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Harmonic Analysis and Tidal Prediction by the Method of Least Squares: A User's Manual

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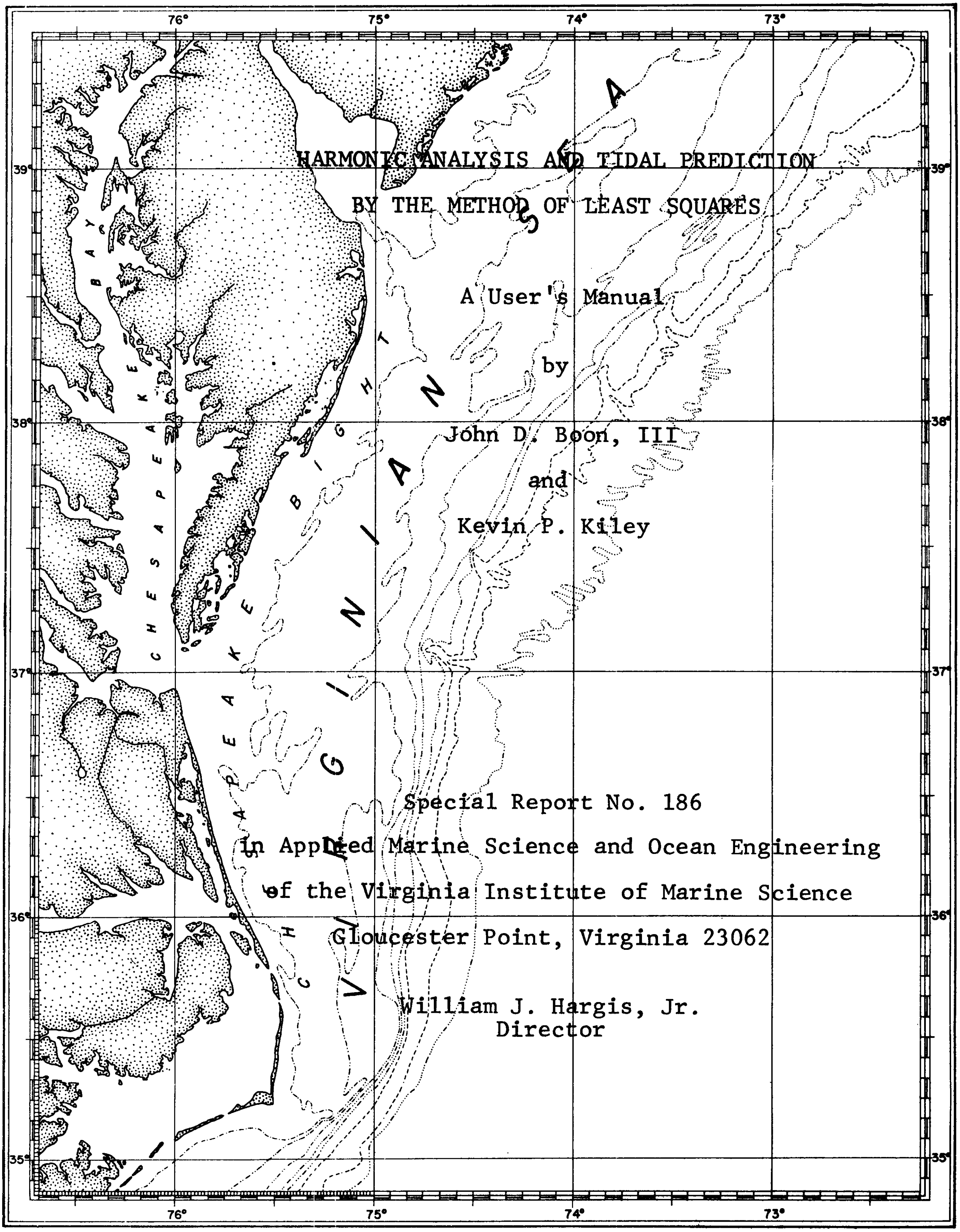

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PREFACE

A common objective in the treatment of measured tides is the mathematical separation by harmonic analysis of certain periodic components which can be used to simulate the astronomical tide. Traditionally, astronomical or predicted tides have been represented by the sum of several such components, each one due to a particular aspect of the tide-producing forces. Once determined, the same set of parameters governing each periodic component may be used to predict the tide during any desired time period at a given location. These parameters are known as the tidal constants for that location.

Although a number of different methods have been developed for determining tidal constants, both before and since the advent of electronic computing techniques, one rarely finds that any of these are intended for practical application by nonspecialists. This is particularly true in the case of the scientist or technician who has a collection of tidal records and would like to utilize them to accurately predict tides by some convenient means. This manual has therefore been written with the general user in mind. It is assumed that the reader has some familiarity with general reference works on the subject of tides.

Introduction

The method of least squares was first applied in the analysis of tides by Horn (1960). An excellent description of its use has been given by Dronkers (1964) who mentions that official tide tables in Germany have since been prepared by this means. The least squares algorithm is exceptionally easy to program on a digital computer and requires very little memory space. It is most efficient when used to obtain a fixed number of tidal constituents from a fixed number of discrete tidal height (or current) measurements representing a standard series length. For the average user, an appropriate series consists of hourly measurements covering the synodic month of approximately 29 days. This series will normally yield ten of the major tidal constituents making up the astronomical tide; several others may be indirectly obtained by the inference formulas of Schureman (1958). For a discussion of the types of constituents normally used in tidal analysis, the reader is referred to Defant (1958).

This manual describes two main programs written in Fortran IV, one for the determination of tidal constants (program HAMELS) and one which utilizes the constants to generate predicted tides (program ASTRO). In addition to the basic derivation of tidal constants, we have included the computational steps necessary to convert these results to a standardized form which enables the prediction of tides for any period within the present century. This is accomplished in both programs by subroutine ORBEL.

Mathematical Development-Method of Least Squares

Let h_t represent a series of tidal height measurements obtained at hourly intervals of time $t = -n, -n+1, \ldots, o, \ldots, n-1, n$ so that $t = o$ at the midpoint of the series. The total number of measurements is then $N = 2n+1$.

The harmonic series used to approximate the measurements using k tidal constituents is

$$h(t) = H_o + \sum_{i=1}^{k} A_i \cos a_i t + \sum_{i=1}^{k} B_i \sin a_i t \qquad (1)$$

Here a_i is the speed of the ith constituent in degrees per mean solar hour and A_i, B_i are coefficients representing the constituent amplitude. H_o represents the mean height of tide in the series. By reducing the measurements to zero mean with no linear trend, H_o may be neglected in equation (1) and we obtain the required least squares fit by choosing coefficients A_i, B_i such that

$$E = \sum_{t=-n}^{n} [h(t) - h_t]^2$$

is a minimum. This will occur when

$$\frac{\partial E}{\partial A_j} = \frac{\partial E}{\partial B_j} = o; \; j = 1, 2, \ldots, k$$

By carrying out the above, 2k simultaneous equations are generated:

$$\sum_{t=-n}^{n} [h(t) - h_t] \cos a_j t = o \qquad (2)$$

$$\sum_{t=-n}^{n} [h(t) - h_t] \sin a_j t = 0 \qquad (3)$$

Substituting equation (1) in the above equations and rearranging terms, the normal equations are

$$\sum_i A_i \sum_t \cos a_i t \cos a_j t + \sum_i B_i \sum_t \sin a_i t \cos a_j t = \sum_t h_t \cos a_j t \qquad (4)$$

$$\sum_i A_i \sum_t \cos a_i t \sin a_j t + \sum_i B_i \sum_t \sin a_i t \sin a_j t = \sum_t h_t \sin a_j t \qquad (5)$$

Due to the selection of a central time origin, we also have

$$\sum_t \sin a_i t \cos a_j t = \sum_t \cos a_i t \sin a_j t = 0$$

as well as the identities

$$\sum_t \cos a_i t \cos a_j t = \tfrac{1}{2}C(a_i - a_j) + \tfrac{1}{2}C(a_i + a_j) = S_{ij}$$

$$\sum_t \sin a_i t \sin a_j t = \tfrac{1}{2}C(a_i - a_j) - \tfrac{1}{2}C(a_i + a_j) = D_{ij}$$

where $C(0) = N$

and $C(\xi) = \dfrac{\sin(N\xi/2)}{\sin(\xi/2)}$, $\xi = a_i + a_j$ or $a_i - a_j \neq 0$

The final set of 2k equations are then

$$\sum_i A_i S_{ij} = \sum_t h_t \cos a_j t \qquad (6)$$

$$\sum_i B_i D_{ij} = \sum_t h_t \sin a_j t \qquad (7)$$

which must be solved for the 2k unknowns A_i and B_i, $i = 1,2,\ldots,k$.

To illustrate, assume that k = 3 tidal constituents are being sought. Written in full, the equations in (6) and (7) are

$$\begin{aligned} A_1 S_{11} + A_2 S_{21} + A_3 S_{31} &= \sum_t h_t \cos a_1 t \\ A_1 S_{12} + A_2 S_{22} + A_3 S_{32} &= \sum_t h_t \cos a_2 t \\ A_1 S_{13} + A_2 S_{23} + A_3 S_{33} &= \sum_t h_t \cos a_3 t \end{aligned} \qquad (6)$$

$$\begin{aligned} B_1 D_{11} + B_2 D_{21} + B_3 D_{31} &= \sum_t h_t \sin a_1 t \\ B_1 D_{12} + B_2 D_{22} + B_3 D_{32} &= \sum_t h_t \sin a_2 t \\ B_1 D_{13} + B_2 D_{23} + B_3 D_{33} &= \sum_t h_t \sin a_3 t \end{aligned} \qquad (7)$$

or in matrix form

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} \Sigma h_t \cos a_1 t \\ \Sigma h_t \cos a_2 t \\ \Sigma h_t \cos a_3 t \end{pmatrix} \qquad (6)$$

$$\begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} \Sigma h_t \sin a_1 t \\ \Sigma h_t \sin a_2 t \\ \Sigma h_t \sin a_3 t \end{pmatrix} \qquad (7)$$

noting that $S_{ij} = S_{ji}$ and $D_{ij} = D_{ji}$ since these symbols represent a summation of cosine and sine products, respectively, in which the order is immaterial. Multiplying both sides of (6) and (7) by the inverse of the respective square matrices, the required coefficients are

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{pmatrix}^{-1} \begin{pmatrix} \Sigma h_t \cos a_1 t \\ \Sigma h_t \cos a_2 t \\ \Sigma h_t \cos a_3 t \end{pmatrix} \quad (6)$$

$$\begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{pmatrix}^{-1} \begin{pmatrix} \Sigma h_t \sin a_1 t \\ \Sigma h_t \sin a_2 t \\ \Sigma h_t \sin a_3 t \end{pmatrix} \quad (7)$$

A computational advantage of the least squares method is now apparent. Given a standard set of k tidal constituents and a standard series containing N tidal measurements, one can obtain the inverse matrices in advance so that only the data summation vectors on the right of (6) and (7) require computation prior to matrix multiplication during a run.

Tidal Constants - Amplitude, Speed and Phase

Having obtained a set of coefficients by the method just described, it is convenient to express each tidal constituent in the form of a simple sine wave of amplitude R_i, speed a_i, and phase ζ_i using

$$h_i(t) = R_i \cos(a_i t - \zeta_i) \quad (8)$$

where

$$R_i = \sqrt{A_i^{\,2} + B_i^{\,2}}$$

$$a_i = 360°/T_i, \; T_i = \text{period of ith constituent}$$

$$\zeta_i = \tan^{-1} (B_i/A_i)$$

The phase angle, ζ_i, fixes the position of the wave form relative to the time origin. In reference to series time, this means that a high water phase occurs at $t = \zeta_i/a_i$ hours after (or if ζ_i is negative, before) zero hour at the midpoint of the series. If standard time is to be used in equation (8) the angle $a_i t_o$ must first be added to ζ_i, t_o being the standard time at the midpoint of the series.

Goodness of Fit Criteria

An important step following an analysis is to ask how well the approximation made with equation (1) represents the input data. One way of answering is through an analysis of variance or sum of squares partitioning among the individual constituents used in the analysis.

The total sum of squares for the tidal measurements is

$$SS_{total} = \sum_t h_t^2 - \left[\sum_t h_t\right]^2/N$$

and the sum of squares contributed by the ith constituent is

$$SS_i = \sum_t h_i(t)^2 - \left[\sum_t h_i(t)\right]^2/N$$

in which $h_i(t)$ values are generated using equation (8) at series times $t = -n, -n+1, \ldots, o, \ldots, n-1, n$. One then obtains the percent sum of squares accounted for by the ith constituent as the fraction $SS_i/SS_{total} \times 100$.

Totaling the percentages for all of the constituents used gives an indication of the strength of that particular model of the tide. It should be noted, however, that a combined sum of less than 100% is to be expected; in an area where the astronomical range is small but weather-related disturbances are pronounced, the combined sum may be less than 50% using any number of astronomical constituents.

