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An Activity-Based Approach to Technology Integration in the Mathematics Classroom

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An Activity-Based Approach to Technology Integration in the Mathematics Classroom

Tim was so learned, that he could name a horse in nine languages. So ignorant, that he bought a cow to ride on.

Benjamin Franklin, 1914, p.54

Integrating Technology into Mathematics Instruction

Becoming a mathematics teacher today can be a challenging endeavor, requiring teachers to learn difficult content and specialized pedagogies, as well as new technological tools and techniques. Pre-service teachers at the secondary level are faced with programs of study that often begin with Calculus, and include abstract topics such as non-Euclidean geometry, discrete mathematics, and modern algebra (NCTM, 2000; U.S. Dept. of Education, 2005). With increased federal mandates in mathematics, today's elementary teachers are also faced with significant mathematics content, involving topics such as number sense, geometry, and probability (NCTM, 2006). At the same time, teachers of mathematics at all levels are encouraged to consider relatively sophisticated strategies for instruction, such as problem-based learning, constructivism, and scaffolding (NCTM, 1999; Fuson, Kalchman, & Bransford, 2005). Technologies such as graphing calculators, symbolic processing programs, mathematical simulations and cross-discipline instructional tools such as robotics kits are becoming ever more available to teachers to aid instruction (Heid, 2005). Such a broad variety of instructional responsibilities and opportunities makes mathematics educators worry about furthering the "mile wide and inch deep" instructional challenges that have plagued mathematics instruction in the past (Kaser, Bourexis, Loucks-Horsley, & Raizen, 1999; Adams, Brower, Hill & Marshall, 2000; NCTM 2005). As Mr. Franklin admonished, how do we help teachers not to become so learned in mathematical concepts, yet so ignorant in modern mathematics instruction, that they essentially "buy a cow to ride on" and thus poorly combine content, pedagogy, and technology to form classroom activities that are instructionally unsound?

One answer may lie in the ways that we consider how content, pedagogy and technology might combine to plan effective instruction in today's quickly evolving mathematics classroom. When recommending ways to teach mathematics, it is important to realize that the discipline is changing rapidly as mathematics educational technologies evolve. This, in turn, is further challenging mathematics teachers to keep up with new content and instructional strategies (Heid, 2005; Peterson, 1988; Sinclair & Crespo, 2006). The concept of fractals is a good example. Computational technologies have helped mathematicians to represent and apply fractional dimensions, investigating and modeling real-life phenomena such as lightening strikes, plant growth, cloud formation, coastline erosion, and blood circulation. Yet the integration of fractals concepts into textbooks and coursework is comparatively new to mathematics teachers, and can be difficult to understand or explain without the use of technology (Falconer, 2003). Technology use is similarly changing the mathematics of statistics, graphing, geometry, matrices, and

probability, to name just a few. Mathematics teachers are encouraged to use a wide range of educational technologies, such as graphing calculators, symbolic processing programs, spreadsheets and simulations, to help their students to learn about such technology-related topics (Heid, 2005; Rosen, 1999). Given the proliferation of new mathematics content, new instructional strategies, and new mathematics-based educational technologies, how can mathematics teachers make optimal choices to support students' learning?

One new framework, called technological pedagogical content knowledge or TPACK (Koehler, & Mishra, 2008) describes the interconnected and interdependent content. pedagogy, and technology knowledge that teachers must have to make and enact such choices. TPACK is built upon Shulman's (1986, 1987) notions of pedagogical content knowledge: the knowledge necessary to teach particular curriculum content, which incorporates both disciplinary and general pedagogical knowledge TPACK designates knowledge of educational technologies – especially how to use these rapidly proliferating tools and resources instructionally, and in varying educational contexts – as teachers' knowledge that requires deliberate examination and development. Mathematics teachers who have well-developed TPACK may demonstrate this knowledge through using an imaginative mix of mathematical content, pedagogies, and technologies within their lessons (Grandgenett, 2008). In some ways, mathematics teachers have a bit of a head start on TPACK development, since the profession's integration of instructional technology to date, when compared with other disciplines, has been relatively strong. For example, the use of graphing calculators in high school mathematics classes has been suggested by several authors as one of the best examples of the successful integration of technologies into teaching and learning (Fuson et. al, 2005; Kaser et. al, 1999; Reece, Dick, Dildine, Smith, Storaasli, Travers, Wotal, & Zygas, 2005). Technology-based applications like Geometer's Sketchpad and Excel, and Web-based resources like the National Library of Virtual Manipulatives are relatively common and well embraced in today's mathematics classroom (Heid, 2005).

