean length and effort data were included (Table 1f), the \% RMSE for all parameters improved, especially for $\hat{q}$ and $\hat{Z}_{2}$. We also simulated scenarios with increasing trend and decreasing trend in effort. In general, these additional simulations are consistent with other scenarios, namely the $q$ and $M$ estimates were correlated but estimates of $Z$ still remained relatively precise and uncorrelated. Results for these additional simulations are documented in the Supplementary Material.

## Application to Norway lobster

The effective effort showed a sharp increase from 1998 until 2001 followed by a gradual decrease over time (Table 2). The fits to the mean length data were similar regardless of whether $M$ was estimated or fixed (Figure 6, top row). For male lobster, a slight reduction and increase in mean lengths was predicted by the model

Table 4. Parameter estimates for the Norway lobster application with $95 \%$ accelerated bias-corrected Cls obtained from bootstrapping in parenthesis.

|  | Female |  |  | Male |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Parameter | Estimate $\boldsymbol{M}$ | Fix $\boldsymbol{M}$ |  | Estimate $\boldsymbol{M}$ | Fix $\boldsymbol{M}$ |
| $9\left(10^{-2}\right)$ | $1.1(0.00022,6.6)$ | $2.3(1.6,2.9)$ |  | 2.7 | $(0.00088,8.1)$ |
| $4.2(3.3,5.4)$ |  |  |  |  |  |
| $M$ | $0.25(0.01,0.31)$ | - |  | $0.36(0.15,0.49)$ | - |
| Correlation | -0.99 | - |  | -0.98 | - |
| $\quad q, M$ |  |  |  |  |  |

Fixed $M$ values $=0.2$ (female) and 0.3 (male).
runs during the time series concurrent with the increase and decrease in effort (Figure 6, top right panel). A similar trend was predicted for the mean lengths of females but was less perceptible (Figure 6, top left panel). When $M$ was estimated for female lobster, $F$ (proportional to the effort) was estimated to be 0.09 at the peak in 2001 and decreased to 0.03 in 2014, the most recent year of the time series, with an estimated $M$ of 0.25 (Table 4). With $M$ fixed at 0.2 , the estimated $F$ s were higher, reaching 0.16 in 2001 and decreasing to 0.05 in 2014.

Mortality rates for males lobster were higher than for females. When $M$ was estimated, the peak $F$ in 2001 was estimated to be 0.20 and decreased to 0.06 by 2014, with an estimated $M$ of 0.36 . With $M$ fixed at 0.30 in the model, the peak $F$ in 2001 was higher at 0.30 and generally decreased over time to 0.10 by 2014. For both sexes, the current $F$ is low relative to $M$.

For both sexes, there was high uncertainty around the point estimate for $q$, and consequently for the year-specific $F$ 's, when $M$ was concurrently estimated (Table 4, Figure 7). However, there
was more certainty in the year-specific $Z$ rates as confidence intervals were narrower (Figure 7). This arose from the highly negative correlation of the estimates of $q$ and $M$ (Table 4). Diagnostic procedures failed to detect problems with the assumptions of a flat-topped selectivity curve (Figure 8) and constant recruitment (not shown).

## Discussion

The model presented in this article extends the application of non-equilibrium length-based estimators by utilizing additional information on fishing effort. Our simulation study of the model showed that the estimates of $q$ and $M$ are highly negatively correlated and their reliability decreases with increasing error in mean length information. This is not a surprising result given that other authors have noted the same difficulty in simultaneous estimation of $q$ and $M$ (e.g. Wang, 1999; Fu and Quinn II, 2000). It still may be possible to obtain reliable estimates of $q$ and $M$ when the mean length data closely reflect the expected true trend in mortality changes, i.e. increasing mean length with decreasing $Z$ and vice versa.

However, even with poorly estimated $q$ and $M$, the corresponding estimates of $Z$ appeared to be reliable. The mean length will respond to a change in $Z$ by equilibrating to the new $Z$ more rapidly when (i) the body growth rate is high, (ii) the magnitude of change in $Z$ is small, and (iii) the change in $Z$ constitutes an increase in $Z$ rather than a decrease of the same magnitude. Hence the proposed extended model appeared to be most promising
when applied to fisheries with such characteristics. These conditions, however, are not prerequisites for reliable $Z$ estimates. Simulation results were also very promising with regard to the model's ability to capture correctly the trend in change of $Z$ rates even with large noise in the mean length data.

The model yields potentially more detailed information than the original Gedamke and Hoenig (2006) model, i.e. year-specific versus of period-specific mortality rates, while reducing the number of parameters ( $q$ and $M$ versus pre- and post-change $Z$ s and year(s) of change for each time period, depending on the assumed number of breaks in the Gedamke and Hoenig model). By using both length and effort data, this approach constitutes a bridge between data-limited models and more complex models and is potentially useful for checking assessment results from complex models.