Tidal Prediction Model

Before accurate predictions can be made, specifically in years other than that of the analytical series, certain adjustments to the tidal constants become necessary. As presented by Schureman (1958), the required harmonic model of the tide is

$$h(t) = H_o + \sum_i f_i H_i \cos[a_i t + (V_o+u)_i - \kappa_i] \qquad (9)$$

where

H_o = height of mean sea level above model datum

f_i = nodal factor for reducing mean amplitude to the required amplitude in the year of prediction

H_i = mean amplitude of ith constituent during 18.6 year-period of lunar node cycle

$(V_o+u)_i$ = the local equilibrium phase of ith constituent in the year of prediction

κ_i = phase of ith constituent relative to the local equilibrium phase

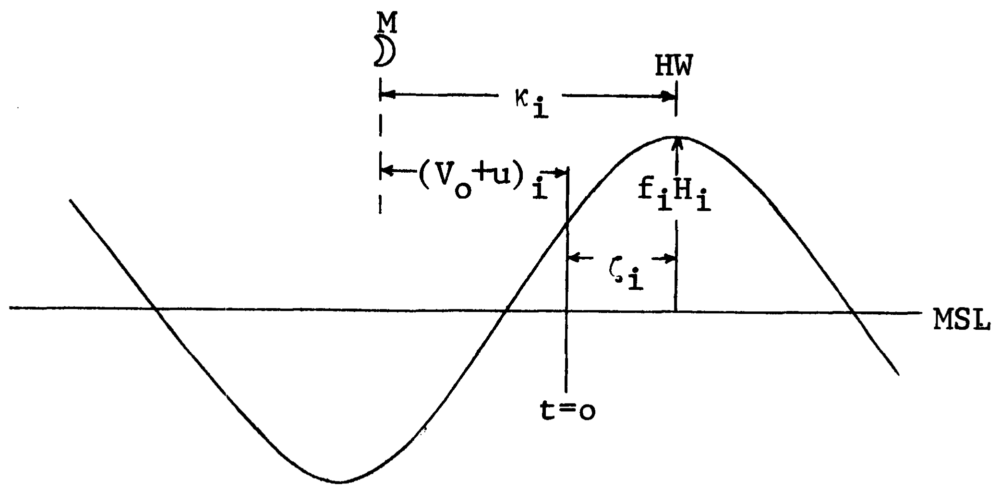
Each of the above arguments associated with equation (9) are essential elements in the tidal analysis and prediction programs of this manual. The following explanations apply to their use in these programs:

H_o - If the predicted heights are to refer to a model datum of mean sea level, H_o must then be zero. Other values depend upon the datum selected. For example, H_o should equal approximately one-half the mean tidal range at the station if the model datum is mean low water. For a discussion of tidal datums, see Marmer (1951), Boon and Lynch (1972), or Swanson (1974).

f_i, H_i - These arguments are related by $R_i = f_i H_i$ where R_i is the expected amplitude of the ith constituent in the year of the prediction. A slight variation in R_i for most (but not all) constituents occurs from one year to the next, the value in any one year depending upon the position of the lunar nodes within an 18.6-year cycle. Subroutine ORBEL computes nodal factors, f_i, to convert R_i to H_i in program HAMELS, H_i to R_i in program ASTRO.

$(V_o+u)_i$ - The equilibrium phase of a constituent is the phase of an imaginary sine curve representing the so-called equilibrium tide in the absence of friction and other factors. In the case of the main lunar constituent, M_2, this would mean that high water occurs just as the moon transits the meridian of the place in question. Thus, the M_2 equilibrium phase simply expresses lunar position (hour angle) in relation to the time origin and meridian in use. Greenwich equilibrium phases refer to Greenwich mean time (G.M.T.) and the prime meridian at Greenwich, England. Subroutine ORBEL computes the latter for up to 37 selected constituents during any year between 1900 and the year 2000.

κ_i - Comparing equations (8) and (9), one sees that $\zeta_i = \kappa_i - (V_o+u)_i$ or $\kappa_i = \zeta_i + (V_o+u)_i$ as illustrated in the following diagram:



sine curve representing ith tidal constituent

The phase angle κ_i is used in equation (9) because it alone behaves as a true constant from one year to the next. Except for certain solar constituents, $(V_o+u)_i$ and hence ζ_i vary in a uniform manner determined by celestial mechanics (the combined symbol for the equilibrium phase signifies that it consists of a slowly changing component and a more rapidly changing component).

κ_i' - A primed kappa symbol indicates that the constituent phase angle, κ_i, has been adjusted so that Greenwich $(V_o+u)_i$ values may be used in place of local $(V_o+u)_i$ in equation (9). This adjustment simply accounts for the difference in longitude between the tide station and the meridian of Greenwich and for the use of local standard time in the predictions. Each κ_i to κ_i' conversion is made according to the formula

$$\kappa_i' = \kappa_i + P_iL - \frac{a_iS}{15}$$

where

L = west longitude in degrees of station at which predictions are to be made

S = west longitude in degrees of time meridian used (e.g., 75°W for U.S. Eastern Standard time)

P_i = number of daily cycles of ith constituent (diurnal constituents = 1, semidiurnal = 2, etc.)

Both κ_i and κ_i' are included in the output of program HAMELS. Only κ_i' values, however, are used as input to program ASTRO.

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APPENDIX A

Program HAMELS

Instructions for using Program HAMELS

I. Preliminary Input

The initial input required by Program HAMELS includes a master list of constituent speeds and other associated parameters for 37 tidal constituents, a set of inference formula constants, and the inverse matrices matching the number and order of the selected constituents. The inverse matrices supplied in this publication must be used with a standard series of 697 observations. Other matrices for a series of different length may be computed using Program LESCO, Appendix C.

The complete card listing of preliminary input data for the standard series is given at the end of this appendix. The array values in each matrix have been multiplied by 1000 to eliminate extraneous leading zeroes and to conform with computational steps taken in the program.

II. Station Control Card

A four-letter station code is used to identify the tide station supplying the tidal data. The latitude and longitude of the station must be given as coded numbers in which the first three digits represent degrees and the remainder minutes and tenths (e.g., 03515.1 = 35° 15.1'). Other control information must be coded as indicated in the user comments listed at the beginning of the program.

III. Tide Data Cards

Twelve hourly heights per card for the first 58 cards (696 heights) followed by the 59th card containing the final height.

A condition of the least squares method is that the total number of heights must be odd.

III. Data Output and Plot

The main output from Program HAMELS consists of a set of harmonic constants obtained by the least squares analysis of the input data. A printer plot subroutine then graphically displays both the observed series of tidal heights and the predicted heights based on the above harmonic constants. Finally a plot of the residual between the observed and computed heights is printed.