However, current integrations of digital technologies, such as graphing calculators or Excel, only scratch the surface of the educational opportunities that these tools and resources make possible in mathematics instruction. Innovative software programs such as *Inspire Data* or the newly enhanced *Mathematica*, or new technologies such as robotics and global positioning systems (GPS) are providing exciting opportunities for the learning of mathematics, as well as creating challenges to both educators and teacher preparation programs. When considering the possibilities for effective technology use in the mathematics classroom, there appear to endless possibilities. Yet, if one considers the learning activities that a mathematics teacher might typically plan,, and the technologies that are generally seen to be useful within that context, a more limited list of possibilities would probably be generated. We suggest that providing teachers with a more comprehensive list of possible mathematics learning activities, along with the specific educational technologies that can best support each, will help both teachers and teacher educators to better integrate educational technologies into mathematics instruction. We believe that regular use of such a taxonomy could contribute significantly to both teachers' TPACK development, and their professional development overall..

We have attempted to generate and categorize this taxonomy of mathematics learning activities, as well as to identify some useful technologies that can support each activity. The taxonomy was generated through collaborative discussion, after a careful review of the technology-based activities represented in the last five years of articles published in the three practitioner journals published by the National Council of Teachers of Mathematics: The Mathematics Teacher, Mathematics Teaching in the Middle School, and Teaching Children Mathematics. In all, more than 180 journal issues were examined. A teacher planning a lesson for a particular mathematical topic or concept can review this taxonomy, selecting several learning activities to combine to structure a lesson, unit, or project plan. We have identified 31 distinct mathematics learning activity types, and acknowledge that some may need to be added, removed, edited, or combined. Considering the rapid evolutions of mathematics, mathematics education, and educational technologies, the list will no doubt also grow and evolve along with advancements in the discipline. Our taxonomy is presented as a beginning point for others to consider and to refine. (We have set up a wiki at http://activitytypes.wmwikis.net/ to facilitate this process.).

As mentioned above, we hope that by creating and sharing this taxonomy of mathematics learning activity types, we can assist the development of "TPACK in action" for mathematics teachers. Even at the most pragmatic level, the taxonomy should be useful to teachers as they select instructional strategies and tools to help students to meet particular curriculum content standards. The individual activity types represent possibilities for teacher instruction, but are conceptualized in terms of students' actions, focusing upon what a teacher might help students to *do* during mathematics instruction. For example, a teacher brainstorming ways to teach the concept of algebraic slope might consider the use of a strategy and a related technology that they might not have initially contemplated, such as "interpreting a phenomenon mathematically" by having students drive an electronic car up different sloped ramps, and then using an interactive graphing program to represent the changing equation of the ramp slope.

Mathematics Learning Activity Types

As mentioned, the purpose of presenting a learning activity types taxonomy for mathematics is to introduce the full range of possible student learning activities for teachers to consider when building lessons that effectively integrate technology, pedagogy, and content. In doing so, we attempt to scaffold teachers' thinking about how to best structure learning activities, best support those activities with educational technologies, and to spark creativity during instructional planning.

Essentially, these mathematics activity types are designed to be *catalysts* to thoughtful and creative instruction by teachers. We have conceptualized seven genres of activity types for mathematics that are derived from the NCTM's process standards. To encourage active engagement by all students, these activity types are expressed using active words to represent the pursuit of a dynamic and student-centered learning environment: Consider, Practice, Interpret, Produce, Apply, Evaluate, and Create. Most of these words are drawn directly from the NCTM standards. Each of the seven genres is presented in a

separate table below that names the activity types included in that genre, defines them briefly, and then provides some example technologies that could support students' learning in each activity well.

The "Consider" Activity Types

When learning mathematics, students are often asked to consider new concepts or information. This request is a familiar one to both students and teachers. Yet, although such learning activities can be very important contributors to student understanding, the "Consider" activity types also often produce some of the lowest levels of student engagement, and are manifested typically using a comparatively direct presentation of foundational knowledge.

