3-1-1998

RDE Model: A Program for Simulating Water Wave Transformation for Harbor Planning

Jerome P.Y. Maa
Virginia Institute of Marine Science

T. W. Hsu
Hydraulics and Ocean Engineering Dept. National Cheng-Kung University

H. H. Hwung
Hydraulics and Ocean Engineering Dept. National Cheng-Kung University

Recommended Citation

Maa, J. P., Hsu, T. W., & Hwung, H. H. (1998) RDE Model: A Program for Simulating Water Wave Transformation for Harbor Planning. VIMS Special Scientific Report No. 136. Virginia Institute of Marine Science, College of William and Mary. https://doi.org/10.25773/hd20-g381

RDE Model: A Program for Simulating Water Wave Transformation for Harbor Planning

Special Scientific Report, No. 136

Jerome P.-Y. Maa

School of Marine Science
Virginia Institute of Marine Science
College of William and Mary
Gloucester Point, VA 23062

T.-W. Hsu and H.-H. Hwung

Hydraulics and Ocean Engineering Dept.
National Cheng-Kung University
Tainan, Taiwan, R.O.C.

March 1998

Abstract

The extended mild slope equation, an elliptic type partial differential equation, was solved directly to simulate water wave refraction, diffraction, reflection, shoaling, and harbor resonance for harbor planning purposes. The finite difference method was used to build a banded matrix equation which was then solved directly by using the Gaussian elimination method with partial pivoting and a newly-developed book-keeping procedure. Because this book-keeping procedure changes the large computer memory requirements into a large hard disk size requirement, this model is capable of handling realistic applications efficiently using small computers. Five verification tests were selected to demonstrate the performance of this numerical model. Comparisons between the model results and available experimental or analytical results indicate a satisfactory agreement. Because of the finite difference method and the direct approach in solving the governing equation, this model is simple to maintain, and more importantly, to upgrade for including other processes, e.g., bottom friction, tidal current influence, and spectrum waves, in the future.

The computer codes for the main program, pre- and post-process software are all included in this report. The above codes are also available in a floppy disk, upon requested. The purpose is letting uses to access this method easily, and of course, provide feed-back if possible.

TABLE OF CONTENTS

TABLE OF FIGURES

Page

INTRODUCTION

Water wave transformation, i.e., the change in wave height and direction caused by irregular bathymetry and/or structures, plays an important role in harbor planning, navigation safety, as well as shore protection and environmental regulations. There are many numerical models available to simulate wave transformation which can be described by the elliptic mild slope equation (Berkhoff 1972; 1976). These models can be divided into three categories: (1) Using a parabolic approximation to simulate wave refraction, weak diffraction, and shoaling for a relatively large study domain, or (2) Solving a hyperbolic, time dependent, mild slope equation, (3) solving the original elliptic, mild slope equation directly.

The first approach imposes some restrictions, but can be solved relatively fast (e.g., REFDIF-1 by Kirby and Dalrymple 1991). Under this category, numerous studies have been conducted during the past decades (e.g., Kirby 1986a; 1986b; 1988; Panchang *et al.* 1988; Dalrymple *et al.* 1989). The second category deals with a transient mild slope equation (Copeland 1985; Madsen and Larsen 1987) and looks for the results at steady state. The computing speed of this approach is approximately the same as to solve the mild slope equation directly (Li 1994b). The advantage is that this approach does not require a huge computer memory. The third category is usually limited to a small study domain, e.g., harbors, because of the huge memory requirement for large domains.

Improvements on the mild slope equation by adding the effects of steep bottom slopes and bottom curvatures have been established recently (Massel 1993; 1995; Chamberlain and Porter 1995; Porter and Staziker 1995). For harbor planning, this extended mild slope equation can more accurately describe the possible drastic changes of water depth in harbors, and thus, it was selected for this study.

Numerical methods developed in the third category can use finite element method (e.g., Behrendt 1985; Chen and Houston, 1987), or finite difference method. Because of the nature of finite element technique, this kind of model is hard to maintain or upgrade. The available finite difference models all used an iteration method (Li and Anastasiou 1992, Li 1994a; 1994b) because of the low computer memory requirements. A major concern about the iteration method is the convergent rate degrades significantly for a complex boundary geometry. The alternative approach, uses the Gaussian elimination method (Dongarra 1979; Mathews 1987) to directly solve the huge banded matrix equation (hereafter called the direct method), was only possible on main-frame computers with immense core memory (on the order of gigabytes or more). For this reason, using the direct method on small computer for harbor planning purposes has never been attempted.

Recently, however, a thrifty banded matrix solver (Maa *et al.* 1997) has been developed. This solver only requires a modest amount of core memory and a large hard disk (which is readily available even for 10 Gigabytes). Thus allowing us to use the direct method to solve the extended mild slope equation with a small computer.

To demonstrate this approach, we first present the governing equation and boundary conditions, followed by the formulation of the finite difference equations (FDE). Applying these FDEs to a study domain resulted in a huge banded matrix equation, which was solved using the thrifty banded matrix solver (Maa *et al.* 1997). Five cases, in which either analytical solutions or experimental results were available, were selected for model verification.

Only simple harmonic waves were used in this first stage study without considering energy loss caused by bottom friction. The more realistic multidirectional random waves as well as other improvements (e.g., bottom friction, identification of wave breaking, third order approximation for the total passing boundary condition, complex wave reflection coefficient to include phase difference) were left for the next stage.

The computer codes for the main programs, all subroutines, and other pre- and post-processing computer codes are all presented in the Appendices. The most-up-to-date source codes are available in floppy disk, upon request.

GOVERNING EQUATION

The extended mild slope equation, Eq. 1, (Massel 1995) was selected for this study. Although Eq. 1 can be transformed to the Helmholtz equation and then solved numerically, it was decided to solve the original form to simplify future upgrades.

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2} + \frac{e_0}{h}\left(\frac{\partial h}{\partial x}\frac{\partial\phi}{\partial x}+\frac{\partial h}{\partial y}\frac{\partial\phi}{\partial y}\right) + k^2(1+\psi)\phi = 0 \qquad (1)$$

where

$$\psi = e_1(kh)\left[\left(\frac{\partial h}{\partial x}\right)^2+\left(\frac{\partial h}{\partial y}\right)^2\right] + \frac{e_2(kh)}{k_o}\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right) \qquad (2)$$

$$e_0 = \frac{kh}{\tanh kh + kh(1-\tanh^2 kh)} * \left(1 - 3\tanh^2 kh + \frac{2\tanh kh}{\tanh kh + kh(1-\tanh^2 kh)}\right) \qquad (3a)$$

$$e_1 = \frac{1}{n\tanh kh}\cdot\frac{1}{24(2kh+\sinh 2kh)^2\cosh^3 kh}\cdot$$

$$\{kh[12+16(kh)^2]\cosh kh + 6kh[\cosh 3kh + \cosh 5kh] +$$

$$[12+84(kh)^2]\sinh kh + 3[1-4(kh)^2]\sinh 3kh - 9\sinh 5kh\} \qquad (3b)$$

$$e_2 = \frac{1}{n}[\frac{-4kh\cosh kh + \sinh kh + (kh)^2\sinh kh + \sinh 3kh}{8(2kh + \sinh 2kh)\cosh^3 kh} - \frac{kh}{2}\frac{\tanh kh}{\cosh^2 kh}] \tag{3c}$$

$$n = \frac{1}{2}\left[1 + \frac{2kh}{\sinh(2kh)}\right] \tag{3d}$$

ϕ is the velocity potential function for a simple harmonic wave flow, $k_o = 4\pi^2/gT^2$ is the deep water wave number, T is the wave period, g is the gravitational acceleration, $k = 2\pi/L$ is the local wave number, L is the local wave length, h is the water depth, $\partial h/\partial x$ and $\partial h/\partial y$ are the bottom slopes in the x and y directions respectively, $\partial^2 h/\partial x^2$ and $\partial^2 h/\partial y^2$ are the bottom curvatures in the x and y direction, respectively, and x and y are the two horizontal coordinates, see Fig. 1.

Using a five-point-approximation, the finite difference equation for Eq. 1 can be written as follows.

$$(1-d_x)\phi_{i-1,j} + r(r-d_y)\phi_{i,j-1} + \lambda\phi_{i,j} + r(r+d_y)\phi_{i,j+1} + (1+d_x)\phi_{i+1,j} = 0 \tag{4}$$

$$d_x = \frac{e_o}{2h}\frac{\partial h}{\partial x}\Delta x \tag{5a}$$

$$d_y = \frac{e_o}{2h}\frac{\partial h}{\partial y}\Delta x \tag{5b}$$

where

$$\lambda = k^2(1+\psi)\Delta x^2 - 2r^2 - 2 \tag{5c}$$

$r=(\Delta x/\Delta y)$, and Δx and Δy are the grid sizes in the x and y directions respectively. Notice that ψ, k, $\partial h/\partial x$, $\partial h/\partial y$, $\partial^2 h/\partial x^2$, and $\partial^2 h/\partial y^2$ can be different at each grid point because of the possible different water depths.

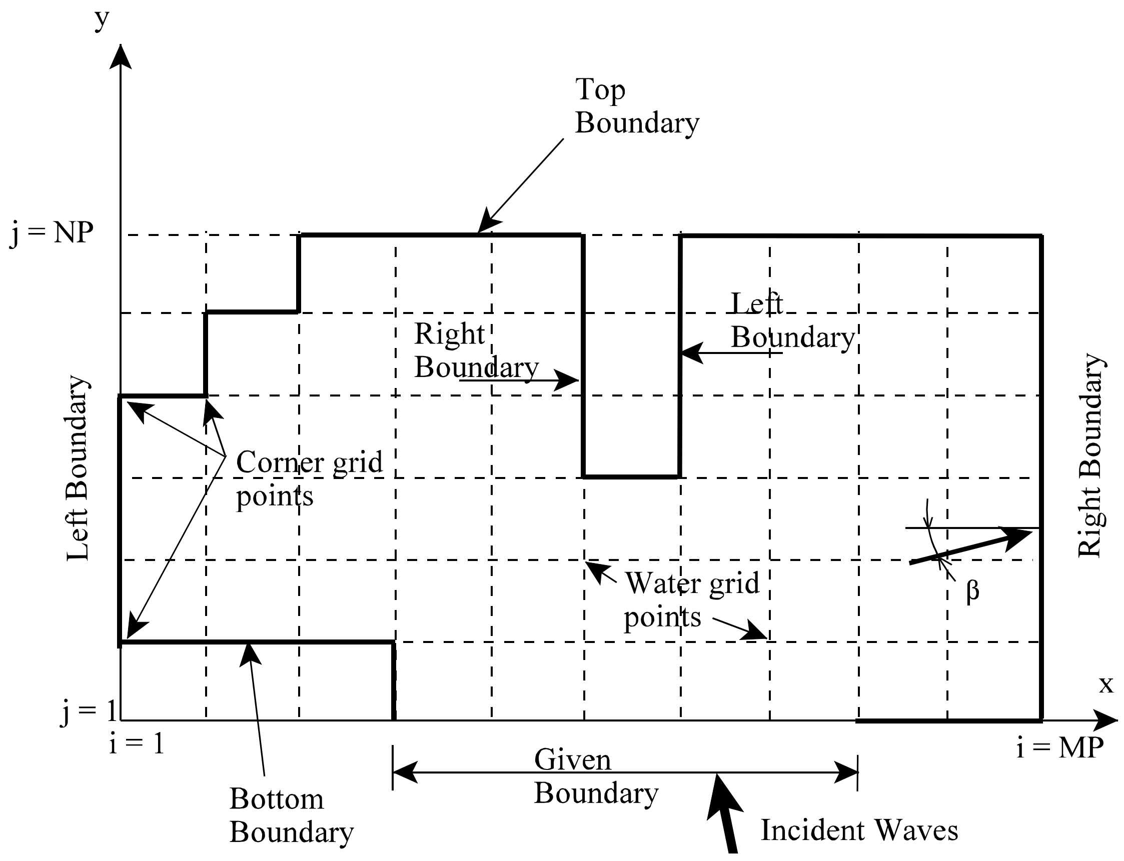


Fig. 1. Coordinate System and Grid Alignment for the Model.

BOUNDARY CONDITIONS

There are only two types of boundary conditions in the simulation of wave transformations: a partial reflection boundary condition and a given boundary condition. These conditions are specified along the border of a study domain, see Fig. 1.

<u>Partial Reflection Boundary Condition</u>: The condition described here (Eqs. 6 and 7) is actually a general condition that can be used for (1) total reflection, (2) partial reflection, or (3) total passing through (Behrendt 1985). This is because there is only one difference (in the selection of a constant coefficient, α, in Eqs. 6 and 7) among these three second-order approximations of the boundary conditions.

$$\frac{\partial\phi}{\partial x} = \pm i\alpha k(\phi + \frac{1}{2k^2}\frac{\partial^2\phi}{\partial y^2}), \qquad \text{on} \pm x \text{ boundary} \tag{6}$$

$$\frac{\partial\phi}{\partial y} = \pm i\alpha k(\phi + \frac{1}{2k^2}\frac{\partial^2\phi}{\partial x^2}), \qquad \text{on} \pm y \text{ boundary} \tag{7}$$

here $i=(-1)^{1/2}$. Equation 6 is applicable to the boundary segments that are perpendicular to the x-axis, where the positive sign is for those segments that have the water grid point on their left side. Equation 7 is applicable to boundary segments that are perpendicular to the y-axis, where the positive sign is for those segments that have the water grid point on the bottom. When $\alpha = 0$, Eqs. 6 and 7 represent total reflection boundary conditions. When $\alpha = 1$, these two equations represent total passing through boundary conditions. For $0 < \alpha < 1$, they represent partial reflection boundary conditions. Notice that Eqs. 6 and 7 are only second-order approximations, and thus, reflection waves will be introduced when the approaching waves deviate more than 30 degrees (i.e., $\beta > 30°$, see Fig. 1) off the normal line of the boundaries, even when $\alpha = 1$. For improvements on this issue, Kirby's third-order approximation may be used (Kirby 1989). Unfortunately, even that approximation has its limitations, and further studies are needed.

In order to use the second-order approximation of the boundary conditions at the boundaries, we used an imaginary grid point that is just one grid outside of the study domain. Here the boundary condition for a positive x boundary is used as an example to demonstrate this issue. Considering that the above boundary condition is specified at the boundary grid, I, in the x direction and the water grid point is located on the left side of this point, the finite difference equation for this boundary condition is

$$\phi_{I+1,j} - \phi_{I-1,j} = \beta_x \phi_{I,j} + \xi_x (\phi_{I,j+1} - 2\phi_{I,j} + \phi_{I,j-1}) \qquad (8)$$

where $\beta_x = 2i\alpha k\Delta x$ and $\xi_x = (i\alpha r^2)/(k\Delta x)$. Notice that $\phi_{I+1,j}$ is outside the study domain. Because the governing equation at this boundary grid is

$$(1-d_x)\phi_{I-1,j} + r(r-d_y)\phi_{I,j-1} + \lambda\phi_{I,j} + r(r+d_y)\phi_{I,j+1} + (1+d_x)\phi_{I+1,j} = 0 \qquad (9)$$

these two equations were used to eliminate $\phi_{I+1,j}$ at the imaginary point and the resulting equation for this kind of boundary is as follows.

$$2\phi_{I-1,j} + [r(r-d_y)+\xi_x(1+d_x)]\phi_{I,j-1} + [\lambda+(\beta_x-2\xi_x)(1+d_x)]\phi_{I,j} + [r(r+d_y)+\xi_x(1+d_x)]\phi_{I,j+1} = 0 \qquad (10)$$

Similarly, using Eq. 4 with either Eq. 6 or 7, the finite difference equations at the negative x, positive y, and negative y boundary points can be obtained (see Appendix I). For a corner grid point, three equations (i.e., Eqs. 4, 6, and 7) are used to obtain the finite difference equation (in Appendix I) that is applicable at that corner point.

<u>Given Boundary Condition</u>: This kind of boundary condition is used at those grid points where input wave information is specified. Because of the possible scatter waves generated from the study domain, the actual velocity potential function is still unknown at these grid points. In other words, there are two velocity potential values at a given boundary grid point, and the outgoing

$$\frac{\partial\phi}{\partial x} = \pm ik(\phi + \frac{1}{2k^2}\frac{\partial^2\phi}{\partial y^2}) + 2ik\phi^g, \qquad \textit{on} \pm x \textit{ boundary} \tag{11a}$$

$$\frac{\partial\phi}{\partial y} = \pm ik(\phi + \frac{1}{2k^2}\frac{\partial^2\phi}{\partial x^2}) + 2ik\phi^g, \qquad \textit{on} \pm y \textit{ boundary} \tag{11b}$$

scatter waves should pass through this boundary and remain unaffected. For this reason, a total passing through boundary condition, eqs. 6 and 7 with $\alpha = 1$, is used together with the given wave velocity potential, ϕ^g, as follows (Behrendt 1985). As an example, we are using the positive x given boundary equation to find the finite difference equation.

Considering that a given boundary is specified at the boundary grid, I, and the water grid point is located on the left of this point, the finite difference equation for this boundary condition, i.e., Eq. 11a with the positive sign, is

$$\phi_{I+1,j}-\phi_{I-1,j} = \beta^1{}_x\phi_{I,j} + 4ik\Delta x(\phi^g)_{I,j} + \xi^1{}_x(\phi_{I,j+1}-2\phi_{I,j}+\phi_{I,j-1}) \tag{12}$$

where $\beta^1{}_x = 2ik\Delta x$ and $\xi^1{}_x = (ir^2)/(k\Delta x)$. Notice that $\phi_{I+1,j}$ is outside the study domain. Because the governing equation at this boundary grid is

$$(1-d_x)\phi_{I-1,j}+ r(r-d_y)\phi_{I,j-1}+ \lambda\phi_{I,j}+ r(r+d_y)\phi_{I,j+1}+ (1+d_x)\phi_{I+1,j}=0 \tag{13}$$

these two equations are used to eliminate $\phi_{I+1,j}$ at the imaginary point and the resulting equation for this kind of boundary is given as follows.

$$2\phi_{I-1,j} + [r(r-d_y)+\xi^1{}_x(1+d_x)]\phi_{I,j-1} + [\lambda+(\beta^1{}_x-2\xi^1{}_x)(1+d_x)]\phi_{I,j} + [r(r+d_y)+\xi^1{}_x(1+d_x)]\phi_{I,j+1} = -4ik\Delta x(1+d_x)(\phi^g)_{I,j} \tag{14}$$

Similarly, one can obtain the finite difference equation (see Appendix I) at the negative x, positive y, and negative y given boundary points.

For a given monochromatic wave with wave height, H, period, T, and direction, θ (reference to the boundary), the given wave velocity potential can be calculated as (Behrendt 1985)

$$\phi^g = Ae^{is} = \frac{igTH}{4\pi}e^{is} \tag{15}$$

where A is the amplitude function and S is the phase function.

For normally incident waves, the phase function should be the same at all entrance grid points. For convenience and without loss of generality, we may choose S = 0 for this condition. For oblique incident waves, see Fig. 2, the phase function can be calculated as follows.

$$S(x_L) = \frac{2\pi\ x_L \sin\theta}{L}\ ,\quad 0 \le S(x_L) \le 2\pi \tag{16}$$

where x_L is the local one-dimensional coordinate, L is the wave length at the boundary location, and θ is the angle between the wave direction and the normal vector of the boundary.

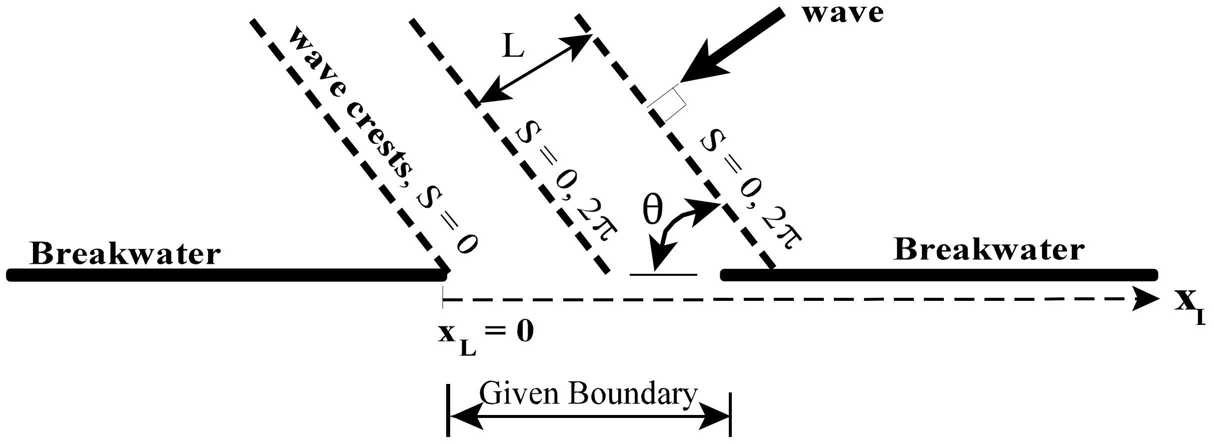


Fig. 2. Calculation of Wave Phase at Given Boundary Grid Points.

NUMERICAL MODEL

Equation 4 and these boundary finite difference equations were applied to all the water grid points in the study domain (Fig. 1), which has MP and NP grids in the x and y directions, respectively. A banded matrix equation can be established as follows

$$B\,X = G \qquad (17)$$

where **B** is a banded matrix with a dimension of M x N, N is the length of this banded matrix (same as the number of water grid points), M is the band width of this matrix, **X** is the unknown column matrix for the wave potential function, and **G** is another column matrix that includes the given boundary conditions. In general, M varies with the grid alignment as well as the geometry of the study domain. The computer codes were written in such a manner that when the x axis is selected parallel with the longer dimension of the study domain, the band width will be a minimum.

As Behrendt (1985)pointed out in his finite element model, the length of each side of an element should not exceed 1/10 of the wave length in order to solve the elliptic equation. A similar requirement holds for this finite difference model, i.e., Δx and Δy should both less than 1/10 of the wave length. Although Δx and Δy are not required to be the same in this model, the less-than-1/10-wave-length requirement practically limits the choice of Δx and Δy. This is because the maximum Δx and Δy is usually desired in practical applications.

The banded matrix equation was solved by using a thrifty banded matrix solver (Maa *et al.* 1997). The traditional banded matrix equation solver usually requires large computer memory space because it stores the entire banded matrix in memory. For practical applications, N is usually on the order of 10^4-10^5 and M is on the order of 10^2 - 10^3. Thus, storing the matrix (using 16 byte complex numbers) alone may require 16 MB to 1.6 GB of memory.

In the thrifty banded matrix solver, the sparse band matrix is stored in two much-smaller matrices (one is a complex matrix, the other is an integer matrix, each with a size of 5 x N). These two small matrices require much less computer memory, *e.g.*, only 1.2 to 12 MB is required for the previously mentioned banded matrix. Furthermore, the solver constructs the banded matrix equation one block at a time, and then, follows the standard Gaussian elimination method with partial pivoting for forward elimination. It saves the results to the hard disk, and then fetches the information from the two smaller matrices to continue the processes (i.e., constructing a block of the banded matrix equation and performing forward elimination). This procedure continues until the entire banded matrix equation is processed. The back substitution begins by reading the last saved data, also one block at a time, to solve the velocity potential. The back substitution also repeats until all the saved blocks are read and processed. Unlike the virtual memory, this process uses minimal disk I/O, leaving computing time for number crunching. For this reason, the efficiency is high. The computing times for the five cases presented next will demonstrate this point.

MODEL VERIFICATIONS

Five cases, in which either the analytical solution or experimental results were available, were selected for model verification.

Case 1. Infinite Long Breakwater with One Gap

Blue and Johnson (1949) provided experimental results for this case study, later generalized by Johnson (1953) and then documented in the Shore Protection Manual. In this case, there was an infinitely long straight breakwater with a gap of 2 wave lengths. Waves normally approached this breakwater.