The scale of the plot may be changed as needed by insertion of new range limits in the calling statements for subroutine PLOT which are found at the end of the main program.

```
C PROGRAM HAMELS- HARMONIC ANALYSIS METHOD OF LEAST SQUARES
C PROGRAM COMPUTES TIDAL CONSTANTS FOR 10 MAIN CONSTITUENTS BY LEAST
C SQUARES PLUS 15 SECONDARY CONSTITUENTS BY INFERENCE FORMULAS OF
C SCHUREMAN. TIDES DUE TO DISTURBING SECONDARY CONSTITUENTS T2,P1,K2
C ARE REMOVED FROM THE RECORD PRIOR TO FINAL DETERMINATION OF CONSTANTS.
C INPUT REQUIRED-
C 1.MASTER LIST OF CONSTITUENT INFORMATION
C   A. 37 TIDAL CONSTITUENT CARDS (I2,1X,A4,1X,F8.4,4X,9F3.1)
C      NOS(I)- NATIONAL OCEAN SURVEY REFERENCE NO. FOR ITH CONSTITUENT
C      SYM(I)- SYMBOL FOR ITH CONSTITUENT (A4)
C      A(I,1)- SPEED OF ITH CONSTITUENT IN DEG/MSH (F8.4)
C      A(I,J)- ORBITAL ELEMENT INDICES FOR ITH CONSTITUENT (8F3.1)
C      CT(I)-  ITH CONSTITUENT TYPE, DIURNAL=1,SEMIDIURNAL=2,ETC. (F3.1)
C   B. 15 INFERENCE FORMULA CARDS    (2F6.3,3I3)
C      SCA(I)- AMPLITUDE PARAMETER, ITH INFERRED CONSTITUENT (F6.3)
C      SCE(I)- PHASE PARAMETER, ITH INFERRED CONSTITUENT (F6.3)
C      IC1(I)- INFERRED PARAMETER INDEX (I3)
C      IC2(I)- INFERRED PARAMETER INDEX (I3)
C      IC3(I)- INFERRED PARAMETER INDEX (I3)
C 2. LEAST SQUARES INVERSE MATRICES - 10 CARDS EACH MATRIX
C      SINV(I,J)X1000 (10F8.6)
C      DINV(I,J)X1000 (10F8.6)
C 3. STATION CONTROL CARD (1X,A4,2F7.1,I3,1X,I4,2I3,F4.1)
C      XIDEN- STATION CODE (A4)
C      XLAT-  STATION LATITUDE (F7.1)
C      XLON-  STATION LONGITUDE (F7.1)
C      LTM-   LONGITUDE OF TIME MERIDIAN (I3)
C      IYR-   YEAR OF OBSERVATIONS (I4)
C      MS-    MONTH STARTING OBSERVATIONS (I3)
C      IDS-   DAY STARTING OBSERVATIONS (I3)
C      TS-    TIME OF FIRST OBSERVATION (F4.1)
C 4. TIDAL DATA CARDS - 59 CARDS FOR STANDARD SERIES OF 29 DAYS
C      HH(I)- HOURLY HEIGHT OF TIDE (32X,12F4.2)
C
C  SUBROUTINES REQUIRED - ORBEL,PLOT
C-----------------------------
      DIMENSION HCS(10),HSN(10),AA(10),BB(10),SINV(10,10),DINV(10,10)
      DIMENSION A(37,9),JS(12),JE(12),HH(697),H(25),R(25),ZETA(25)
      DIMENSION VOU(37),F(37),NOS(37),SCA(25),SCE(25),T(697),PHH(697)
      DIMENSION IC1(25),IC2(25),IC3(25),SS(25),SYM(37),CT(37)
      REAL KAPA(25),KAPR(25),KPRMK(25)
      NHH=697
      NPH=1
      M=10
      N=15
C---------------------------------------------
C READ MASTER LIST OF CONSTITUENT SPEEDS, ORBITAL ELEMENT INDICES,
C SECONDARY CONSTITUENT INDICES.
C---------------------------------------------
      N1=M+1
      N2=M+N
      READ(5,1)(NOS(I),SYM(I),(A(I,J),J=1,9),CT(I),I=1,37)
    1 FORMAT(I2,1X,A4,1X,F8.4,4X,9F3.1)
      READ(5,46)(SCA(I),SCE(I),IC1(I),IC2(I),IC3(I),I=N1,N2)
   46 FORMAT(2F6.3,3I3)
C---------------------------------------------
C READ LEAST SQUARES INVERSE MATRICES(X1000)
C---------------------------------------------
```

```
      READ(5,45)((SINV(I,J),J=1,M),I=1,M)
      READ(5,45)((DINV(I,J),J=1,M),I=1,M)
   45 FORMAT(10F8.6)
C------------------------
C READ STATION CONTROL CARD
C READ HOURLY HEIGHTS
C------------------------
      READ(5,2) XIDEN,XLAT,XL,ON,LTM,IYR,MS,IDS,TS
      XLON=XL+ON/60.
    2 FORMAT(1X,A4,F7.1,F3.0,F4.1,I3,1X,I4,2I3,F4.1)
      READ(5,43)(HH(I),I=1,NHH)
   43 FORMAT(32X,12F4.2)
C---------------------------------------------------------------
C COMPUTE JULIAN START AND MIDPOINT DATES, ZULU START AND MIDPOINT TIMES
C COMPUTE LEAP YEAR ADJUSTMENT
C---------------------------------------------------------------
      DEGRAD=57.29577951
      ZIP=0.1
      LY=1
      LEAP=(2000.-IYR)/4.
      XLEAP=(2000.-IYR)/4.-LEAP
      IF(XLEAP.GT.ZIP)LY=0
      JS(1)=1
      JE(1)=31
      JS(2)=32
      JE(2)=59+LY
      K=1
      DO 5 I=3,12
      JS(I)=JE(I-1)+1
      JE(I)=JS(I)+29+K
      K=1-K
      IF(I.EQ.7)K=1
    5 CONTINUE
      TZ=LTM/15
      ZS=TS+TZ
      ZDS=(JS(MS)+IDS-2)*24.+ZS
      ZDM=ZDS+(NHH/NPH-1)/2
      JDS=ZDS/24+1
      JDM=ZDM/24+1
      ZM=ZDM-(JDM-1)*24
      CALL ORBEL(IYR,JDS,JDM,ZS,ZM,NOS,A,VOU,F)
C----------------------------------------------
C REMOVE LINEAR TREND, ADJUST DATA TO ZERO MEAN
C COMPUTE TOTAL SUMS OF SQUARES FOR SERIES
C----------------------------------------------
      JUMP=0
      SX=0.0
      SX2=0.0
      SY=0.0
      SXY=0.0
      DO 7 I=1,NHH
      SX=SX+I
      SX2=SX2+I*I
      SY=SY+HH(I)
      SXY=SXY+I*HH(I)
    7 CONTINUE
  600 XBAR=SX/NHH
      YBAR=SY/NHH
```

```
      SXY=SXY-XBAR*SY
      SX2=SX2-XBAR*SX
      BZ=SXY/SX2
      CZ=YBAR-BZ*XBAR
      DO 8 I=1,NHH
      T(I)=I-1
      PHH(I)=0.0
      HH(I)=HH(I)-BZ*I-CZ
    8 CONTINUE
  602 TSS=0.0
      DO 11 I=1,NHH
      TSS=TSS+HH(I)*HH(I)
   11 CONTINUE
      GO TO 15
   16 SY=0.0
      DO 17 I=1,NHH
      SY=SY+HH(I)
   17 CONTINUE
      YBR=SY/NHH
      DO 18 I=1,NHH
      HH(I)=HH(I)-YBR
   18 CONTINUE
C-----------------------------------------------
C COMPUTE COSINE AND SINE SUMMATION VECTORS
C-----------------------------------------------
  606 JUMP=1
   15 CONTINUE
      DO 13 J=1,M
      HCS(J)=0.0
      HSN(J)=0.0
      II=-(NHH-1)/2
      ARG1=A(J,1)/DEGRAD
      DO 13 K=1,NHH
      ARG=ARG1*II
      HCS(J)=HCS(J)+HH(K)*COS(ARG)
      HSN(J)=HSN(J)+HH(K)*SIN(ARG)
      II=II+1
   13 CONTINUE
C-----------------------------------
C COMPUTE LEAST SQUARES COEFFICIENTS
C-----------------------------------
      DO 10 I=1,M
      AA(I)=0.0
      BB(I)=0.0
      DO 30 J=1,M
      AA(I)=AA(I)+SINV(I,J)*HCS(J)
      BB(I)=BB(I)+DINV(I,J)*HSN(J)
   30 CONTINUE
      AA(I)=AA(I)/1000.
      BB(I)=BB(I)/1000.
   10 CONTINUE
C-------------------------------------------
C COMPUTE CONSTITUENT AMPLITUDES AND PHASES
C-------------------------------------------
      DO 14 I=1,M
      ZETA(I)=ATAN2(BB(I),AA(I))*DEGRAD
      ZETA(I)=ZETA(I)+A(I,1)*(NHH/NPH-1)/2
      IDUMP=ZETA(I)/360
```

```
      ZETA(I)=ZETA(I)-IDUMP*360
      IF(ZETA(I).LT.0.0)ZETA(I)=ZETA(I)+360.
      ARG=AA(I)*AA(I)+BB(I)*BB(I)
      R(I)=SQRT(ARG)
   14 CONTINUE
      IF(JUMP.GT.0)GO TO 19
C-----------------------------------------------------
C COMPUTE INFERRED SECCNDARY CONSTITUENTS T2,P1,K2
C REMOVE T2,P1,K2 FRCM DATA
C-----------------------------------------------------
      DO 4 I=1,6
      KAPA(I)=VOU(I)-CT(I)*XLON+A(I,1)*TZ+ZETA(I)
      H(I)=R(I)/F(I)
    4 CONTINUE
      H(20)=H(2)*SCA(20)
      KAPA(20)=KAPA(2)+SCE(20)*(KAPA(2)-KAPA(1))
      H(23)=H(4)*SCA(23)
      KAPA(23)=KAPA(4)+SCE(23)*(KAPA(4)-KAPA(6))
      H(25)=H(2)*SCA(25)
      KAPA(25)=KAPA(2)+SCE(25)*(KAPA(2)-KAPA(1))
      ZETA(20)=KAPA(20)-VOU(27)+2.*XLON-A(20,1)*TZ
      ZETA(23)=KAPA(23)-VOU(30)+XLON-A(23,1)*TZ
      ZETA(25)=KAPA(25)-VOU(35)+2.*XLON-A(25,1)*TZ
      II=0
      DO 6 I=1,NHH
      ARG1=(A(20,1)*II-ZETA(20))/DEGRAD
      ARG2=(A(23,1)*II-ZETA(23))/DEGRAD
      ARG3=(A(25,1)*II-ZETA(25))/DEGRAD
      HH(I)=HH(I)-F(27)*H(20)*COS(ARG1)-F(30)*H(23)*COS(ARG2)
      HH(I)=HH(I)-F(35)*H(25)*CCS(ARG3)
      II=II+1
    6 CONTINUE
  604 IF(JUMP.EQ.0)GO TO 16
C---------------------------------------
C COMPUTE REMAININC SECCNDARY CONSTITUENTS
C---------------------------------------
   19 DO 20 I=1,M
      J=NOS(I)
      KAPA(I)=VOU(J)-CT(I)*XLON+A(I,1)*TZ+ZETA(I)
      L=KAPA(I)/360.
      KAPA(I)=KAPA(I)-L*360.
      KAPR(I)=VOU(J)+ZETA(I)
      L=KAPR(I)/360.
      KAPR(I)=KAPR(I)-L*360.
      KPRMK(I)=KAPR(I)-KAPA(I)
      H(I)=R(I)/F(J)
   20 CONTINUE
      DO 21 I=N1,N2
      J=IC1(I)
      K=IC2(I)
      L=IC3(I)
      KAPA(I)=KAPA(K)+SCE(I)*(KAPA(K)-KAPA(L))
      LL=KAPA(I)/360.
      KAPA(I)=KAPA(I)-LL*360.
      IF(KAPA(I).LT.0.0)KAPA(I)=KAPA(I)+360.
      KAPR(I)=KAPA(I)+CT(I)*XLON-A(I,1)*TZ
      KPRMK(I)=KAPR(I)-KAPA(I)
      H(I)=H(J)*SCA(I)
```

```
      ISUBC=NOS(I)
      ZETA(I)=KAPA(I)-VOU(ISUBC)+CT(I)*XLON-A(I,1)*TZ
   21 CONTINUE
C-----------------------------------------------------
C COMPUTE PERCENT SUMS OF SQUARES FOR EACH CONSTITUENT
C-----------------------------------------------------
      SST=0.0
      DO 22 J=1,N2
      L=NOS(J)
      SS(J)=0.0
      II=0
      ARG1=A(J,1)/DEGRAD
      ZETA1=ZETA(J)/DEGRAD
      DO 23 K=1,NHH
      ARG=ARG1*II-ZETA1
      Y=F(L)*H(J)*CCS(ARG)
      SY=SY+Y
      SS(J)=SS(J)+Y*Y
      PHH(K)=PHH(K)+Y
      II=II+1
   23 CONTINUE
      SS(J)=(SS(J)-SY*SY/NHH)*100./TSS
      SST=SST+SS(J)
      SS(J)=SS(J)+ 1.E-4
   22 CONTINUE
C---------------------
C PRINT RESULTS
C---------------------
      XLON=XL*100.+ON
      WRITE(6,24) XIDEN,XLAT,XLON,LTM,IYR,MS,IDS,TS,NHH
   24 FORMAT('1',/////1X,'HAMELS- HARMONIC ANALYSIS METHOD OF LEAST SQUA
     1RES',//1X,'STATION ',A4,2F7.1,I3,'W',3X,'YEAR ',I4,/1X,'29 DAY SER
     2IES STARTING ',I3,'-',I2,3X,F5.1,' HRS',4X,I4,' OBSERVATIONS',//1X
     3,'NOS NO.  CONST.',1X,'SPEED',5X,'H',5X,'KAPPA',2X,'KPRIME',5X,
     4'KPR-K',2X,'0/0 TSS')
      WRITE(6,3)(NOS(I),SYM(I),A(I,1),H(I),KAPA(I),KAPR(I),KPRMK(I),
     1SS(I),I=1,N2)
    3 FORMAT(4X,I2,3X,A4,2X,F8.4,F7.3,F8.2,F8.1,2X,F8.2,2X,F7.2)
      WRITE(6,26) YBAR,SST
   26 FORMAT(/1X,'SERIES MSL ',F6.2,41X,F6.2)
C---------------
C PLOT RESULTS
C---------------
      HMIN=-5.0
      HMAX=5.0
      TMIN=0.0
      TINCR=1.0
      WRITE(6,35) XIDEN,TS,MS,IDS,IYR
   35 FORMAT('1'//1X,A4,/1X,'OBSERVED HOURLY HEIGHTS(X) AND PREDICTED HO
     1URLY HEIGHTS(+) VERSUS TIME',/1X,'29-DAY SERIES STARTING AT ',F4.1
     2,'HRS',1X,I2,'-',I2,1X,I4//1X)
      CALL PLOT(NHH,HH,PHH,T,HMIN,HMAX,HMIN,HMAX,TMIN,TINCR)
      WRITE(6,36) XIDEN,TS,MS,IDS,IYR
   36 FORMAT('1'//1X,A4,/1X,'RESIDUAL(X)= OBSERVED MINUS PREDICTED HOURL
     1Y HEIGHTS VERSUS TIME',/1X,'29-DAY SERIES STARTING AT ',F4.1,' HRS
     2',1X,I2,'-',I2,1X,I4,5X,'DATUM IS SERIES MSL'//1X)
      DO 37 I=1,NHH
      HH(I)=HH(I)-PHH(I)
   37 PHH(I)=0.0
      CALL PLOT(NHH,HH,PHH,T,HMIN,HMAX,HMIN,HMAX,TMIN,TINCR)
      STOP
      END
```

```
      SUBROUTINE ORBEL(IYR,JS,JM,ZS,ZM,NOS,A,VOU,F)
C-----------------------------------------------------------------
C SUBROUTINE COMPUTES GREENWICH EQUILIBRIUM PHASES AND NODAL FACTORS
C FOR ANY OR ALL OF 37 TIDAL CONSTITUENTS USED IN STANDARD HARMONIC
C ANALYSIS AND TIDAL PREDICTION. COMPUTATIONS ARE BASED ON ORBITAL ELEMENT
C FORMULAE BY SCHUREMAN(1958) FOR EITHER HARMONIC ANALYSIS OR TIDAL PREDICTIONS
C 1. INPUT VARIABLES
C     IYR- YEAR OF OBSERVATIONS/PREDICTIONS
C     JS- JULIAN DAY BEGINNING THE MONTH IN WHICH SERIES STARTS
C     JM- JULIAN DAY CONTAINING MIDPOINT OF SERIES
C     ZS- GREENWICH MEAN (ZULU) TIME AT START OF SERIES
C     ZM- GREENWICH MEAN (ZULU) TIME AT MIDPOINT OF SERIES
C     NOS(I)- NATIONAL OCEAN SURVEY REFERENCE NUMBER FOR ITH CONSTITUENT
C     A(I,1)- SPEED OF ITH CONSTITUENT IN DEG/MSH
C     A(I,J)- ORBITAL ELEMENT INDICES (J=2,9) FOR ITH CONSTITUENT
C 2. OUTPUT VARIABLES
C     VOU(I)- GREENWICH EQUILIBRIUM PHASE FOR ITH CONSTITUENT
C     F(I)- NODAL FACTOR FOR ITH CONSTITUENT
C-----------------------------------------------
      DIMENSION VOU(37),F(37),A(37,9),NOS(37)
      DEGRAD=57.29577951
      LYC=(IYR-1901)*.25
      S=263.849224+129.38482*(IYR-1900)+13.176397*(JS+LYC)+.549016*ZS
      IDUMP=S/360
      S=S-IDUMP*360
      P=334.272317+40.662466*(IYR-1900)+.111404*(JS+LYC)+.0046418*ZS
      IDUMP=P/360
      P=P-IDUMP*360
      H=279.203854-.238725*(IYR-1900)+.985647*(JS+LYC)+.041069*ZS
      P1=281.220810+.017178*(IYR-1900)+.0000471*(JS+LYC)+.000002*ZS
      PM=334.272321+40.662466*(IYR-1900)+.111404*(JM+LYC)+.0046418*ZM
      IDUMP=PM/360
      PM=PM-IDUMP*360
      PM=PM/DEGRAD
      XN=259.209010-19.328186*(IYR-1900)-.052954*(JM+LYC)-.0022064*ZM
      XN=XN/DEGRAD
      XN2=2.0*XN
      ARG=.9136949-.0356926*COS(XN)
      OI=ARCOS(ARG)
      OI2=2.0*OI
      ARG=.0897056*SIN(XN)/SIN(OI)
      XNU=ARSIN(ARG)
      XNU2=2.0*XNU
      ARG=.206727*SIN(XN)*(1.-.0194926*COS(XN))
      ARG2=.9979852+.206727*COS(XN)-.0020148*COS(XN2)
      XI=ATAN2(ARG,ARG2)
      ARG=SIN(XNU)
      ARG2=COS(XNU)+.334766/SIN(OI2)
      XNUP=ATAN2(ARG,ARG2)*DEGRAD
      ARG=SIN(XNU2)
      ARG2=COS(XNU2)+.0726184/(SIN(OI)*SIN(OI))
      XNDP=ATAN2(ARG,ARG2)*DEGRAD
      PP=PM-XI
      PP2=2.0*PP
      XI=XI*DEGRAD
      XNU=XNU*DEGRAD
      ARG=SIN(PP2)
      ARG3=0.5*OI
```

```
      ARG2=(COS(ARG3)*COS(ARG3)/(6.*SIN(ARG3)*SIN(ARG3)))-COS(PP2)
      R=ATAN2(ARG,ARG2)*DEGRAD
      UL2=2.0*(XI-XNU)-R
      ARG=SIN(PP)*(5.0*COS(OI)-1.)
      ARG2=COS(PP)*(7.0*COS(OI)+1.)
      Q=ATAN2(ARG,ARG2)*DEGRAD
      UM1=(XI-XNU)+Q
      DO 1 J=1,37
      I=NOS(J)
      VOU(I)=A(J,2)*S+A(J,3)*H+A(J,4)*P+A(J,5)*P1+A(J,6)*90.+A(J,7)*XI+A
     1(J,8)*XNU+A(J,9)*XNUP
      IDUMP=VOU(I)/360
      VOU(I)=VOU(I)-IDUMP*360
      IF(VOU(I))3,1,1
    3 VOU(I)=VOU(I)+360.
    1 CONTINUE
      VOU(18)=VOU(18)+UM1
      VOU(33)=VOU(33)+UL2
      VOU(35)=VOU(35)-XNDP
      XNU=XNU/DEGRAD
      PP2=COS(PP2)
      XNU=COS(XNU)
      XNU2=COS(XNU2)
      SQS=SIN(ARG3)*SIN(ARG3)
      SQC=COS(ARG3)*COS(ARG3)
      QT1=12.0*SQS*PP2/SQC
      QT2=36.0*SQS*SQS/(SQC*SQC)
      SQ1=SIN(OI)
      SQ2=SQ1*SQ1
      SQ3=SIN(OI2)
      DO 2 I=1,37
      F(I)=1.000
    2 CONTINUE
      F(1)=SQC*SQC/.91544
      F(3)=F(1)
      F(4)=SQRT(.8965*SQ3*SQ3+.6001*SQ3*XNU+.1006)
      F(5)=F(1)*F(1)
      F(6)=SQ1*SQC/.37988
      F(7)=F(1)*F(1)*F(1)
      F(8)=F(1)*F(4)
      F(10)=F(5)
      F(11)=F(1)
      F(13)=F(1)
      F(14)=F(1)
      F(15)=SQ1*SQS/.016358
      F(16)=F(1)
      F(18)=SQRT(2.31+1.435*PP2)*F(6)
      F(19)=SQ3/.72137
      F(20)=1.327757-1.991635*SQ2
      F(23)=F(1)
      F(24)=SQ2/.1578
      F(25)=F(6)
      F(26)=F(6)
      F(29)=F(6)
      F(31)=F(1)
      F(32)=SQC*SQC*SQC/.8758
      F(33)=SQRT(1.-QT1+QT2)*F(1)
      F(34)=F(1)*F(1)*F(4)
      F(35)=SQRT(19.0444*SQ2*SQ2+2.7702*SQ2*XNU2+.0981)
      F(36)=F(5)*F(5)
      F(37)=F(5)
      RETURN
      END
```

```
      SUBROUTINE PLOT(L,ORD1,ORD2,AB,FMIN1,FMAX1,FMIN2,FMAX2,AMIN,ANCR)
C-----------------------------------------------------------------
C SUBROUTINE PLOTS ON PRINTER ONE OR TWO ARRAYS OF Y VALUES (ACROSS PAGE)
C AGAINST ONE ARRAY OF X VALUES (DOWN PAGE)
C-----------------------------------------------------------------
C USAGE - CALL PLOT(N,Y1,Y2,X,Y1MIN,Y1MAX,Y2MIN,Y2MAX,XMIN,XINCR)
C-----------------------------------------------------------------
C DESCRIPTION OF PARAMETERS
C     N       - NUMBER OF POINTS TO BE PLOTTED
C     Y1      - ARRAY CONTAINING FIRST SET OF Y VALUES (LENGTH N)
C     Y2      - ARRAY CONTAINING SECOND SET OF Y VALUES (LENGTH N)
C     X       - ARRAY CONTAINING SET OF X VALUES (LENGTH N)
C     Y1MIN   - MINIMUM VALUE CHOSEN FOR Y1 SCALE
C     Y1MAX   - MAXIMUM VALUE CHOSEN FOR Y1 SCALE
C     Y2MIN   - MINIMUM VALUE CHOSEN FOR Y2 SCALE
C     Y2MAX   - MAXIMUM VALUE CHOSEN FOR Y2 SCALE
C     XMIN    - MINIMUM VALUE OF THE X ARRAY
C     XINCR   - INCREMENT FOR EACH VALUE OF X
C-----------------------------------------------------------------
C PLOT SYMBOLS   Y1- X       Y2- +      COINCIDENT POINTS- *
C-----------------------------------------------------------------
C
C NOTE: Y SCALE RESOLUTION IS 100 COLUMNS FULL SCALE
C IF ONLY ONE Y ARRAY IS TO BE PLOTTED THEN THE SAME ARRAY NAME
C MUST BE USED FOR BOTH Y1 AND Y2
C-----------------------------------------------------------------
      DIMENSION ORD1(1),ORD2(1),AB(1),IPLT(105),YCALE(11)
      DATA  IPLT/105*' '/,ISYMB/' '/,ISYMO/'O'/,ISYMX/'X'/,ISYMY/'+'/
      DATA  ISYMA/'*'/,ISYMP/'+'/,ISYML/'|'/
      LA=6
      YNCR1=(FMAX1-FMIN1)/10.
      YNCR2=(FMAX2-FMIN2)/10.
      FMINA=AMIN
      YNCRA=ANCR*10.
    5 LATCH=1
      LINE=1
      K=2
      IF(YNCR2-YNCR1)105,104,105
  104 IF(FMIN2-FMIN1)105,103,105
  103 K=1
  105 M=ISYMX
      YNCRO=YNCR1
      FMINO=FMIN1
      DO 110 J=1,K
      YCALE(1)=FMINO
      DO 17 I=1,11
   17 YCALE(I)=FMINO+YNCRO*(I-1)+5.0E-5
      WRITE(LA,14) M,YCALE
   14 FORMAT('0',2X,'(',A1,')',1X,11F10.2,/' ',13X,'*',21('....*'))
      M=ISYMY
      YNCRO=YNCR2
  110 FMINO=FMIN2
      FINC1=YNCR1/10.
      FINC2=YNCR2/10.
      FINL=YNCRA/10.
      FI=FMINA
      ICON=ISYMB
      IZERO1=(0-FMIN1)/FINC1+1.5
```

```
      IZERO2=(0-FMIN2)/FINC2+1.5
      DO 7 J=1,L
      IPLT(IZERO1)=ISYML
      IPLT(IZERO2)=ISYML
      IPT1=(ORD1(J)-FMIN1)/FINC1+1.5
      IPT2=(ORD2(J)-FMIN2)/FINC2+1.5
      IF(IPT1-105)60,64,61
   61 IPT1=105
      IPLT(IPT1)=ISYMO
      GO TO 65
   60 IF(IPT1)63,63,64
   63 IPT1=1
      IPLT(IPT1)=ISYMO
      GO TO 65
   64 IPLT(IPT1)=ISYMX
   65 IF(IPT2-105)40,44,41
   41 IPT2=105
      IPLT(IPT2)=ISYMO
      GO TO 45
   40 IF(IPT2)43,43,44
   43 IPT2=1
      IPLT(IPT2)=ISYMO
      GO TO 45
   44 IPLT(IPT2)=ISYMY
   45 IF(IPT1-IPT2)50,46,50
   46 IF(IPLT(IPT1)-ISYMO)49,50,49
   49 IPLT(IPT1)=ISYMA
   50 IPT3=(AB(J)-FMINA)/FINL+1.5
   77 IF(IPT3-LATCH)70,71,72
   71 GO TO (8,9),LINE
    8 IIFI=FI/1
      TEMFI=(FI-IIFI)*24.
      ITEMFI=TEMFI/1
      TEMFI=(TEMFI-ITEMFI)*.006
      FI=IIFI+(ITEMFI*.01)+TEMFI
      WRITE(LA,10) FI,IPLT
   10 FORMAT(' ',1XF10.2,' *',105A1)
      ICT=1
      LINE=2
      GO TO 12
    9 WRITE(LA,11) IPLT
   11 FORMAT(' ',11X,' +',105A1)
      ICT=ICT+1
      IF(ICT-6)12,13,12
   13 LINE=1
   12 IPLT(IPT1)=ISYMB
      IPLT(IPT2)=ISYMB
      FI=FMINA+LATCH*FINL+5.0E-6
      LATCH=LATCH+1
      GO TO 7
   72 GO TO (73,74),LINE
   73 WRITE(LA,75) FI
   75 FORMAT(' ',1XF10.2,' *')
      ICT=1
      LINE=2
      GO TO 79
   74 WRITE(LA,76)
   76 FORMAT(' ',11X,' +')
      ICT=ICT+1
      IF(ICT-5)79,78,79
```

```
78 LINE=1
79 FI=FMINA+LATCH*FINL+5.0E-6
   LATCH=LATCH+1
   GO TO 77
70 IF(J-1)81,80,81
81 ICON=ISYMP
80 WRITE(LA,30) ICON,IPLT
30 FORMAT(A1,13X,105A1)
   IPLT(IPT1)=ISYMB
   IPLT(IPT2)=ISYMB
 7 CONTINUE
   WRITE(LA,94)
94 FORMAT(' ',13X,'*',21('....*'))
   RETURN
   END
```