Although the assumptions of constant $q$ and $M$ are rather tenuous, especially for stocks with a long history of exploitation and diverse fishing patterns, the primary application of length-based estimators is in the realm of data-poor fisheries where the ability to obtain reliable estimates of time-varying $q$ and $M$ is unlikely. Others have shown that, even with underlying time-varying $M$ in the fishery dynamics, the assumption and estimation of a constant $M$ in the model can be a useful and viable option (Deroba and Schueller, 2013; Johnson et al., 2015).

In the absence of direct fishing effort data, one could estimate effort from total commercial catch and a catch rate series. In practice, it may be difficult to obtain a time series of standardized fishing effort because of multiple and changing fishing gears. The


Figure 7. Estimates of fishing (F, grey squares) and total mortality rates (Z, black triangles) with 95\% Cls (dot-and-dash lines) when the natural mortality rate is estimated.


Figure 8. Effect of varying the values of $L_{c}$ on the resulting estimates of $M$ and $q$ for female and male Nephrops. The vertical line on the left represents the value of $L_{c}$ adopted for the stock assessment while the vertical line on the right represents the value of $L_{c}$ corresponding to the use of $13 \%$ of the length observations on the right.
availability of research survey effort data presents a potential solution to this problem via the equation of $\frac{\text { Total Catch } \text { commercial }^{\text {CPUE }} \text { surrey }}{}$ $=$ Effective Effort where effective effort is defined as the fishing effort required by the commercial fleet to obtain the given catch if all the fishers are fishing in a similar manner as that of the survey boat. Hence, the corresponding estimated $q$ in the model will be the survey catchability coefficient.
As mentioned earlier, the extent to which the constant $q$ assumption is met is dependent on the ability to standardize effort. Typically, covariates that affect $q$ are identified and their effects are removed using generalized linear models (Lo et al., 1992; Maunder and Punt, 2004). This standardization procedure is important in situations where the effect of the covariates is not constant and the behaviour of the fishery has changed over time. For example, improvements in fishing efficiency, such as the use of more efficient hook types (Stewart and Martell, 2014) or the relative experience of active fishers (van Poorten et al., 2016), can additively increase the effective $q$ of a nominal unit of fishing effort over time. Standardization relies on identifying effort associated with and without these covariates. However, if these data are not collected, the effect of covariates cannot be de-trended from the overall effort time series. Similarly, if reporting has changed over time, then the trend in nominal effort is likely to be biased. It is important to note that even with proper standardization of effort, the CPUE may still not be proportional to abundance (Walters, 2003; Ellis and Wang, 2006).
In a data-limited model, one can explore alternative hypotheses to evaluate the importance of this constant $q$ assumption. If one conceivably has working knowledge of the year or time period in which $q$ in the fishery has shifted, one could partition the dataset into two parts and fit the model with different $q$ estimates for each time period. Similarly, the division of the time series with break points in $q$ can decrease error associated with a trend in $q$ over time. One can then compare the models with and without breakpoints using a likelihood-ratio test for parsimony and model selection. It is recommended that this comparison be conducted routinely as a diagnostic procedure given that the time series is sufficiently long. On the other hand, our simulation studies showed that it is relatively harder to obtain reliable estimates of $q$ than $M$ when estimated simultaneously; hence one could reduce the number of parameters by fixing the value of $M$ in the model in the interest of estimating $q$.

Due to the strict functional form of $Z$ as a linear function of effort, the overall trend of the mean lengths and effort can be informative and be resilient to occasional strong cohorts. On the other hand, a strong trend in recruitment can create conflicting signals between the mean length and effort. For example, effort in the North Sea dab (Limanda limanda) trawl fishery was estimated to have decreased, which suggested a decrease in mortality over time (Section 4.6.4.2 of ICES, 2016), yet a lack of trend in mean length was observed. The mean length with effort model would have produced conspicuous residual patterns. However, a linear increase in recruitment (fourfold over the observed time series) was also observed in the recruitment index. The increased recruitment balanced the decreased effort to keep mean length constant. In this case, the use of the recruitment index rather than the constant recruitment assumption resulted in better model performance.

Other factors such as the effects of sex-specific differences in growth and varying recruitment were not considered in the simulations for this article, but can be incorporated when such
information is readily available as in the Norway lobster example (discussed further below). An analyst with knowledge of the fishery can parameterize the model specifically to the fishery for further simulation studies of the model performance.

