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# **Simulated performance of catch curve methods for estimating total mortality rate**

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**Data Report 60**

**October 2012**



## **Introduction**

This document has been issued as VIMS Data Report 60 and provides additional simulation results for Smith et al. (2012) published in the North American Journal of Fisheries Management.

Catch curve methods are a basic tool of population dynamics for estimating total mortality rate from age composition. There are a number of methodological issues which remain unresolved. Smith et al. (2012) attempts to provide guidelines on the use of these methods based on extensive Monte Carlo simulations. This report presents additional simulation results to supplement the results in the journal article. The estimators, evaluation criteria, simulation procedures, and conditions simulated are given in the journal article.

## **Reference**

- Chapman, D. G., and D. S. Robson. 1960. The analysis of a catch curve. *Biometrics* 16:354-368.
- Smith, M. W., A. Y. Then, C. Wor, G. Ralph, K. H. Pollock and J. M. Hoenig. (2012). Recommendations for catch curve analysis. *North American Journal of Fisheries Management*, 32:5, 956-967.

## Heincke's estimator

The following section contains plots of the percent bias (%BIAS) and the percent root mean square error (%RMSE) of the estimated total mortality rate ( $Z$ ) and the estimated standard error of  $Z$  for various combinations of sample size ( $N = 200$  or  $600$ ), non-sampling error ( $\sigma = 0.3$  or  $0.6$ ), selectivity/partial recruitment curve (steep or gradual), and natural mortality rate ( $M = 0.2$  or  $0.5$ ).