MASTER LIST OF CONSTITUENT INFORMATION - PROGRAM HAMELS

A. TIDAL CONSTITUENT CARDS

1	M2	28.9841042	-2	2	0	0	0	2	-2	0	2
2	S2	30.0000000	0	0	0	0	0	0	0	0	2
3	N2	28.4397296	-3	2	1	0	0	2	-2	0	2
4	K1	15.0410686	0	1	0	0	1	0	0	-1	1
5	M4	57.9682084	-4	4	0	0	0	4	-4	0	4
6	O1	13.9430356	-2	1	0	0	-1	2	-1	0	1
7	M6	86.9523126	-6	6	0	0	0	6	-6	0	6
9	S4	60.0000000	0	0	0	0	0	0	0	0	4
12	S6	90.0000000	0	0	0	0	0	0	0	0	6
36	M8	115.9364169	-8	8	0	0	0	8	-8	0	8
11	NU2	28.5125830	-3	4	-1	0	0	2	-2	0	2
13	MU2	27.9682084	-4	4	0	0	0	2	-2	0	2
14	2N2	27.8953548	-4	2	2	0	0	2	-2	0	2
15	OO1	16.1391017	2	1	0	0	1	-2	-1	0	1
16	LAM2	29.4556254	-1	0	1	0	2	2	-2	0	2
18	M1	14.4966939	-1	1	0	0	1	0	0	0	1
19	J1	15.5854433	1	1	-1	0	1	0	-1	0	1
25	RHO1	13.4715145	-3	3	-1	0	-1	2	-1	0	1
26	Q1	13.3986609	-3	1	1	0	-1	2	-1	0	1
27	T2	29.9589333	0	-1	0	1	0	0	0	0	2
28	R2	30.0410667	0	1	0	-1	2	0	0	0	2
29	2Q1	12.8542862	-4	1	2	0	-1	2	-1	0	1
30	P1	14.9589314	0	-1	0	0	-1	0	0	0	1
33	L2	29.5284789	-1	2	-1	0	2	0	0	0	2
35	K2	30.0821373	0	2	0	0	0	0	0	0	2
8	MK3	44.0251728	-2	3	0	0	1	2	-2	-1	3
10	MN4	57.4238338	-5	4	1	0	0	4	-4	0	4
17	S1	15.0000000	0	0	0	0	2	0	0	0	1
20	MM	0.5443747	1	0	-1	0	0	0	0	0	0
21	SSA	0.0821373	0	2	0	0	0	0	0	0	0
22	SA	0.0410686	0	1	0	0	0	0	0	0	0
23	MSF	1.0158958	2	-2	0	0	0	0	0	0	0
24	MF	1.0980331	2	0	0	0	0	-2	0	0	0
31	2SM2	31.0158958	2	-2	0	0	0	-2	2	0	2
32	M3	43.4761563	-3	3	0	0	2	3	-3	0	3
34	2MK3	42.9271398	-4	3	0	0	-1	4	-4	1	3
37	MS4	58.9841042	-2	2	0	0	0	2	-2	0	4

B. INFERENCE FORMULA CARDS

```
0.038-1.464 01 02 01       11 11
0.024-2.000 01 02 01       13 12
0.026-2.072 01 02 01       14 13
0.043 1.000 06 04 06       15 14
0.007-0.536 01 02 01       16 15
0.071-0.500 06 04 06       18 16
0.079 0.496 06 04 06       19 17
0.038-1.429 06 04 06       25 18
0.194-1.496 06 04 06       26 19
0.059-0.040 02 02 01       27 20
0.008 0.040 02 02 01       28 21
0.026-1.992 06 04 06       29 22
0.331-0.075 04 04 06       30 23
0.028-0.464 01 02 01       33 24
0.272 0.081 02 02 01       35 25
```