The given wave condition was specified along the gap with the same phase. Along the breakwater, a total reflection boundary condition (Eqs. 6 and 7 with $\alpha = 0$) was specified. On the other three sides, a total passing through boundary condition was assigned ($\alpha = 1$). The grid size, wave parameters, and other parameters used in the model are given in Table 1.

For comparison, the calculated wave heights were normalized by the incident wave height, and then plotted on the left hand side of Fig. 3. The results documented in the Shore Protection Manual were duplicated on the right hand side of Fig. 3. It is obvious that there is an excellent agreement between these two, except at the gap. More discussion of this difference is given in the Discussion section.

The calculated contours of a specified wave phase value (e.g., $S = 3$), which may be used to represent the wave crest lines, were also plotted in Fig. 3 to compare with those documented in the Shore Protection Manual. Excellent agreement between these two are also obvious.

To insure that the computer codes were correct, we assigned the breakwater and the gap located on the left hand side, right hand side, top side, as well as bottom side of the study domain. The results were identical.

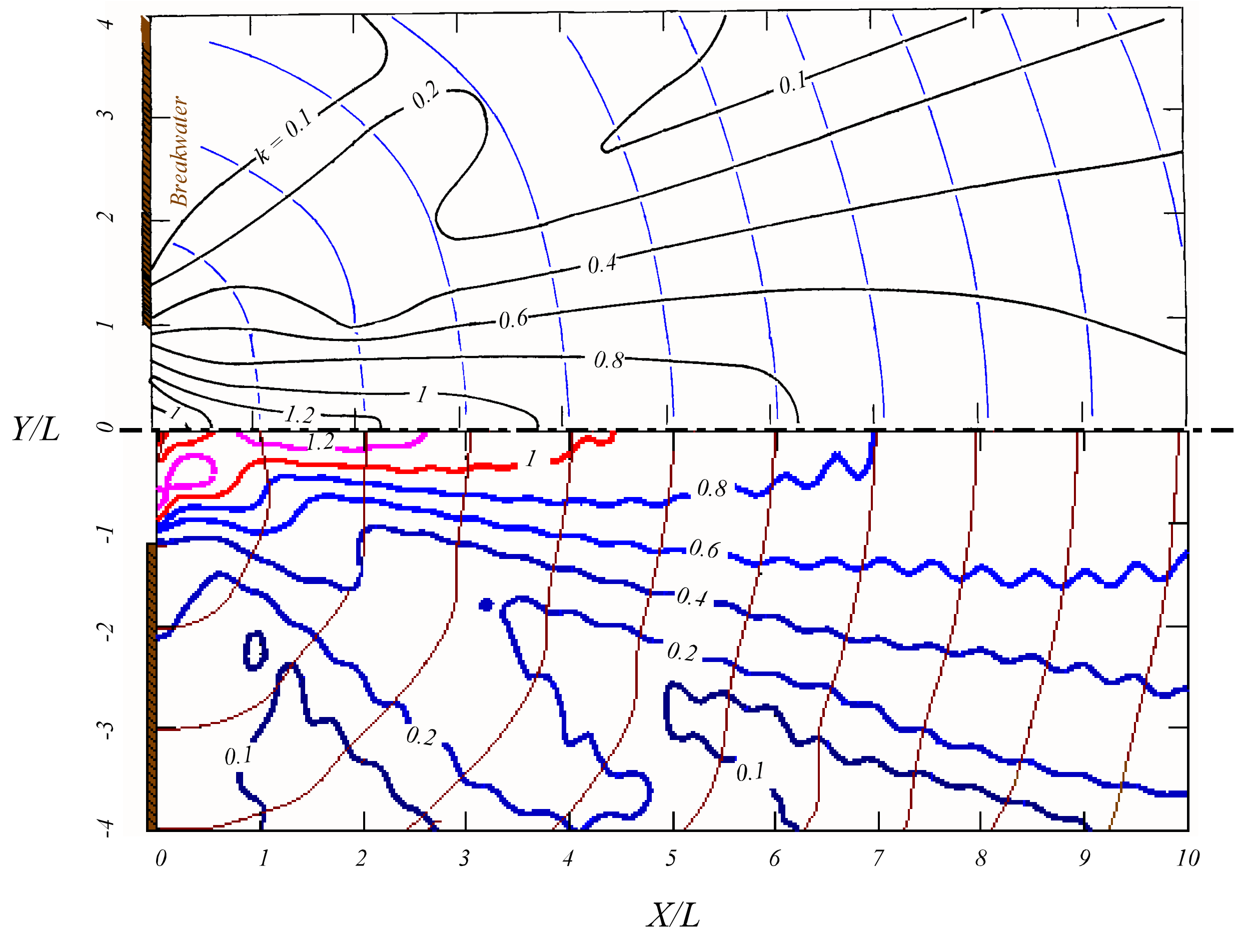


Fig. 3. Comparison of Wave Height Contours and Wave Crests for an Infinite-Long Breakwater with a Gap. The left half is a copy from the Shore Protection Manual, the right half is the model results

Table 1, Parameters used and Results in the Model Verifications

Parameters	Case 1	Case 2	Case 3	Case 4	Case 5
H (m)	0.05	0.041	0.01	0.01	0.01
T (s)	0.8	0.9	varies	varies	0.73
θ (deg)	0	60	0	0	0
h (m)	0.5	0.4	0.2573	0.1563	0.25-0.102
Δx (m)	0.1	0.1	0.003212	0.05	0.02
Δy (m)	0.1	0.1	0.003025	0.05	0.02
W x L (m)	10x8	22x9.6	0.4497 x 0.1845	10x1	2.2x1.2
MP x NP	101x81	221x97	141x62	201x21	111x61
M	163 (203)	195	125	43	123
N	8181	21437	4601	4221	6769
Computing time (s)	42 (62)	180	41	32	24

The computing time is based on a Pentium Pro Personal computer with 200 Mhz speed, 64 MB of memory, and Windows NT operation system. The memory requirement of this model to run the above five cases is about 7 MB. For Case 1, the different computing times represent the results of a different band width caused by selecting x axis parallel to the long (or short) side of the study domain.

W, L are the width and length of the study domain.
MP, NP are the grid number in x and y direction, respectively, see Fig. 1.

Case 2, A Finite-length Single Breakwater

Goda *et al.* (1971) presented their analytical solution for this case study: a two-wave-length long straight breakwater was located in a constant water depth environment. The incident waves approached the breakwater with an angle of 60 degrees; and the analytical solution of normalized wave height distribution is plotted in Fig. 4a. For numerical modeling, the given wave condition was specified along the left border, except at the portion for the breakwater, where a total reflection boundary condition (α = 0) was used. On the other three sides, the total passing through boundary condition (α = 1) was used. All parameters used to simulate this case are also given in Table 1. The model results (contours of the normalized wave height) are plotted in Fig. 4b. Reasonably good agreement can be found at the near field after the breakwater. For the far field, however, the model results are about 20% higher.

It is necessary to point out that the computing domain in the x direction was much larger than that displayed in Fig. 4b. The computing domain in the y direction was also slightly larger than that displayed in Fig. 4b (see Table 1 for the size). The selection of this computing domain was because the two possible boundary conditions on the open boundaries (α = 0 for the total reflection boundary condition, or α = 1 for the total passing through boundary condition) both required a much large simulation domain to minimize the influence of reflected waves. Although $\alpha = 1$ was specified along the three open boundaries, reflected waves were introduced because of the second order approximation of the passing through boundary condition and the large incident waves angle (30 degrees off from the normal vectors of the boundaries). Therefore, the unwanted reflected waves propagate into the computing domain and cause error at the far field. It is not clear at this time, however, how much error is contributed from the grid size selection inherent from the finite difference method.

To improve the passing through open boundary condition, a third order approximate (Kirby 1989) is necessary, and that will be the goal for our next stage modifications.

Wave Height Contours

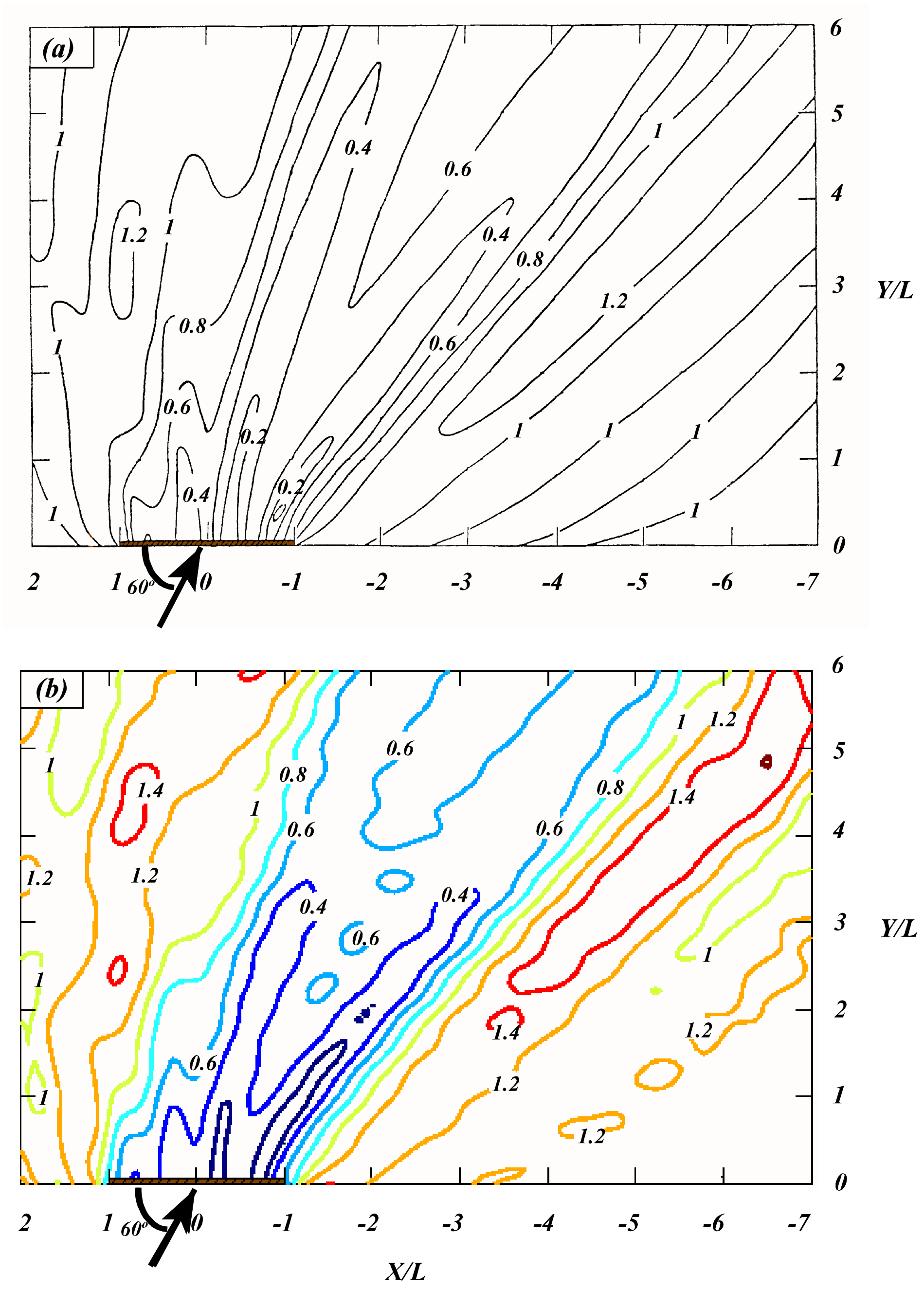


Fig. 4. Comparison of Wave Height Contours for a Finite-Length Breakwater with Oblique Incident Waves. (a) Analytical solution from Goda *et al.* (1971); (b) Model results.

Case 3. Resonance in a Rectangular Harbor

Many studies of this case have been conducted. In addition to the analytical solution given by Unluata and Mei (1973), there have been many physical model studies (e.g., Ippen and Goda 1963; Lee 1971) as well as numerical studies (e.g., Chen and Mei 1974; Lee 1971; Behrendt 1985). The geometry of this harbor is given in Fig. 5 with harbor length E = 0.3212 m, harbor width B = 0.0605 m, and a constant water depth of 0.2573 m.

In the current model simulation, the entire computation domain is given in Fig. 5. On the left hand side of this domain, the given boundary condition was specified. Along the harbor perimeter, the total reflection boundary condition ($\alpha = 0$) was specified. At the top and bottom open boundaries, the total passing through boundary condition was assigned ($\alpha = 1$). If this harbor does not exist, the wave height would be doubled at the right side border (x = 0.1285 m, see Fig. 5) for any given wave condition because of the total reflection boundary condition specified there. This doubled wave height ($2H_i$) was used to normalize the calculated wave heights at the harbor end (x=0.4497 m) shown in Fig. 6. For the harbor geometry given in Fig. 5, Unluata and Mei found that the wave height at the harbor end became very large ($H/2H_g \rightarrow 8$, see Fig. 6) when the ratio of harbor length to wave length approached 1/4 ($2\pi E/L = 1.324$). This is the first resonance; the second resonance occurred at

2π

E/

L

=

4.

20

4.

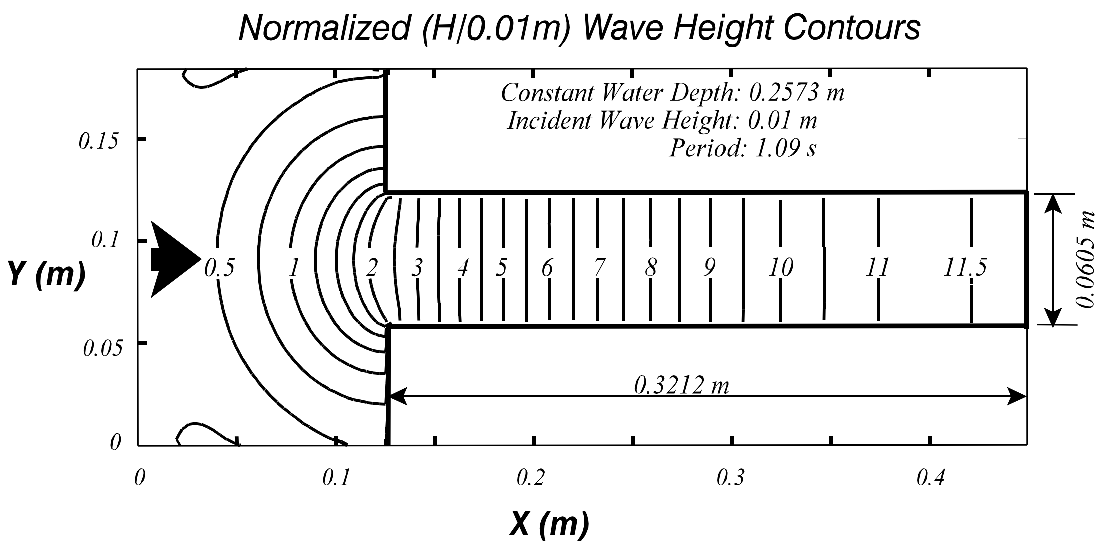


Fig. 5. The Typical Rectangular Harbor. The Displayed Normalized Wave Height Contours is the Response near Harbor Resonance.

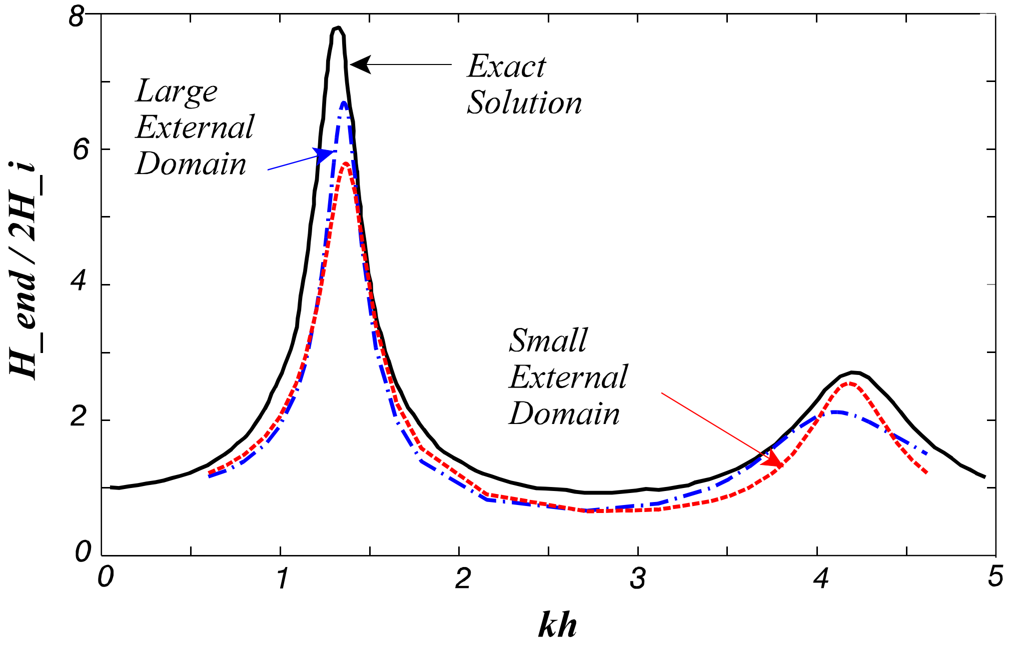


Fig. 6. Comparison with Analytical Solution (Solid line) and Model-Calculated Normalized Wave Height at the Harbor End. The dashed line is the model results with the open ocean boundary domain shown in Fig. 5 (12.85 x 18.45 cm). The dashed-dotted line is the model results when the open Ocean boundary domain increased to 19.27 x 30.15 cm.

Because an approximation (i.e., the rectangular open boundary) of the exact geometry for the open boundary was used, the model results (dashed line in Fig. 6) cannot be as accurate as the analytical solutions (solid line in Fig. 6). When the size of the open boundary domain (on the left hand side of Fig. 5) increased from 12.85 cm x 18.45 cm to 19.27 cm x 30.15 cm, the performance of this model also changed; see the dash-dotted line in Fig. 6. As pointed out by Chen (personnel communication 1997), a semi-circle open boundary is needed for an accurate simulation of the resonance. The calculated contours of the normalized wave height at a near resonance frequency (see Fig. 5) further demonstrated that the scatter waves propagated outward from the harbor entrance and also showed why a semi-circular open boundary is needed.

Although the performance of this model is not perfect at this time, it does indicate the resonance near the two resonant frequencies (Fig. 6). As the results of the first stage study, we will leave the model as is. We may find a better way to implement the semi-circle open boundary requirement in the next stage study.

Case 4, Influence of bottom slope and Curvature

Davies and Heathershaw (1984) presented the results of a physical model study and analytical solution for evaluating the influence of bottom curvature and steep slope on wave reflection and transmission. In their physical model study, monochromic waves were generated at one end of a one-dimensional wave flume. At the middle of this flume, there was a selected number (1, 2, 4, and 10) of sinusoidal bedform. The average water depth within the sinusoidal bed area was the same as the water depth before and after the sinusoidal bedform. The sinusoidal bedform had an amplitude, b, of 0.05 m and length, L_s, of 1.0 m. When waves encountered this kind of bed, some of the wave energy reflected back and some passed through (e.g., see Fig. 7). They measured the reflection coefficients, k_r (see Fig. 8 for four sinusoidal bedform), in terms of wave period, water depth, and number of sinusoidal bed forms.

Although this is a one-dimensional case, we can use the two-dimensional model to simulate the wave transformation along this flume. The positive x direction was selected as the wave propagation direction to reduce the band width of the matrix **B**. In the y direction, an arbitrary number of grid points (21 grid points) was selected to represent the channel width. At x = 0, the given wave boundary condition was specified. At the flume end, a total passing through boundary condition was specified.

On both the two Y-directional borders, a total reflection boundary condition was used. All other parameters used in the model are given in Table 1.

The model results clearly demonstrated the progress and reflective waves before the bed forms as well as the transmitted waves after the bed forms (Fig. 7). Because of the small k_r (0.083) in this particular case, the normalized transmitted wave height was also close to 1. Wave heights were larger than the incident waves at many places where the bed form were located.

The model also performed very well in identifying the maximum wave reflection; see Fig. 8. This demonstrates a satisfactory performance for this model when encountering irregular bathymetry.

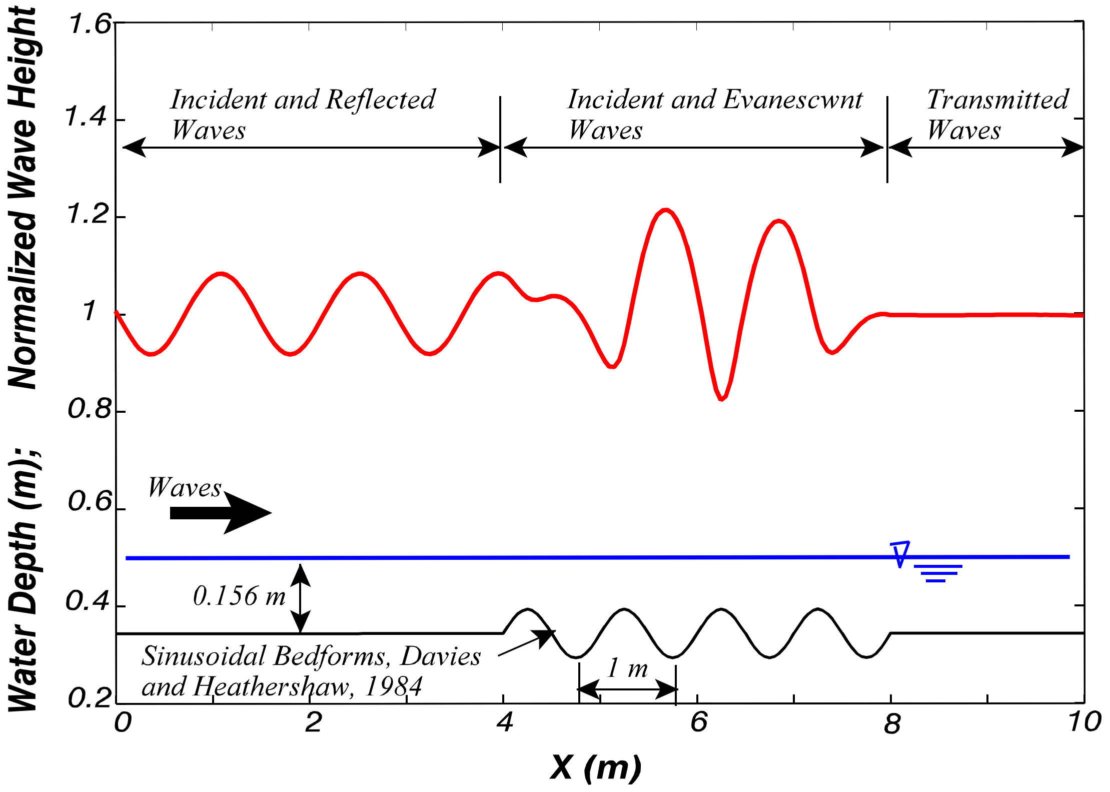


Fig. 7. Calculated Wave Height Profile along a Wave Flume with Four Sinusoidal Bed forms, which are plotted on the bottom.