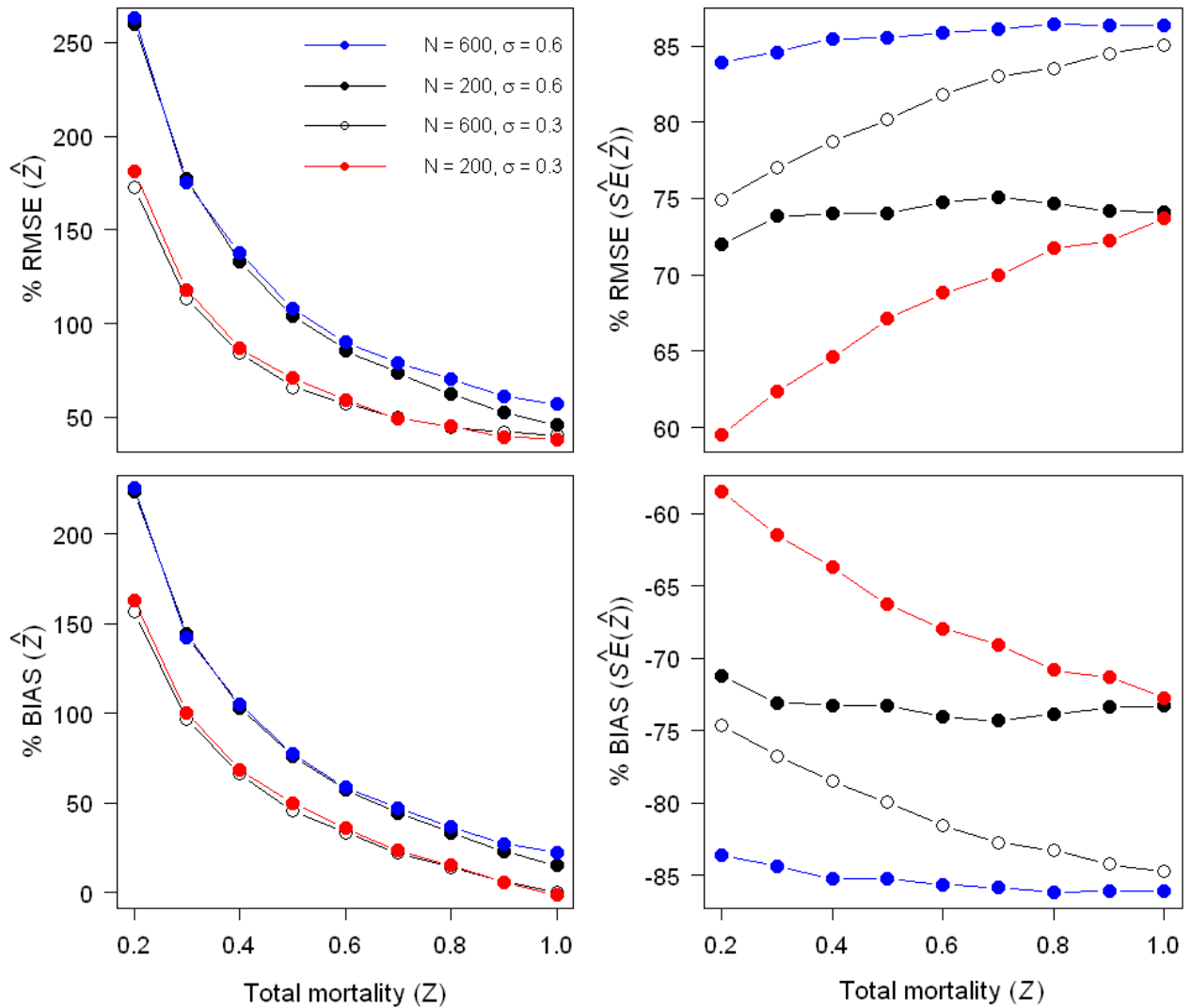


Figure 1. Percent bias (%BIAS) and percent root mean square error (%RMSE) of Heincke's estimator of total mortality rate ( $Z$ ) (**left column**) and of the estimator of standard error (**right column**). Results are for scenarios where the natural mortality rate ( $M$ ) is  $0.2\text{year}^{-1}$ , the selectivity/partial recruitment curve is steep and sample size ( $N$ ) and non-sampling error ( $\sigma$ ) are as indicated in the legend.

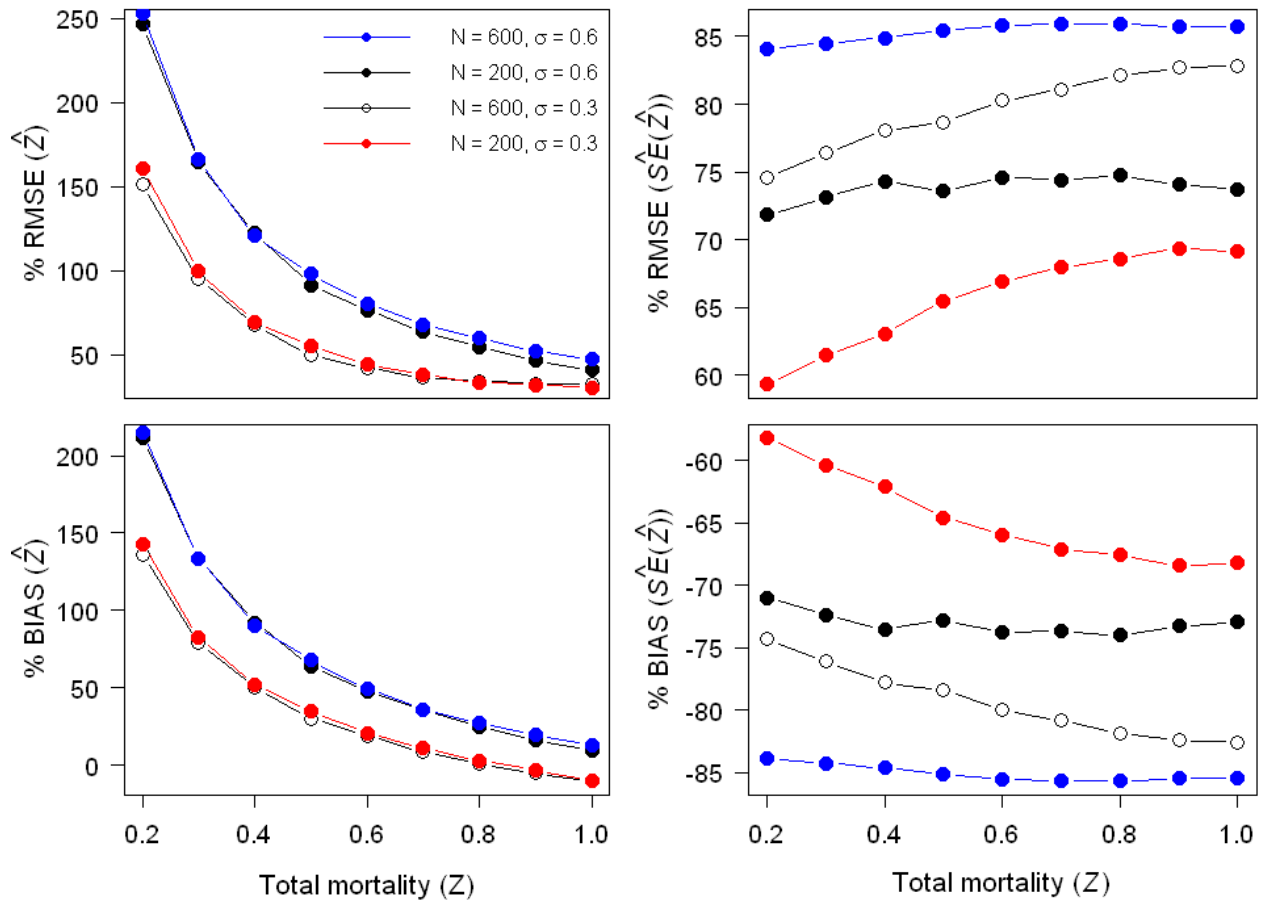


Figure 2. Percent bias (%BIAS) and percent root mean square error (%RMSE) of Heincke's estimator of total mortality rate ( $Z$ ) (**left column**) and of the estimator of standard error (**right column**). Results are for scenarios where the natural mortality rate ( $M$ ) is  $0.2 \text{ year}^{-1}$ , the selectivity/partial recruitment curve is gradual and sample size ( $N$ ) and non-sampling error ( $\sigma$ ) are as indicated in the legend.