LEAST SQUARES INVERSE MATRICES - 29 DAY SERIES

(697 OBSERVATIONS OF HOURLY HEIGHT OF TIDE)

SINV (x1000)

2872605	0044749	0150952	0005986	-0010657	0014762	-0010024	-0006646	-0007359	-0008942
0044749	2866435	0025608	0002003	-0012694	0010597	-0011132	-0008285	-0008294	-0009744
0150952	0025608	2874475	0001416	0012372	-0007908	0010632	0008379	0008029	0009250
0005986	0002003	0001416	2868870	0009336	-0171260	0009211	0006344	0006963	0008346
-0010657	-0012694	0012372	0009336	2865584	0011398	-0010085	0043529	-0005138	-0008925
0014762	0010597	-0007908	-0171260	0011398	2880493	0010251	0008320	0007985	0009028
-0010024	-0011132	0010632	0009211	-0010085	0010251	2867082	-0013953	0043759	-0008149
-0006646	-0008285	0008379	0006344	0043529	0008320	-0013953	2866225	-0008339	-0010765
-0007359	-0008294	0008029	0006963	-0005138	0007985	0043759	-0008339	2866211	-0014721
-0008942	-0009744	0009250	0008346	-0008925	0009028	-0008149	-0010765	-0014721	2868785

DINV (x1000)

2883245	0054310	0142765	-0008060	-0000386	0008169	-0000766	0002218	0001089	-0001118
0054310	2874729	0018739	-0010656	-0002727	0006077	-0001682	0000020	0000004	-0001390
0142765	0018739	2879982	0012442	0003405	-0005063	0001875	0001124	0000564	0001305
-0008060	-0010656	0012442	2889535	-0002567	-0160830	-0000772	-0003975	-0002206	0000347
-0000386	-0002727	0003405	-0002567	2875263	0003027	-0001742	0052723	0003466	-0002402
0008169	0006077	-0005063	-0160830	0003027	2877929	0001541	0002340	0001309	0000727
-0000766	-0001682	0001875	-0000772	-0001742	0001541	2873735	-0005343	0051613	-0003725
0002218	0000020	0001124	-0003975	0052723	0002340	-0005343	2874553	-0000018	-0003363
0001089	0000004	0000564	-0002206	0003466	0001309	0051613	-0000018	2874521	-0008300
-0001118	-0001390	0001305	0000347	-0002402	0000727	-0003725	-0003363	-0008300	2870453

SAMPLE OUTPUT - PROGRAM HAMELS

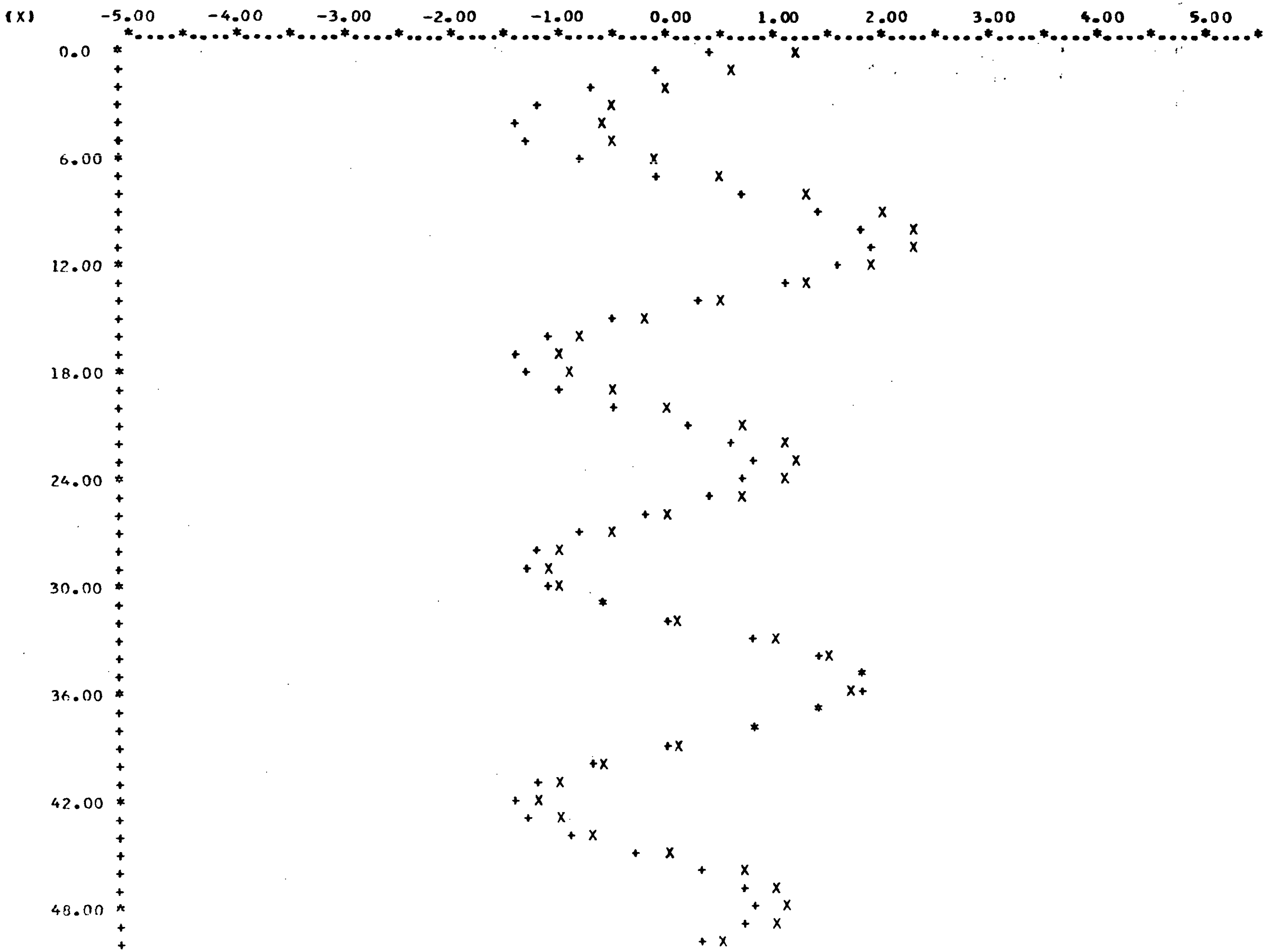
HAMELS- HARMONIC ANALYSIS METHOD OF LEAST SQUARES

STATION HRVA 3656.8 7619.9 75W YEAR 1970
29 DAY SERIES STARTING 12- 1 0.0 HRS 697 OBSERVATIONS

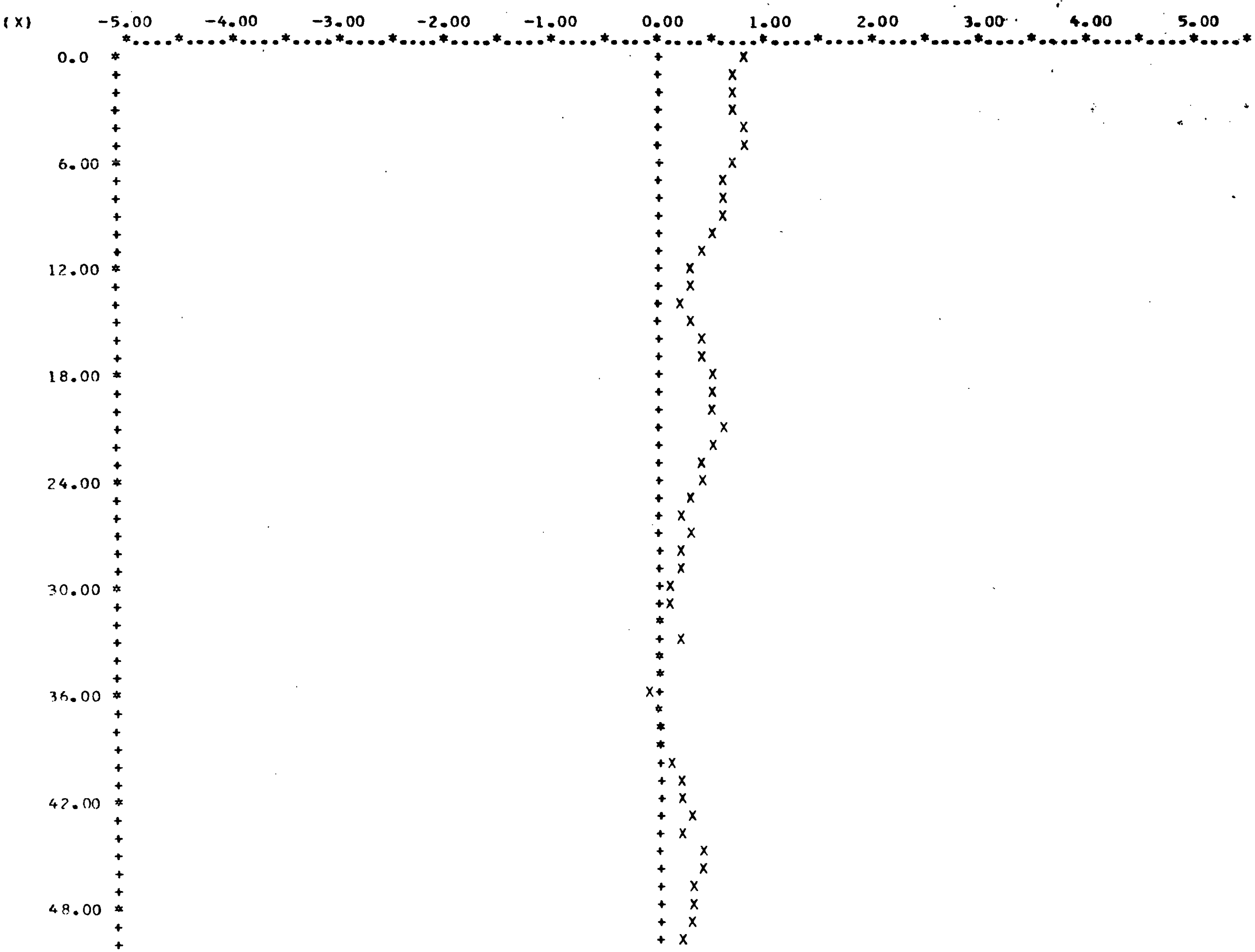
NOS NO.	CONST.	SPEED	H	KAPPA	KPRIME	KPR-K	0/0 TSS
1	M2	28.9841	1.175	251.68	259.4	7.74	66.89
2	S2	30.0000	0.155	256.44	259.1	2.66	1.24
3	N2	28.4397	0.174	238.07	248.5	10.46	1.46
4	K1	15.0410	0.176	124.93	126.1	1.13	1.92
5	M4	57.9682	0.022	341.14	356.6	15.49	0.02
6	O1	13.9430	0.145	130.54	137.2	6.62	1.46
7	M6	86.9523	0.017	272.09	295.3	23.23	0.01
9	S4	60.0000	0.026	13.01	18.3	5.33	0.03
12	S6	90.0000	0.009	97.37	105.4	7.99	0.00
36	M8	115.9364	0.007	33.14	64.1	30.97	0.00
11	NU2	28.5125	0.045	249.48	259.6	10.10	0.10
13	MU2	27.9682	0.028	246.93	259.8	12.82	0.04
14	2N2	27.8953	0.031	246.59	259.8	13.19	0.05
15	OO1	16.1391	0.006	119.32	115.0	-4.36	0.01
16	LAM2	29.4556	0.008	253.89	259.3	5.39	0.00
18	M1	14.4966	0.010	124.93	128.8	3.85	0.03
19	J1	15.5854	0.011	119.32	117.7	-1.60	0.01
25	RHO1	13.4715	0.006	113.71	122.7	8.97	0.00
26	Q1	13.3986	0.028	108.10	117.4	9.34	0.06
27	T2	29.9589	0.009	275.46	278.3	2.87	0.00
28	R2	30.0410	0.001	280.21	282.7	2.46	0.00
29	2Q1	12.8542	0.004	91.27	103.3	12.06	0.00
30	P1	14.9589	0.058	85.66	87.2	1.54	0.17
33	L2	29.5284	0.033	294.48	299.5	5.02	0.02
35	K2	30.0821	0.042	299.23	301.5	2.25	0.15

SERIES MSL 5.37 73.68

HRVA
OBSERVED HOURLY HEIGHTS(X) AND PREDICTED HOURLY HEIGHTS(+) VERSUS TIME
29-DAY SERIES STARTING AT 0.0HRS 12- 1 1970



HRVA
RESIDUAL(X)= OBSERVED MINUS PREDICTED HOURLY HEIGHTS VERSUS TIME
29-DAY SERIES STARTING AT 0.0 HRS 12- 1 1970



SAMPLE OUTPUT - PROGRAM HAMELS

APPENDIX B

Program ASTRO

Instructions for using Program ASTRO

I. Preliminary Input

As in Program HAMELS, a master list of constituent information is initially required as input to the program. The list includes 37 tidal constituents normally used by the National Ocean Survey in official U.S. tide table publications. The list is presented at the end of this appendix.