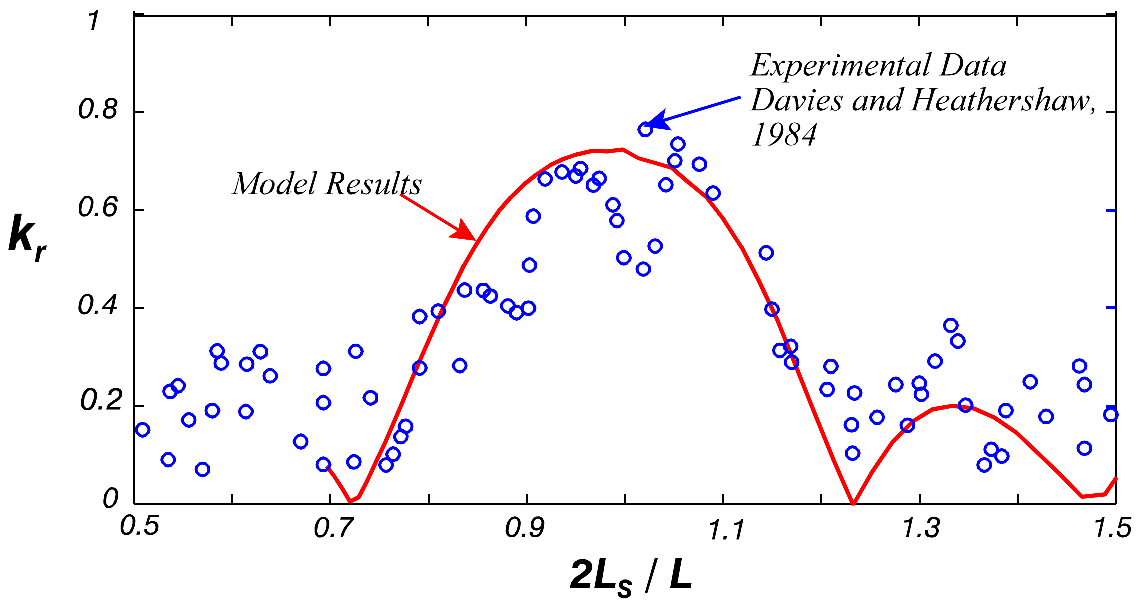


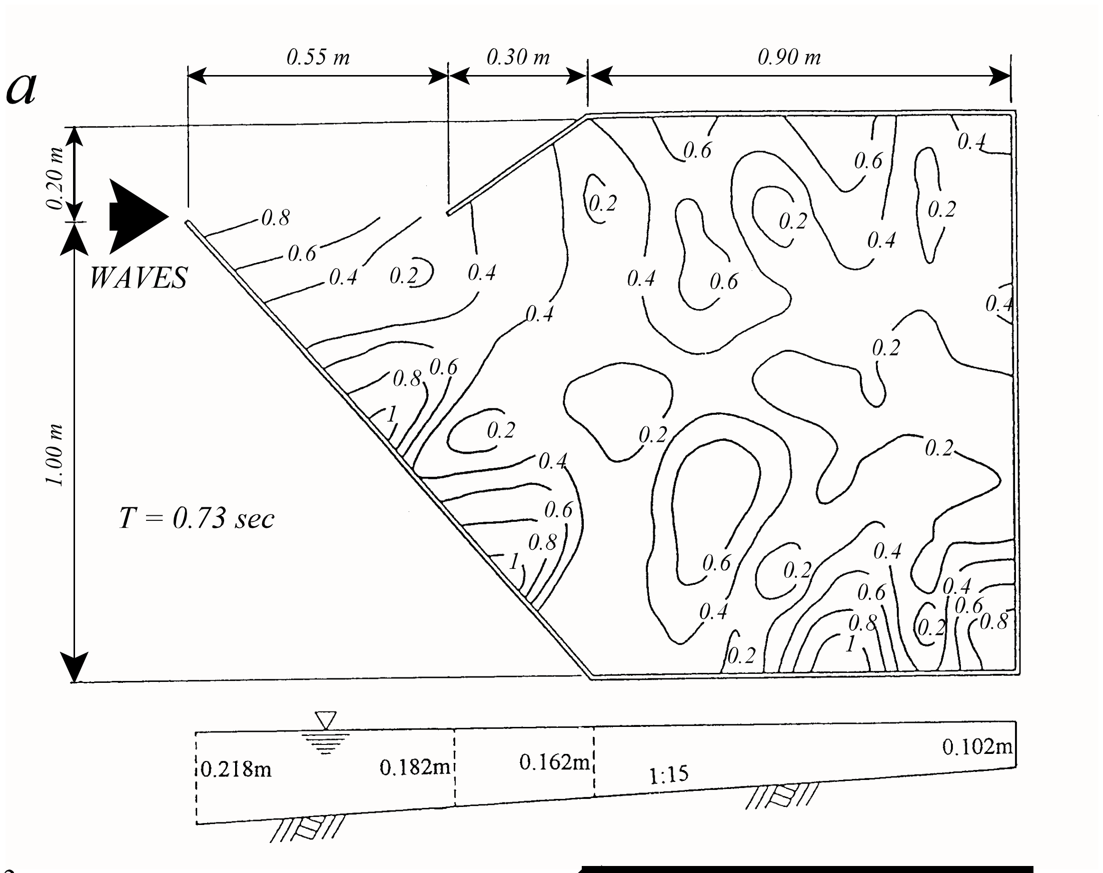
Fig. 8. Comparison of Calculated and Measured Wave Reflection Coefficient for the Four Sinusoidal Bed forms.

Case 5, A Simple but Practical Harbor

Sato *et al.* (1990) presented the results (ratio of local and input wave heights) of a physical model study on a small harbor (Fig. 9a). This case was selected because of the small study domain, complicated boundary conditions, and realistic harbor geometry.

The incident wave period was 0.73 s with the wave direction given in Fig. 9a. The given incident wave height was not given but that was immaterial because the measurements were normalized.

Crashed stones were placed on the seaward side of the breakwater in order to dissipate wave energy. For this reason, α was selected as 0.8 to represent a nearly total passing through boundary condition on the seaward face of the breakwater. Total passing through boundary conditions ($\alpha = 1$) were specified for the open sea boundary in both y directional borders. Within the harbor, the vertical wall was capable of reflecting most of the wave energy, and thus, a small α should be selected. It was not documented, however, what the α should be because no experimental data was available. The model results with $\alpha = 0.03$ (Fig. 9b) were quite close to the measurements. When $\alpha = 0$, however, the resulting wave heights at the node points were about 2 times larger than those when $\alpha = 0.03$. Here the node points referred to those locations where the normalized wave heights were large. When using $\alpha = 0.1$, on the other hand, the resulting wave heights were too small to compare with the measurements.



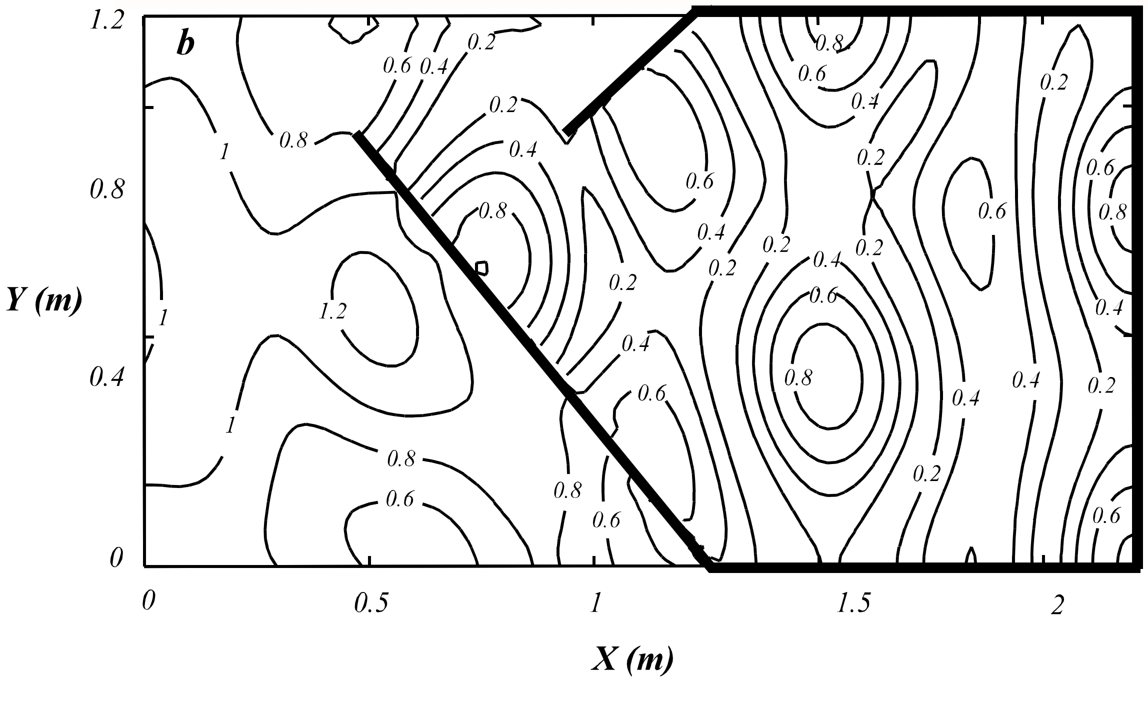


Fig. 9. Comparison of Wave Height Contours in a Simple Harbor (a)Laboratory measurements by Sato *et al.* (1990); (b) Model Results.

It may be argued that the selection of $\alpha = 0$ with wave energy loss caused by bottom friction may also be able to give results similar to those given in Fig. 9a. This is possible because of the small α used in this case. This argument will be examined in our next stage study when the energy loss caused by bottom friction will be implemented.

DISCUSSION AND CONCLUSIONS

Requiring the grid size to be less than 1/10 of the wave length is the major disadvantage of solving the elliptic equation. Because of the small grid size, the band matrix, $\mathbf{B}$, can easily become very large, and thus, require more computing time. For this reason, this method is not recommended when there is no reflection and only weak diffraction. A model using the parabolic approximation should be used instead. The inability to simulate wave reflection and strong wave diffraction, however, prohibits the use of that approach for harbor planning purposes.

The requirement of a small grid size (i.e., less than 1/10 of the wave length) improves the feasibility of using the finite difference method to simulate a complicated harbor geometry. The advantage in using the finite element method, on the other hand, diminishes as the grid size decreases.

One advantage of using the direct method to solve the elliptic equation is that there is no convergence concern even for a very complicated geometry. In general, convergence is one of the major concerns for irregular geometries using iteration methods.

The most important advantage of using the direct method (i.e., forming a banded matrix equation and solving it using the Gaussian elimination method) to simulate wave transformation processes is depicted next. For a given harbor geometry, different wave conditions only change the column matrix **G**, and does not affect the banded matrix **B** in eq. 17. This is important because the computing times for the forward elimination and back substitution are proportional to N^3 and N^2, respectively (Mathews 1987), where N is the length of the banded matrix. Since the given wave conditions do not affect **B**, only a one-time computation of the forward elimination is needed. This can save significant computing time if many wave conditions are involved (usually it is true for harbor planning) for a harbor geometry. To examine the more realistic spectrum waves, this advantage is also important because it can improve the computing efficiency significantly. Improving the computer codes for this purpose, however, is rather straightforward, and will be given in the next stage study.

In conclusion, by using the finite difference method and a simple book-keeping procedure to relax the huge memory requirements, a much simple numerical model for simulating wave reflection, refraction, diffraction, shoaling, and harbor resonance for harbor planning purposes has been developed. This model can simulate these processes using small computers with reasonable speed. With a more powerful computer, the time required for modeling these processes will be much less.

The effects of bottom friction are not yet included in this model. It can easily be added later because it only slightly affects the construction of the banded matrix equation, while the procedures for obtaining the solution remain the same.

Wave breaking, another important phenomenon, is not examined in this study. This model, however, can give unrealistically large wave heights at the breaking line. This feature can be used in post-processing software to identify the breaking line. After breaking, unfortunately, this model will not be able to continue the simulation because the theory is no longer valid.

ACKNOWLEDGMENTS

Financial support from the U.S. Department of the Interior, Minerals Management Service, Office of International and Marine Minerals, under Cooperative Agreement No 14-35-001-30740 is sincerely acknowledged. Support from the National Science Council of the Republic of China, Contract Number NSC 86-2611-E-006-019 is also sincerely acknowledged. Mr. M.-T. Maa helps to prepare diagrams and proofread the manuscript.

REFERENCES

Berkhoff, J. C. W., 1972, "Computation of Combined Refraction-Diffraction." Proceedings, 13th International Conf. on Coastal Engrg., ASCE, 1, 471-490.

Berkhoff, J. C. W., 1976, Mathematic Models for Simple Harmonic Linear Water Waves, Wave Diffraction and Refraction, Publication No. 1963, Delft Hydraulics Laboratory, Delft, The Netherlands.

Behrendt, L., 1985, A Finite Element Model for Water Wave Diffraction Including Boundary Absorption and Bottom Friction, Series Paper 37, Institute of Hydrodynamics and Hydraulic Engrg., Technical Univ. of Denmark.

Blue, F. L. Jr. and J. W. Johnson, 1949, "Diffraction of Water Waves passing Through a Breakwater Gap," Transactions, American Geophysical Union, 30(5), 705-718.

Chamberlain, P.G. and D. Porter, 1995, "The Modified Mild-slope Equation," J. Fluid Mech., 291, 393-407.

Chen, H.S. and C.C. Mei, 1974, Oscillations and Wave Forces in an Offshore Harbor, Rep. No. 190, Parsons Lab. MIT, Cambridge, Mass.

Chen, H.S. and J.R. Houston, 1987, Calculation of Water Oscillation in Coastal Harbors, HARBS and HARBD User's Manual, Instruction Report CERC-87-2, CERC, Dept. of the Army, WES, Corps of Engineers, Vicksburg, Mississippi 39180-0631.

Copeland, G.J.M., 1985, "A practical Alternative to the 'Mild-slope' Wave Equation", Coastal Engrg., 9, 125-149.

Dalrymple, R. A., Suh, K. D., Kirby, J. T., and Chae, J. W., 1989, "Models for Very Wide-angle Water Waves and Wave Diffraction, Part 2, Irregular Bathymetry," J. Fluid Mechanics, 201, 299-322.

Davies, A.G. and A.D. Heathershaw, 1984, "Surface-wave Propagation over Sinusoidally Varying Topography," J. Fluid Mechanics, 144, 419-443.

Dongarra, J.J., J.R. Bunch, C.B. Moler, and G.W. Stewwart, 1979, Linpack Users Guide, SIAM.

Goda Y, Yoshimura T., Ito, M., 1971, Report of the Port and Harbor Research Institute, 10, (2).

Ippen, A.T. and Goda, Y., 1963, Wave Induced Oscillations in Harbors: The Solution for a Rectangular Harbor Connected to the Open Sea, Rep. No. 59, Hydrodynamics Lab, MIT, Cambridge, MASS.

Johnson, J. W., 1953, "Generalized Wave Diffraction Diagrams," Proceedings of the second Conference on Coastal Engineering, ASCE, 6-23.

Kirby, J. T., 1986a, "Higher-order Approximations in the Parabolic Equation Method for Water Waves," J. of Geophysical Research, 91 (C1), 933-952.

Kirby, J. T., 1986b, "Rational Approximations in the Parabolic Equation Method for Water Waves," Coastal Engrg., 10, 355-378.

Kirby, J. T., 1988, "Parabolic Waves Computations in Non-orthogonal Coordinate Systems," J. of Waterway, Port, Coastal, and Ocean Engrg., ASCE, 114(6), 673-685.

Kirby, J. T., 1989, "Note on Parabolic Radiation boundary conditions for Elliptic wave Calculation," Coastal Engrg., 13, 211-218.

Kirby, J. T. and Dalrymple, R. A., 1991, User's Manual, Combined Reffraction/Diffraction Model, REF/DIF 1, Ver 2.3, Center for Applied Coastal Research, Dept. Of Civil Engineering, Univ. Of Delaware, Newark, DE 19716.

Lee, J.-J., 1971, "Wave-induced Oscillations in Harbors of Arbitrary geometry,"" J. Fluid. Mechanics, 45(2), 375-394.

Li, B. and K. Anastasiou, 1992, "Efficient Elliptic Solvers for the Mild-slope Equation using the Multigrid Technique," Coastal Engrg., 245-266.

Li, B., 1994a, "A Generalized Conjugate Gradient Model for the Mild Slope Equation," Coastal Engrg., 23, 215-225.

Li, B., 1994b, "An Evolution Equation for Water Waves," Coastal Engrg., 23, 227-242.

Maa, J. P.-Y., M.-H. Maa, C. Li, and Q. He, 1997, Using the Gaussian Elimination Method for Large Banded Matrix Equations, Special Scientific Report, No. 135, Virginia Institute of Marine Science, Gloucester Point, VA 23062.

Madsen, P.A. and Larsen, J., 1987, "An Efficient Finite-difference Approach to the Mild-slope Equation," Coastal Engrg., 11, 329-351

Massel, S. R., 1993, "Extended Refraction-diffraction Equation for Surface Waves," Coastal Engineering, 19, 97-126.

Massel, S. R., 1995, Ocean Surface Waves: Their Physics and Prediction, World Scientific Publ., Singapore.

Mathews, J. H., 1987, Numerical Methods for Computer Science, Engineering, and Mathematics, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 507pp.

Panchang, V. G., Cushman-Roisin, B., and Pearce, B.R. (1988). "Combined Refraction-diffraction of Short Waves in Large Coastal Regions," Coastal Engrg., 12(2), 133-156.

Porter, D. and D.J. Staziker, 1995, "Extensions of the Mild-slope Equation," J. Fluid Mech., 300, 367-382.

Sato, N., M. Isobe, and T. Izumiya, 1990, "A Numerical Model for Calculating Wave Height Distribution in a Harbor of Arbitrary Shape,"" Coastal Engineering in Japan, 33(2), 119-132.

Shore Protection Manual, 1977, Coastal Engineering Research Center, 3nd editions, U.S. Army, Corps of Engineers.

Unluate U. and C.C. Mei, 1973, Long Wave Excitation in Harbors - an Anslytical Study, Rep. No 171, Parsons Lab. MIT, Cambridge, MASS.

Appendix I. Finite Difference Equations for Boundary Conditions.

The extended mild slope equation (Eq. 4) and the second-order approximation of the partial reflection boundary conditions (Eqs. 6 and 7, in finite difference form) are combined together to form the finite difference equation at these boundaries. Notice that the word "right," "left," "top," or "bottom" are used to indicate that the boundary grid point (with an upper case index either I, or J) is on the right, left, top, or bottom side of the water cells. The origin of the Cartesian coordinates is located at the lower-left corner. Equation 10 is repeated here as Eq. I-5 for convenience.

(1) Top side partial reflection boundary.
Considering that this boundary condition is specified at the boundary grid, J, in the y direction and the water grid point is located on the lower side of this point, the finite difference equation for Eq. 7, with a positive sign, is

$$\phi_{i,J+1}-\phi_{i,J-1} = \beta_y\phi_{i,J} + \xi_y(\phi_{i+1,J}-2\phi_{i,J}+\phi_{i-1,J}) \tag{I-1}$$

where $\beta_y = 2i\alpha k\Delta y$ and $\xi_y = (i\alpha)/(kr^2\Delta y)$. With Eq. 4 specified for grid point (i,J), the resulting finite difference equation can be found as follows.

$$[r(r+d_y)\xi_y+(1-d_x)]\phi_{i-1,J}+ 2r^2\phi_{i,J-1}+ [\lambda+(\beta_y-2\xi_y)r(r+d_y)]\phi_{i,J}$$
$$+ [r(r+d_y)\xi_y+(1+d_x)]\phi_{i+1,J}=0 \tag{I-2}$$

(2) Bottom side partial reflection boundary.

$$[r(r-d_y)\xi_y+(1-d_x)]\phi_{i-1,J}+ [\lambda+(\beta_y-2\xi_y)r(r-d_y)]\phi_{i,J}$$
$$+ 2r^2\phi_{i,J+1}+ [r(r-d_y)\xi_y+(1+d_x)]\phi_{i+1,J}=0 \tag{I-3}$$

(3) Left hand side partial reflection boundary.

$$[r(r-d_y)+\xi_x(1-d_x)]\phi_{I,j-1}+ [\lambda+(\beta_x-2\xi_x)(1-d_x)]\phi_{I,j}$$
$$+ [r(r+d_y)+\xi_x(1-d_x)]\phi_{I,j+1}+ 2\phi_{I+1,j}=0 \tag{I-4}$$

(4) Right hand side partial reflection boundary.

$$2\phi_{I-1,j}+ [r(r-d_y)+\xi_x(1+d_x)]\phi_{I,j-1}+ [\lambda+(\beta_x-2\xi_x)(1+d_x)]\phi_{I,j}$$

$$+ [r(r+d_y)+\xi_x(1+d_x)]\phi_{I,j+1}=0 \qquad \text{(I-5)}$$

(5) Bottom-Left corner partial reflection boundary.

Considering that this boundary condition is specified at the boundary grid (I,J), and the water grid point is located on the up right side of this point, using Eqs. 6, 7, and I-3 to eliminate the two velocity potential ($\phi_{I,J-1}$ and $\phi_{I-1,J}$) and the resulting finite difference equation is

$$(\lambda+(\beta_x-2\xi_x)(1-d_x)+ [r(r-d_y)+\xi_x(1-d_x)]\frac{\beta_y-(2+2\xi_x-\beta_x)\xi_y}{1-\xi_x\xi_y})\ \phi_{I,J}+$$

$$([r(r+d_y)+\xi_x(1-d_x)]+[r(r-d_y)+\xi_x(1-d_x)]\frac{1+\xi_x\xi_y}{1-\xi_x\xi_y})\ \phi_{I,J+1}+$$

$$2(1+\frac{\xi_y[\xi_x(1-d_x)+r(r-d_y)]}{1-\xi_x\xi_y})\ \phi_{I+1,J}= 0 \qquad \text{(I-6)}$$

(6) Top-left corner partial reflection boundary.

$$(\xi_x(1-d_x)+r(r-d_y)+\frac{1+\xi_x\xi_y}{1-\xi_x\xi_y}[\xi_x(1-d_x)+r(r+d_y)])\ \phi_{I,J-1}+$$

$$(\lambda+(\beta_x-2\xi_x)(1-d_x)+ [r(r+d_y)+\xi_x(1-d_x)]\frac{\beta_y+\xi_y(\beta_x-2-2\xi_x)}{1-\xi_x\xi_y})\ \phi_{I,J}+$$

$$2(1+\frac{\xi_y[\xi_x(1-d_x)+r(r+d_y)]}{1-\xi_x\xi_y})\ \phi_{I+1,J} = 0 \qquad \text{(I-7)}$$

(7) Top-right corner partial reflection boundary.

$$2\left(1+\frac{\xi_y[r(r+d_y)+\xi_x(1+d_x)]}{1-\xi_x\xi_y}\right)\phi_{I-1,J}+$$

$$\left([r(r-d_y)+\xi_x(1+d_x)]+[r(r+d_y)+\xi_x(1+d_x)]\frac{1+\xi_x\xi_y}{1-\xi_x\xi_y}\right)\phi_{I,J-1}+(\lambda+$$

$$(\beta_x-2\xi_x)(1+d_x)+[r(r+d_y)+\xi_x(1+d_x)]\frac{\beta_y+\xi_y(\beta_x-2-2\xi_x)}{1-\xi_x\xi_y})\phi_{I,J}=0 \qquad \text{(I-8)}$$

(8) Bottom-right corner partial reflection boundary.

$$2\left(1+\frac{\xi_y[r(r-d_y)+\xi_x(1+d_x)]}{1-\xi_x\xi_y}\right)\phi_{I-1,J}+$$

$$\left(\lambda+(\beta_x-2\xi_x)(1+d_x)+[r(r-d_y)+\xi_x(1+d_x)]\frac{\beta_y+\xi_y(\beta_x-2-2\xi_x)}{1-\xi_x\xi_y}\right)\phi_{I,J}+$$

$$\left([r(r+d_y)+\xi_x(1+d_x)]+[r(r-d_y)+\xi_x(1+d_x)]\frac{1+\xi_x\xi_y}{1-\xi_x\xi_y}\right)\phi_{I,J+1}=0 \qquad \text{(I-9)}$$

(9) Top side given boundary

$$[r(r+d_y)\xi^1{}_y+(1-d_x)]\phi_{i-1,J}+2r^2\phi_{i,J-1}+[\lambda+(\beta^1{}_y-2\xi^1{}_y)r(r+d_y)]\phi_{i,J}$$

$$+[r(r+d_y)\xi^1{}_y+(1+d_x)]\phi_{i+1,J}=-4ik\Delta yr(r+d_y)(\phi^g)_{i,J} \qquad \text{(I-10)}$$

(10) Bottom side given boundary.