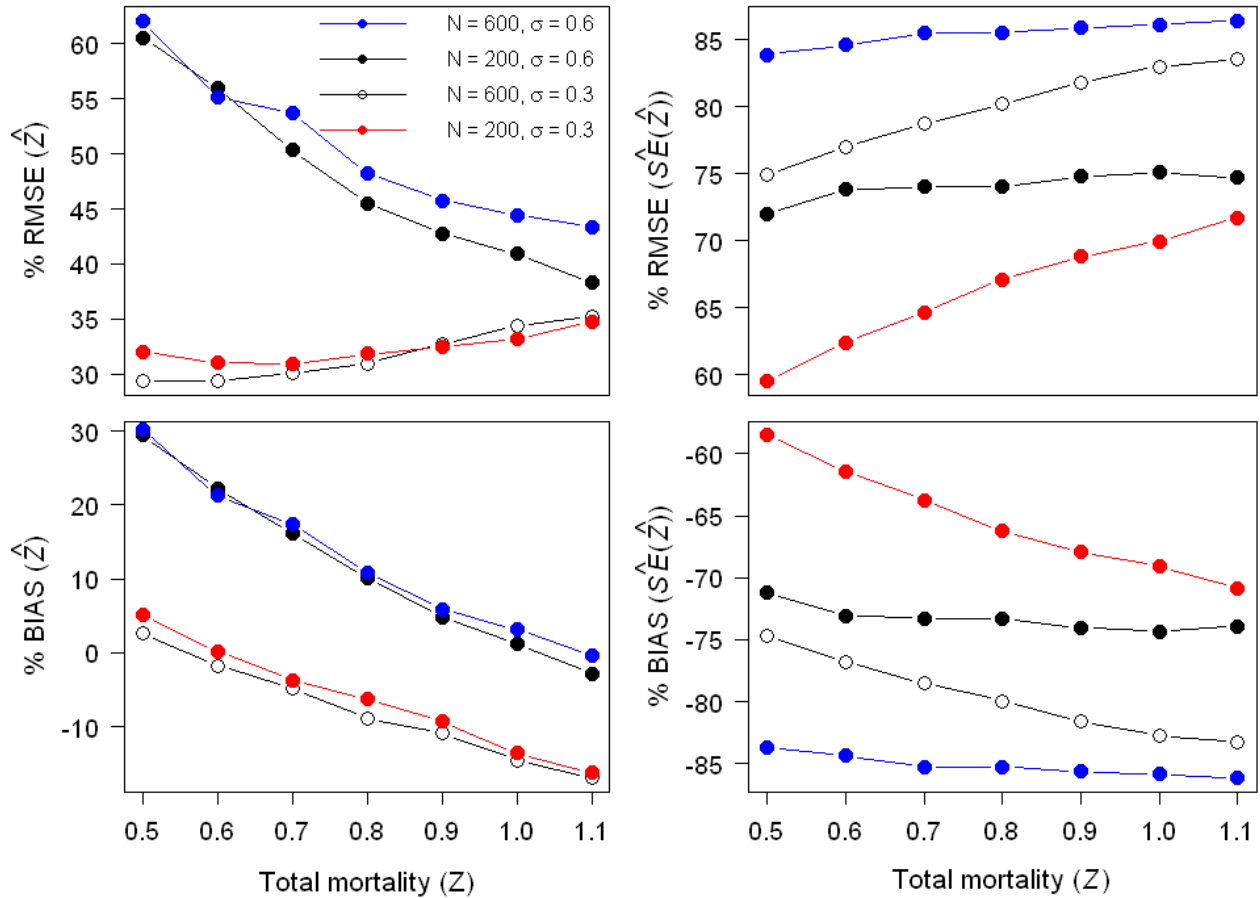


Figure 3. Percent bias (%BIAS) and percent root mean square error (%RMSE) of Heincke's estimator of total mortality rate ( $Z$ ) (**left column**) and of the estimator of standard error (**right column**). Results are for scenarios where the natural mortality rate ( $M$ ) is  $0.5 \text{ year}^{-1}$ , the selectivity/partial recruitment curve is steep and sample size ( $N$ ) and non-sampling error ( $\sigma$ ) are as indicated in the legend.