Table 1: The "Consider" Activity Types

Activity Type	Brief Description	Example Technologies
	The student gains information from a teacher or	Powerpoint, YouTube, document
Attend to a Demonstration	student presentation, videoclip, animation,	camera, interactive whiteboard,
	interactive whiteboard or other display media.	videoconferencing, or other display media
	The student extracts information from textbooks,	Electronic textbooks, websites (i.e.
Read Text	or other written materials, in either print or digital	the Math Forum), informational
	form.	.pdfs
	The student discusses a concept or process with a	Ask-an-expert sites (e.g., Ask Dr.
Discuss	teacher, other students, or an external expert.	Math), online discussion groups,
		videoconferencing
	The student examines a pattern presented and	Graphing calculators, virtual
Recognize a Pattern	attempts to more fully understand the pattern.	manipulative sites (e.g., the National
Recognize a Fattern		Library of Virtual Manipulatives),
		spreadsheets
	The student explores or investigates a concept	Web searching, informational
Investigate a Concept	(such as fractals), perhaps by use of the Internet or	databases (Wikipedia), virtual
	other research-related resources.	worlds (Second Life), simulations
Understand or Define a	The student strives to understand the context of a	Web searching, concept mapping
Problem	stated problem or to define the mathematical	software, ill-structured problem
FIODICIII	characteristics of a problem.	media (i.e. Jasper Woodbury)

The "Practice" Activity Types

In the learning of mathematics, it is often important for students to be able to practice computational techniques or other algorithm-based strategies, so that these skills can be automated for later and higher-level mathematical application. Some educational technologies can be used to assist these processes. The table below offers both the range of practice-based learning activities and examples of how technology can assist in their implementation.

Table 2: The "Practice" Activity Types

	The student undertakes computation-based	Scientific calculators, graphing
Do Computation	strategies using numeric or symbolic processing.	calculators, spreadsheets,
		Mathematica
	The student rehearses a mathematical strategy or	Mathblaster drill and practice
Drill and Practice	technique, and perhaps uses computer-aided	software, online textbook
	repetition and feedback in the practice process.	supplements, online homework help
		websites (WebMath).
	The student carries out a mathematical strategy or	Virtual manipulatives, Web-based
Solve a Puzzle	technique within the context of solving an	puzzles (magic squares), brainteaser
	engaging puzzle, which may be facilitated or	Web sites (CoolMath)
	posed by the technology.	

The "Interpret" Activity Types

In the discipline of mathematics, concepts and relationships can be quite abstract, representing, at times, a bit of a mystery to students. They often need to spend time deducing and explaining these relationships to internalize them. Educational technologies can be used to help students investigate concepts and relationships more actively, and to assist with interpretion of what they observe. This table displays activity types that can support such interpretive processes, and provides examples of available technologies that can be used to support their formation.

Table 3: The "Interpret" Activity Types

Activity Type	Brief Description	Example Technologies
	The student poses a conjecture, perhaps using	Dynamic geometry software
Pose a Conjecture	dynamic software to display relationships.	(Geometer's Sketchpad), widgets
		(Explore Learning), e-mail
	The student develops a mathematical argument	Concept mapping software
Develop an Argument	related to why they think that something is true.	(Inspiration), presentation software,
Develop all Argument	Technology may help to form and to display that	blogs, specialized word processing
	argument (e.g. a proof).	software (Theorist), e-mail
	The student attempts to examine a concept or	Database software (Microsoft
Categorize	relationship in order to categorize it into a set of	Access), online databases, concept
	known categories.	mapping software, drawing software
	The student explains the relationships apparent	Data visualization software (Inspire
	from a mathematical representation (table,	Data), 2D and 3D animations, video
Interpret a Representation	formula, chart, diagram, graph, picture, model,	(iMovie), Global Positioning
	animation, etc.).	Devices (GPS), engineering
		visualization software (MathCad)
	The student attempts to approximate some	Scientific calculator, graphing
Estimate	mathematical value, by further examining	calculator, spreadsheets, student
	relationships using supportive technologies.	response systems (Clickers)
	Assisted by technology as needed, the student	Digital cameras, video, computer-
Interpret a Phenomenon Mathematically	examines a mathematics related phenomenon	aided laboratory equipment,
	(such as velocity, acceleration, the Golden Ratio,	interactive graphing program,
	gravity, etc.).	specialized word processing,
		robotics, electronics kits

When students are actively engaged in the study of mathematics, they can become motivated producers of mathematical works, rather than just passive consumers of prepared materials. Educational technologies can serve as excellent "partners" in this production process, aiding in the refinement and formalization of student products, as well as helping the student to share the fruits of their mathematical labors. The activity types listed below suggest technology-assisted efforts in which this occurs.