$$[r(r-d_y)\xi^1{}_y+(1-d_x)]\phi_{i-1,J}+ [\lambda+(\beta^1{}_y-2\xi^1{}_y)r(r-d_y)]\phi_{i,J} + 2r^2\phi_{i,J+1}$$

$$+ [r(r-d_y)\xi^1{}_y+(1+d_x)]\phi_{i+1,J} = 4ik\Delta yr(r-d_y)(\phi^g)_{i,J} \qquad \text{(I-11)}$$

(11) Left hand side given boundary.

$$[r(r-d_y)+\xi^1{}_x(1-d_x)]\phi_{I,j-1}+ [\lambda+(\beta^1{}_x-2\xi^1{}_x)(1-d_x)]\phi_{I,j}+$$

$$+ [r(r+d_y)+\xi^1{}_x(1-d_x)]\phi_{I,j+1}+ 2\phi_{I+1,j} = 4ik\Delta x(1-d_x)(\phi^g)_{I,j} \qquad \text{(I-12)}$$

(12) Right hand side given boundary.
This equation is the same as Eq. 14, which is repeated here for convenience.

$$2\phi_{I-1,j}+ [r(r-d_y)+\xi^1{}_x(1+d_x)]\phi_{I,j-1}+ [\lambda+(\beta^1{}_x-2\xi^1{}_x)(1+d_x)]\phi_{I,j} +$$

$$+ [r(r+d_y)+\xi^1{}_x(1+d_x)]\phi_{I,j+1} = -4ik\Delta x(1+d_x)(\phi^g)_{i,J} \qquad \text{(I-13)}$$