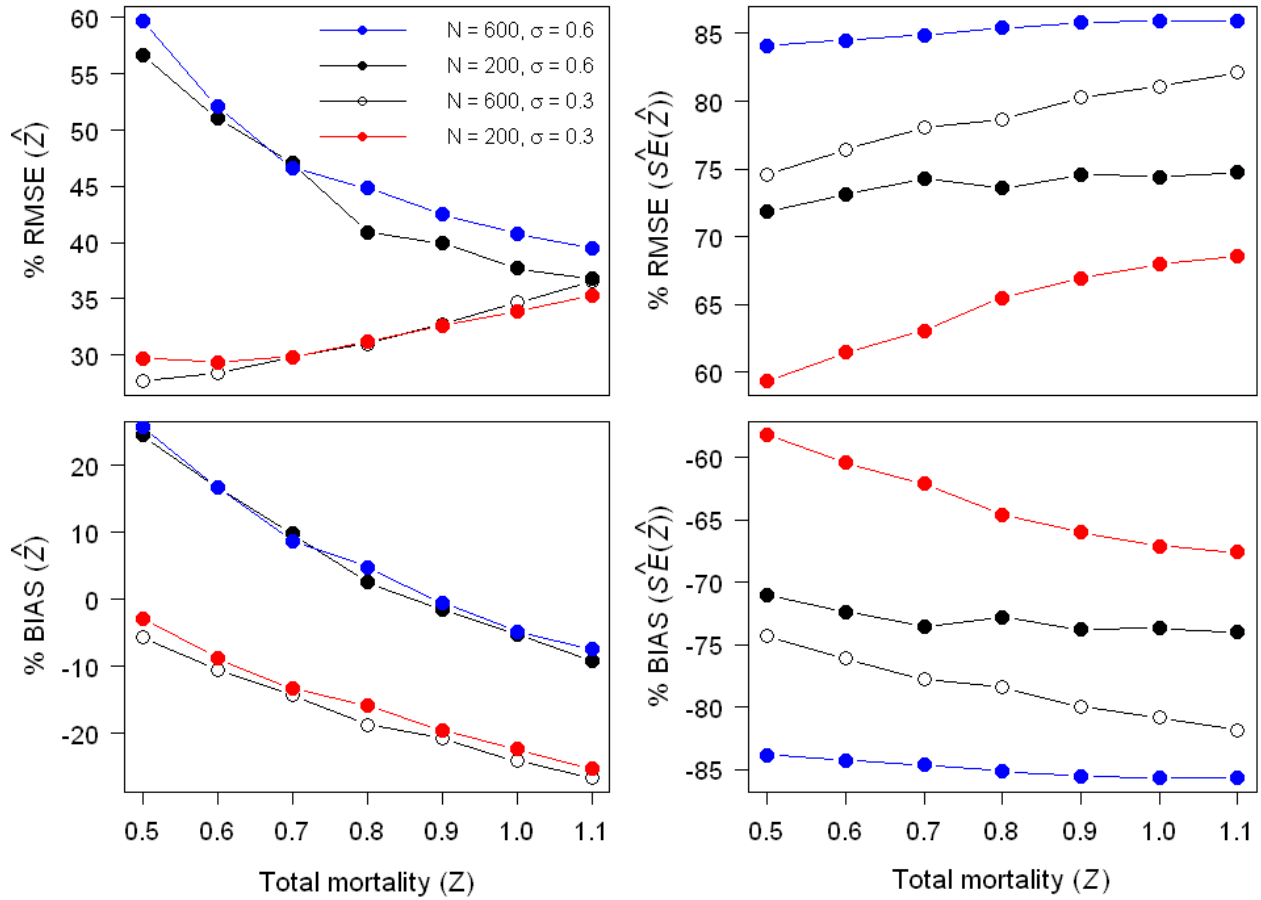


Figure 4. Percent bias (%BIAS) and percent root mean square error (%RMSE) of Heincke's estimator of total mortality rate ( $Z$ ) (**left column**) and of the estimator of standard error (**right column**). Results are for scenarios where the natural mortality rate ( $M$ ) is  $0.5 \text{ year}^{-1}$ , the selectivity/partial recruitment curve is gradual and sample size ( $N$ ) and non-sampling error ( $\sigma$ ) are as indicated in the legend.



## **Effect of different levels of $\alpha$ on the z-test age of full recruitment selection criterion**

A statistical approach to estimating the first fully recruited age group to use in catch curve analysis was proposed by Chapman and Robson 1960. However, no guidance was given on the level of alpha to use in the z-test. The following section shows plots of %RMSE of the estimated  $Z$  when  $M = 0.2 \text{ yr}^{-1}$ , partial recruitment/selectivity curve is steep,  $N = 200$  and  $\sigma = 0.6$ . Alpha was fixed at 0.01, 0.025, 0.05, 0.075, and 0.1. As shown below little difference in performance was obtained at different levels of  $\alpha$  and as a result  $\alpha = 0.05$  was used for all subsequent analysis.