Table 4: The "Produce" Activity Types

Activity Type	Brief Description	Example Technologies
	The student demonstrates a topic or concept to	Interactive whiteboard, video
Do a Demonstration	show their understanding of a mathematical idea	(YouTube), document camera,
Do a Demonstration	or process. Technology may assist in the	presentation software, podcasts
	development or presentation of the product.	
	The student produces a report, annotation,	Specialized word processing (Math
Generate Text	explanation, journal entry or document, to	Type), collaborative documents
Generate Text	illustrate their understanding.	(Google docs), blogs, online
		discussion groups
	Technology may assist in the description or	Engineering visualization software,
Describe an Object or	documentation process, as the student produces a	concept mapping software,
Concept Mathematically	mathematical explanation of an object or concept.	specialized word processing,
		Mathematica
	The student develops a mathematical	Spreadsheet, virtual manipulatives
Produce a Representation	representation (table, formula, chart, diagram,	(digital geoboard), spreadsheets,
	graph, picture, model, animation, etc.) using	Inspire Data, concept mapping
	technology for production assistance, if necessary.	software, graphing calculator
	The student poses a mathematical problem that is	Word processing, online discussion
Develop a Problem	illustrative of some mathematical concept,	groups, Wikipedia, Web searching,
	relationship, or investigative question.	e-mail

The "Apply" Activity Types

The utility of mathematics in the physical world can be found in its authentic applications. Educational technologies can be used to help students to apply mathematics in the world, and to link mathematical concepts to real-world phenomena. The technologies essentially become students' assistants in their mathematical work, helping them to link mathematical concepts to the realities in which they live.

Table 5: The "Apply" Activity Types

Activity Type	Brief Description	Example Technologies
	The student reviews or selects a mathematics	Online help sites (WebMath, Math
	related strategy for a particular context or	Forum), Inspire Data, dynamic
Choose a Strategy	application.	geometry/algebra software
		(Geometry Expressions),
		Mathematica, MathCAD
	The student applies their mathematical knowledg	e Test-taking software, Blackboard,
Take a Test	within the context of a testing environment, such	survey software, student response
	as with computer-assisted testing software.	systems

	The student applies a mathematical representation	Spreadsheet, robotics, graphing
Apply a Representation	to a real life situation (table, formula, chart,	calculator, computer-aided
	diagram, graph, picture, model, animation, etc.).	laboratories, virtual manipulatives
		(algebra tiles)

The "Evaluate" Activity Types

When students evaluate the mathematical work of others, or self-evaluate their own, they utilize and develop a comparatively sophisticated understanding of mathematical concepts and processes. Educational technologies can become valuable allies in this effort, assisting students in the evaluation process by helping them to undertake concept comparisons, test solutions or conjectures, and/or integrate feedback from other individuals into revisions of their own work. The following table lists some of these evaluation-related mathematics learning activities.

Table 6: The "Evaluate" Activity Types

Activity Type	Brief Description	Example Technologies
Compare and Contrast	The student compares and contrasts different	Inspiration, Web searches,
	mathematical strategies or concepts, to see which	Mathematica, MathCad
	is more appropriate for a particular situation.	
	The student systematically tests a solution, and	Scientific calculator, graphing
Test a Solution	examines whether it makes sense based upon	calculator, spreadsheet,
rest a Solution	systematic feedback, which might be assisted by	Mathematica, Geometry
	technology.	Expressions
	The student poses a specific conjecture and then	Geometer Sketchpad, statistical
Test a Conjecture	examines the feedback of any interactive results	packages (e.g/, SPSS, Fathom),
	to potentially refine the conjecture.	online calculators, robotics
Evaluate Mathematical Work	The student evaluates a body of mathematical	Online discussion groups, blogs,
	work, through the use of peer or technology-	Mathematica, MathCad, Inspire
	aided feedback.	Data