Appendix II. FORTRAN Source Codes for the main program RDE.FOR

```
      Program RDE
c
c This program solves the extended mild slope equation for simulating
c water wave refraction, diffraction, reflection, and shoaling.
c
c It uses second order approximation on the governing and boundary
c conditions.  This program uses a direct method to solve the huge
c Banded matrix equation, which was obtained by doing finite
c difference on the elliptic equation.
c
c  iq,jq : parameters specified for reserving space for 2-D arrays
c          They should be at least the same as  mp and np.
c  mp,np : number of grid points in x and y direction, respectively.
c          In applications, the x-axis should be parallel to the
c          longer dimension of the water body.  The y axis should
c          parallel to the short dimension of the water body.  This
c          allignment will generate a smaller banded width.
c  dx,dy : grid sizes in x and y direction, respectively.  In this
c          program, only the equally spaced grid is allowed.  But
c          dx do not have to equal to dy.
c  dlevel: water elev. that added to the bathymetric depth for
c          storm tide.
c  dmin  : min. water depth at a point that this point will be
c          considered as water.
c  kq   : the larger dimension of the banded coefficient matrix.
c         The maximum kq equals to mp*np if all points in the
c         x-y plane are water points.  Practically, it should be
c         much smaller (equal to the length of the total water
c         cells in the study domain because of land cells.
c  nbc  : number of grid points that are assigned as either a
c         passing through, a total reflection, or a partial
c         reflection boundaries.
c  ngc  : number of grid points that are assigned to have a given
c         boundary, e.g., wave height, period, and direction.
c  id   : a 2-D array to store the identification for each grid
c         point. Detail explanation is given in the program.
c
c  The following logical numbers are used to read or write files
c
c  1 : CONFILE for Control parameters.
c  9 : OUTFILE for storage output.  This file contains calculation
c                  results for all cases.  So it may be very big.
c  11: CHKFILE for storage data for checking purposes.  This file
c                  erease itself when completed for each case and
c                  reused to save disk space.
c  15: BCDFILE for input ID codes and boundary conditions (given
c                  boundary as well as radiation boundary.
c  16: DEPFILE for input water grid.
c
```

```
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502,
     *           iw=1000, jw=1700)
c
      character*70 title
      character*20 outfile, confile, chkfile, depfile, bcdfile
      character*11 result
      character*1 id
c
      common /files/ depfile, bcdfile
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl, yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /works/ zw(iw,jw), ipvt(kq)
      common /given/ nog, ig(ngc), jg(ngc)
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
      dimension period_g(85), waveH_g(85), angle0_g(85), dlevel_g(85)
c
      zi=(0.0, 1.0)
      grav=9.8
      pi=3.1415926
c
  5   continue
      print*,'Key in the Control file name <XXXXX.CON> : '
      read(*,'(a20)') confile
      open(1, file=confile, form='formatted',status='old', err=10)
      go to 20
 10   print*,'File not found, or, type not matched '
      go to 5
 20   read(1,'(a70)') title
      print*, title
      call time(result)
c
c  name   : a 16 characters string to store the path, name of file.
c           Later, three file types (chk, lis, grd) will be appended
c           to open an input depth grid file and two output files.
c  xorient: The angle of x-axis, counted clockwise from The North.
c  xl, yl : The global x and y coordinates of the origin of the grid
c
c  mp,np : Max. grid numbers in x and y direction, respectively.
c  dx,dy : grid sizes in x and y direction, respectively.
c  ncase : Number of cases (wave height, period, angle) in this
c          control file
c  dlevel: water elevation that added to the bathymetric depth,
c          e.g. storm tide
c  dmin  : minimum water depth that will be considered as water
c
      read(1,'(a)') depfile
c
```

```
c  the following statement is needed to remove the space on right
c  hand, since MS PowerStation uses all the space declared
c
      read(1,'(a)') bcdfile
      read(1,'(a)') outfile
      read(1,'(a)') chkfile
c
      read(1,*) yorient, xl, yl
      read(1,*) mp,np,dx,dy, ncase, dmin
      if(mp .gt. iq) then
       print*,'IQ is smaller than MP, change IQ to ',mp
       stop
       end if
      if(np .gt. jq) then
       print*,'JQ is smaller than NP, change JQ to ',np
       stop
       end if
c
      r = dx/dy
c
c wave angle: counted clockwise, from North.  For example,
c if waves come from NE, then the angle will be 45+180=225 degres.
c For wave coming from S, the angle will be 0 degrees.
c For waves coming from E, teh angle will be 90 degrees.
c The angle given is always reference to North.  The angle that
c
      print*,'The following   waveH, period, and Angle are included'
      do ic=1,ncase
       read(1,*) waveH_g(ic),period_g(ic),angle0_g(ic),dlevel_g(ic)
       write(*,52) ic, waveH_g(ic), period_g(ic), angle0_g(ic),
     *                   dlevel_g(ic)
 52   format(' i, waveH(m), T(s), Ang(d),Surge(m)=',i4,2f8.2,2f7.1)
       end do
c
c
      print*,'Output data goes to ', outfile
      open(9,file=outfile,form='formatted',status='unknown')
c
      print*,'Water depth grid input from ',depfile
      call depth_in
c
      print*,'Boundary condition input from ',bcdfile
      call bound_in
c
      do 200 ic=1,ncase
c
      print*,'Check data goes to ', chkfile
      open(11, file=chkfile, form='formatted',status='unknown')
c
      print*,'case ',ic
      period=period_g(ic)
      waveH=waveh_g(ic)
      angle0=angle0_g(ic)
      dlevel=dlevel_g(ic)
c
c  change water depth, not now, wait until operational
c
c       do i=1,mp
c          do j=1,np
c             dep(i,j)=dep(i,j)+dlevel
c             end do
c          end do
c
c  wk0 : wave number at deep water
c
      wk0=4.0*pi*pi/(grav*period*period)
      write(11,'(a70)') title
      write(11,55)  waveH, period, angle0, dlevel
 55   format(' waveH (m)=',f8.2/'     T (s)=',f8.2/
     *       ' Ang (deg)=',f8.1/' Surge (m)=',f8.1/
     *       ' Given boundary conditions are specified at'/
     *       '     i       j           zph ')
c
c  calculate the given velocity potential based on wave height,
c  period, direction, and grid locations
c
      zpot=-zi*waveh*grav*period/(4.0*pi)
      do k=1,nog
       zp(k)=zpot*exp(zi*phas(k) )
       write(11,60) ig(k),jg(k),zp(k)
 60      format(2i7,2f12.8)
       end do
c
c construct the unknown COLUMN matrix ZX, in the eq. ZA*ZX=ZB
c Get each unknown"s location: imap(map),jmap(map)
c
c map : the number of total unknown, or the length of X.  later,
c       it is reassigned as N
c
      map=0
      do i=1,mp
       do j=1,np
         if(id(i,j) .ne. 'e' ) then
            map=map+1
               imap(map)=i
            jmap(map)=j
            end if
         end do
       end do
        if(map .gt. kq) then
           print*,'Increase KQ to ', map
        stop
        end if
c
```

```
c       if(n .lt. 0.9*kq) then
c         print*
c         print*,'-----------------------------------------------'
c         print*,'Its better to reduce the length of the Matrix,'
c         print*,'KQ, & increase the length of matrix ZW: IW,'
c         print*,'in order to reduce disk I/O'
c         print*,'-----------------------------------------------'
c         end if
c       print*,'Length of the Banded Matrix is',n
c       print*,'The parameter KQ has been declared as',kq
c
c  n: length of the banded matrix
c
        n=map
c
c  Set up the storage matrices ZA & IA and find out the band width
c
       mu=0
       mL=0
       do 80 map=1,n
      i=imap(map)
      j=jmap(map)
c
      if(id(i,j) .eq. '0') then
         call domain(i,j,map)
         go to 70
         end if
c
      if(id(i,j) .eq. 'g') then
         call given_bc(i,j,map)
         go to 70
         end if
c
      if(id(i,j) .eq. '1') then
         call bound_t(i,j,map)
         go to 70
         end if
      if(id(i,j) .eq. '2') then
         call bound_b(i,j,map)
         go to 70
         end if
      if(id(i,j) .eq. '3') then
         call bound_l(i,j,map)
         go to 70
         end if
      if(id(i,j) .eq. '4') then
         call bound_r(i,j,map)
         go to 70
         end if
      if(id(i,j).eq.'5' .or. id(i,j).eq.'o' .or. id(i,j).eq.'p') then
         call corner_lb(i,j,map)
         go to 70
         end if
      if(id(i,j).eq.'6' .or. id(i,j).eq.'q' .or. id(i,j).eq.'r') then
         call corner_lt(i,j,map)
         go to 70
         end if
      if(id(i,j).eq.'7' .or. id(i,j).eq.'u' .or. id(i,j).eq.'v') then
         call corner_rb(i,j,map)
         go to 70
         end if
      if(id(i,j).eq.'8' .or. id(i,j).eq.'s' .or. id(i,j).eq.'t') then
         call corner_rt(i,j,map)
         go to 70
         end if
c
 70   if(id(i,j).ne.'4' .and. id(i,j) .ne. '7' .and. id(i,j).ne.'8'
     *  .and. id(i,j) .ne. 'c' .and. id(i,j) .ne. 'd'  ) then
          mu_c=ia(map,5) - ia(map,3)
        else
          mu_c=0
        end if
      if(mu_c .gt. mu) mu=mu_c
      if(id(i,j) .ne. '3' .and. id(i,j) .ne. '5'.and.id(i,j).ne. '6'
     *   .and. id(i,j) .ne. 'a' .and. id(i,j) .ne. 'b'  ) then
           mL_c=ia(map,3) - ia(map,1)
        else
           mL_c=0
        end if
      if(mL_c .gt. mL) mL=mL_c
      write(11,75) i,j, ia(map,1),ia(map,2),ia(map,3),
     *               ia(map,4),ia(map,5), za(map,1), za(map,2),
     *               za(map,3), za(map,4),za(map,5), zb(map)
 75   format(1x,2i5,3x,5i6,2x,2f7.3, 2x,2f7.3, 2x,2f7.3,
     *                       2x,2f7.3, 2x,2f7.3, 2x,2f7.3 )
      if(ia(map,1) .lt. 0 .or. ia(map,2) .lt. 0 .or. ia(map,3) .lt. 0
     *     .or. ia(map,4) .lt. 0 .or. ia(map,5) .lt. 0) then
          write(*,78) map,i,j, id(i,j), (ia(map,kk), kk=1,5)
 78      format(' Map, i,j,id=',i10,2i5,a5/' ia(map,k)=',5i8/
     *          ' All 5 IAs should be non-negative, Check !')
          stop
          end if
c
 80      continue
c
      close(11)
      m=mL+mu+1
      Lda=m+mL
      print*,'The band width & length =', m, n
      print*,'Upper band width=',mu
      print*,'Lower band width=',mL
      print*,'Lda for BMSOLVER=',Lda
      if(Lda .gt. iw) then
       write(*,90) Lda
 90   format('Please change parameter IW to >=',i6,' and re-run')
       stop
       end if
c
c  uses the complex band matrix solver to solve ZA * ZX = ZB
c
      call bmsolver(ier_code)
c
c  output results
c
      if( ier_code .eq. 0) then
c
       write(9,'(a60)') title
       write(9,100) mp,np, dx,dy
 100     format(2i5,2f10.6,2f10.3)
       write(9,110) yorient,xl,yl
 110     format(f10.2,2f10.5)
       write(9,120) waveh,period,angle0, dlevel, n
 120     format(4f10.3,i10/'   i   j    id     dep     real  imagin')
c
       do k=1,n
          i=imap(k)
          j=jmap(k)
          write(9,140) i, j, id(i,j), dep(i,j), zb(k)
 140        format(2x,i5,1x,i5,a3,f10.4,2d18.10)
          end do
c
       end if
 200  continue
      close(9)
c
      print*,result
      call time(result)
      print*,result
c
      stop
      end
c
c-----------------------------------------------------
c
      subroutine depth_in
c
c  This sub. reads ASCII depth grids, ID codes
c
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, nbc=1500, ngc=502)
c
      character*70 dtitle,label
      character*20 depfile, bcdfile
      character*1 id
c
      common /files/ depfile, bcdfile
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /gener/ mp, np, dx,dy, r, yorient, xl, yl, wk0
c
      open(16, file=depfile, form='formatted',status='old')
      read(16,25) dtitle
      print*, dtitle
      read(16,25) label
      print*, label
      read(16,*) m_dep, n_dep, dx_d,dy_d, xl,yl,czmin,czmax,noz,yorii
      write(*,5) czmin, czmax, noz
  5   format(' The min., max., and # of raw survey are ',2f8.3,i8)
      if(m_dep .eq. mp .and. n_dep .eq. np .and. dx_d .eq. dx .and.
     *   dy_d .eq. dy .and. yorii .eq. yorient) go to 20
      write(*,10) m_dep, n_dep, dx_d, dy_d, yorii,
     *               mp, np, dx, dy, yorient
 10   format(' The following two rows should be the same '/
     *       5x,2i7, 2f10.3, f10.1/5x, 2i7, 2f10.3, f10.1/
     *       ' Found when reading depth grid file.'/' Usually this',
     *       ' means the data file assignment is wrong.')
      stop
 20   continue
      read(16,25) label
      write(*,25) label
 25   format(a70)
      read(16,25) label
      do i=1,mp
       read(16,*) ins
       if(ins .eq. i) then
          read(16,*) (dep(i,j),j=1,np)
        else
          write(*,30) i,ins
 30         format(' Sequence wrong for depth input at',2i5)
          stop
        end if
       end do
      print*,'Completed reading water depth matrix, leave depth_in'
      close(16)
c
      return
      end
c
c-----------------------------------------------------
c
      subroutine bound_in
c
c  This sub. reads ASCII given and radiation boundary conditions
c
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, nbc=1500, ngc=502)
c
      character*70 dtitle,label
      character*20 bcdfile, depfile
      character*1 id
c
      common /files/ depfile, bcdfile
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /gener/ mp, np, dx,dy, r, yorient, xl, yl, wk0
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /given/ nog, ig(ngc), jg(ngc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
      open(15, file=bcdfile, form='formatted',status='old')
c
      read(15,2) dtitle
  2   format(a70)
      print*, dtitle
      read(15,2) label
      print*, label
      read(15,*) m_bcd, n_bcd, dx_b,dy_b, xl,yl,czmin,czmax,noz,yorii
      write(*,5) czmin, czmax, noz
  5   format(' The min., max., and # of raw survey are ',2f8.3,i8)
c
      if(m_bcd .eq. mp .and. n_bcd .eq. np .and. dx_b .eq. dx .and.
     *   dy_b .eq. dy .and. yorii .eq. yorient) go to 20
      write(*,10) m_bcd, n_bcd, dx_b, dy_b, yorii,
     *               mp, np, dx, dy, yorient
 10   format(' The following two rows should be the same '/
     *       5x,2i7, 2f10.3, f10.1/5x, 2i7, 2f10.3, f10.1/
     *       ' Found when reading depth grid file.'/' Usually this',
     *       ' means the data file assignment is wrong.')
      stop
 20   continue
c
c ID code for each grid point
c
c      : 0, water interior point
c      : 1, upper boundary condition
c      : 2, lower boundary condition
c      : 3, left boundary condition
c      : 4, right boundary condition
c      : 5, left bottom corner B.C.
c      : 6, left top corner B.C.
c      : 7, right bottom corner B.C.
c      : 8, right top corner B.C.
c      : g, grid point that have a given wave condition
c      : e, land point
c      : o, same as 5, also a given B.C. for wave coming from bottom
c      : p, same as 5, also a given B.C. for wave coming from left
c      : q, same as 6, also a given B.C. for wave coming from left
c      : r, same as 6, also a given B.C. for wave coming from top
c      : s, same as 7, also a given B.C. for wave coming from top
c      : t, same as 7, also a given B.C. for wave coming from right
c      : u, same as 8, also a given B.C. for wave coming from right
c      : v, same as 8, also a given B.C. for wave coming from bottom
```

```
c
c  The following is an example
c
c    j (y)
c    ^          rgggggggggggggggggggggggggggggs
c    |          3000000000000000000000000000004
c    |          3000000000000000000000000000004
c    |          ...............................
c    |                     or
c    |
c    |     np   eeeee6111111111111111111111111118     .gggs     .111t
c    |          q1111000000000000000000000000004     .0004     .000g
c    |          g0000000000000000000000000000004     .0004     .000g
c    |          g0000000000000000000000000000004     .0004     .000g
c    |          30000000000000000000000000000004     .0004     .000g
c    |          52200000000000000000000000222227  or .0004  or .000g
c    |          eee3000000000000000000000004eeeeee     .0004     .000g
c    |          eee3000000000000000000000004eeeeee     .0004     .000g
c    |          eee3000000000000000000000004eeeeee     .2227     .000g
c    |      1   eeeogggggggggggggggggggggveeeeee     .eeee     .222u
c    |          1                              mp                 (x)
c    |------------------------------------------------------------> i
c
      read(15,2) label
      print*,label
      nblk=np/75 + 1
      do nb=1,nblk
       read(15,2) label
       nb1=(nb-1)*75 + 1
       nb2=nb1+74
       if(nb2 .gt. np) nb2=np
       do i=1,mp
          read(15,30) ins1, (id(i,j),j=nb1,nb2)
 30         format(i5,75a1)
          if(ins1 .ne. i) then
            write(*,35) i,ins1
 35            format(' Sequence wrong for ID input at',2i5)
            stop
            end if
          end do
       end do
      print*,'Completed reading ID code matrix'
c
c  nog   : number of grid points that a given wave boundary
c          conditions are specified.
c  ig,jg: one-D arrays to store the grid number of the boundary
c          in I and J directions.
c  read the grid number for given boundary points
c
      read(15,*) nog
      if(nog .gt. ngc) then
       print*,'NOG > NGC, change PARAMETER  NGC to ',nog
```

```
          stop
          end if
         write(*,40) nog
 40      format(i5,' given boundary conditions')
         read(15,2) label
         print*,label
         do i=1,nog
          read(15,50) ins,ig(i),jg(i), id(ig(i),jg(i)), phas(i)
 50          format(1x,3i5,4x,a1,f10.5)
          if(ins .le. 3) then
             write(*,50) ins,ig(i),jg(i),id(ig(i),jg(i)), phas(i)
             end if
          end do
         print*,'Completed reading Given B.C.s'
c
c  nobc    : number of grid points that a radiational B.C. is given
c  ibc,jbc: one-D arrays to store the grid number of the
c           boundary in I and J directions
c  zalp    : one-D array to store wave reflection coefficient
c           at the boundary.  If only one side is subject to
c           wave reflection (e.g., ID(i,j)=1, 2, 3, or 4), then
c           only the real part will be used.
c           If both sides are subject to wave reflection (e.g.,
c           ID(i,j)=x, y), then the reflection coefficeint of
c           the second side will be stored as the imaginary part.
c           At a corner boundary point (e.g., ID=5,6,7,8), the
c           reflection coefficient in x direction is store as the
c           real part.  The reflection coefficient in y direction
c           is stored in the imaginary part.
c
c read the radiational boundary points and the reflection coeff.
c
         read(15,*) nobc
         if(nobc .gt. nbc) then
            print*,'NOBC > NBC, change parameter NBC  to ',nobc
            stop
            end if
         write(*,60) nobc
 60      format(i5,' radiational Boundary conditions')
         read(15,2) label
         print*, label
         zi=(0.0, 1.0)
         do i=1, nobc
          read(15,*) ins, ibc(i), jbc(i), id_bc, alp_r, alp_i
          zalp(i)=alp_r + zi*alp_i
          if(id_bc.ge.1 .and. id_bc.le.4 .and. alp_i.eq.0.0) go to 70
          if(id_bc.ge.5 .and. id_bc.le.8) go to 70
          write(*,65) i,ibc(i),jbc(i),id_bc, alp_i
  65         format(' Boundary condition wrong at   i, ibc,jbc= ',3i5/
     *              ' ID=',i3,' alp_i should = 0, but it is ',f8.3)
          stop
  70         continue
```

```
        if(alp_r .gt. 1.0 .or. alp_r .lt. 0.0 .or. alp_i .lt. 0.0
     *                  .or. alp_i .gt. 1.0) then
          write(*,80) i, ibc(i),jbc(i), alp_r, alp_i
  80        format(' Boundary condition wrong at   i, ibc,jbc= ',3i5/
     *             ' They should >= 0.0 and <=1.0, But they =',2f8.3)
          stop
          end if
        end do
       print*,'Completed reading radiational B.C.s, leaving BOUND_IN'
       close(15)
c
       return
       end
c
c---------------------------------------------------------------------
c
       subroutine corner_lb(i,j,map)
       implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
       character*1 id
       parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
       common /waves/ period, waveH, angle0
       common /bottm/ dep(iq,jq), id(iq,jq)
       common /matrx/ za(kq,5), ia(kq,5), zb(kq)
       common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
       common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
       common /given/ nog, ig(ngc), jg(ngc)
       common /bound/ nobc, ibc(nbc), jbc(nbc)
       common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
c  It processes the left-bottom corner related 5 elements.
c  possible ID layout are as follows:
c    j
c    ^
c    |     3 0 0 0        3 0 0 0 0       3 0 0 0 0 0 0
c    |     3 0 0 0        3 0 0 0 0       3 0 0 0 0 0 0
c    |     3 0 0 0        5 2 0 0 0       5 2 2 0 0 0 0
c    |     5 2 2 2        e e 3 0 0       e e e 5 0 0 0
c    |                    e e 5 2 2       e e e e 5 2 2
c    |---------------------------------------------------> i
c
c  The assignment of 5 points follows the following pattern:
c  1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)
c
c  give the reflection coefficient, 1.0, from the boundary conditions
c
       zi=(0.0,1.0)
       ick=1
       if(id(i,j) .eq. '5') then
        do it=1,nobc
          if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
             alpha_x= real(zalp(it) )
             alpha_y=aimag(zalp(it) )
             ick=0
             go to 10
             end if
          end do
        else
       if(id(i,j) .eq. 'o' ) then
          alpha_x= 0.0
          alpha_y= 1.0
          ick=0
          end if
       if(id(i,j) .eq. 'p' ) then
          alpha_x= 1.0
          alpha_y= 0.0
          ick=0
          end if
        end if
 10   continue
      if( ick .eq. 1) then
       write(*,20) i,j, id(i,j)
 20      format(' Refl. coef. not found in CORNER_LB at',2i5,a4)
       stop
       end if
c
      dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*( dep(i+1,j)-dep(i,j) )/(2.0*dep(i,j) )
      d_y=e0*r*( dep(i,j+1)-dep(i,j) )/(2.0*dep(i,j) )
c
      dhdx2=( (dep(i+1,j)-dep(i,j) )/dx )**2
      dhdy2=( (dep(i,j+1)-dep(i,j) )/dy )**2
      d2hdx2= 0.0
      d2hdy2= 0.0
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
      zbetax=2*zi*alpha_x*dk1*dx
      zxix=zi*alpha_x*r*r/(dk1*dx)
      zbetay=2*zi*alpha_y*dk1*dy
      zxiy=zi*alpha_y/(dk1*r*r*dy)
c
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
c
      za(map,1)=0.0
      za(map,2)=0.0
      za(map,3)=cnmd + (zbetax-2.0*zxix)*(1.0-d_x) +
     1                 (r*(r-d_y)+zxix*(1.0-d_x) )*
     2          (zbetay+zxiy*(zbetax-2*zxix-2.0))/(1.0-zxix*zxiy)
      za(map,4)=r*(r+d_y)+zxix*(1.0-d_x) +
     1    (r*(r-d_y)+zxix*(1.0-d_x) )*(1.0+zxix*zxiy)/(1.0-zxix*zxiy)
      za(map,5)=2.0*(1.0 + zxiy*(zxix*(1.0-d_x) + r*(r-d_y) )/
     1                                       (1.0 - zxix*zxiy) )
      ia(map,1)=0
      ia(map,2)=0
      ia(map,3)=map
      ia(map,4)=map+1
      ia(map,5)=map+idistr(i,j)-1
      if(id(i,j) .eq. '5' ) zb(map)=0.0
      if(id(i,j) .eq. 'o' .or. id(i,j) .eq. 'p' ) then
       do it=1,nog
          if(ig(it) .eq. i .and. jg(it) .eq. j) then
             zpotent=zp(it)
             go to 30
             end if
          end do
       write(*,25) i,j
 25       format(' Given b.c. location not matched, at LB, i,j=', 2i5)
       return
 30   if(id(i,j).eq.'p') zb(map) = 4.0*zi*dk1*dx*(1.0+d_x)*zpotent
       if(id(i,j).eq.'o') zb(map) = 4.0*zi*dk1*dy*r*(r-d_y)*zpotent
       end if
c
      return
      end
c
c-------------------------------------------------------------
c
      subroutine corner_lt(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      character*1 id
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /given/ nog, ig(ngc), jg(ngc)
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
c  Give the corner related 5 elements.  possible allignment are
c  as follows:
c    j
c    ^
c    |     6 1 1 1 .       e e 6 1 1 . .    e e e e e 6 1
c    |     3 0 0 0 .       e e 3 0 0 . .    e e e e 6 0 .
c    |     3 0 0 0 .       6 1 0 0 0 . .    e e e 6 0 0 .
c    |     3 0 0 0 .       3 0 0 0 0 . .    e 6 1 0 0 0 .
c    |     3 0 0 0 .       3 0 0 0 0 . .    6 0 0 0 0 0 .
c    |     . . . . .       3 0 0 0 0 . .    3 0 0 0 0 0 .
c    |                                      . . . . . . .
c    |-------------------------------------------------> i
c
c  The assignment of 5 points follows the following pattern:
c  1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)
```

```
c
      zi=(0.0,1.0)
c
c  find the reflection coefficient from the boundary conditions
c
      ick=1
      if( id(i,j) .eq. '6') then
       do it=1,nobc
          if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
           alpha_x= real(zalp(it) )
           alpha_y=aimag(zalp(it) )
           ick=0
           go to 10
           end if
          end do
       else
         if(id(i,j) .eq. 'q' ) then
            alpha_x= 1.0
            alpha_y= 0.0
            ick=0
            end if
         if(id(i,j) .eq. 'r') then
            alpha_x= 0.0
            alpha_y= 1.0
            ick=0
            end if
       end if
 10   continue
      if( ick .eq. 1) then
       write(*,20) i, j, id(i,j)
 20   format(' Refl. Coef. not found in CORNER_LT at',2i4,a4)
       stop
       end if
c
      dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*( dep(i+1,j)-dep(i,j) )/(2.0*dep(i,j) )
      d_y=e0*r*( dep(i,j)-dep(i,j-1) )/(2.0*dep(i,j) )
c
      dhdx2=( (dep(i+1,j)-dep(i,j) )/dx )**2
      dhdy2=( (dep(i,j)-dep(i,j-1) )/dy )**2
      d2hdx2= 0.0
      d2hdy2= 0.0
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
      zbetax=2*zi*alpha_x*dk1*dx
      zxix=zi*alpha_x*r*r/(dk1*dx)
      zbetay=2*zi*alpha_y*dk1*dy
      zxiy=zi*alpha_y/(dk1*r*r*dy)
c
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
      za(map,1)=0.0
      za(map,2)=zxix*(1.0-d_x)+r*(r-d_y) +
     1     (zxix*(1.0-d_x)+r*(r+d_y))*(1.0+zxix*zxiy)/(1.0-zxix*zxiy)
      za(map,3)=cnmd + (zbetax-2.0*zxix)*(1.0-d_x) +
     1         (r*(r+d_y) + zxix*(1.0-d_x) )*
     2         (zbetay + zxiy*(zbetax-2.0*zxix-2.0) )/(1.0-zxix*zxiy)
      za(map,4)=0.0
      za(map,5)=2.0*(1.0 +
     1             zxiy*(zxix*(1.0-d_x)+r*(r+d_y) )/(1.0-zxix*zxiy) )
c
      ia(map,1)=0
      ia(map,2)=map-1
      ia(map,3)=map
      ia(map,4)=0
      ia(map,5)=map+idistr(i,j)-1
      if( id(i,j) .eq. '6') zb(map)=0.0
      if( id(i,j) .eq. 'q' .or. id(i,j) .eq. 'r') then
       do it=1,nog
          if(ig(it) .eq. i .and. jg(it) .eq. j) then
             zpotent=zp(it)
             go to 30
             end if
          end do
       write(*,25) i,j
 25   format(' Given b.c. location not matched, at LT, i,j=', 2i5)
       return
 30   if(id(i,j).eq.'q') zb(map) = 4.0*zi*dk1*dx*(1.0+d_x)*zpotent
       if(id(i,j).eq.'r') zb(map) = -4.0*zi*dk1*dy*r*(r-d_y)*zpotent
       end if
c
      return
      end
c
c-----------------------------------------------------------
c
      subroutine corner_rb(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      character*1 id
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda,imap(kq), jmap(kq)
      common /given/ nog, ig(ngc), jg(ngc)
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
c  Give the right-bottom corner related 5 elements:
c    j
c    ^    . . . . .       . . . . . .       . . . . . . .
c    |    . 0 0 0 4       . 0 0 0 0 4       . 0 0 0 0 0 4
```

```
c    |   . 0 0 0 4       . 0 0 0 2 7       . 0 0 0 0 0 7
c    |   . 0 0 0 4       . 0 0 4 e e       . 0 0 0 0 7 e
c    |   . 0 0 0 4       . 0 0 4 e e       . 0 0 0 7 e e
c    |   . 2 2 2 7       . 0 0 4 e e       . 0 0 7 e e e
c    |                   . 2 2 7 e e       . 2 7 e e e e
c    |-----------------------------------------------------> i
c
c  The assignment of each 5 points follows the following pattern:
c  1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)
c
      zi=(0.0,1.0)
      ick=1
c
c  find the reflection coefficient from the boundary conditions
c
      if( id(i,j) .eq. '7') then
       do it=1,nobc
          if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
             alpha_x= real(zalp(it) )
             alpha_y=aimag(zalp(it) )
             ick=0
             go to 10
             end if
          end do
     else
       if( id(i,j) .eq. 'u') then
          alpha_x=1.0
          alpha_y=0.0
          ick=0
          end if
       if( id(i,j) .eq. 'v') then
          alpha_x=0.0
          alpha_y=1.0
          ick=0
          end if
     end if
c
 10   if(ick .eq. 1) then
       write(*,20) i,j, id(i,j)
 20      format(' Refl. coef. not found in CORNER_RB at',2i5,a5)
       stop
       end if
c
      dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*( dep(i,j)-dep(i-1,j) )/(2.0*dep(i,j) )
      d_y=e0*r*( dep(i,j+1)-dep(i,j) )/(2.0*dep(i,j) )
      dhdx2=( (dep(i,j)-dep(i-1,j) )/dx )**2
      dhdy2=( (dep(i,j+1)-dep(i,j) )/dy )**2
      d2hdx2= 0.0
      d2hdy2= 0.0
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
      zbetax=2*zi*alpha_x*dk1*dx
      zxix=zi*alpha_x*r*r/(dk1*dx)
      zbetay=2*zi*alpha_y*dk1*dy
      zxiy=zi*alpha_y/(dk1*r*r*dy)
c       psi=0.0
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
c
      za(map,1)=2*(1.0+zxiy*(r*(r-d_y)+zxix*(1+d_x))/(1.0-zxix*zxiy))
      za(map,2)=0.0
      za(map,3)=cnmd + (zbetax-2.0*zxix)*(1.0+d_x) +
     1          (r*(r-d_y) + zxix*(1.0+d_x) )*
     2          (zbetay + zxiy*(zbetax-2.0*zxix-2.0))/(1.0-zxix*zxiy)
      za(map,4)=r*(r+d_y) + zxix*(1.0+d_x) +
     1    (r*(r-d_y)+zxix*(1.0+d_x) )*(1.0+zxix*zxiy)/(1.0-zxix*zxiy)
      za(map,5)=0.0
c
      ia(map,1)=map-idistl(i,j)+1
      ia(map,2)=0
      ia(map,3)=map
      ia(map,4)=map+1
      ia(map,5)=0
      if( id(i,j) .eq. '7') zb(map)=0.0
      if( id(i,j) .eq. 'u' .or. id(i,j) .eq. 'v') then
       do it=1,nog
          if(ig(it) .eq. i .and. jg(it) .eq. j) then
             zpotent=zp(it)
             go to 30
             end if
          end do
       write(*,25) i,j
 25   format(' Given b.c. location not matched, at RB, i,j=', 2i5)
       return
 30   if(id(i,j).eq.'u') zb(map) = -4.0*zi*dk1*dx*(1.0+d_x)*zpotent
       if(id(i,j).eq.'v') zb(map) = 4.0*zi*dk1*dy*r*(r-d_y)*zpotent
       end if
c
      return
      end
c
c------------------------------------------------------------
c
      subroutine corner_rt(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      character*1 id
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /given/ nog, ig(ngc), jg(ngc)
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
c  Gives the right-top corner related 5 elements.  We have 2
c  possibles as follows:
c     j
c    ^
c    |    1 1 1 8       . 1 1 8 e e e e   . 1 1 1 8 e e e e
c    |    0 0 0 4       . 0 0 4 e e e e   . 0 0 0 0 8 e e e
c    |    0 0 0 4       . 0 0 4 e e e e   . 0 0 0 0 0 8 e e
c    |    0 0 0 4       . 0 0 0 1 1 1 8   . 0 0 0 0 0 4 e e
c    |    . . . .       . 0 0 0 0 0 0 4   . 0 0 0 0 0 4 e e
c    |------------------------------------------------------> i
c
c  The assignment of each 5 points follows the following pattern:
c  1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)
c
      zi=(0.0, 1.0)
      ick=1
c
c  find the reflection coefficient from the boundary conditions
c
      if( id(i,j) .eq. '8') then
       do it=1,nobc
          if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
             alpha_x= real(zalp(it) )
             alpha_y=aimag(zalp(it) )
             ick=0
             go to 10
             end if
          end do
     else
       if( id(i,j) .eq. 't') then
          alpha_x=1.0
          alpha_y=0.0
          ick=0
          end if
       if( id(i,j) .eq. 's') then
          alpha_x=0.0
          alpha_y=1.0
          ick=0
          end if
     end if
c
 10   continue
      if( ick .eq. 1) then
       write(*,20) i,j, id(i,j)
 20      format(' Refl. coef. not found in CORNER_RT at',2i5,a5)
       stop
       end if
c
```

```
      dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*( dep(i,j)-dep(i-1,j) )/(2.0*dep(i,j) )
      d_y=e0*r*( dep(i,j)-dep(i,j-1) )/(2.0*dep(i,j) )
      dhdx2=( (dep(i,j)-dep(i-1,j) )/dx )**2
      dhdy2=( (dep(i,j)-dep(i,j-1) )/dy )**2
      d2hdx2= 0
      d2hdy2= 0
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
      zbetax=2*zi*alpha_x*dk1*dx
      zxix=zi*alpha_x*r*r/(dk1*dx)
      zbetay=2*zi*alpha_y*dk1*dy
      zxiy=zi*alpha_y/(dk1*r*r*dy)
c
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
c
      za(map,1)=2*(1.0+zxiy*(r*(r+d_y)+zxix*(1+d_x))/(1.0-zxix*zxiy))
      za(map,2)=r*(r-d_y)+zxix*(1.0+d_x) +
     1    (r*(r+d_y)+zxix*(1.0+d_x) )*(1.0+zxix*zxiy)/(1.0-zxix*zxiy)
      za(map,3)=cnmd + (zbetax - 2.0*zxix)*(1.0+d_x) +
     1         (r*(r+d_y)+zxix*(1.0+d_x) )*
     2         (zbetay+zxiy*(zbetax-2.0*zxix-2.0) )/(1.0-zxix*zxiy)
      za(map,4)=0.0
      za(map,5)=0.0
c
      ia(map,1)=map-idistl(i,j)+1
      ia(map,2)=map-1
      ia(map,3)=map
      ia(map,4)=0
      ia(map,5)=0
      if( id(i,j) .eq. '8') zb(map)=0.0
      if( id(i,j) .eq. 's' .or. id(i,j) .eq. 't') then
       do it=1,nog
          if(ig(it) .eq. i .and. jg(it) .eq. j) then
             zpotent=zp(it)
             go to 30
             end if
          end do
       write(*,25) i,j
 25   format(' Given b.c. location not matched, at RT, i,j=', 2i5)
       return
 30   if(id(i,j).eq.'t') zb(map) = -4.0*zi*dk1*dx*(1.0+d_x)*zpotent
       if(id(i,j).eq.'s') zb(map) = -4.0*zi*dk1*dy*r*(r+d_y)*zpotent
       end if
c
      return
      end
c
c-----------------------------------------------------------------
c
```

```
      subroutine Given_BC(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
      character*1 id
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /given/ nog, ig(ngc), jg(ngc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
c processes a water cell that is also a given B.C., find
c ZA and IA and the coefficient column matrix zb(map).
c
c  j    Top           bottom         left          right
c  |  ..g g e e..   ..0 0 0 0 0 ..  g 0 0 0 .    . 0 0 0 g
c  |  ..0 0 0 0..   ..0 0 0 0 0 ..  g 0 0 0 .    . 0 0 0 g
c  |  ..0 0 0 0..   ..g g g e e ..  g 0 0 0 .    . 0 0 0 g
c  |------------------------------------------------------> i
c
c here, the coefficient for zp(i-1,j), zp(i,j-1), zp(i,j),
c zp(i,j+1), and zp(i+1,j) are stored in (map,1), (map,2), (map,3),
c (map,4) and (map,5), respectively.
c
      zi=(0.0, 1.0)
      dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
c
      do it=1,nog
       if(ig(it) .eq. i .and. jg(it) .eq. j) then
          zpotent=zp(it)
          go to 10
          end if
       end do
      write(*,8) i,j
  8   format(' Given boundary location not matched, at i,j=', 2i5)
      stop
c
c  left hand side given B.C.
c
 10   continue
      if(id(i+1,j) .eq. '0') then
       d_x=e0*( dep(i+1,j)-dep(i,j) )/(2.0*dep(i,j) )
       d_y=e0*r*( dep(i,j+1)-dep(i,j-1) )/(4.0*dep(i,j) )
       dhdx2=( (dep(i+1,j)-dep(i,j) )/dx )**2
       dhdy2=( (dep(i,j+1)-dep(i,j-1) )/(2.0*dy) )**2
       d2hdx2= 0.0
       d2hdy2= 0.0
       psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
```

```
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2.0
      zbetax=2*zi*dk1*dx
      zxix=zi*r*r/(dk1*dx)
      za(map,1)=0.0
      za(map,2)=r*(r-d_y) + zxix*(1.0-d_x)
      za(map,3)=cnmd + (zbetax-2.0*zxix)*(1.0-d_x)
      za(map,4)=r*(r+d_y) + zxix*(1.0-d_x)
      za(map,5)=2.0
      ia(map,1)=0
      ia(map,2)=map-1
      ia(map,3)=map
      ia(map,4)=map+1
      ia(map,5)=map + idistr(i,j)-1
      zb(map)= 4.0*zi*dk1*dx*(1.0-d_x)*zpotent
      go to 20
      end if
c
c  right hand side given B.C.
c
     if(id(i-1,j) .eq. '0') then
      d_x=e0*( dep(i,j)-dep(i-1,j) )/(2.0*dep(i,j) )
      d_y=e0*r*( dep(i,j+1)-dep(i,j-1) )/(4.0*dep(i,j) )
      zbetax=2*zi*dk1*dx
      zxix=zi*r*r/(dk1*dx)
      dhdx2=( (dep(i,j)-dep(i-1,j) )/dx )**2
      dhdy2=( (dep(i,j+1)-dep(i,j-1) )/(2.0*dy) )**2
      d2hdx2= 0.0
      d2hdy2= 0.0
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2.0
      za(map,1)=2.0
      za(map,2)=r*(r - d_y) + zxix*(1.0 + d_x)
      za(map,3)=cnmd + (zbetax - 2.0*zxix)*(1.0 + d_x)
      za(map,4)=r*(r + d_y) + zxix*(1.0 + d_x)
      za(map,5)=0.0
      ia(map,1)=map-idistl(i,j)+1
      ia(map,2)=map-1
      ia(map,3)=map
      ia(map,4)=map+1
      ia(map,5)=0
      zb(map) = -4.0*zi*dk1*dx*(1.0+d_x)*zpotent
      go to 20
      end if
c
c  bottom side given B.C.
c
     if(id(i,j+1) .eq. '0') then
      d_x=e0*(dep(i+1,j)-dep(i-1,j) )/(4.0*dep(i,j) )
      d_y=e0*r*(dep(i,j+1)-dep(i,j) )/(2.0*dep(i,j) )
      zbetay=2*zi*dk1*dy
      zxiy=zi/(dk1*r*r*dy)
```

```
       dhdx2=( (dep(i+1,j)-dep(i-1,j) )/(2.0*dx) )**2
       dhdy2=( (dep(i,j+1)-dep(i,j) )/dy )**2
       d2hdx2= 0.0
       d2hdy2= 0.0
       psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
       cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2.0
       za(map,1)=r*(r-d_y)*zxiy +(1.0-d_x)
       za(map,2)=0.0
       za(map,3)=cnmd + (zbetay-2.0*zxiy)*r*(r-d_y)
       za(map,4)=2.0*r*r
       za(map,5)=r*(r-d_y)*zxiy + (1.0+d_x)
       ia(map,1)=map-idistl(i,j)+1
       ia(map,2)=0
       ia(map,3)=map
       ia(map,4)=map+1
       ia(map,5)=map+idistr(i,j)-1
       zb(map) = 4.0*zi*dk1*dy*r*(r-d_y)*zpotent
       go to 20
       end if
c
c  top side given B.C.
c
      if(id(i,j-1) .eq. '0') then
       d_x=e0*( dep(i+1,j)-dep(i-1,j) )/(4.0*dep(i,j) )
       d_y=e0*r*( dep(i,j)-dep(i,j-1) )/(2.0*dep(i,j) )
       zbetay=2*zi*dk1*dy
       zxiy=zi/(dk1*r*r*dy)
       dhdx2=( (dep(i+1,j)-dep(i-1,j) )/(2.0*dx) )**2
       dhdy2=( (dep(i,j)-dep(i,j-1) )/dy )**2
       d2hdx2= 0.0
       d2hdy2= 0.0
       psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
       cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2.0
c
       za(map,1)=r*(r+d_y)*zxiy + (1.0-d_x)
       za(map,2)=2.0*r*r
       za(map,3)=cnmd + (zbetay-2.0*zxiy)*r*(r+d_y)
       za(map,4)=0.0
       za(map,5)=r*(r+d_y)*zxiy + (1.0+d_x)
       ia(map,1)=map-idistl(i,j)+1
       ia(map,2)=map-1
       ia(map,3)=map
       ia(map,4)=0
       ia(map,5)=map+idistr(i,j)-1
       zb(map)= -4.0*zi*dk1*dy*r*(r+d_y)*zpotent
       go to 20
       end if
c
      print*,'Cannot find where to apply this B.C., at i,j=', i,j
   20 continue
```

```
c
      print*,'i,j,map, ia(map,3)=',i,j,map,ia(map,3)
      return
      end
c
c-------------------------------------------------------------------
c
      subroutine bound_r(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
      character*1 id
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
c  Processes the right side boundary related 4 ZA and IA.
c  find the reflection coefficient from the B.C.
c
c  ^ j
c  |  ..11117       ...11117eee
c  |  ..00004        ..00004eee
c  |  ..00004        ..00004eee
c  |  ..00004        ..00000117
c  |  ..00004        ..00000004
c  |  ..22228       ...22222228
c  |----------------------------------> i
c
c  The assignment of each 5 points follows the following pattern:
c  1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)
c
      zi=(0.0,1.0)
      do it=1,nobc
       if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
         alpha= real(zalp(it) )
         go to 10
         end if
       end do
      write(*,8) i,j
  8   format(' Reflection coefficient not found in BOUND_R at',2i5)
      return
c
 10   dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*( dep(i,j)-dep(i-1,j) )/(2.0*dep(i,j) )
      d_y=e0*r*( dep(i,j+1)-dep(i,j-1) )/(4.0*dep(i,j) )
      dhdx2=( (dep(i,j)-dep(i-1,j) )/dx )**2
      dhdy2=( (dep(i,j+1)-dep(i,j-1) )/(2.0*dy) )**2
      d2hdx2= ( dep(i,j)-2.0*dep(i-1,j)+dep(i-2,j) )/(2.0*dx*dx )
      d2hdy2= ( dep(i,j+1)-2.0*dep(i,j)+dep(i,j-1) )/(2.0*dy*dy )
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
      zbetax=2*zi*alpha*dk1*dx
      zxix=zi*alpha*r*r/(dk1*dx)
c
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2.0
      za(map,1)=2.0
      za(map,2)=r*(r - d_y) + zxix*(1.0 + d_x)
      za(map,3)=cnmd + (zbetax - 2.0*zxix)*(1.0 + d_x)
      za(map,4)=r*(r + d_y) + zxix*(1.0 + d_x)
      za(map,5)=0.0
      ia(map,1)=map-idistl(i,j)+1
      ia(map,2)=map-1
      ia(map,3)=map
      ia(map,4)=map+1
      ia(map,5)=0
c
      zb(map)=0.0
      return
      end
c
c--------------------------------------------------------------
c
      subroutine bound_l(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
      character*1 id
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp,np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
c  Construct the Trifty storage matrix ZA, & IA when the left
c  hand point (i-1,j) is a Neumann type boundary point.
c
c  Assignment of the 5 points follows the following pattern:
c  1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)
c  find the reflection coefficient from the B.C.
c
      zi=(0.0,1.0)
      do it=1,nobc
       if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
          alpha= real(zalp(it) )
          go to 10
          end if
       end do
      write(*,8) i,j
  8   format(' Reflection coefficient not found in BOUND_L at',2i5)
```

```
      return
c
 10   dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*(dep(i+1,j)-dep(i,j) )/(2.0*dep(i,j) )
      d_y=e0*r*(dep(i,j+1)-dep(i,j-1) )/(4.0*dep(i,j) )
      dhdx2=( (dep(i+1,j)-dep(i,j) )/dx )**2
      dhdy2=( (dep(i,j+1)-dep(i,j-1) )/(2.0*dy) )**2
      d2hdx2= ( dep(i+2,j)-2.0*dep(i+1,j)+dep(i,j) )/(2.0*dx*dx )
      d2hdy2= ( dep(i,j+1)-2.0*dep(i,j)+dep(i,j-1) )/(2.0*dy*dy )
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
      zbetax=2*zi*alpha*dk1*dx
      zxix=zi*alpha*r*r/(dk1*dx)
c
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
      za(map,1)=0.0
      za(map,2)=r*(r-d_y) + zxix*(1.0-d_x)
      za(map,3)=cnmd + (zbetax-2.0*zxix)*(1.0-d_x)
      za(map,4)=r*(r+d_y) + zxix*(1.0-d_x)
      za(map,5)=2.0
      ia(map,1)=0
      ia(map,2)=map-1
      ia(map,3)=map
      ia(map,4)=map+1
      ia(map,5)=map+idistr(i,j)-1
      zb(map)=(0.0, 0.0)
c
      return
      end
c
c------------------------------------------------------------------
c
      subroutine bound_b(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
      character*1 id
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
c
c  Give the bottom side boundary related 5 ZA and IA.
c  find the reflection coefficient from the boundary conditions
c
c  The assignment of each 5 points follows the following pattern:
c  1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)
c
      zi=(0.0,1.0)
      do it=1,nobc
       if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
          alpha= real(zalp(it) )
          go to 10
          end if
       end do
      write(*,8) i,j
  8   format(' Reflection coefficient not found in BOUND_B',2i5)
      return
c
 10   dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*(dep(i+1,j)-dep(i-1,j) )/(4.0*dep(i,j) )
      d_y=e0*r*(dep(i,j+1)-dep(i,j) )/(2.0*dep(i,j) )
      dhdx2=( (dep(i+1,j)-dep(i-1,j) )/(2.0*dx) )**2
      dhdy2=( (dep(i,j+1)-dep(i,j) )/dy )**2
      d2hdx2= ( dep(i+1,j)-2.0*dep(i,j)+dep(i-1,j) )/(2.0*dx*dx )
      d2hdy2= ( dep(i,j+2)-2.0*dep(i,j+1)+dep(i,j) )/(2.0*dy*dy )
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
      zbetay=2*zi*alpha*dk1*dy
      zxiy=zi*alpha/(dk1*r*r*dy)
c
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
      za(map,1)=r*(r-d_y)*zxiy +(1.0-d_x)
      za(map,2)=0.0
      za(map,3)=cnmd + (zbetay-2.0*zxiy)*r*(r-d_y)
      za(map,4)=2.0*r*r
      za(map,5)=r*(r-d_y)*zxiy + (1.0+d_x)
      ia(map,1)=map-idistl(i,j)+1
      ia(map,2)=0
      ia(map,3)=map
      ia(map,4)=map+1
      ia(map,5)=map+idistr(i,j)-1
      zb(map)=0.0
c
      return
      end
c
c-------------------------------------------------------------
c
      subroutine bound_t(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, kq=80000, nbc=1500, ngc=502)
      character*1 id
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /bound/ nobc, ibc(nbc), jbc(nbc)
      common /compx/ zp(ngc), zalp(nbc), phas(ngc)
```

```
c
c  Give the top side boundary related 5 ZA ans IA.
c  find the reflection coefficient from the boundary conditions
c
c  The assignment of each 5 points follows the following pattern:
c  1: (i-1,j),  2: (i,j-1),  3: (i,j),  4: (i,j+1),  5: (i+1,j)
c
      zi=(0.0,1.0)
      do it=1,nobc
       if(ibc(it) .eq. i .and. jbc(it) .eq. j) then
          alpha = real(zalp(it) )
          go to 10
          end if
       end do
      write(*,8) i,j
  8   format(' Reflection coefficient not found in BOUND_T at',2i5)
      return
c
 10   dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*( dep(i+1,j)-dep(i-1,j) )/(4.0*dep(i,j) )
      d_y=e0*r*( dep(i,j)-dep(i,j-1) )/(2.0*dep(i,j) )
      dhdx2=( (dep(i+1,j)-dep(i-1,j) )/(2.0*dx) )**2
      dhdy2=( (dep(i,j)-dep(i,j-1) )/dy )**2
      d2hdx2= ( dep(i+1,j)-2.0*dep(i,j)+dep(i-1,j) )/(2.0*dx*dx )
      d2hdy2= ( dep(i,j)-2.0*dep(i,j-1)+dep(i,j-2) )/(2.0*dy*dy )
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
      zbetay=2*zi*alpha*dk1*dy
      zxiy=zi*alpha/(dk1*r*r*dy)
c
      cnmd=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
      za(map,1)=r*(r+d_y)*zxiy + (1.0-d_x)
      za(map,2)=2.0*r*r
      za(map,3)=cnmd + (zbetay-2.0*zxiy)*r*(r+d_y)
      za(map,4)=0.0
      za(map,5)=r*(r+d_y)*zxiy + (1.0+d_x)
      ia(map,1)=map-idistl(i,j)+1
      ia(map,2)=map-1
      ia(map,3)=map
      ia(map,4)=0
      ia(map,5)=map+idistr(i,j)-1
      zb(map)=0.0
c
      return
      end
c
c
      subroutine domain(i,j,map)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, kq=80000)
      character*1 id
      common /waves/ period, waveH, angle0
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
c
c for interior grid points
c In this sub., the coeffieient matrix zb(map) = 0.
c In this system, the coefficient of zp(i-1,j) is stored in (map,1),
c                 the coefficient of zp(i,j-1) is stored in (map,2),
c                 the coefficient of zp(i,j) is stored in (map,3),
c                 the coefficient of zp(i,j+1) is stored in (map,4),
c             and the coefficient of zp(i+1,j) is stored in (map,5)
c
c  coefficients
c
      dk1=wn(dep(i,j), period)
      dkh=dk1*dep(i,j)
      call e012(dkh, e0, e1, e2)
      d_x=e0*( dep(i+1,j)-dep(i-1,j) )/(4.0*dep(i,j) )
      d_y=e0*r*( dep(i,j+1)-dep(i,j-1) )/(4.0*dep(i,j) )
      dhdx2=( (dep(i+1,j)-dep(i-1,j) )/(2.0*dx) )**2
      dhdy2=( (dep(i,j+1)-dep(i,j-1) )/(2.0*dy) )**2
      d2hdx2= ( dep(i+1,j)-2.0*dep(i,j)+dep(i-1,j) )/(dx*dx )
      d2hdy2= ( dep(i,j+1)-2.0*dep(i,j)+dep(i,j-1) )/(dy*dy )
      psi=e1*(dhdx2+dhdy2) + e2/wk0*(d2hdx2 + d2hdy2)
c
      za(map,1)=1.0-d_x
      za(map,2)=r*(r-d_y)
      za(map,3)=dk1*dk1*(1.0+psi)*dx*dx -2.0*r*r - 2
      za(map,4)=r*(r+d_y)
      za(map,5)=1.0+d_x
c
c relative location
c
      ia(map,1)=map-idistl(i,j)+1
      ia(map,2)=map-1
      ia(map,3)=map
      ia(map,4)=map+1
      ia(map,5)=map+idistr(i,j)-1
c
c   rigth hand side
c
      zb(map)=(0.0, 0.0)
c
      return
      end
c
c-------------------------------------------------------------------
c
      integer function idistr(i,j)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401)
      character*1 id
c
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /bottm/ dep(iq,jq), id(iq,jq)
c
c  Calculates the distance between points (i,j) & (i+1,j).  It show
c  many grid points (must be a water point, a radiational boundary
c  point, or a given boundary point) in between. It counts vertically
c  from the point(i,j) and up (i,j+1), (i,j+2), ..., (i,np), (i+1,1),
c  (i+1,2), ..., to (i+1,j).  In the matrix equation, this distance
c  represents the up band width at that particular diagonal element
c (i,j).
c  If this point is a radiational B.C. point, then the distance is
c  zero.
C
      idistr=0
      if(id(i,j).eq.'4' .or. id(i,j).eq.'7' .or. id(i,j).eq.'8') then
       return
        else
       do 10 jk=1,np
          if( id(i,jk) .eq. 'e' .or. jk .le. j ) go to 10
          idistr=idistr+1
  10       continue
c
       do 20 jk=1,np
          if( id(i+1,jk) .eq. 'e' .or. jk .gt. j ) go to 20
          idistr=idistr+1
  20       continue
c
       idistr=idistr+1
c
      end if
c
      return
      end
c
c
      integer function idistl(i,j)
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401)
      character*1 id
c
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /bottm/ dep(iq,jq), id(iq,jq)
c
c  Calculate the distance between point (i-1,j) & (i,j). The distance
c  means how many grid points, which must be a water or a boundary
c  point, are in between. It counts vertically from (i,j) downward
c  (i,j-1), (i,j-2)...(i,1) and restarted from previous i line
c  (i-1,np), (i-1,np-1)... to(i-1,j).
c
c  In the matrix equation, the distance represents the lower band
```

```
c  width at that particular diagonal element (i,j).
c  If it is a left side radiational boundary point, then distance is
c  zero
c
      idistl=0
      if(id(i,j).eq.'3' .or. id(i,j).eq.'5' .or. id(i,j).eq.'6') then
       return
        else
       do 10 jk=1,np
          if( id(i-1,jk) .eq. 'e' .or. jk .lt. j ) go to 10
          idistl=idistl+1
  10     continue

       do 20 jk=1,np
          if( id(i,jk) .eq. 'e' .or. jk .ge. j ) go to 20
          idistl=idistl+1
  20     continue
c
       idistl=idistl+1
c
      end if
c
      return
      end
c
c
      subroutine BMsolver(ierr_code)
c
c It solves a complex banded matrix equation ZAF * ZX = ZB
c where ZAF is a complex band matrix with dimension (m * n)
c       ZX and ZB are two complex column matrices with length (n).
c
c Because of the hudge size of ZAF, e.g., (300 x 50000), it is
c designed to solve this problem using the following two steps.
c
c First, don't use the full size of ZAF, instead, uses two small
c matrices: ZA and IA that each only uses 5 x 50000 to save space.
c
c ZA : a complex matrix (5 x n) to store the coefficent matrix in
c      a matrix equation ZAF * ZX = ZB.  Because of using the finite
c      difference method to solve an elliptic equation, there are
c      only 5 elements to be saved for the coefficient matrix.  The
c      band width, however, is much much large than 5 with a lot of
c      "zero."  By doing so, we need another matrix
c IA : to save the corresponding locations.
c
c Second, uses a working matrix, ZW(IW,JW), and do a systematical
c swap between a hard disk and memory.  For this reason, be sure
c that you do have enough space in your hard disk.  For example, a
c complex matrix with size of (300 x 50000) requires 120 MB for
c storage, if using 4 byte for a real number.  If using 8 bytes,
c then, 240 MB is needed.
```

```
c
c IW,JW: IW should be >=  m+ml, where ml is the lower band width.
c        The size of JW depends on the available computer memory.
c        In general, the large the JW, the less the disk IO, and
c        thus, the faster the computing speed.
c        As a rule of thumb, you may select JW = 2*IW and tried to
c        see if your computer has enough memory to run the program.
c
c The procedures follows that given in the subroutine CGBFA & CGBSL
c from LINPACK.  The major difference is just doing it one block at
c a time, stores the results in hard disk sequently.  After forward
c elimination, reverse the process by reading the data from hard disk
c and do back substitute for the solution.
c
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      parameter (iq=501, jq=401, kq=80000, iw=1000, jw=1700)
c
      character*8 tmpfile
      character*3 chrc
      character*1 id
c
      common /bottm/ dep(iq,jq), id(iq,jq)
      common /matrx/ za(kq,5), ia(kq,5), zb(kq)
      common /gener/ mp, np, dx,dy, r, yorient, xl,yl, wk0
      common /unknw/ n, mu, mL, m, Lda, imap(kq), jmap(kq)
      common /works/ zw(iw,jw), ipvt(kq)
c
      cabs1(zdum)=abs(real(zdum)) + abs(aimag(zdum))
c
c  Gaussian elimination with partial pivoting
c
c  NK: # of columns for the working matrix that will be saved.
c  NW: The number of column needed for working, nw=m-1+nk
c  NS: An index to show the number of working matrix.
c
c  check working matrix dimension
c
      nk=450
      nw=m+nk-1
c
      write(*,5) iw, jw, Lda, nw
  5   format(' The working matrix, ZW(iw,jw) is  ZW(',2i5,')'/
     *       ' The best condition should be iw ~> Lda= ',i5/
     *       '                            and jw ~> nw = ',i5)
      if(nw .gt. jw) then
       write(*,10) Lda, nw
 10       format(' IW shoult be >= ',i5, ' and JW must be >=', i5/
     *          ' Please change program and re-run .'//
     *          ' If no memory, you can reduce the NK')
       ierr_code=1
       return
       end if
```

```
c
c  clear up the working matrix
c
      do i=1,iw
       do j=1,nw
          zw(i,j)=(0.0, 0.0)
          end do
       end do
c
c construct the first working matrix, ZW, which has a size of
c (Lda x nw).  The thrify storaged matrices are expanded, only
c to the first (nk x 5) block.
c
c The flag is used to record, only onec, when ZA data starts lost.
c Later it will be used to re-fetch into the working matrix
c
      ns=0
      map1=0
      iflag=1
      ne=nw
      if(ne .gt. n) ne=n
c
      do map=1,ne
       do i=1,5
          j=ia(map,i)
          if(j .ne. 0) then
             if(j .le. nk+m-1) then
              ids=j-map
              zw(m-ids,j)=za(map,i)
            else
              if(iflag .eq. 1) then
                 map1=map
                 iflag=0
                 end if
            end if
             end if
          end do
       end do
c
c Gaussian elimination with partial pivoting
c
      j1=min0(n,m)-1
      jz=j1
      ju=0
      nt=0
      index=0
c
  100 continue
c
      if(ju .ne. 0) ju=ju-nk
      if(jz .ne. j1) jz=jz-nk
c
```

```
c  index=0,  the first, 2nd, ..., block of matrix.
c       =1,  the last block of matrix
c
      if(index .eq. 0 .and. ne .ne. n) then
       nc=nk
      else
       nc=n-ns*nk-1
      end if
c
      do k=1,nc
       kp1=k+1
       kr=ns*nk + k
c
c  find L = pivot index
c
       Lm=min0(mL,n-ns*nk-k)
       L=icamax(Lm+1, zw(m,k), 1) + m -1
       ipvt(kr)=L+kr-m
c
c  zero pivot implies that this column are all zeros, a singular
c  matrix.
c
       if(cabs1(zw(L,k)) .lt. 0.10e-8) goto 120
c
c  interchange if necessary
c
       if(L .ne. m) then
          zt=zw(L,k)
          zw(L,k)=zw(m,k)
          zw(m,k)=zt
c            print*,'***********pivoting'
          end if
c
c  compute multipliers
c
       zt=-(1.0d0,0.0d0)/zw(m,k)
       call cscal(Lm, zt, zw(m+1,k), 1)
c
c  swap ZB array, if necessary
c
       Lp=ipvt(kr)
       zt=zb(Lp)
       if(Lp .ne. kr) then
          zb(Lp)=zb(kr)
          zb(kr)=zt
          end if
       call caxpy(Lm, zt, zw(m+1,k), 1, zb(kr+1), 1)
c
c  row elimenation with column indexing
c
       ju=min0(max0(ju, mu+ipvt(kr)-ns*nk), n-ns*nk)
       mm=m
      if(ju .ge. kp1) then
         do j=kp1,ju
            L=L-1
            mm=mm-1
            zt=zw(L,j)
            if(L .ne. mm) then
             zw(L,j)=zw(mm,j)
             zw(mm,j)=zt
             end if
            call caxpy(Lm, zt, zw(m+1,k), 1, zw(mm+1,j), 1)
            end do
         end if
c
      go to 150
  120    continue
      print*,'Zero diagonal element at (L,k)=', L, k
      stop
  150 continue
c
      end do
c
c  Eexcept the last working matrix, write the upper triangular
c  matrix (from j=1,nk) into hard disk.  For the last one, i.e.,
c  nc<>nk, go to back substitute directly.
c
      if(nc .eq. nk) then
      nt=nt+1
      print*,'Writing tem. file #', nt
      tmpfile='t'//chrc(nt)//'.tmp'
      open(12,file=tmpfile,form='unformatted')
      write(12) ((zw(i,j),i=1,m),j=1,nk)
      close(12)
c
c  moving the rest working matrix forward to the beginning
c
      do j=nk+1, nk+m-1
         L=j-nk
         do i=1,Lda
            zw(i,L)=zw(i,j)
            end do
         end do
c
c  clear the rest working area for reading new working matrix
c
      do j=m, nk+m-1
         do i=1,Lda
            zw(i,j)=(0.0,0.0)
            end do
         end do
c
c read in a full block of ZA and IA, by two steps
c
```

```
            ns=ns+1
            if( (ns+1)*nk+m-1 .lt. n) then
               ne=nk
               index=0
c
c  For intermediate blocks, read in ZA and IA by two steps.
c  First read in the upper triangular matrix that were cut off
c  at the previous time when constructing the working matrix.
c
               if(map1 .ne. 0) then
                  do map=map1, m-1+ns*nk
                   do i=4,5
                      j=ia(map,i)
                      if(j .ne. 0) then
                       if(j .gt. m-1+ns*nk) then
                          ids=j-map
                          zw(m-ids,j-ns*nk)=za(map,i)
                          end if
                       end if
                      end do
                   end do
                  end if
c
c  now read in next block of ZA and IA
c
               iflag=1
               map1=0
               do k=1,ne
                  map=k + ns*nk + m-1
                  do i=1,5
                   j=ia(map,i)
                   if(j .ne. 0) then
                      if(j .le. ns*nk+ne+m-1) then
                       ids=j-map
                       zw(m-ids,j-ns*nk)=za(map,i)
                         else
                       if(iflag .eq. 1) then
                          map1=map
                          iflag=0
                          end if
                         end if
                      end if
                   end do
                  end do
c
c  End of read in a block of ZA and IA.
c
              else
c
c  This is to read the last block of ZA and IA
c
               ne=n-(m-1+ns*nk)
```

```
          index=1
          if(map1 .ne. 0) then
             do map=map1,m-1+ns*nk
             do i=4,5
                j=ia(map,i)
                if(j .ne. 0) then
                 if(j .gt. m-1+ns*nk) then
                    ids=j-map
                    zw(m-ids,j-ns*nk)=za(map,i)
                    end if
                 end if
                end do
             end do
             end if
c
c now read the last block of ZA and IA
c
          iflag=1
          map1=0
          do k=1,ne
             map=m-1+ns*nk+k
             do i=1,5
            j=ia(map,i)
            if(j .ne. 0) then
                 if(j .le. m-1+ns*nk+ne) then
             ids=j-map
             zw(m-ids,j-ns*nk)=za(map,i)
                   else
             ierr_code=2
             write(*,180)
 180        format(' If this happen, it is wrong.  At the last ',
     *                  'block,'/' it should have enough space to ',
     *                  'include all the remaining matrix')
             return
            end if
            end if
           end do
             end do
          iflag=1
          end if
c
       goto 100
c
      else
c
c   backward substitution from the last submatrix
c
        if( nt .eq. 0) nc=n-1
        do kb=1,nc+1
         kr=n+1-kb
         k=nc+2-kb
         zb(kr)=zb(kr)/zw(m,k)
         Lm=min0(kr,m)-1
         La=m-Lm
         Lb=kr-Lm
         zt=-zb(kr)
         call caxpy(Lm, zt, zw(La,k), 1, zb(Lb), 1)
         end do
c
c  If one loop can include all elements, i.e., for a small banded
c  matrix, just stops after this.
c
        if( nt .eq. 0 ) go to 400
       end if
c
c  complete the rest backward substitute
c
       ns=0
 200   continue
c
c  clear the working matrix for reading new submatrix from disk.
c
       do j=1,nk+m-1
        do i=1,m
           zw(i,j)=(0.0,0.0)
           end do
        end do
c
       print*,'Reading tem. file #', nt
       open(14,file=tmpfile,form='unformatted')
       read(14) ((zw(i,j),i=1,m),j=1,nk)
       close(14,status='delete')
c
       do kb=1,nk
        kr=n+1-(nc+1)-ns*nk-kb
        k=nk+1-kb
        zb(kr)=zb(kr)/zw(m,k)
        Lm=min0(kr,m)-1
        La=m-Lm
        Lb=kr-Lm
        zt=-zb(kr)
        call caxpy(Lm, zt, zw(La,k), 1, zb(Lb), 1)
        end do
       ns=ns+1
       nt=nt-1
       tmpfile='t'//chrc(nt)//'.tmp'
       if(nt .gt. 0) goto 200

 400   continue
c
c  To restore the original sequence of ZB.  Since it starts pivoting
c  at the 2nd.  we started at 2nd too for restoring.
c
       print*,'Restore the original sequency'
```

```
      do 1000 kb=2,n
       k=n+1-kb
       L=ipvt(k)
       zt=zb(L)
       if( L .eq. k ) go to 1000
       print*,'pivoting at L,k,kb=',L,k,kb
       zb(L)=zb(k)
       zb(k)=zt
 1000    continue
c
      return
      end
c
c
      character*3 function chrc(nt)
c
c It changes an input integer number NT to character
c CHANGQING LI 06/94
c
      character*1 c1
      character*2 c2
      character*3 c3
c
      i1=nt
      i2=i1/10
      if(i2 .gt. 0) then
       i3=i2/10
       if(i3 .gt. 0) then
         i4=i3/10
         if(i4 .gt. 0) then
           print*,'-----------------------------------------'
           print*,'The given integer is > 999, not allowed.'
           print*,'-----------------------------------------'
           chrc='-1'
        else
          c3=char(48+i3)//char(48+i2-i3*10)//char(48+i1-i2*10)
          chrc=c3
c            write(11,*) c3
        end if
     else
        c2=char(48+i2)//char(48+i1-i2*10)
        chrc=c2
c          write(11,*) c2
     end if
      else
     c1=char(48+i1)
     chrc=c1
c       write(11,*) c1
      end if
c
      return
      end
```

```
c
c
      SUBROUTINE CSCAL(N,ZCA,ZCX,INCX)
C
C  PURPOSE  Complex vector scale x = a*x
C  WRITTEN on Oct. 1, 79, REVISION on Aug. 01, 82
C  CATEGORY NO.  D1A6
C  KEYWORDS  BLAS,COMPLEX,LINEAR ALGEBRA,SCALE,VECTOR
C  AUTHORS  LAWSON, C. L., (JPL),   HANSON, R. J., (SNLA)
C           KINCAID, D. R., (U. OF TEXAS), and KROGH, F. T., (JPL)
C
C  DESCRIPTION
C
C                B L A S  Subprogram
C    Description of Parameters
C
C     --Input--
C        N  number of elements in input vector(s)
C       CA  complex scale factor
C       CX  complex vector with N elements
C     INCX  storage spacing between elements of CX
C
C     --Output--
C    CSCAL  complex result (unchanged if N .LE. 0)
C
C     replace complex CX by complex CA*CX.
C     For I = 0 to N-1, replace CX(1+I*INCX) with  CA * CX(1+I*INCX)
C  REFERENCES: LAWSON C.L., HANSON R.J., KINCAID D.R., KROGH F.T.,
C              "BASIC LINEAR ALGEBRA SUBPROGRAMS FOR FORTRAN USAGE,"
C              ALGORITHM NO. 539, TRANSACTIONS ON MATHEMATICAL
C              SOFTWARE, VOLUME 5, NUMBER 3, SEPTEMBER 1979, 308-323
C  ROUTINES CALLED  (NONE)
C
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      dimension zcx(1)
C      COMPLEX*8 CA, CX(1)
C  FIRST EXECUTABLE STATEMENT  CSCAL
      IF(N .LE. 0) RETURN
      NS = N*INCX
       DO I = 1,NS,INCX
          ZCX(I) = ZCA*ZCX(I)
          end do
      RETURN
      END
c
c
      SUBROUTINE CAXPY(N,ZCA,ZCX,INCX,ZCY,INCY)
C
C  WRITTEN on Oct. 01, 79, REVISION on April 25, 84
C  CATEGORY NO.  D1A7
C  KEYWORDS  BLAS,COMPLEX,LINEAR ALGEBRA,TRIAD,VECTOR
C  AUTHOR  LAWSON, C. L., (JPL),  HANSON, R. J., (SNLA)
```

```
C          KINCAID, D. R., (U. OF TEXAS), KROGH, F. T., (JPL)
C  PURPOSE  Complex computation y = a*x + y
C  DESCRIPTION
C
C                B L A S  Subprogram
C    Description of Parameters
C
C     --Input--
C        N  number of elements in input vector(s)
C      ZCA  complex scalar multiplier
C      ZCX  complex vector with N elements
C     INCX  storage spacing between elements of CX
C      ZCY  complex vector with N elements
C     INCY  storage spacing between elements of CY
C
C     --Output--
C      ZCY  complex result (unchanged if N .LE. 0)
C
C     Overwrite complex ZCY with complex  ZCA*ZCX + ZCY.
C     For I = 0 to N-1, replace
c        ZCY(LY+I*INCY) with ZCA*ZCX(LX+I*INCX) + ZCY(LY+I*INCY), where
c        LX = 1 if INCX .GE. 0, else LX = (-INCX)*N
C        and LY is defined in a similar way using INCY.
C  REFERENCES
C         LAWSON C.L., HANSON R.J., KINCAID D.R., KROGH F.T.,
C         *BASIC LINEAR ALGEBRA SUBPROGRAMS FOR FORTRAN USAGE*,
C         ALGORITHM NO. 539, TRANSACTIONS ON MATHEMATICAL
C         SOFTWARE, VOLUME 5, NUMBER 3, SEPTEMBER 1979, 308-323
C  ROUTINES CALLED  (NONE)
C
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      dimension zcx(1), zcy(1)
C      COMPLEX CX(1),CY(1),CA
C  FIRST EXECUTABLE STATEMENT  CAXPY
      CANORM = ABS(REAL(ZCA)) + ABS(AIMAG(ZCA))
      IF(N.LE.0 .OR. CANORM.EQ.0.E0) RETURN
      IF(INCX.EQ.INCY.AND.INCX.GT.0) GO TO 20
      KX = 1
      KY = 1
      IF(INCX.LT.0) KX = 1+(1-N)*INCX
      IF(INCY.LT.0) KY = 1+(1-N)*INCY
       DO 10 I = 1,N
       ZCY(KY) = ZCY(KY) + ZCA*ZCX(KX)
       KX = KX + INCX
       KY = KY + INCY
   10 CONTINUE
      RETURN
   20 CONTINUE
      NS = N*INCX
       DO 30 I=1,NS,INCX
       ZCY(I) = ZCA*ZCX(I) + ZCY(I)
   30     CONTINUE
```

```
      RETURN
      END
c
c
      INTEGER FUNCTION ICAMAX(N,zCX,INCX)
c
C  WRITTEN on Oct. 01, 79, REVISed on Aug. 01, 82
C  CATEGORY NO.  D1A2
C  KEYWORDS  BLAS,COMPLEX,LINEAR ALGEBRA,MAXIMUM COMPONENT,VECTOR
C  AUTHOR  LAWSON, C. L., (JPL), HANSON, R. J., (SNLA)
C          KINCAID, D. R., (U. OF TEXAS), KROGH, F. T., (JPL)
C  PURPOSE  Find the location (or index) of the largest component
C           of a complex vector
C  DESCRIPTION
C
C                B L A S  Subprogram
C    Description of Parameters
C
C     --Input--
C        N  number of elements in input vector(s)
C      zCX  complex vector with N elements
C     INCX  storage spacing between elements of CX
C
C     --Output--
C   ICAMAX  smallest index (zero if N .LE. 0)
C
C      Returns the index of the component of CX having the
C      largest sum of magnitudes of real and imaginary parts.
C     ICAMAX = first I, I = 1 to N, to minimize
C     ABS(REAL(CX(1-INCX+I*INCX))) + ABS(IMAG(CX(1-INCX+I*INCX)))
C  REFERENCES
C         LAWSON C.L., HANSON R.J., KINCAID D.R., KROGH F.T.,
C         *BASIC LINEAR ALGEBRA SUBPROGRAMS FOR FORTRAN USAGE*,
C          ALGORITHM NO. 539, TRANSACTIONS ON MATHEMATICAL
C          SOFTWARE, VOLUME 5, NUMBER 3, SEPTEMBER 1979, 308-323
C  ROUTINES CALLED  (NONE)
C
C
      implicit real*8 (a-h, o-y), integer*4 (i-n), complex*16 (z)
      dimension zcx(1)
C     COMPLEX CX(1)
C***FIRST EXECUTABLE STATEMENT  ICAMAX
      ICAMAX = 0
      IF(N .LE. 0) RETURN
      ICAMAX = 1
      IF(N .LE. 1) RETURN
      NS = N*INCX
      II = 1
      SUMMAX = ABS(REAL(zCX(1))) + ABS(AIMAG(zCX(1)))
      DO 20 I=1,NS,INCX
       SUMRI = ABS(REAL(zCX(I))) + ABS(AIMAG(zCX(I)))
       IF(SUMMAX .GE. SUMRI) GO TO 10
```

```
         SUMMAX = SUMRI
         ICAMAX = II
   10      II = II + 1
   20      CONTINUE
        RETURN
        END
c
c
c
        function wn(depth, period)
c
c  This function calculate the airy's wave number
c  period : in second.
c  depth  : in meter.
c  wavek  : in meter^-1
c
        implicit real*8 (a-h, o-y)
        grav=9.8
        dol0=depth/(1.56*period*period)
        if (dol0 .ge. 0.7) then
         wavel=1.56*period*period
         wavekh=6.2832*depth/wavel
          else
         if (dol0 .ge. 0.01) then
            cgm=6.2832/period
            y=cgm*cgm*depth/grav
            sum=1.0 + 0.6666666666*y + 0.355555555*y*y +
      *          0.1608465608*y*y*y + 0.0632098765*y*y*y*y +
      *          0.0217540484*y*y*y*y*y + 0.00654078*y*y*y*y*y*y
            wavekh=sqrt(y*y + y/sum)
           else
c
c  long wave
c
            c=sqrt(grav*depth)
            wavel=c*period
            wavekh=6.2832*depth/wavel
           end if
          end if
c
        wn=wavekh/depth
        return
        end
c
c
        subroutine e012(x,e0,e1,e2)
c
c Massel's subroutine for calculating the three coefficients
c e0 is for bottom slope term.
c e1 is for slope square term.
c e2 is for curvature term.
c x = kh, is the only input parameter.
```

```
c
       implicit real*8 (a-h, o-y)
       sh=sinh(x)
       ch=cosh(x)
       th=tanh(x)
       dum=th + x*(1.0 - th*th)
       e0=x/dum*(1.0 - 3.0*th*th + 2.0*th/dum)
c
       sh2=sinh(2.0*x)
       sh3=sinh(3.0*x)
       ch3=cosh(3.0*x)
       sh5=sinh(5.0*x)
       ch5=cosh(5.0*x)
       p=0.5*(1.0+2.0*x/sh2)
       t1w=1.0/(24.0*ch**3*(2.0*x+sh2)**2)* (x*(12.0+16.0*x**2)*ch +
      *    6.0*x*(ch3+ch5) + sh*(12.0+84.0*x**2) +
      *    3.0*(1.0-4.0*x**2)*sh3 -9.0*sh5)
       t2=0.125*(-4.0*x*ch + sh +8.0*x**2*sh + sh3)/ch**3/(2.0*x+sh2) -
      *    x/2.0*th/ch**2
       e1=t1w/p/th
       e2=t2/p
       return
       end
c
```