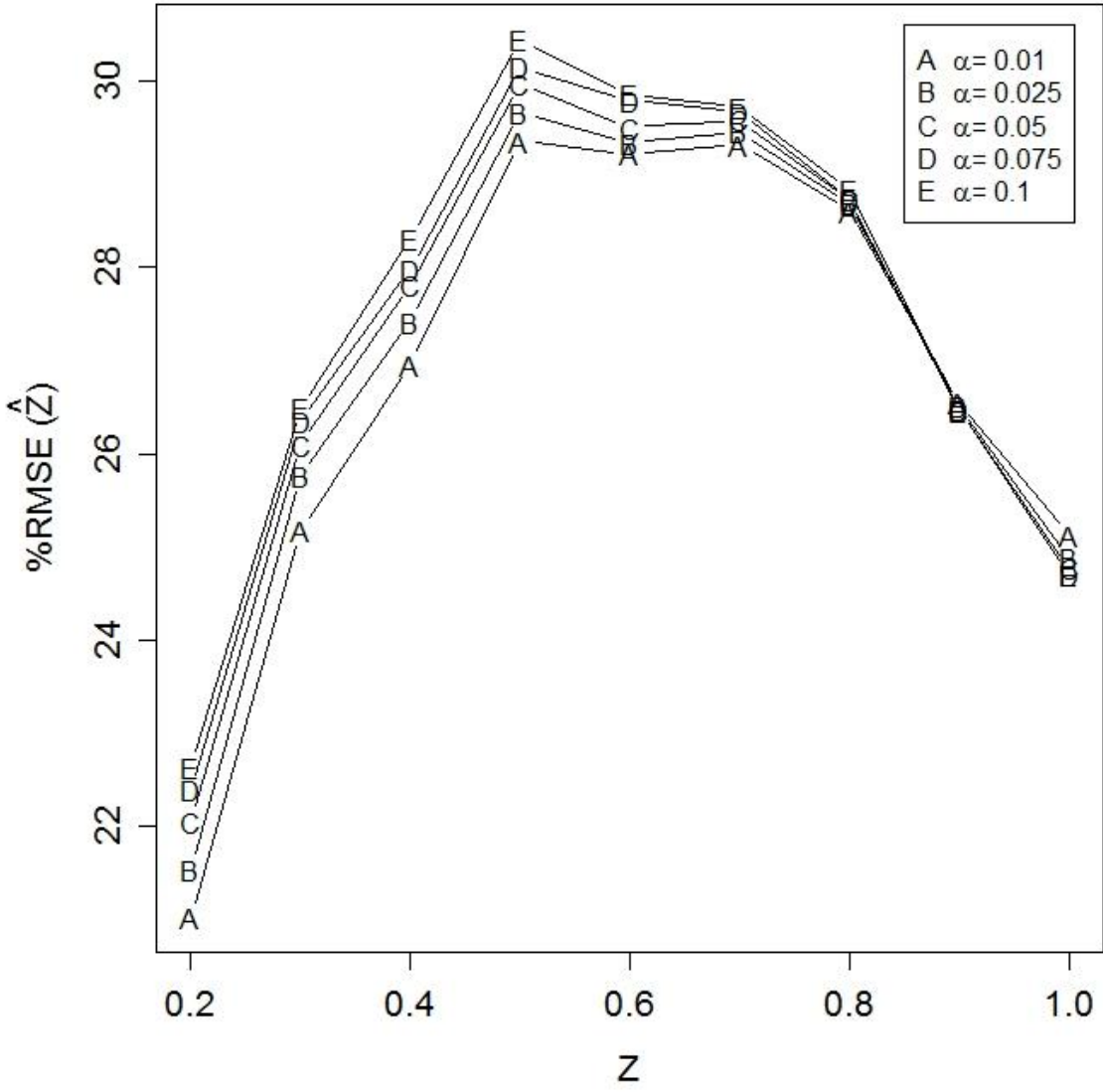


Figure 5. Comparison of the %RMSE obtained from the Chapman – Robson estimator with z-test age of full recruitment selection criterion rejecting deficient ages at 5 levels of  $\alpha$ .