## The Norway lobster population

The application of the mean length and effort model to Norway lobster generally supports the simulation results regarding the reliability of the estimates of $Z$. For both sexes, similar model fits to the mean lengths are observed when $M$ is estimated and fixed (Figure 6). There is little trend in the observed mean lengths over time which suggests that $Z$ has not changed much. Despite this, $Z$ 's can be estimated well from the mean length data and are consistent regardless of whether $M$ is estimated or fixed (Figure 6). With a fixed $M$, the estimated $q$ is small so that the estimated $F$ 's are small relative to $M$. On the other hand, the estimate of $q$ used to estimate the magnitude of $F$ is much less precise due to the low contrast in $Z$. As a result, there is low precision in the year specific $F$ s based on the wide confidence intervals. An improved estimate of $q$ would require more precision and contrast in the mean length data or a reliable estimate of an externally obtained $M$.

Additionally, the availability of sex-specific length composition and life history data allowed for the analysis of sex-specific mortality rates. Higher $M$ and $F$ s were estimated for males relative to females and the estimated $M$ rates were similar to the values previously assumed in the ICES assessment (ICES, 2014). This is consistent with the life history of the stock, with decreased vulnerability to fishing pressure and predation risks for females due to their burrowing behaviour.

## Further methodological developments

Guiding principles for the development of new methods for assessing data-limited stocks may be expressed in terms of the following desiderata:
(i) Relaxed assumptions
(ii) Diagnostics
(iii) Model flexibility and the integration of diverse data types
(iv) Bridges to (more) data-rich methods

Relaxed assumptions refers to models that require fewer assumptions or are robust to failures of assumption, or can handle failures of assumption, e.g. by estimating extra parameters. For the method described in this article, a concern may be that catchability is assumed constant over time. However, technological improvements may result in increasing gear efficiency (effort creep) or a change in survey vessel can result in a change in catchability. Approaches to address this issue had been discussed earlier. In general, there are enough degrees of freedom to allow for more than one catchability parameter to be estimated so that separate $q$ 's can be estimated for each of several time stanzas or a functional form for the variability in $q$ over time can be imposed. Similarly, variation in natural mortality rate with time or age can be modelled. However, it is not possible to let $q$ and $M$ vary together without external information to inform the parameter estimation.

The quantification of uncertainty in assessment results is extremely important but is usually conditional on the underlying model being correct. Consequently, diagnostic procedures are
extremely important in identifying areas of uncertainty. The assumptions of constant recruitment and knife edge selectivity in the model presented here can be checked using the diagnostic procedures described in this manuscript. In general, the incorporation of additional types of data into the model requires additional assumptions to be made. However, by incorporating additional data, one will likely have enhanced diagnostic capabilities. For example, Huynh et al. (2017a) considered a model with observations on mean length and aggregate catch rate over time. They noted that variable recruitment tended to cause the length residuals and catch rate residuals to be of opposite sign-this afforded the opportunity to detect the effects of an extreme recruitment event.

It is an accepted principle that a stock assessment should consider all information available even if some information is discounted as less reliable than other information. Thus, an assessment model should afford the opportunity to incorporate various types of information. A rich set of models can be developed from the basic non-equilibrium model for mean length described by Gedamke and Hoenig (2006) and the extended model utilizing effort data presented here. If an index of recruitment is available, it can be incorporated into either model to relax the assumption that recruitment is constant over time (see Gedamke et al., 2008; ICES, 2016). Huynh et al. (2017a) incorporated an aggregate catch rate over time. Models have also been developed for multi-species and multi-stock inference (Punt et al., 2011). In the former, changes in mortality rate can be made synchronous as, for example, changes in fishing effort in a trap fishery; the effort change can be expected to affect the entire assemblage of fish simultaneously (Huynh et al., 2017b). In the latter, parameters such as natural mortality rate can vary among stocks but may be assumed to be drawn from a distribution.

The last desideratum refers to the evolution of a stock assessment from data-poor to data-rich methods. The method presented in this article uses mean length and effort observations over time; effort might be obtained by dividing total catch by catch per unit effort. Catch and effort data can form the basis of a surplus production model if the time series is long enough and has enough contrast. The use of the model described in this article can thus serve as a bridge between the data-poor and datarich worlds, providing immediate benefit from the collection of catch and effort (or catch rate) data while leading in the long term to the integration of a production model with mean length observations.

## Summary

The extended, non-equilibrium, mean length-based estimator presented in this article appears to be a promising tool in assessing relatively data-limited stocks. Even if one cannot obtain reliable estimates of the parameters $q$ and $M$ because considerable noise is present in the mean length data, the corresponding estimates of $Z$ may be useful. If fisheries and stock assessors are interested primarily in trends in changes of $Z$ over the time series, this method appears to be able to capture this information accurately, given that the other assumptions of the method are met.

## Supplementary data

Supplementary material is available at the ICESJMS online version of this article.

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