Appendix III. FORTRAN program to generate the input data file for plotting. The input file for this program is the output file generated by RDE.FOR

```
      program outform
c
c This program is the first post-process program for the program
c RDE.FOR.  It converts the one-dimensional output to a two
c dimensional format for a faster graphic using RDPLT.M
c
c In this program, the conversion of x and y coodnated are NOT
c non-dimensionalized.  This is for the convenience of comparison.
c
c  iq,jq : parameters specified for reserving space for 2-D arrays.
c          They should be at least the same as  mp and np.
c  mp,np : number of grid points in x and y direction, respectively.
c          In applications, the x-axis should be parallel to the
c          longer dimension of the water body.  The y axis should
c          parallel to the short dimension of the water body.  This
c          allignment will generate a smaller banded width.
c  dx,dy : grid sizes in x and y direction, respectively.  In this
c          program, only the equally spaced grid is allowed.  But
c          dx do not have to equal to dy.
c  tide : water elev. that added to the bathymetric depth for
c          storm tide.
c  dmin : min. water depth at a point that this point will be
c          considered as water.
c  id   : a 2-D array to store the identification for each grid
c         point. 0 for water cell; 1 for land cell.
c
      parameter (iq=330, jq=263)
c
      implicit  real*8 (a-h,o-z)
      character*70 title,label
      character*5 name
      character*12 infile
      character*20 outfile
      character*1 idc
      character*2 chrc
      integer*1 id(iq,jq)
c
      dimension wh(iq,jq), s(iq,jq), dep(iq,jq), dir(iq,jq)
c
      pi=3.1415926
      twopi=2.0*pi
      grav=9.8
c
c   Input bathemetry and incident wave condition
c
      print*,' Key in the input filename <XXXXX.lis as XXXXX >: '
      read(*,'(a)') name
c
```

```
      infile=name // '.lis'
      print*,'Input file name : ', infile
      open(1, file=infile, form='formatted',status='old')
c
      icase=0
 25   continue
      icase=icase+1
c
      read(1,'(a70)', end=1000) title
      print*, title
c
c  mp,np : Max. grid numbers in x and y direction, respectively.
c  dx,dy : grid sizes in x and y direction, respectively.
c  tide  : water elev. that added to the bathymetric depth,
c          e.g. storm tide
c  dmin  : min. water depth that will be considered as water
c
      read(1,*) mp,np,dx,dy
      if(mp .gt. iq) then
       print*,'IQ is smaller than MP, change IQ to ',mp
       stop
       end if
      if(np .gt. jq) then
       print*,'JQ is smaller than NP, change JQ to ',np
       stop
       end if
c
c  clear the matrices
c
      print*,'Clean the matrices...'
      do i=1,mp
       do j=1,np
          id(i,j)=1
          dep(i,j)=0.0
          wh(i,j)=0.0
          s(i,j)=0.0
          dir(i,j)=0.0
          end do
       end do
c
c  yorient: The azimuth of y-axis related to the True North
c  xl, yl : The x and y coordinates of the origin of the grid
c
      read(1,*) yorient, xl, yl
c
      read(1,*) waveH, period, angle0, tide, map
      write(*,30) waveH, period, angle0, tide
 30   format(' wave height(m)=',f8.3/'     Period (s)=',f8.2/
     *       '     Angle(deg)=',f8.1/'     Tide  (m) =',f8.2)
c
      read(1,35) label
      write(*,35) label
 35   format(a50)
      ph_g=grav*period*waveh/(4.0*pi)
c
      do n=1,map
       read(1,65) i,j,idc,depth,phr,phi
 65      format(2x,i5,1x,i5,a3,f10.4,2d18.10)
       dep(i,j)=depth
       h=sqrt(phr*phr + phi*phi)
       wh(i,j)=h/ph_g
       s(i,j)=datan2(phi, phr)
       if( s(i,j) .lt. 0.0) s(i,j)=s(i,j) + twopi
       if(idc .ne. 'e') id(i,j)=0
       if(i .eq. mp .and. j .eq. np) go to 70
       end do
c
 70   mb=1
      me=mp
      outfile=name // '_' // chrc(icase) // '.out'
      print*,'outfile name = ', outfile
      open(12,file=outfile,form='formatted',status='unknown')
      write(12,'(a60)') title
      write(12,80) mp,np, dx,dy, dmin, yorient,xl,yl
 80   format(2i5,2f10.5,2f10.3/f10.2,2f10.5)
      write(12,90) waveh,period,angle0, tide
 90   format(4f10.3)
c
c  write the ID code first
c
      write(12,125)
 125  format(' I.D. code for each grid point'/' +-------------> i')
      do j=1,np
       jj=np-j+1
       write(12,130) jj, (id(i,jj),i=mb,me)
 130     format(i5/(40i2))
       end do
c
c  write the wave height
c
      write(12,135)
 135  format(' Normalized wave height at each grid point'/
     *       ' +------------> i')
      do j=1,np
       jj=np-j+1
       write(12,140) jj, (wh(i,jj),i=mb,me)
 140     format(i5/(10f8.5))
       end do
c
      write(12,145)
 145  format(' Wave phase for each grid point'/' +-------------> i')
      do j=1,np
       jj=np-j+1
       write(12,150) jj, (s(i,jj),i=mb,me)
 150     format(i5/(10f8.5))
        end do
c
       pi=3.1415926
       twopi=2.0*pi
c
c Calculate wave directions. Notice that the phase value only changes
c from 0 to 2*pi. When it is more than 2*pi, it drop back to a little c
more than 0. This cause problem for calculate wave direction
c because it is not a continuous function (it is, but computer
c can not understand it. So we set up a criterion: when the change of
c phase is more than 4.0, it means the two grid points just across a
c drop. We simply change the small phase S to S+2*PI and continue the
c calculation.
c
       do i=2, mp-1
        do j=2, np-1
           dss= s(i,j+1) - s(i,j-1)
           if( dss .lt. -4.0) dss=dss+twopi
           if( dss .gt.  4.0) dss=dss-twopi
           dsdy=dss/(2*dy)
           dss= s(i+1,j) - s(i-1,j)
           if( dss .lt. -4.0) dss=dss+twopi
           if( dss .gt.  4.0) dss=dss-twopi
           dsdx=dss/(2*dx)
           dir(i,j)=atan2(dsdy, dsdx)
           end do
        end do
c
       do i=2, mp-1
        dss= s(i,np) - s(i,np-1)
        if( dss .lt. -4.0) dss=dss+twopi
        if( dss .gt.  4.0) dss=dss-twopi
        dsdy=dss/dy
        dss= s(i+1,np) - s(i-1,np)
        if( dss .lt. -4.0) dss=dss+twopi
        if( dss .gt.  4.0) dss=dss-twopi
        dsdx=dss/(2.0*dx)
        dir(i,np)=atan2(dsdy, dsdx)
        end do
c
       do i=2, mp-1
        dss= s(i,2) - s(i,1)
        if( dss .lt. -4.0) dss=dss+twopi
        if( dss .gt.  4.0) dss=dss-twopi
        dsdy=dss/dy
        dss= s(i+1,1) - s(i-1,1)
        if( dss .lt. -4.0) dss=dss+twopi
        if( dss .gt.  4.0) dss=dss-twopi
        dsdx=dss/(2.0*dx)
        dir(i,1)=atan2(dsdy, dsdx)
        end do
```

```
c
      do j=2, np-1
       dss= s(1,j+1) - s(1,j-1)
       if( dss .lt. -4.0) dss=dss+twopi
       if( dss .gt.  4.0) dss=dss-twopi
       dsdy=dss/(2.0*dy)
       dss= s(2,j) - s(1,j)
       if( dss .lt. -4.0) dss=dss+twopi
       if( dss .gt.  4.0) dss=dss-twopi
       dsdx=dss/dx
       dir(1,j)=atan2(dsdy, dsdx)
       end do
c
      do j=2, np-1
       dss= s(mp,j+1) - s(mp,j-1)
       if( dss .lt. -4.0) dss=dss+twopi
       if( dss .gt.  4.0) dss=dss-twopi
       dsdy=dss/(2.0*dy)
       dss= s(mp,j) - s(mp-1,j)
       if( dss .lt. -4.0) dss=dss+twopi
       if( dss .gt.  4.0) dss=dss-twopi
       dsdx=dss/dx
       dir(mp,j)=atan2(dsdy, dsdx)
       end do
c
      write(12,155)
 155  format(' Wave direction at each grid point'/' +------------> i')
      do j=1,np
       jj=np-j+1
       write(12,160) jj, (dir(i,jj),i=mb,me)
 160      format(i5/(10f8.4))
       end do
c
      write(12,195)
 195  format(' Water depth for each grid point'/' +------------> i')
      do j=1,np
       jj=np-j+1
       write(12,200) jj, (dep(i,jj),i=mb,me)
 200      format(i5/(10f8.3))
       end do

      close(12)
c
      go to 25
1000  continue
      stop
      end
c
c
      character*2 function chrc(nt)
c
c It changes an input integer number NT to character
```

```
c
      character*2 c1
      character*2 c2
c
      i1=nt
      i2=i1/10
      if(i2 .gt. 0) then
       i3=i2/10
       if(i3 .gt. 0) then
         print*,'-----------------------------------------'
         print*,'The given integer is > 99, not allowed.'
         print*,'-----------------------------------------'
         chrc='-1'
        else
         c2=char(48+i2)//char(48+i1-i2*10)
         chrc=c2
c            write(11,*) c2
        end if
      else
        c1=char(48+i1)
        chrc='0' // c1
c          write(11,*) c1
      end if
c
      return
      end
```