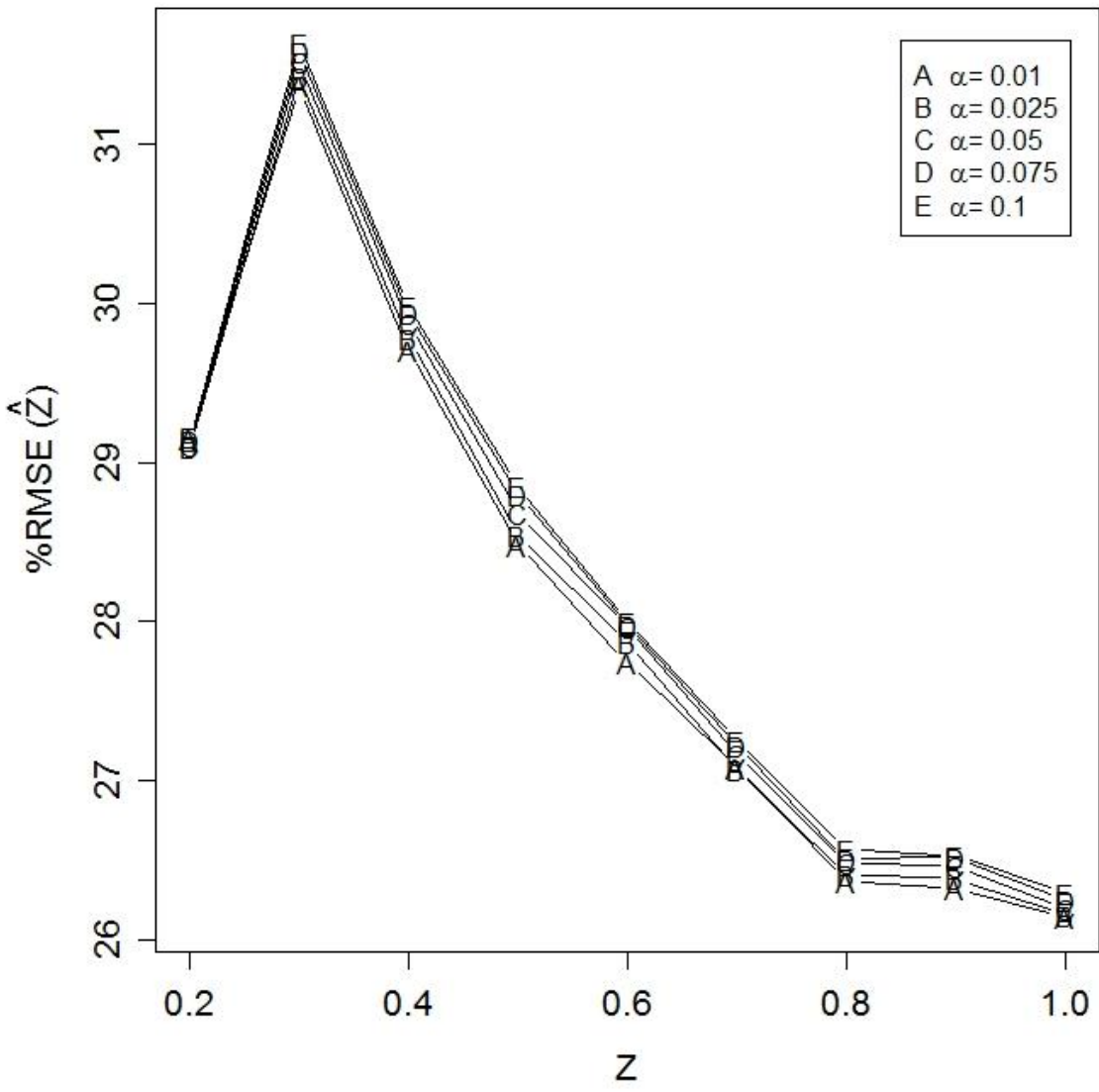


Figure 6. Comparison of the %RMSE obtained from the regression (RG) estimator with z-test age of full recruitment selection criterion rejecting deficient ages at 5 levels of  $\alpha$ .