II. Station ID Card, Tidal Datum Card

The station ID card identifies the tide station, its location, and the local time meridian used. The tidal datum card names the datum used (for labeling purposes), fixes its position relative to mean sea level, and gives the number of tidal constituents to be used in the predictions.

III. Tidal Constants Cards

Each tidal constituent to be used in the predictions must be represented by a tidal constants card. These may include any combination of the 37 constituents referred to in the master list. Cards containing the sets of tidal constants (amplitude and phase) may be entered in any order as long as the correct NOS reference number appears on each card.

IV. Station Control Card

This card specifies the year and the month series in which predictions are wanted as well as their type (hourly heights or times and heights of highs and lows). Various combinations may be achieved for a given station by entering one or more control cards, followed by a blank card to terminate the program.

```
C **PROGRAM ASTRO** ASTRONOMICAL TIDE PREDICTION PROGRAM FOR
C  COMPUTATION OF HOURLY HEIGHTS AND/OR TIMES AND HEIGHTS OF HIGH AND
C  LOW TIDE AT STATIONS HAVING KNOWN TIDAL CONSTANTS
C---------------------------------------------------------
C **INPUT VARIABLE LIST**
C  XIDEN- STATION IDENTIFIER CODE (A4)
C  XLAT,XLON- STATION LATITUDE AND LONGITUDE  (2F7.1)
C  LTM- LONGITUDE OF TIME MERIDIAN USED AT STATION  (I3)
C  IYR- YEAR OF REQUIRED PREDICTIONS  (I4)
C  MS,ME- STARTING MONTH, ENDING MONTH  (2I3)
C  ITYPE- SELECTS HOURLY HEIGHTS(01) OR HIGHS AND LOWS(02)    (I3)
C  DATNM- NAME OF MODEL DATUM (I4)
C  HO- HEIGHT OF LOCAL MEAN SEA LEVEL ABOVE MODEL DATUM  (F5.3)
C  NTC- NUMBER OF TIDAL CONSTITUENTS USED  (I3)
C  NOS(I)- NATIONAL OCEAN SURVEY REFERENCE NO. FOR ITH CONSTITUENT
C  A(I,1)- SPEED OF ITH TIDAL CONSTITUENT IN DEG/MSH (F8.4)
C  A(I,J)- ORBITAL ELEMENT INDICES FOR ITH CONSTITUENT (8F3.1)
C  H(I)- MEAN AMPLITUDE OF ITH TIDAL CONSTITUENT  (F5.3)
C  XKP(I)- PHASE OF ITH CONSTITUENT ADJUSTED FOR STATION LOCATION (F5.1)
C--------------------------------------
C **CALLED VARIABLES**
C  F(I)- NODAL FACTOR REDUCING H TO YEAR OF PREDICTION
C  VOU(I)- GREENWICH EQUILIBRIUM PHASE FOR ITH CONSTITUENT
C---------------------------------------------------
C **INPUT SEQUENCE AND FORMAT**
C 1. STATION ID CARD- XIDEN,XLAT,XLON,LTM (1X,A4,2F7.1,I3)
C 2. TIDAL DATUM CARD- DATNM,HO,NTC (1X,A4,1X,F5.3,I3)
C 3. TIDAL CONSTANTS CARDS- H(I),XKP(I) (1X,F5.3,1X,F5.1)
C 4. STATION CONTROL CARD- IYR,MS,ME,ITYPE (1X,I4,2I3,4X,I3)
C 5. ADDITIONAL STATION CONTROL CARDS IF DESIRED
C 6. BLANK CARD TO TERMINATE JOB
C-------------------------------------------------
C  SUBROUTINES REQUIRED- ORBEL,CONSUM
C-------------------------------------------------
      DIMENSION F(37),H(37),VOU(37),XKP(37),HH(366,24),XLW(366,2)
      DIMENSION JS(12),JE(12),ND(367),THW(366,2),HW(366,2),TLW(366,2)
      DIMENSION A(37,9),DMTL(366),DMN(366),NOS(37),KNOS(37)
C---------------------------------------------------
C READ MASTER LIST OF CONSTITUENT SPEEDS, ORBITAL ELEMENT INDICES
C---------------------------------------------------
      READ(5,44)(NOS(I),(A(I,J),J=1,9),I=1,37)
   44 FORMAT(I2,6X,F8.4,4X,8F3.1)
C----------------------------------
C READ STATION ID CARD
C READ TIDAL DATUM CARD
C READ TIDAL CONSTANTS CARDS
C----------------------------------
      READ(5,1) XIDEN,XLAT,XLON,LTM
    1 FORMAT(1X,A4,2F7.1,1X,I2)
      READ(5,3) DATNM,HO,NTC
    3 FORMAT(1X,A4,1X,F5.3,I3)
      READ(5,4) (KNOS(J),H(J),XKP(J),J=1,NTC)
    4 FORMAT(I2,2X,F5.3,2X,F5.1)
C-------------------------------
C READ STATION CONTROL CARD
C-------------------------------
  200 READ(5,2) IYR,MS,ME,ITYPE
    2 FORMAT(1X,I4,2I3,4X,I3)
```

```
      IF(IYR.EQ.0)GO TO 101
C-----------------------------------------------------
C COMPUTE JULIAN DATES STARTING AND ENDING EACH MONTH
C ADJUST FOR LEAP YEAR, NON-LEAP YEAR
C-----------------------------------------------------
      DEGRAD=57.29577951
      ZIP=0.1
      LY=1
      LEAP=(2000.-IYR)/4.
      XLEAP=(2000.-IYR)/4.-LEAP
      IF(XLEAP.GT.ZIP)LY=0
      JS(1)=1
      JE(1)=31
      JS(2)=32
      JE(2)=59+LY
      K=1
      DO 5 I=3,12
      JS(I)=JE(I-1)+1
      JE(I)=JS(I)+29+K
      K=1-K
      IF(I.EQ.7)K=1
    5 CONTINUE
      ZM=12.*(LY+1)
      CALL ORBEL(IYR,1,183,0.0,ZM,NOS,A,VOU,F)
      IF(ITYPE-1)6,6,7
C-----------------------
C COMPUTE HOURLY HEIGHTS
C-----------------------
    6 DO 8 K=MS,ME
      J1=JS(K)
      J2=JE(K)
      ND(J1)=1
      DO 9 I=J1,J2
      ND(I+1)=ND(I)+1
      DO 10 J=1,24
      JT=((I-1)*24)+J-1
      HH(I,J)=HO
      DO 11 M=1,NTC
      L=KNOS(M)
      ARG=(A(L,1)*JT+VOU(L)-XKP(M))/DEGRAD
      HH(I,J)=HH(I,J)+F(L)*H(M)*COS(ARG)
   11 CONTINUE
   10 CONTINUE
    9 CONTINUE
C-----------------------
C PRINT HOURLY HEIGHTS
C-----------------------
      WRITE(6,12) XIDEN,XLAT,XLON,LTM,IYR,K,NTC
   12 FORMAT('1',/1X,A4,2F7.1,2X,'T.M.',I3,'W',/1X,'YEAR ',I4,2X,'MONTH'
     1,I3,2X,'NO. CONST. ',I2)
      WRITE(6,13) DATNM
   13 FORMAT(//1X,'PREDICTED HOURLY HEIGHT OF TIDE IN FEET ABOVE ',A4,'
     1DATUM')
      IF(HO.EQ.0.0)GO TO 115
      WRITE(6,94) DATNM,HO
   94 FORMAT(/1X,A4,' DATUM IS ',F5.3,' FEET BELOW MSL')
  115 J3=J1+22
      WRITE(6,39)(ND(I),I=J1,J3)
```

```
   39 FORMAT(/1X,'D/HR    0',23I4)
      WRITE(6,14)(ND(I),(HH(I,J),J=1,24),I=J1,J2)
   14 FORMAT(31(1X,I2,2X,24F4.1/))
    8 CONTINUE
      GO TO 200
C-----------------------------------------
C COMPUTE TIMES AND HEIGHTS OF HIGHS AND LOWS
C-----------------------------------------
    7 DO 15 K=MS,ME
      J1=JS(K)
      J2=JE(K)
      ND(J1)=1
      DO 16 I=J1,J2
      THW(I,2)=1000000.
      TLW(I,2)=1000000.
      HW(I,2)=1000000.
      XLW(I,2)=1000000.
      N=1
      M=1
      NSKIP=1
      ND(I+1)=ND(I)+1
      XJT=(I-1)*24
      CALL CONSUM(KNOS,XJT,A,VOU,XKP,F,HO,H,NTC,SUM,DSUM)
      SUMO=SUM
      DSUMO=DSUM
      IF(DSUMO)20,17,20
   17 XJT=XJT+1.
      CALL CONSUM(KNOS,XJT,A,VOU,XKP,F,HO,H,NTC,SUM,DSUM)
      IF(DSUMO-DSUM)19,20,18
   18 THW(I,N)=0.0
      HW(I,N)=SUMO
      DSUMO=-1.
      N=N+1
      NSKIP=5
      GO TO 20
   19 TLW(I,M)=0.0
      XLW(I,M)=SUMO
      DSUMO=1.
      M=M+1
      NSKIP=5
   20 NQ=0
      DO 25 J=1,24
      NSKIP=NSKIP-1
      IF(NSKIP.GT.0)GO TO 25
      XJT=((I-1)*24)+J
      CALL CONSUM(KNOS,XJT,A,VOU,XKP,F,HO,H,NTC,SUM,DSUM)
      SAVE=DSUMO
      IF(DSUM*DSUMO)23,22,28
   22 IF(J.EQ.24)GO TO 25
      IF(SAVE-DSUM)27,26,26
   26 THW(I,N)=J
      HW(I,N)=SUM
      DSUMO=-1.
      N=N+1
      GO TO 21
   27 TLW(I,M)=J
      XLW(I,M)=SUM
      DSUMO=1.
```

```
      M=M+1
      GO TO 21
   23 NQ=NQ+1
      XJ=(J-1)+0.1*NQ
      XJT=((I-1)*24)+XJ
      CALL CONSUM(KNOS,XJT,A,VOU,XKP,F,HO,H,NTC,SUM,DSUM)
      IF(DSUM*DSUMO)30,30,28
   30 DIFF=ABS(DSUM)-ABS(DSUMO)
      IF(DIFF)31,29,29
   29 SUM=SUMO
      XJ=XJ-0.1
   31 IF(SAVE-DSUM)33,32,32
   32 THW(I,N)=XJ
      HW(I,N)=SUM
      DSUMO=-1.
      N=N+1
      GO TO 21
   33 TLW(I,M)=XJ
      XLW(I,M)=SUM
      DSUMO=1.
      M=M+1
   21 NQ=0
      NSKIP=5
      GO TO 25
   28 DSUMO=DSUM
      SUMO=SUM
      IF(NQ.GT.0)GO TO 23
   25 CONTINUE
   16 CONTINUE
C---------------------------------------------
C PRINT TIMES AND HEIGHTS OF HIGHS AND LOWS
C---------------------------------------------
   55 WRITE(6,12) XIDEN,XLAT,XLON,LTM,IYR,K,NTC
      WRITE(6,34) DATNM
   34 FORMAT(//1X,'PREDICTED TIMES AND HEIGHTS OF HIGH AND LOW TIDE',/1X
     1,'TIME IN HRS',5X,'HEIGHT IN FEET ABOVE ',A4,' DATUM')
      IF(HO.EQ.0.0)GO TO 336
      WRITE(6,337) DATNM,HO
  337 FORMAT(/1X,A4,' DATUM IS ',F5.3,' FEET BELOW MSL')
  336 WRITE(6,338)
  338 FORMAT(/1X,'DAY',3X,'THW',3X,'HW',2X,'TLW',3X,'LW (AM)',2X,'THW',3
     1X,'HW',2X,'TLW',3X,'LW (PM)')
      WRITE(6,35)(ND(I),(THW(I,J),HW(I,J),TLW(I,J),XLW(I,J),J=1,2),I=J1,
     1J2)
   35 FORMAT(31(2X,I2,1X,4(1X,F4.1),5X,4(1X,F4.1)/))
   15 CONTINUE
      GO TO 200
  101 STOP
      END
```

```
      SUBROUTINE CONSUM(KNOS,XJT,A,VOU,XKP,F,HO,H,NTC,SUM,DSUM)
      DIMENSION A(37,9),VOU(37),F(37),H(37),XKP(37),KNOS(37)
      DEGRAD=57.29577951
      SUM=HO
      DSUM=0.0
      DO 1 J=1,NTC
      I=KNOS(J)
      ARG=(A(I,1)*XJT+VOU(I)-XKP(J))/DEGRAD
      SUM=SUM+F(I)*H(J)*COS(ARG)
      DSUM=DSUM-A(I,1)*F(I)*H(J)*SIN(ARG)
    1 CONTINUE
      RETURN
      END
```