Appendix IV. Matlab program (RDEPLOT.M) to generate graphics as the final output of the program RDE.FOR The input data files are from the program OUTFORM.FOR.

```
% Program RDEPLOT.M
% MathLab M file for plotting the outputs of RDE.FOR
% It uses MatLab ver. 4.2C.1 for WINDOWS
%
% Program developed by Jerome P.-Y. Maa at
%       Virginia Institute of Marine Science, Feb. 1996
%
clc;
%
% The directory (c:\break\*.out) may change for different application
%
[name,path]=uigetfile('c:\break\*.out','Available Data files');
file=[path,name];
[fid, message]=fopen(file);
if fid == -1
   message
   end
%
% read contour lines information
%
[name,path]=uigetfile('c:\break\*.lvl','Available contour level
files');
file_lvl=[path,name];
[fid_lvl, message]=fopen(file_lvl);
if fid_lvl == -1
   message
   end
ftitle=fgetl(fid_lvl);
disp(ftitle);
nh=fscanf(fid_lvl, '%d', 1);
v_h=fscanf(fid_lvl, '%f', nh);
ns=fscanf(fid_lvl, '%d', 1);
v_s=fscanf(fid_lvl, '%f', ns);
ndep=fscanf(fid_lvl, '%d', 1);
v_dep=fscanf(fid_lvl, '%f', ndep);
fclose(fid_lvl);
%
% read data
%
   ftitle=fgetl(fid);
   disp(ftitle);
   mp=fscanf(fid, '%d', 1);
   np=fscanf(fid, '%d', 1);
   dx=fscanf(fid, '%f', 1);
   dy=fscanf(fid, '%f', 1);
   dmin=fscanf(fid, '%f', 1);
   dlevel=fscanf(fid, '%f', 1);
   disp(['mp= ',int2str(mp), '  np= ',int2str(np)] );
   disp(['dx= ',num2str(dx), '  dy= ',num2str(dy)] );
   xl=fscanf(fid, '%f', 1);
   yl=fscanf(fid, '%f', 1);
   yorient=fscanf(fid, '%f', 1);
   disp(['Yorient= ', num2str(yorient)] );
%
   waveh=fscanf(fid, '%f', 1);
   period=fscanf(fid, '%f', 1);
   angle0=fscanf(fid, '%f', 1);
%
   disp(fgetl(fid) );
   disp(fgetl(fid) );
   fgetl(fid);
%
% read ID codes
%
   id=[];
   for j=1:np
      jj=np-j+1;
      jnk=fscanf(fid, '%d', 1);
      if jnk ~= jj
      disp(['ID Seq. error, j=',int2str(jnk),'<> jj=',int2str(jj)] );
         disp('paused');
         pause;
         end
      a=fscanf(fid, '%d', mp);
      id(jj, 1:mp)=a';
      end
   clear a;
%
% read normalized wave heights
%
   disp(fgetl(fid) );
   disp(fgetl(fid) );
   fgetl(fid);
   waveh=[];
   for j=1:np
      jj=np-j+1;
      nk=fscanf(fid, '%d', 1);
      if nk ~= jj
      disp(['WaveH Seq. error, j=',int2str(nk),'<>jj=',int2str(jj)]);
         disp('paused');
         pause;
         end
      a=fscanf(fid,'%f', mp);
      waveh(jj, 1:mp)=a';
      end
%
% read phases
%
   disp(fgetl(fid) );
   disp(fgetl(fid) );
   fgetl(fid);
   s=[];
   for j=1:np
      jj=np-j+1;
      nk=fscanf(fid, '%d', 1);
      if nk ~= jj
      disp(['Phase Seq. error, j=',int2str(nk),'<>jj=',int2str(jj)]);
         disp('paused');
         pause;
         end
      a=fscanf(fid,'%f', mp);
      s(jj, 1:mp)=a';
      end
%
% read wave direction
%
   disp(fgetl(fid) );
   disp(fgetl(fid) );;
   fgetl(fid);
   dir=[];
   for j=1:np
      jj=np-j+1;
      nk=fscanf(fid, '%d', 1);
      if nk ~= jj
      disp(['Dir Seq. error, j=',int2str(nk),'<>jj=',int2str(jj)] );
         disp('paused');
         pause;
         end
      a=fscanf(fid,'%f', mp);
%
%  Because x axis is vertical and y axis is horizontal when plotting
%  The angle is calculated to reflect this
%
      dir(jj, 1:mp)=a';
      end
%
% read water depth
%
   disp(fgetl(fid) );
   disp(fgetl(fid) );;
   fgetl(fid);
   d=[];
   for j=1:np
      jj=np-j+1;
      nk=fscanf(fid, '%d', 1);
      if nk ~= jj
      disp(['Depth Seq. error, j=',int2str(nk),'<>jj=',int2str(jj)]);
         disp('paused');
         pause;
         end
      a=fscanf(fid,'%f', mp);
      d(jj, 1:mp)=a';
      end
   clear a;
   fclose(fid);
%
%  find the coordinates of x & y
%
   xmax=(mp-1)*dx;
   xmin=0;
   ymin=0;
   ymax=(np-1)*dy;
%
   axis_ratio=(xmax-xmin)/(ymax-ymin);
%
   clc;
   dep_sel=0.5;
   disp('After plotted, you may drag a area for zoom in');
   disp(' or use left mouse bottom to zoom in 2 times');
   disp('        right mouse bottom to zoom out');
   disp('The following options are available :');
   disp('   1. select plot area ');
   disp('   2. plot wave height contours');
   disp('   3. plot wave vectors');
   disp('   4. plot water depth contours');
   disp('   5. plot wave phases');
   disp('   6. plot a profile parallel to X axis');
   disp('   7. plot a profile parallel to Y axis');
   disp('  -1. exit');
   iop=input('please select : ');
   if size(iop) == 0 iop=0;  end
%
   while (iop+1)
%
      if iop == 1
         disp(['X axis ranged from ',num2str(xmin), ' to ',
          num2str(xmax),' km']);
         xs1=input('Select the min. of X plotting domain : ');
         xs2=input('Select the max. of X plotting domain : ');
%  select Y dmain for plotting
         disp(['Y axis ranges from ', num2str(ymin),' to ',
          num2str(ymax),' km']);
         ys1=input('Select the min. of Y plotting domain : ');
         ys2=input('Select the max. of Y plotting domain : ');
         xc=xs1:dx:xs2;
         yc=ys1:dy:ys2;
         end
%
% Plot the normalized wave height contours
%
      if iop == 2
         figure(1);
         clf;
         zoom off;
         axis equal;
         set(gca,'linewidth',1);
         [cl,hhh]=contour(xc, yc, waveh, v_h);
         set(hhh, 'linewidth',2);
         clabel(cl, 'manual');
         xlabel('x'); ylabel('y');
         title('Wave Height Contours', 'fontsize',18);
         ans1=input('To: 0.hpgl file;  1.BMP file;  2.gif file : ');
         if ans1 == 0   print figure1 -dhpgl;  end
         if ans1 == 1   print figure1 -dbmp256;  end
         if ans1 == 2   print figure1 -dgif8;  end
         zoom on;
         end
%
%  plots the vectors
%
      if iop == 3
% select X domain for plotting
         ans2=input('Want depth contour plots underlay the phase plot
           <y/n> ? ', 's');
         format short;
         clc;
         nx=input('to skip how many vectors in X domain <3-6>: ');
         dxn=nx*dx;
         is1=xs1/dx+1;
         is2=xs2/dx+1;
         disp(['Selected area from i = ',int2str(is1),' to ',
           int2str(is2) ]);
%  select Y dmain for plotting
         ny=input('to skip how many vectors in y domain <3-6>: ');
         dyn=dy*ny;
         js1=ys1/dy+1;
         js2=ys2/dy+1;
         disp(['Selected area from j = ',int2str(js1),' to ',
           int2str(js2) ]);
         xc=xs1:dxn:xs2;
         yc=ys1:dyn:ys2;
% find the sub_matrix
         figure(2);
         clf;
         zoom off;
         waveh_s=[];
         dir_s=[];
         dep_s=[];
         axis equal;
         mv_a=size(xc');
         nv_a=size(yc');
         mv=mv_a(1);
         nv=nv_a(1);
         for i=1: mv
            iold=is1+(i-1)*nx;
            for j=1:nv
```

```
                jold=js1+(j-1)*ny;
                waveh_s(j,i)=waveh(jold, iold);
                dir_s(j,i)=dir(jold, iold);
                dep_s(j,i)=d(jold, iold);
                end
             end
          u=waveh_s.*cos(dir_s);
          v=waveh_s.*sin(dir_s);
          quiver(xc, yc, u, v, 2);
          set(gca,'aspectratio',[axis_ratio,1]);
          set(gca,'xlim',[xs1,xs2], 'ylim',[ys1,ys2]);
          xlabel('X (m)'); ylabel('Y (m)');
          title('Wave Vectors');
          if ans2 =='y'
             hold on;
             cl=contour(xc, yc, dep_s, v_dep);
             end
          ans2_2=input('To: 0.hpgl file;  1.BMP file;  2.gif file :');
          if ans2_2 == 0   print figure2 -dhpgl;  end
          if ans2_2 == 1   print figure2 -dbmp256;  end
          if ans2_2 == 2   print figure2 -dgif8;  end
          zoom on;
          end
%
%  plot the depth contours
%
       if iop == 4
          figure(3);    clf;
          zoom off;     grid off;
          xc=xmin:dx:xmax;
          yc=ymin:dy:ymax;
          axis equal;
          [cl,hhh]=contour(xc, yc, d, v_dep);
          set(hhh, 'linewidth',3)
          set(gca,'aspectratio',[axis_ratio,1]);
          set(gca,'xlim',[xmin,xmax],'ylim',[ymin,ymax]);
          clabel(cl, 'manual');
          xlabel('X (m)'); ylabel('Y (m)');
          title('Water Depth Contours');
% check if need other type of output
          ans3=input('To:  0.return;  1.gif file;  2.BMP file :  ');
          if ans3 == 1   print figure3 -dgif8;    end
          if ans3 == 2   print figure3 -dbmp256;  end
          zoom on;
          end
%
%  plot the phase contours
%
       if iop == 5
          ans4=input('Want depth contour plots underlay the phase plot
            <y/n> ? ', 's');
          figure(4);
```

```
         clf;
         zoom off;
         axis equal;
         xc=xmin:dx:xmax;
         yc=ymin:dy:ymax;
         cl=contour(xc, yc, s, v_s);
         set(gca,'aspectratio',[axis_ratio,1]);
         set(gca,'xlim',[xmin,xmax], 'ylim',[ymin,ymax]);
         clabel(cl, 'manual');
         xlabel('X (m)'); ylabel('Y (m)');
         title('Wave Phase Contours');
         if ans4 =='y'
            hold on;
            cl=contour(xc, yc, d, v_dep);
            end
         ans4_2=input('To: 0.return;  1.BMP;  2.gif; 3.HPGL file: ');
         if ans4_2 == 1   print figure4 -dbmp256;  end
         if ans4_2 == 2   print figure4 -dgif8;  end
         if ans4_2 == 3   print figure4 -dhpgl;  end
         zoom on;
         end
%
%  plot wave height profile parallel to x axis
%
      if iop == 6
         ks5=input(['Select a y grid number <0-',int2str(np),'>: ']);
         for i=1:mp
            yas(i)=waveh(ks5,i);
            das(i)=dep_sel - d(ks5,i);
            end
         ans51=input('Experimental data file name : ', 's');
         namei=['\break\berk\', ans51, '.txt'];
         nameo=['\break\', ans51,'.dat'];
         disp(['Data coming from ', namei]);
         [fidi, message]=fopen(namei, 'r');
            if fidi == -1
               message
               end
         ftitle=fgetl(fidi);
         disp(ftitle);
         ndata=fscanf(fidi, '%d', 1);
         ftitle=fgetl(fidi);
         ftitle=fgetl(fidi);
         disp(ftitle);
         xd1=[]; yd=[];

         for j=1:ndata
            a=fscanf(fid,'%f', 2);
            xd1(j)=a(1);
            yd(j)=a(2);
            end
         fclose(fidi);
         xd=25.5-xd1
         xw(1)=xc(3); xw(2)=xc(mp-3);  yw(1)=dep_sel;  yw(2)=dep_sel;
%
         figure(5)
         clf;
         zoom off;
         plot(xc, yas, 'r-', 'linewidth',3);
         hold on;
         plot(xc,das,'k-','linewidth',2);
         hold on;
         plot(xw,yw, 'g-', 'linewidth', 2);
         hold on;
         plot(xd,yd, 'ro', 'linewidth',3);
         set(gca, 'xlim', [4, 15]);
         set(gca, 'ylim', [0, 2.5]);
         ans52=input('To:  0.HPGL file; 1.BMP file;  2.gif file : ');
         if ans52 == 0   print figure5 -dhpgl;  end
         if ans52 == 1   print figure5 -dbmp256;  end
         if ans52 == 2   print figure5 -dgif8;  end
         zoom on;
         ans52=input('Save in an ASCII file XPROFILE <y/n> : ', 's');
         if ans52 == 'y'
            [fido, message]=fopen(nameo,'wt');
            if fid == -1
               message
               end
            for i=1:mp
                fprintf(fido,'%10.4f %10.4f %10.4f\n',xc(i), das(i),
                 yas(i) );
               end
            fclose(fido);
            disp(['data is in file ', nameo]);
            end
         end
%
%  plot wave height profile paralel to y axis
%
      if iop == 7
         ks6=input(['Select a x grid number <0-',int2str(mp),'>:']);
         for j=1:np
            yas2(j)=waveh(j,ks6);
            das2(j)=dep_sel - d(j,ks6);
            end
% save data to a disk file
         ans51=input('Experimental data file name : ', 's');
         nameo=['\break\', ans51,'.dat'];
%
         namei=['\break\berk\', ans51, '.txt'];
         disp(['Data coming from ', namei]);
         [fidi, message]=fopen(namei, 'r');
         if fidi == -1
               message
               end
         ftitle=fgetl(fidi);
         disp(ftitle);
         ndata=fscanf(fidi, '%d', 1);
         ftitle=fgetl(fidi);
         ftitle=fgetl(fidi);
         disp(ftitle);
         xd1=[]; yd=[];

         for j=1:ndata
               a=fscanf(fid,'%f', 2);
               xd1(j)=a(1);
               yd(j)=a(2);
               end
         fclose(fidi);
         xd=25.5-xd1
%
         ans61=input('Save in an ASCII file YPROFILE <y/n> : ', 's');
         if ans61 == 'y'
            outdata=[yc; das2; yas2];
            [fido, message]=fopen(nameo,'wt');
            if fido == -1
               message
               end
            fprintf(fido,'%10.4f %10.4f %10.4f\n',outdata);
            fclose(fido);
            disp(['Data is in file : ', nameo]);
            end;
%
         figure(6);
         clf;
         zoom off;
         axis equal;
         plot(yc, yas2, 'r-', 'linewidth',3);
         xw(1)=yc(2); xw(2)=yc(np-2);  yw(1)=dep_sel;  yw(2)=dep_sel;
         hold on;
         plot(xw,yw, 'g-', 'linewidth', 2);
         xlabel('Y (m)', 'fontsize', 15);
         ylabel('Wave Height (cm)', 'fontsize', 15);
         hold on;
         plot(yc,das2,'k-','linewidth',2);
         ans62=input('To: 0.return; 1.BMP file; 2.gif file : ');
         if ans62 == 1   print figure6 -dbmp256;  end
         if ans62 == 2   print figure6 -dgif8;  end
         zoom on;
         end
%
%  plot wave trajectories
%
      if iop == 8
         q81=input('Want Depth contours overlap ? <y/n> : ', 's');
         q82=input('Want plot every trajectories? <y/n> : ', 's');
```

```
         if q82 == 'n'
            jsk=input(' Skips how many grid points < 2, 3, 4>: ');
            else
            jsk=1;
            end
         ntraj=fix((np-5)/jsk);
         figure(8);
         clf;
         axis equal;
         zoom off;
         set(gca,'aspectratio',[axis_ratio,1]);
         set(gca,'xlim',[xmin,xmax], 'ylim',[ymin,ymax]);
         xlabel('X (m)'); ylabel('Y (m)');
         title('Wave Trajectory Plot');
         hold on;
% compute locations of every point in a trajectory
         for j=1: ntraj
            xt=[];
            yt=[];
            k=1;
            kp=(j-1)*jsk + 10;
            if kp < (np-10)
               x1=xmax;
               y1=yc(kp);
               xt(k)=x1;
               yt(k)=y1;
               disp(['Traj. line #',int2str(j),', j=',int2str(kp)] );
               angle=dir(kp,mp);
               for i=1:mp-1
                  ii=mp-i;
                  k=k+1;
                  x2=x1 - dx;
                  y2=y1 + dx*sin(angle );
% find where the intersect point located in next x-grid level
                  for L=2:np-1
                    if yc(L) <= y2
                       Lup=L+1;
                       Ldown=L;
                       end
                    end
%  use linear interpretation to find wave angle and water depth
                  dir1=dir(Ldown,ii);
                  dir2=dir(Lup,ii);
                  d1=y2 - yc(Ldown);
                  slope=abs(dir2- dir1)/dy;
                  if dir2 > dir1
                      angle= dir1 + slope*d1;
                     else
                      angle= dir2 + slope*(dy - d1);
                     end
%
                  dep1=d(Ldown, ii);
                  dep2=d(Lup,ii);
                  slope=abs(dep2- dep1)/dy;
                  if dep2 > dep1
                      dep = dep1 + slope*d1;
                     else
                      dep = dep2 + slope*(dy - d1);
                     end
%
                  if dep >= 0.0
                     xt(k)=x2;
                     yt(k)=y2;
                     x1=x2;
                     y1=y2;
                     end
                  end
%  completed one trajectory line
               plot(xt,yt,'-');
               hold on;
               end
%  move to the start point of next trajectory line
            end
%
         hold on;
         if q81 == 'y'
            contour(xc, yc, d, v_dep, 'k');
            end
% check if need other type of output
         ans83=input('To:  0.return;  1.gif file;  2.HPGL file : ');
         if ans83 == 1
            print figure8 -dbmp256;
            end
         if ans8_3 == 2
            print figure8 -dhpgl;
            end
         zoom on;
         end
%
%
      clc;
      disp('The following options are available :');
      disp('   1. Select plot area');
      disp('   2. plot wave height contours');
      disp('   3. plot wave vectors');
      disp('   4. plot water depth contours');
      disp('   5. plot wave phases');
      disp('   6. plot a profile parallel to X axis');
      disp('   7. plot a profile parallel to Y axis');
      disp('   8. trajectory plot ');
      disp('  -1. exit');
      iop=input('please select : ');
      if size(iop) == 0 iop=0;  end
%
```

```
        end
%
end
```