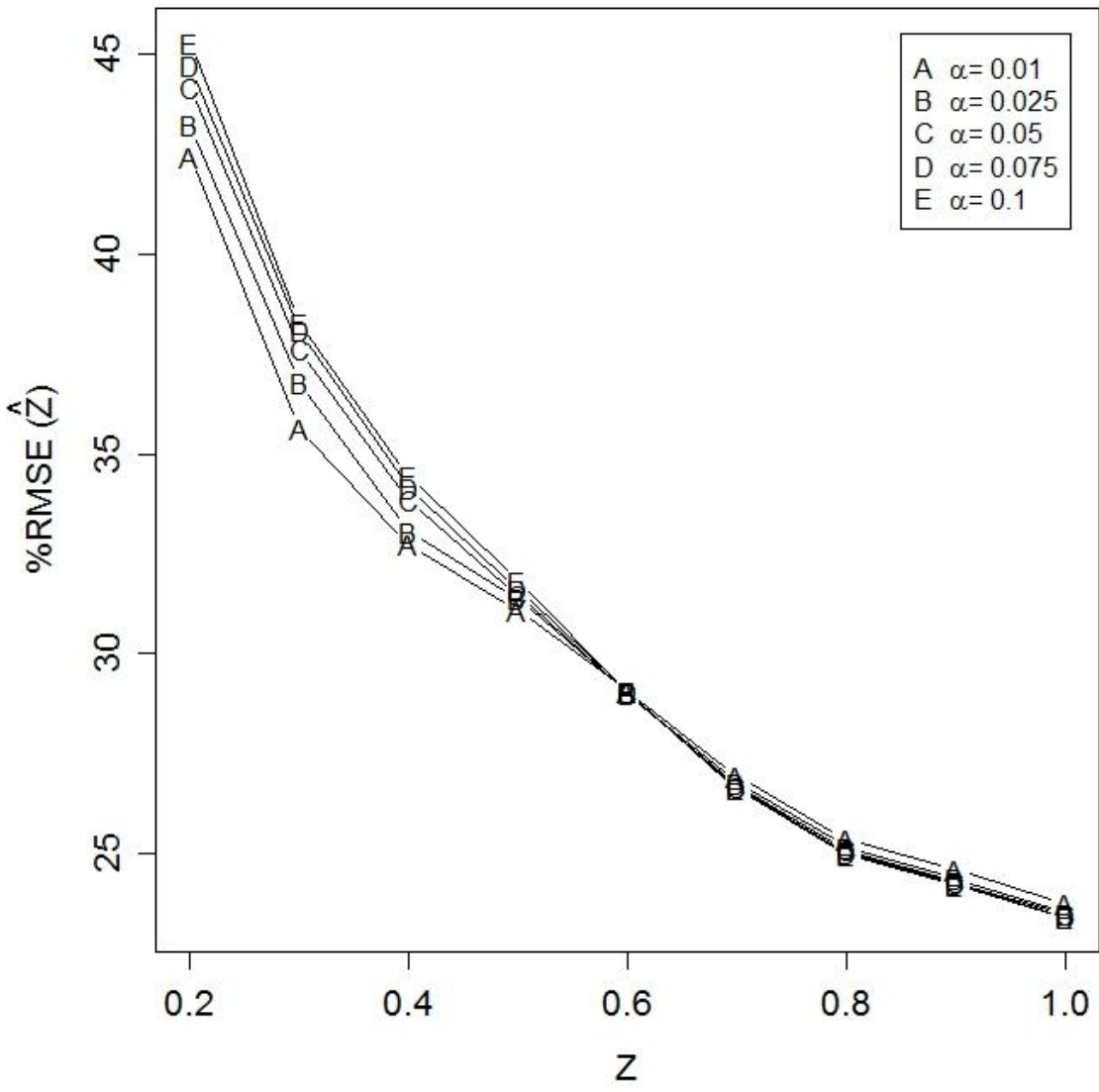


Figure 7. Comparison of the %RMSE obtained from the regression (R1) estimator with z-test age of full recruitment selection criterion rejecting deficient ages at 5 levels of  $\alpha$ .

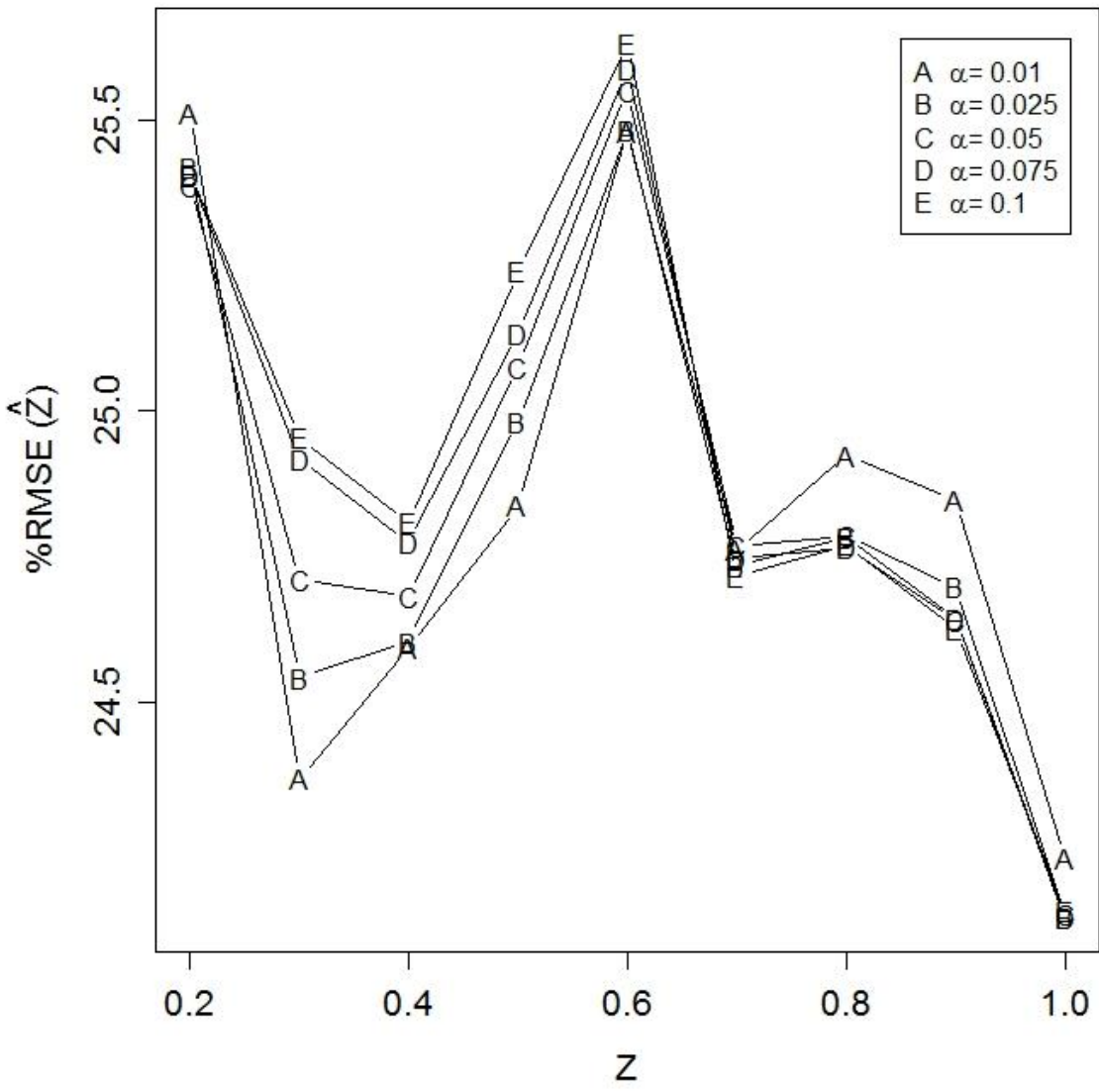


Figure 8. Comparison of the %RMSE obtained from the regression ( $R1'$ ) estimator with z-test age of full recruitment selection criterion rejecting deficient ages at 5 levels of  $\alpha$ .