The "Create" Activity Types

When students are involved in some of the highest levels of mathematics learning, they are often engaged in very creative and imaginative thinking processes. Albert Einstein once implied that imagination was more important than knowledge when it came to mathematics (Priwer & Phillips, 2003). It is said that this idea was consistent with his strong belief that mathematics is a very inventive, inspired, and imaginative endeavor. Educational technologies can be used to help students to be creative in their mathematical work, and to assist other students' mathematics learning. The activity types below represent these creative elements and processes in students' mathematical learning and interaction.

Table 7: The "Create" Activity Types

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Activity Type	Brief Description	Example Technologies

	The student develops and delivers a lesson on a	Presentation software, interactive
Teach a Lesson	particular mathematics concept, strategy, or	video, video, podcasts
	problem.	
	The student develops a systematic plan to address	Concept mapping software,
Create a Plan	some mathematical problem or task.	collaborative writing software,
		MathCad, Mathematica
	The student imaginatively engages in the	Word processor, animation tools,
Create a Product	development of a student project, invention, or	MathCad, Mathematica, Geometer
	artifact, such as a new fractal, tessellation, or	Sketchpad
	other creative product.	
	The student creates a mathematical process that	Computer programming, robotics,
Create a Process	others might use, test or replicate, essentially	Mathematica, MathCad, Inspire
	engaging in mathematical creativity.	Data, iMovie

Combinations of Activity Types

A creative lesson plan by a teacher usually combines two or more activity types into a varied and engaging learning experience. In fact, when learning activities are combined and integrated, they may better resemble the complexity of real-life applications of mathematics and help the problem solver to practice solving problems that are more realistic than the relatively artificial problems often found in textbooks (Checkly, 2006; Fuson et. al, 2005). Combining activity types may also provide opportunities for students to develop more divergent ways of thinking, such as in mathematical modeling work, which uses mathematical language, such as equations, to describe systems (Aris, 1994; Gershenfeld, 1998). Here are a few examples of mathematics learning plans, including a simple combination and two more complex combinations of mathematics activity types.

Example 1: Recognizing and Researching the Fibonacci Series.

A common mathematics topic for teachers to assign for student research in the middle school mathematics classroom is the remarkable Fibonacci Series. This series, in which each term is created by summing the two terms that appear before it (i.e. 1, 2, 3, 5, 8, 13, 21, 34, etc.) is found very commonly in nature, in such items as the spiraled skin of pineapples, the stems of conifer trees, the curved edges in sea shells, and even the family trees of honeybees (Cook, 1979). A simple combination of activity types that might be used by teachers for building student understanding of the Fibonacci Series, is to first ask students to "recognize [the] pattern" (from the "Consider" activity types) by displaying it on a chalkboard or spreadsheet, to see if students can add terms to the sequence correctly. Then the students might "investigate [the] concept" (also from the "Consider" activity types) by doing a Web search on the Fibonacci series to discover where it might be represented in the physical world. Students are often amazed at the many diverse examples of this series that can be found in the natural world.

Example 2: Defining, Representing, and Solving a Paper Folding Problem

An interesting problem that is sometimes posed to elementary students who are studying exponential numbers, is how thick a piece of newspaper might be if it is folded in half, a total of 10 times. To help students to define the problem, they are often given a single

large piece of newsprint to attempt the folding. However, the thickness of the folded paper soon creates an impossible situation, and students find that they need to move to computational strategies to solve the problem. They are essentially faced with the rapidly expanding exponential sequence of 1, 2, 4, 8, 16, 32, etc., that doesn't allow them to fold the paper more than 6 or 7 times. At this point, a teacher might encourage students to use a spreadsheet to mathematically "represent" (from the "Produce" activity types) the problem in columns on the spreadsheet, where each fold numerically describes measurements of the growing thickness. Students might then be encouraged to "test a conjecture" (from the "Evaluate" activity types) or solution (also from the "Evaluate" activity types) to this problem, which relates to powers of 2, and eventually determine that the solution is most likely 2 to the 10th power, or 1024 units of thickness. If the teacher realizes that this same problem has been showcased on the television program "MythBusters," s/he could end the lesson by asking students to "attend to" (from the "Consider" activity types) the Mythbusters clip. In this video, the show's hosts jokingly use a sheet of paper the size of a football field, and modern construction equipment, to see if the size of the paper makes any difference in how many folds they are able to make while trying to solve the problem. (They eventually get to 12.)