MASTER LIST OF CONSTITUENT INFORMATION - PROGRAM ASTRO

1	M2	28.9841042	-2	2	0	0	0	2	-2	0	2
2	S2	30.0000000	0	0	0	0	0	0	0	0	2
3	N2	28.4397296	-3	2	1	0	0	2	-2	0	2
4	K1	15.0410686	0	1	0	0	1	0	0	-1	1
5	M4	57.9682084	-4	4	0	0	0	4	-4	0	4
6	O1	13.9430356	-2	1	0	0	-1	2	-1	0	1
7	M6	86.9523126	-6	6	0	0	0	6	-6	0	6
8	MK3	44.0251728	-2	3	0	0	1	2	-2	-1	3
9	S4	60.0000000	0	0	0	0	0	0	0	0	4
10	MN4	57.4238338	-5	4	1	0	0	4	-4	0	4
11	NU2	28.5125830	-3	4	-1	0	0	2	-2	0	2
12	S6	90.0000000	0	0	0	0	0	0	0	0	6
13	MU2	27.9682084	-4	4	0	0	0	2	-2	0	2
14	2N2	27.8953548	-4	2	2	0	0	2	-2	0	2
15	OO1	16.1391017	2	1	0	0	1	-2	-1	0	1
16	LAM2	29.4556254	-1	0	1	0	2	2	-2	0	2
17	S1	15.0000000	0	0	0	0	2	0	0	0	1
18	M1	14.4966939	-1	1	0	0	1	0	0	0	1
19	J1	15.5854433	1	1	-1	0	1	0	-1	0	1
20	MM	0.5443747	1	0	-1	0	0	0	0	0	0
21	SSA	0.0821373	0	2	0	0	0	0	0	0	0
22	SA	0.0410686	0	1	0	0	0	0	0	0	0
23	MSF	1.0158958	2	-2	0	0	0	0	0	0	0
24	MF	1.0980331	2	0	0	0	0	-2	0	0	0
25	RHO1	13.4715145	-3	3	-1	0	-1	2	-1	0	1
26	Q1	13.3986609	-3	1	1	0	-1	2	-1	0	1
27	T2	29.9589333	0	-1	0	1	0	0	0	0	2
28	R2	30.0410667	0	1	0	-1	2	0	0	0	2
29	2Q1	12.8542862	-4	1	2	0	-1	2	-1	0	1
30	P1	14.9589314	0	-1	0	0	-1	0	0	0	1
31	2SM2	31.0158958	2	-2	0	0	0	-2	2	0	2
32	M3	43.4761563	-3	3	0	0	2	3	-3	0	3
33	L2	29.5284789	-1	2	-1	0	2	0	0	0	2
34	2MK3	42.9271398	-4	3	0	0	-1	4	-4	1	3
35	K2	30.0821373	0	2	0	0	0	0	0	0	2
36	M8	115.9364169	-8	8	0	0	0	8	-8	0	8
37	MS4	58.9841042	-2	2	0	0	0	2	-2	0	4

SAMPLE OUTPUT - PROGRAM ASTRO

HRVA 3656.8 7619.9 T.M. 75W
YEAR 1970 MONTH 12 NO. CONST. 25

PREDICTED TIMES AND HEIGHTS OF HIGH AND LOW TIDE
TIME IN HRS HEIGHT IN FEET ABOVE MSL DATUM

DAY	THW	HW	TLW	LW (AM)	THW	HW	TLW	LW (PM)
1	10.5	1.6	3.9	-1.4	22.8	0.8	16.8	-1.2
2	11.4	1.6	4.7	-1.3	23.7	0.8	17.8	-1.2
3	12.2	1.5	5.7	-1.2	****	****	18.7	-1.1
4	0.6	0.9	6.8	-1.1	13.0	1.4	19.7	-1.2
5	1.5	0.9	7.8	-1.1	13.9	1.2	20.5	-1.2
6	2.7	1.0	8.9	-1.1	15.1	1.1	21.5	-1.2
7	4.0	1.1	10.0	-1.1	16.3	1.0	22.3	-1.3
8	5.0	1.3	11.0	-1.1	17.2	1.0	23.1	-1.3
9	5.8	1.5	12.0	-1.1	18.0	0.9	24.0	-1.3
10	6.6	1.6	13.0	-1.2	18.8	0.9	****	****
11	7.4	1.6	0.8	-1.4	19.6	0.9	13.9	-1.2
12	8.2	1.6	1.7	-1.4	20.4	0.9	14.7	-1.3
13	8.9	1.6	2.4	-1.4	21.1	0.9	15.3	-1.3
14	9.6	1.6	3.1	-1.4	21.9	0.8	15.9	-1.2
15	10.4	1.5	3.8	-1.3	22.7	0.8	16.6	-1.2
16	11.1	1.4	4.5	-1.2	23.4	0.8	17.4	-1.1
17	11.9	1.3	5.3	-1.1	****	****	18.2	-1.0
18	0.2	0.8	6.2	-1.0	12.5	1.2	19.0	-1.0
19	0.9	0.8	7.1	-0.9	13.2	1.1	19.8	-1.0
20	1.6	0.8	8.0	-0.9	13.9	0.9	20.5	-1.0
21	2.6	0.8	8.9	-0.9	14.9	0.8	21.3	-1.0
22	3.8	0.9	9.9	-0.9	16.1	0.7	22.1	-1.0
23	4.8	1.1	10.8	-0.9	17.0	0.8	22.9	-1.1
24	5.5	1.2	11.7	-1.0	17.7	0.8	23.7	-1.1
25	6.2	1.4	12.7	-1.0	18.5	0.8	****	****
26	7.0	1.5	0.5	-1.2	19.3	0.8	13.6	-1.1
27	7.8	1.6	1.4	-1.3	20.1	0.9	14.4	-1.3
28	8.6	1.7	2.2	-1.4	20.9	0.9	15.1	-1.3
29	9.4	1.7	3.0	-1.5	21.7	1.0	15.8	-1.4
30	10.3	1.7	3.7	-1.5	22.6	1.0	16.6	-1.4
31	11.1	1.7	4.5	-1.5	23.5	1.1	17.4	-1.3

SAMPLE OUTPUT - PROGRAM ASTRO

HRVA 3656.8 7619.9 T.M. 75W
YEAR 1970 MONTH 12 NO. CONST. 25

PREDICTED HOURLY HEIGHT OF TIDE IN FFET ABOVE MSL DATUM

D/HR	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	0.4	-0.1	-0.7	-1.2	-1.4	-1.2	-0.7	-0.0	0.7	1.3	1.6	1.6	1.3	0.7	0.1	-0.6	-1.1	-1.2	-1.0	-0.7	-0.1	0.4	0.8	0.8
2	0.7	0.3	-0.3	-0.8	-1.2	-1.3	-1.0	-0.6	0.0	0.7	1.3	1.5	1.5	1.2	0.6	-0.1	-0.7	-1.1	-1.2	-1.0	-0.6	-0.1	0.5	0.8
3	0.8	0.6	0.2	-0.3	-0.8	-1.1	-1.2	-1.0	-0.5	0.1	0.8	1.3	1.5	1.4	1.0	0.5	-0.2	-0.7	-1.1	-1.1	-0.9	-0.5	-0.0	0.5
4	0.8	0.8	0.6	0.2	-0.3	-0.7	-1.1	-1.1	-0.9	-0.5	0.1	0.7	1.2	1.4	1.2	0.9	0.4	-0.2	-0.7	-1.1	-1.1	-0.9	-0.5	0.1
5	0.6	0.9	0.9	0.7	0.3	-0.1	-0.6	-1.0	-1.1	-0.9	-0.5	0.1	0.7	1.1	1.2	1.1	0.8	0.3	-0.2	-0.8	-1.1	-1.1	-0.9	-0.5
6	0.1	0.6	0.9	0.9	0.8	0.5	-0.0	-0.5	-0.9	-1.1	-0.9	-0.5	0.0	0.6	0.9	1.1	1.0	0.7	0.2	-0.3	-0.9	-1.2	-1.2	-0.9
7	-0.4	0.2	0.7	1.0	1.1	1.0	0.6	0.1	-0.5	-0.9	-1.1	-0.9	-0.6	-0.1	0.5	0.8	1.0	0.9	0.6	0.1	-0.5	-1.0	-1.2	-1.2
8	-0.9	-0.3	0.3	0.8	1.2	1.3	1.1	0.7	0.1	-0.5	-0.9	-1.1	-1.0	-0.6	-0.1	0.4	0.8	1.0	0.9	0.5	0.0	-0.6	-1.1	-1.3
9	-1.2	-0.8	-0.3	0.4	1.0	1.4	1.5	1.2	0.8	0.1	-0.5	-1.0	-1.1	-1.0	-0.7	-0.2	0.4	0.8	0.9	0.8	0.4	-0.1	-0.7	-1.2
10	-1.3	-1.2	-0.8	-0.2	0.6	1.2	1.5	1.5	1.3	0.7	0.1	-0.6	-1.0	-1.2	-1.1	-0.7	-0.1	0.5	0.8	0.9	0.7	0.3	-0.3	-0.8
11	-1.2	-1.3	-1.2	-0.7	0.0	0.8	1.3	1.6	1.6	1.2	0.6	-0.1	-0.7	-1.1	-1.2	-1.1	-0.6	0.0	0.5	0.8	0.9	0.6	0.2	-0.4
12	-0.9	-1.3	-1.4	-1.1	-0.6	0.2	1.0	1.4	1.6	1.5	1.1	0.5	-0.2	-0.8	-1.2	-1.3	-1.0	-0.4	0.2	0.6	0.9	0.8	0.5	0.1
13	-0.5	-1.1	-1.4	-1.4	-1.0	-0.3	0.4	1.1	1.5	1.6	1.4	1.0	0.3	-0.4	-1.0	-1.3	-1.2	-0.8	-0.3	0.3	0.7	0.9	0.8	0.4
14	-0.1	-0.6	-1.2	-1.4	-1.2	-0.8	-0.1	0.6	1.2	1.5	1.5	1.3	0.8	0.2	-0.6	-1.1	-1.2	-1.0	-0.7	-0.1	0.4	0.7	0.8	0.7
15	0.3	-0.2	-0.8	-1.2	-1.3	-1.1	-0.6	0.0	0.7	1.2	1.5	1.4	1.1	0.6	-0.1	-0.7	-1.1	-1.1	-0.9	-0.5	0.0	0.5	0.8	0.8
16	0.6	0.2	-0.3	-0.9	-1.2	-1.2	-0.9	-0.4	0.2	0.8	1.2	1.4	1.3	1.0	0.4	-0.3	-0.8	-1.1	-1.0	-0.8	-0.4	0.2	0.6	0.8
17	0.8	0.5	0.1	-0.4	-0.9	-1.1	-1.1	-0.8	-0.3	0.3	0.9	1.2	1.3	1.1	0.7	0.1	-0.4	-0.8	-1.0	-1.0	-0.7	-0.2	0.3	0.6
18	0.8	0.7	0.4	-0.0	-0.5	-0.8	-1.0	-0.9	-0.6	-0.2	0.4	0.9	1.2	1.2	0.9	0.5	-0.0	-0.5	-0.9	-1.0	-0.9	-0.6	-0.1	0.4
19	0.7	0.8	0.7	0.4	-0.0	-0.4	-0.8	-0.9	-0.8	-0.5	-0.1	0.4	0.9	1.1	1.0	0.7	0.3	-0.1	-0.6	-0.9	-1.0	-0.8	-0.5	-0.0
20	0.5	0.8	0.8	0.7	0.4	0.0	-0.4	-0.7	-0.9	-0.8	-0.5	-0.0	0.5	0.8	0.9	0.8	0.6	0.2	-0.2	-0.7	-0.9	-1.0	-0.7	-0.4
21	0.1	0.6	0.8	0.8	0.7	0.4	0.1	-0.4	-0.7	-0.9	-0.7	-0.4	-0.0	0.5	0.7	0.8	0.7	0.5	0.1	-0.3	-0.8	-1.0	-0.9	-0.7
22	-0.3	0.2	0.7	0.9	0.9	0.8	0.5	0.1	-0.4	-0.8	-0.9	-0.7	-0.4	-0.0	0.4	0.7	0.7	0.7	0.4	0.0	-0.5	-0.9	-1.0	-0.9
23	-0.6	-0.2	0.3	0.8	1.0	1.1	0.9	0.5	0.0	-0.5	-0.8	-0.9	-0.7	-0.4	-0.0	0.4	0.6	0.8	0.6	0.3	-0.1	-0.6	-1.0	-1.1
24	-0.9	-0.5	-0.1	0.5	0.9	1.2	1.2	0.9	0.5	-0.0	-0.5	-0.9	-0.9	-0.8	-0.5	-0.0	0.4	0.7	0.8	0.6	0.2	-0.2	-0.7	-1.1
25	-1.1	-0.9	-0.5	0.0	0.7	1.1	1.4	1.3	1.0	0.5	-0.1	-0.7	-1.0	-1.0	-0.9	-0.5	0.0	0.5	0.8	0.8	0.5	0.1	-0.4	-0.9
26	-1.2	-1.2	-0.9	-0.5	0.2	0.9	1.3	1.5	1.4	1.0	0.4	-0.2	-0.8	-1.1	-1.1	-0.9	-0.4	0.2	0.6	0.8	0.8	0.5	0.0	-0.5
27	-1.0	-1.3	-1.3	-0.9	-0.3	0.4	1.1	1.5	1.6	1.4	0.9	0.3	-0.4	-0.9	-1.2	-1.2	-0.9	-0.3	0.3	0.7	0.9	0.8	0.4	-0.1
28	-0.6	-1.1	-1.4	-1.3	-0.9	-0.2	0.6	1.3	1.6	1.7	1.4	0.9	0.2	-0.5	-1.1	-1.3	-1.2	-0.8	-0.2	0.4	0.8	0.9	0.8	0.4
29	-0.2	-0.8	-1.3	-1.5	-1.3	-0.8	-0.0	0.8	1.4	1.7	1.7	1.4	0.8	0.0	-0.7	-1.2	-1.4	-1.1	-0.7	-0.1	0.5	0.9	1.0	0.8
30	0.3	-0.3	-0.9	-1.4	-1.5	-1.2	-0.6	0.1	0.9	1.5	1.7	1.6	1.3	0.6	-0.2	-0.9	-1.3	-1.3	-1.1	-0.6	0.1	0.6	1.0	1.0
31	0.8	0.3	-0.4	-1.0	-1.4	-1.4	-1.1	-0.5	0.2	0.9	1.5	1.7	1.5	1.1	0.4	-0.3	-1.0	-1.3	-1.3	-1.0	-0.5	0.2	0.7	1.0

