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MINKOWSKI PRODUCT OF CONVEX SETS AND PRODUCT NUMERICAL RANGE

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Abstract. Let K_1, K_2 be two compact convex sets in $\mathbf{C}$. Their Minkowski product is the set $K_1K_2 = \{ab : a \in K_1, b \in K_2\}$. We show that the set K_1K_2 is star-shaped if K_1 is a line segment or a circular disk. Examples for K_1 and K_2 are given so that K_1 and K_2 are triangles (including interior) and K_1K_2 is not star-shaped. This gives a negative answer to a conjecture by Puchala et. al concerning the product numerical range in the study of quantum information science. Additional results and open problems are presented.

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