Example 3: Interpreting, Producing, and Testing a Garbage Pickup Model

In high school discrete mathematics, a common combination of activities is focused upon modeling how a garbage truck might efficiently move through a system of streets to pick up the garbage each week. A teacher might encourage students to "interpret a representation" (from the "Interpret" activity types) by examining maps of local streets, or perhaps viewing a satellite image of their area using Google Maps. The students could then be asked to "understand or define a problem" and decide upon the parameters for efficient garbage pickup. Typically, students describe various considerations, such as the need to conserve gas by not retracing a route once the truck has already traveled a street. The students are then encouraged to "produce a representation" (from the "Produce" activity types) of the streets as a "network" of line segments for the streets and nodes for the street intersections. They are then asked to "create a plan" (from the "Create" activity types) for an efficient garbage pickup route using this mathematical representation of their neighborhood. Often, students prefer to use some sort of computer-assisted drawing capabilities, such as the drawing utilities in Microsoft Word, or the more sophisticated MathCad, to depict the system of nodes and connecting line segments, and to formalize their planning. Soon students also realize that "odd or even" nodes (which are named according to the number of line segments at a street intersection), is also a consideration for planning the most efficient route.

Finally, students might be asked to "compare and contrast" their routes (from the "Evaluate" activity types) by creating some sort of numerical index for their route (perhaps with a spreadsheet chart) that might compute the number of miles traveled and/or the amount of gasoline used. In doing so, they are encouraged to "evaluate [their own and others'] mathematical work" (from the "Evaluate" activity types) to create maximally efficient routes. This particular mathematical challenge illustrates a realistic use of mathematical modeling, while also exemplifying a combination of mathematical

learning activity types that encourages flexibility, creativity, and pedagogically appropriate technology use.

Final Thoughts

As in the mathematical modeling process, the discipline of mathematics itself is a very creative, exciting and dynamic endeavor. As defined by Steen, mathematics "involves observing, representing and investigating patterns and relationships in social and physical phenomena and between mathematical objects themselves" (1998, page 16). The mathematics learning activity types presented in this article can hopefully help teachers to better engage and motivate students in their classrooms, perhaps involving these students more fully in the creativity of real-life mathematics, as well as helping them to appreciate the great utility of mathematics in understanding our natural world. Most importantly, if used effectively, the learning activity types might help a teacher to make mathematics instruction more meaningful and relevant, by encouraging students to see authentic applications and growing potential for this important discipline, which is becoming increasingly linked to technology.

If we are to help teachers to develop their TPACK, so that they can combine technology, pedagogy, and content successfully in their classrooms, we will no doubt need to do so via clear, instructionally sound strategies and examples. When we have separated content, pedagogy, and technology in our pre-service and in-service programs in the past, we have risked teachers' very superficial understanding of the instructional power of their successful combination, with less-than-optimal instructional results. We need instead to carefully and consciously scaffold the development of teachers' TPACK, so that they can make thoughtful and maximally effective instructional choices that combine technology, pedagogy, and content into lessons that are characterized by an engaging and authentic blend of both the theoretical and application perspectives of mathematics.

Such integration can be done, and done well, if we give teachers the support and encouragement they need to be creative designers of classroom instruction. Ben Franklin would no doubt be in agreement with this approach, since he had an uncommon passion for applying knowledge to the world in which he lived, advocating for an educational system that was both philosophically and practically sound. The taxonomy of mathematics learning activity types shared here is a vehicle that can be used to support teachers who are trying to make mathematics more comprehensible and practical for their students, while also building interest and motivation through diverse, engaging and technology-rich learning activities. If we are successful in such efforts -- in the words of Mr. Franklin -- our students will be less likely to "buy a cow to ride on" and instead be able to apply mathematics effectively to real-world situations,, while continuing to grow in their understanding and appreciation of mathematics and its many uses..

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