Appendix V. Grid Generation and Control Parameter File Generation Program. The output files are used to run the program RDE.FOR This is an example to show how to generate data file. For a more complicated cases, a comprehensive program is needed.

```
      program bathymetric_matrix_6
c
c  This program generates the input water depth matrix, I.D. codes
c  for each cell and boundary conditions that are needed for the wave
c  refraction and diffraction program RDE.FOR
c
c  the classic rectangular harbor, Chen and Mei's configuration
c
      parameter (iq=400,jq=200, kq=500, Lq=2000)
      character*1 id
      character*13 name
      character*17 grdfile, bcdfile, confile,lisfile, chkfile
      character*30 header
      character*80 title
      complex zalp
c
      common/const/ mp, np, dx, dy, period, waveh, angle, r, dep_c
      common/array/ depth(iq,jq), id(iq,jq)
      common/bound/ nobc, ibc(Lq), jbc(Lq), zalp(Lq)
      common/given/ nog, ig(kq), jg(kq)
c
c  mp     : mesh number in x direction
c  np     : mesh number in y direction
c  period : input wave period
c  waveh  : input wave height
c  angle  : input wave angle
c  wlength: input wave length
c
c   dx    : spatial grid size in x-direction
c   dy    : spatial grid size in y-direction
c   r     : dx/dy
c
c  ID_code for water depth
c       : 0, water interior point
c
c       : 1, upper boundary condition point
c       : 2, lower boundary condition point
c       : 3, left boundary condition point
c       : 4, right boundary condition point
c
c       : 5, left bottom corner B.C. point
c       : 6, left top corner B.C. point
c       : 7, right bottom corner B.C. point
c       : 8, right top corner B.C. point
c
c       : e, land point
```

```
c      : g, grid point that has a given wave condition
c
c  harbor length h=0.3212m, width=0.0605m,
c
  5   print*,'Select 1. mei121;  2. mei122;  3. mei141;  4. Mei160: '
      read(*,*) iption
      go to (10, 12, 14, 16) iption
 10   name='\break\Mei121'
      title=' Classic rectanglar harbor, Chen and Mei, MP=121'
      mp=121
      m1=20
      np=62
      n1=21
      n2=41
      go to 20
 12   name='\break\mei122'
      title=' Classic rectanglar harbor, Chen and Mei, Mp=122'
      mp=122
      m1=20
      np=62
      n1=21
      n2=41
      go to 20
 14   name='\break\mei141'
      title=' Classic rectanglar harbor, Chen and Mei, Mp=141'
      mp=141
      m1=40
      np=62
      n1=21
      n2=41
      go to 20
 16   name='\break\mei161'
      title=' Classic rectanglar harbor, Chen and Mei, Mp=161'
      mp=161
      m1=60
      np=101
      n1=41
      n2=61
 20   dep_c=0.2573
      period=2.2
      waveh=0.01
      angle=0.0
      dx=0.003212
      dy=0.003025
      call Meil_bc(m1,n1,n2)
c
c  check array size
c
      if(mp .gt. iq .or. np .gt. jq) then
        write(*,70) mp,np, iq,jq
 70     format(' Given array (mp,np)=',2i5,' is larger than ',
     *          'allocated (iq,jq)=',2i5)
```

```
        stop
          end if
c
       grdfile=name // '.grd'
       open(8,file=grdfile,form='formatted', status='unknown')
       write(8,'(a80)') title
       write(8,100) mp, np, dx, dy
 100   format('   M     N     dx    dy      xL      yL      zmin  ',
      *       '   zmax   noz   yorient'/ 2i5, 2f10.6,'   0.000   0.000',
      *       '   0.10   0.10     0   0.0')
c
       header=' Water depth metrix (meter)    '
       write(8,'(a30)') header
       header=' +------------------------> j'
       write(8,'(a30)') header
       do i=1,mp
        write(8,120) i,(depth(i,j), j=1,np)
120        format(i5/(10f8.4))
        end do
       close(8)
c
       bcdfile=name // '.bcd'
       open(8,file=bcdfile,form='formatted', status='unknown')
       write(8,'(a80)') title
       write(8,100) mp, np, dx, dy
       header=' I.D. code for each grid point'
       write(8,'(a30)') header
       header=' +------------------------> j'
       iblock=np/75 + 1
       do ib=1,iblock
          write(8,'(a30)') header
          numb1=(ib-1)*75 + 1
        numb2=numb1+75-1
          if( numb2 .gt. np) numb2 = np
        do i=1,mp
           write(8,140) i,(id(i,j), j=numb1,numb2)
 140          format(i4,1x,75a1)
           end do
        end do
c
       header='Given B.C. at following points'
       phase=0.0
       write(8,'(i5,2x,a30)') nog, header
       write(8,150)
 150   format('   n    i     j     id    phase')
       do i=1,nog
        write(8,155) i, ig(i),jg(i), id(ig(i),jg(i)), phase
        end do
 155   format(1x,3i5, 4x,a1, f10.6)
c
       header='Radiation B. C. at points...  '
       write(8,'(i5,2x,a30)') nobc, header
       write(8,160)
 160  format('    n    i    j     id     r_alp     i_alp')
       do i=1,nobc
        write(8,170) i, ibc(i), jbc(i), id(ibc(i),jbc(i)), zalp(i)
        end do
 170     format(1x,3i5,4x,a1,2f8.2)
       close(8)
c
c open an control file for control parameters
c
       confile=name // '.con'
       lisfile=name // '.lis'
       chkfile=name // '.chk'
       icase=46
       open(8,file=confile,form='formatted')
       write(8,'(a80)') title
       write(8,190) grdfile, bcdfile, lisfile, chkfile
 190  format(a18/a18/a18/a18/'  0.0    0.0    0.0')
       write(8,200) mp, np, dx, dy, icase
 200  format(2i5, 2f10.6,i5, '  0.0')
       write(8,210)
 210  format(' 0.01    2.20    0.0   0.0'/' 0.010   1.90    0.0   0.0'/
     *       ' 0.010   1.70    0.0   0.0'/' 0.010   1.50    0.0   0.0'/
     *       ' 0.010   1.40    0.0   0.0'/' 0.010   1.30    0.0   0.0'/
     *       ' 0.010   1.25    0.0   0.0'/' 0.010   1.20    0.0   0.0'/
     *       ' 0.010   1.18    0.0   0.0'/' 0.010   1.16    0.0   0.0'/
     *       ' 0.010   1.15    0.0   0.0'/' 0.010   1.14    0.0   0.0'/
     *       ' 0.010   1.13    0.0   0.0'/' 0.010   1.12    0.0   0.0'/
     *       ' 0.010   1.11    0.0   0.0'/' 0.010   1.105   0.0   0.0'/
     *       ' 0.010   1.10    0.0   0.0'/' 0.010   1.095   0.0   0.0'/
     *       ' 0.010   1.09    0.0   0.0'/' 0.010   1.085   0.0   0.0'/
     *       ' 0.010   1.08    0.0   0.0'/' 0.010   1.06    0.0   0.0'/
     *       ' 0.010   1.04    0.0   0.0'/' 0.010   1.00    0.0   0.0'/
     *       ' 0.010   0.95    0.0   0.0'/' 0.010   0.90    0.0   0.0'/
     *       ' 0.010   0.80    0.0   0.0'/' 0.010   0.70    0.0   0.0'/
     *       ' 0.010   0.65    0.0   0.0'/' 0.010   0.62    0.0   0.0'/
     *       ' 0.010   0.61    0.0   0.0'/' 0.010   0.60    0.0   0.0'/
     *       ' 0.010   0.59    0.0   0.0'/' 0.010   0.588   0.0   0.0'/
     *       ' 0.010   0.58    0.0   0.0'/' 0.010   0.57    0.0   0.0'/
     *       ' 0.010   0.564   0.0   0.0'/' 0.010   0.562   0.0   0.0'/
     *       ' 0.010   0.56    0.0   0.0'/' 0.010   0.558   0.0   0.0'/
     *       ' 0.010   0.555   0.0   0.0'/' 0.010   0.553   0.0   0.0'/
     *       ' 0.010   0.55    0.0   0.0'/' 0.010   0.54    0.0   0.0'/
     *       ' 0.010   0.535   0.0   0.0'/' 0.010   0.53    0.0   0.0')
       close(8)
       stop
       end
c
c-----------------------------------------------------------------
       subroutine MeiL_BC(m1,n1,n2)
c
c  generates the water and ID depth matrix
```

```
c
      parameter (iq=400,jq=200, kq=500, Lq=2000)
      character*1 id
      complex zalp
      common/const/ mp, np, dx, dy, period, waveh, angle, r, dep_c
      common/array/ depth(iq,jq), id(iq,jq)
      common/bound/ nobc, ibc(Lq), jbc(Lq), zalp(Lq)
      common/given/ nog, ig(kq), jg(kq)
c
c first establish the ID code and water depth for the entire grid point
c
      do j=1,np
       do i=1,mp
       id(i,j)='0'
       depth(i,j)=dep_c
       end do
       end do
c
c  given boundary at the left hand side
c
      nog=0
      do j=2,np-1
       nog=nog+1
       ig(nog)=1
       jg(nog)=j
       id(ig(nog),jg(nog) )='g'
       end do
      nog=nog+1
      ig(nog)=1
      jg(nog)=1
      id(ig(nog),jg(nog) )='p'
      nog=nog+1
      ig(nog)=1
      jg(nog)=np
      id(ig(nog),jg(nog) )='q'

c
c  Check the Array size
c
      if(nog .gt. kq) then
       write(*,10) nog, kq, nog
10        format(' The # of point for given B.C.=',i6,' > Kq=',i6/
     *        ' Change the statement to Kq=',i6,' and recompile')
       stop
       end if

c
c  NOBC : number of boundary points
c
      nobc=0
c
c top side B.C.
```

```
c
      do i=2,m1-1
       nobc=nobc+1
       ibc(nobc)=i
       jbc(nobc)=np
       id(ibc(nobc), jbc(nobc) )='1'
       zalp(nobc)=1.0
       end do
      do i=m1+1,mp-1
       nobc=nobc+1
       ibc(nobc)=i
       jbc(nobc)=n2
       id(ibc(nobc), jbc(nobc))='1'
       zalp(nobc)=0.0
       end do
c
c bottom side B.C.
c
      do i=2,m1-1
       nobc=nobc+1
       ibc(nobc)=i
       jbc(nobc)=1
       id(ibc(nobc), jbc(nobc))='2'
       zalp(nobc)=1.0
       end do
      do i=m1+1,mp-1
       nobc=nobc+1
       ibc(nobc)=i
       jbc(nobc)=n1
       id(ibc(nobc), jbc(nobc) )='2'
       zalp(nobc)=0.0
       end do
c
c right side total reflection B.C.
c
      do j=n1+1,n2-1
       nobc=nobc+1
       ibc(nobc)=mp
       jbc(nobc)=j
       id(ibc(nobc), jbc(nobc) )='4'
       zalp(nobc)=0.0
       end do
      do j=2,n1-1
       nobc=nobc+1
       ibc(nobc)=m1
       jbc(nobc)=j
       id(ibc(nobc), jbc(nobc) )='4'
       zalp(nobc)=0.0
       end do
      do j=n2+1,np-1
       nobc=nobc+1
       ibc(nobc)=m1
       jbc(nobc)=j
       id(ibc(nobc), jbc(nobc) )='4'
       zalp(nobc)=0.0
       end do
c
c  land points
c
      do i=m1+1,mp
       do j=1,n1-1
          id(i,j)='e'
          end do
       do j=n2+1,np
          id(i,j)='e'
          end do
       end do
c
c right bottom corner b.c.
c
      nobc=nobc+1
      ibc(nobc)=m1
      jbc(nobc)=1
      id(ibc(nobc), jbc(nobc) )='7'
      zalp(nobc)=(0.0, 1.0)
      nobc=nobc+1
      ibc(nobc)=mp
      jbc(nobc)=n1
      id(ibc(nobc), jbc(nobc) )='7'
      zalp(nobc)=(0.0, 0.0)
c
c right top corner b.c.
c
      nobc=nobc+1
      ibc(nobc)=m1
      jbc(nobc)=np
      id(ibc(nobc), jbc(nobc) ) ='8'
      zalp(nobc)=(0.0, 1.0)
      nobc=nobc+1
      ibc(nobc)=mp
      jbc(nobc)=n2
      id(ibc(nobc), jbc(nobc) ) ='8'
      zalp(nobc)=(0.0, 0.0)
c
c check the array assignment
c
      if(nobc .gt. Lq) then
       write(*,20) nobc, Lq, nobc
20        format(' The # of point for Radiation B.C.=',i6,' > Lq=',i6/
     *           ' Change the statement to Lq=',i6,' and recompile ')
       stop
       end if
      return
      end
```