APPENDIX C

Program LESCO

Instructions for using Program LESCO

I. Purpose

Program LESCO is used to compute the inverse of the least squares coefficient matrices required by Program HAMELS. The inverse matrices may be computed for an observational series of virtually any length but the program is limited to a maximum of ten tidal constituents (two 10 X 10 matrices) in present form. If more than ten constituents are needed, both the main program and subroutines INVER and MAPROD must be modified accordingly.

II. Input Required

The required information must be entered on cards in the following order:

A. Data information card (I3, I4, I3)

M - number of tidal constituents (I3)
N - total number of observations (I4)
NPD - number of observations per day (I3)

B. Constituent Cards (A3, 2X, F13.10)

C - constituent symbol (A3)
W - constituent speed (F13.10)

Note: The order of the constituent cards determines the order in which the constituents must appear in Program HAMELS.

III. Output

A listing of the cosine summation (S) and sine summation (D) matrices are printed along with their respective inverse matrices and a set of verification (identity) matrices obtained as the products S x SINV and D x DINV.

The inverse matrices are multiplied by a factor of 1000 to eliminate extraneous leading zeroes and allow retention of significant figures in the lesser elements of the array output. The remainder of the output consists of a deck of punched cards containing

SINV x 1000 (10F8.6)

DINV x 1000 (10F8.6)

```
C-----------------------------------------------------------------------
C PROGRAM LESCO - GENERATES COEFFICIENT MATRICIES AND COMPUTES THE
C INVERSE MATRICIES FOR THE LEAST SQUARES METHOD OF HARMONIC
C ANALYSIS.  THE INVERSE MATRICIES ARE MULTIPLIED BY 1000 DUE TO THE
C SMALL VALUES OF THE NON-PIVOT ELEMENTS.
C INPUT REQUIRED
C 1. MASTER LIST OF CONSTITUENT AND INPUT DATA INFORMATION
C     M= ORDER OF COEFFICIENT MATRIX (MAXIMUM = 10)
C     N= NUMBER OF OBSERVATIONS IN TIDAL SERIES ( N MUST BE ODD)
C     NPD= NO. OBSERVATIONS PER DAY
C     C(I)= CONSTITUENT SYMBOL FOR ITH CONSTITUENT
C     W(I)= ITH CONSTITUENT SPEED (DEG/MSH)
C 2. SUBROUTINES REQUIRED - INVERS, MAPROD
C-----------------------------------------------------------------------
      DOUBLE PRECISION W,S,D,SINV,DINV,SSINV,DDINV,WS,WSN,SFS,WD,WDN,SFD
      DIMENSION W(10),C(10),S(10,10),D(10,10),SINV(10,10)
      DIMENSION DINV(10,10),SSINV(10,10),DDINV(10,10)
      DATA IR/5/,IW/6/,IP/7/
C---------------------------------------------------------
C READ MATRIX ORDER(M), SERIES LENTH
C READ M,N,NPD, CONSTITUENT SYMBOLS AND CONSTITUENT SPEEDS
C---------------------------------------------------------
      READ(IR,1) M,N,NPD
    1 FORMAT(I3,I4,I3)
      NTEST=N/2
      NTEST= N-NTEST*2
      IF(NTEST)101,101,30
  101 WRITE(IW,102)
  102 FORMAT('1',//1X,'N EVEN, PROGRAM CANCELLED')
      GO TO 100
   30 READ(IR,2) (C(I),W(I),I=1,M)
    2 FORMAT(A3,2X,F13.10)
      ND=N/NPD
      WRITE(IW,18) ND,N,M
   18 FORMAT('1',/1X,'MATRIX COEFFICIENTS - PROGRAM LESCO',//1X,'RECORD
     1LENGTH- ',I3,'DAYS',/1X,'TOTAL NO. OBS.- ',I4,/1X,
     2    'NO. CONSTITUENTS- ',I3,//1X,'CONSTITUENT SPEEDS (DEG/MSH)')
      WRITE(IW,19)(C(I),W(I),I=1,M)
   19 FORMAT(/1X,A4,2X,F8.4)
C------------------------------------------
C GENERATE COEFFICIENT MATRIX
C------------------------------------------
      DO 3 I=1,M
      DO 3 J=1,M
C
      IF (I-J) 45,45,43
   43 S(I,J)=S(J,I)
      D(I,J)=D(J,I)
```

```
         GO TO 3
      45 WS=(W(I)+W(J))/114.591559
         WSN=WS*N
         SFS=DSIN(WSN)/DSIN(WS)
         IF(I-J)5,4,5
       4 S(I,J)=(N+SFS)/2.
         D(I,J)=(N-SFS)/2.
         GO TO 3
       5 WD=(W(I)-W(J))/114.591559
         WDN=WD*N
         SFD=DSIN(WDN)/DSIN(WD)
         S(I,J)=(SFD+SFS)/2.
         D(I,J)=(SFD-SFS)/2.
       3 CONTINUE
C-----------------------------------------------------------------------
C WRITE MATRIX, INVERSE MATRIX, MATRIX PRODUCTS
C-----------------------------------------------------------------------
         WRITE(IW,6)
       6 FORMAT('1',//1X,'S',/1X)
         DO 30 I=1,M
      80 WRITE(IW,7)(S(I,J),J=1,M)
         WRITE(IW,8)
       7 FORMAT(' ',10F12.6)
       8 FORMAT('1',//1X,'D',/1X)
         DO 82 I=1,M
      82 WRITE(IW,7)(D(I,J),J=1,M)
         CALL INVERS(M,S,SINV)
         IF (M.EQ.0) GO TO 100
         CALL INVERS(M,D,DINV)
         IF (M.EQ.0) GO TO 100
         WRITE(IW,9)
       9 FORMAT('1',//1X,'SINV X 1000',/1X)
         DO 84 I=1,M
      84 WRITE(IW,7) (SINV(I,J),J=1,M)
         WRITE(IW,10)
      10 FORMAT('1',//1X,'DINV X 1000',/1X)
         DO 86 I=1,M
      86 WRITE(IW,7) (DINV(I,J),J=1,M)
         DO 64 I=1,M
      64 WRITE(IP,66) (SINV(I,J),J=1,M)
      66 FORMAT(10F8.6)
         DO 70 I=1,M
      70 WRITE(IP,66) (DINV(I,J),J=1,M)
         WRITE(IW,20)
      20 FORMAT('1',//1X,'CHECK S*SINV',/1X)
         CALL MAPROD(M,S,SINV)
         WRITE(IW,21)
      21 FORMAT('1',//1X,'CHECK D*DINV',/1X)
         CALL MAPROD(M,D,DINV)
     100 STOP
         END
```

```
      SUBROUTINE INVERS(M,F,A)
C-----------------------------------------------------------------------
C SUBROUTINE INVERS - COMPUTES THE INVERSE OF THE M X M MATRIX IN
C PLACE.  THE METHOD IS GAUSS-JORDEN ELIMINATION USING MAXIMUM PIVOT
C STRATEGY.  IF THE PIVOT IS LESS THAN .00000001, THE PROGRAM IS
C TERMINATED.
C 1. INPUT VARIABLES
C       M - ORDER OF COEFFICIENT MATRIX
C       F - COEFFICIENT MATRIX
C 2. OUTPUT VARIABLES
C       A - INVERSE OF COEFFICIENT MATRIX
C-----------------------------------------------------------------------
      REAL*8 A,F,PIVOT
      DIMENSION IROW(10),JCOL(10),JORD(10),Y(13),A(10,10),F(13,10)
      DATA IR/5/,IW/6/,IP/7/
      EPS=.00000001
      DO 2 I=1,M
      DO 2 J=1,M
    2 A(I,J)=F(I,J)
      IF (M.LE.10) GO TO 5
      WRITE(IW,200)
      M=0
      RETURN                                                            660
    5 DO 18 K=1,M
      KM1=K-1                                                           710
      PIVOT=0.                                                          750
      DO 11 I=1,M
      DO 11 J=1,M
      IF (K.EQ.1) GO TO 9                                               800
      DO 8 ISCAN=1,KM1                                                  810
      DO 8 JSCAN=1,KM1                                                  820
      IF (I.EQ.IROW(ISCAN)) GO TO 11                                    830
      IF (J.EQ.JCOL(JSCAN)) GO TO 11                                    840
    8 CONTINUE                                                          850
    9 IF (DABS(A(I,J)).LE.DABS(PIVOT)) GO TO 11                         860
      PIVOT=A(I,J)                                                      870
      IROW(K)=I                                                         880
      JCOL(K)=J                                                         890
   11 CONTINUE                                                          900
      IF (DABS(PIVOT).GT.EPS) GO TO 13                                  930
      WRITE(IW,205)
      M=0
      RETURN                                                            960
   13 IROWK=IROW(K)                                                     990
      JCOLK=JCOL(K)                                                    1000
      DO 14 J=1,M
   14 A(IR3WK,J)=A(IROWK,J)/PIVOT                                      1050
      A(IROWK,JCOLK)=1./PIVOT                                          1080
```

```
      DO 18 I=1,M
      AIJCK=A(I,JCOLK)
      IF (I.EQ.IROWK) GO TO 18
      A(I,JCOLK)=-AIJCK/PIVOT
      DO 17 J=1,M
   17 IF (J.NE.JCOLK) A(I,J)=A(I,J)-AIJCK*A(IROWK,J)
   18 CONTINUE
      DO 20 I=1,M
      IROWI=IROW(I)
      JCOLI=JCOL(I)
   20 JORD(IROWI)=JCOLI
      DO 28 J=1,M
      DO 27 I=1,M
      IROWI=IROW(I)
      JCOLI=JCOL(I)
   27 Y(JCOLI)=A(IROWI,J)
      DO 28 I=1,M
   28 A(I,J)=Y(I)
      DO 30 I=1,M
      DO 29 J=1,M
      IROWJ=IROW(J)
      JCOLJ=JCOL(J)
   29 Y(IROWJ)=A(I,JCOLJ)
      DO 30 J=1,M
   30 A(I,J)=Y(J)*1000.
      RETURN
  200 FORMAT('0','N TOO BIG')
  205 FORMAT('0','MAGNITUDE OF PIVOT IS LESS THAN EPS')
      END
```

```
      SUBROUTINE MAPROD(M,A,AINV)
C-----------------------------------------------------------------------
C SUBROUTINE MAPROD - MULTIPLIES THE COEFFICIENT MATRIX BY ITS INVERSE
C MATRIX TO INSURE THE PRODUCT IS THE IDENTITY MATRIX.
C 1. INPUT VARIABLES
C      M - ORDER OF COEFFICIENT MATRIX
C      A - COEFFICIENT MATRIX
C      AINV - INVERSE OF COEFFICIENT MATRIX
C 2. OUTPUT VARIABLES
C      AAINV - PRODUCT OF A AND AINV (THIS SHOULD BE THE IDENTITY MATRIX)
C-----------------------------------------------------------------------
      DOUBLE PRECISION A,AINV,AAINV
      DIMENSION A(10,10),AINV(10,10),AAINV(10,10)
      DATA IR/5/,IW/6/,IP/7/
      DO 10 I=1,M
      DO 10 J=1,M
      AAINV(I,J)=0.
      DO 10 K=1,M
      AAINV(I,J)=AAINV(I,J)+A(I,K)*AINV(K,J)/1000.
   10 CONTINUE
      DO 15 I=1,M
   15 WRITE(IW,11)(AAINV(I,J),J=1,M)
   11 FORMAT('0',10F12.7)
      RETURN
      END
```