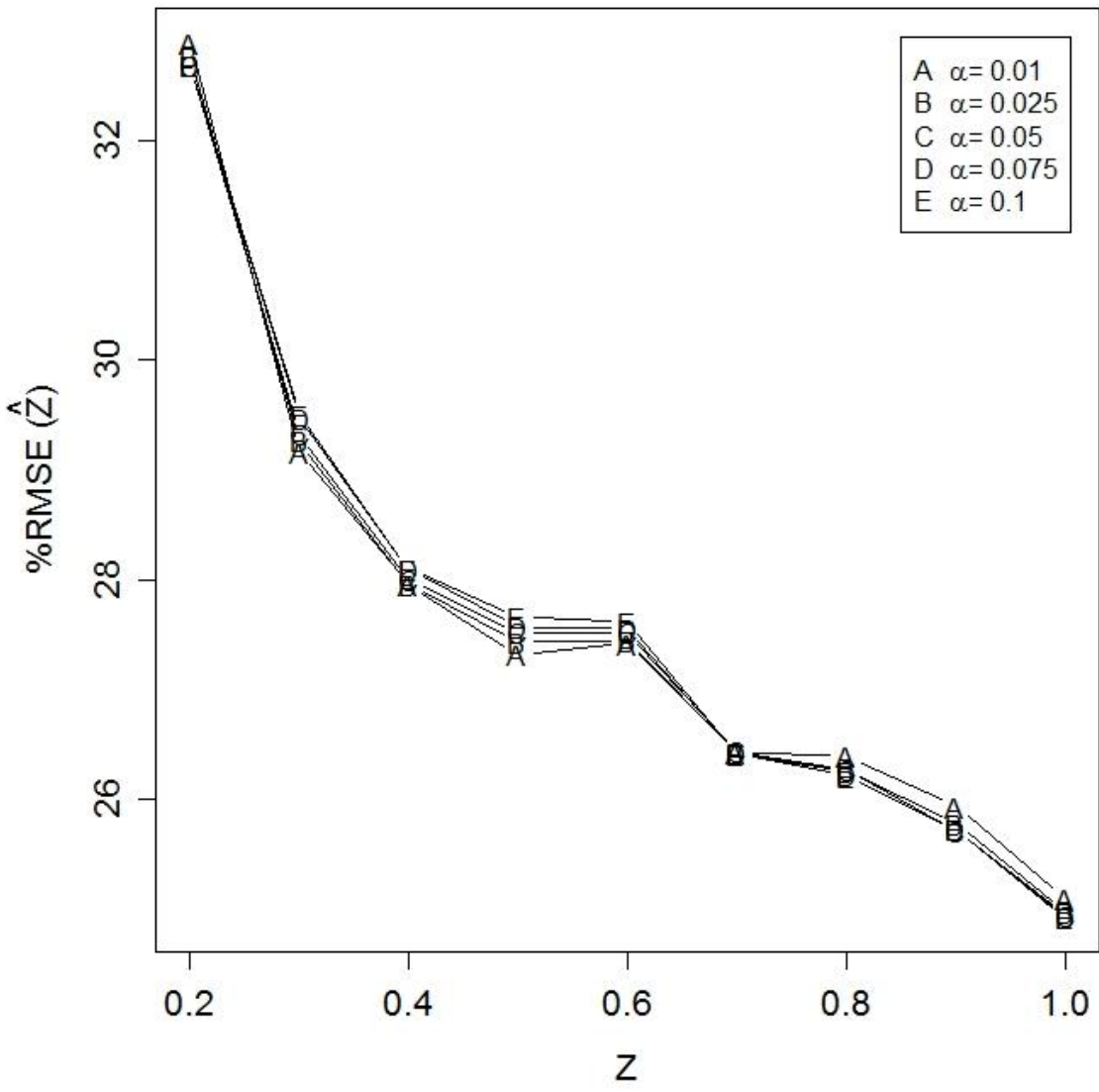


Figure 9. Comparison of the %RMSE obtained from the regression ( $R1''$ ) estimator with z-test age of full recruitment selection criterion rejecting deficient ages at 5 levels of  $\alpha$ .

### **Comparison of %RMSE for the unweighted, simple weighted, and iteratively reweighted regression estimators.**

In all cases the unweighted result is shown with a solid black line, the simple weighted result is shown with a dotted blue line, and the iteratively reweighted result is shown with a dashed red line. Only results obtained using the RG right truncation method are shown because it was the best performing right truncation method when weighting was used. As shown below, in most cases differences between the simple weighted and iteratively reweighted methods were slight; however, occasionally the simple weighted method noticeable outperformed the iterative method and was therefore preferred for all subsequent analysis.

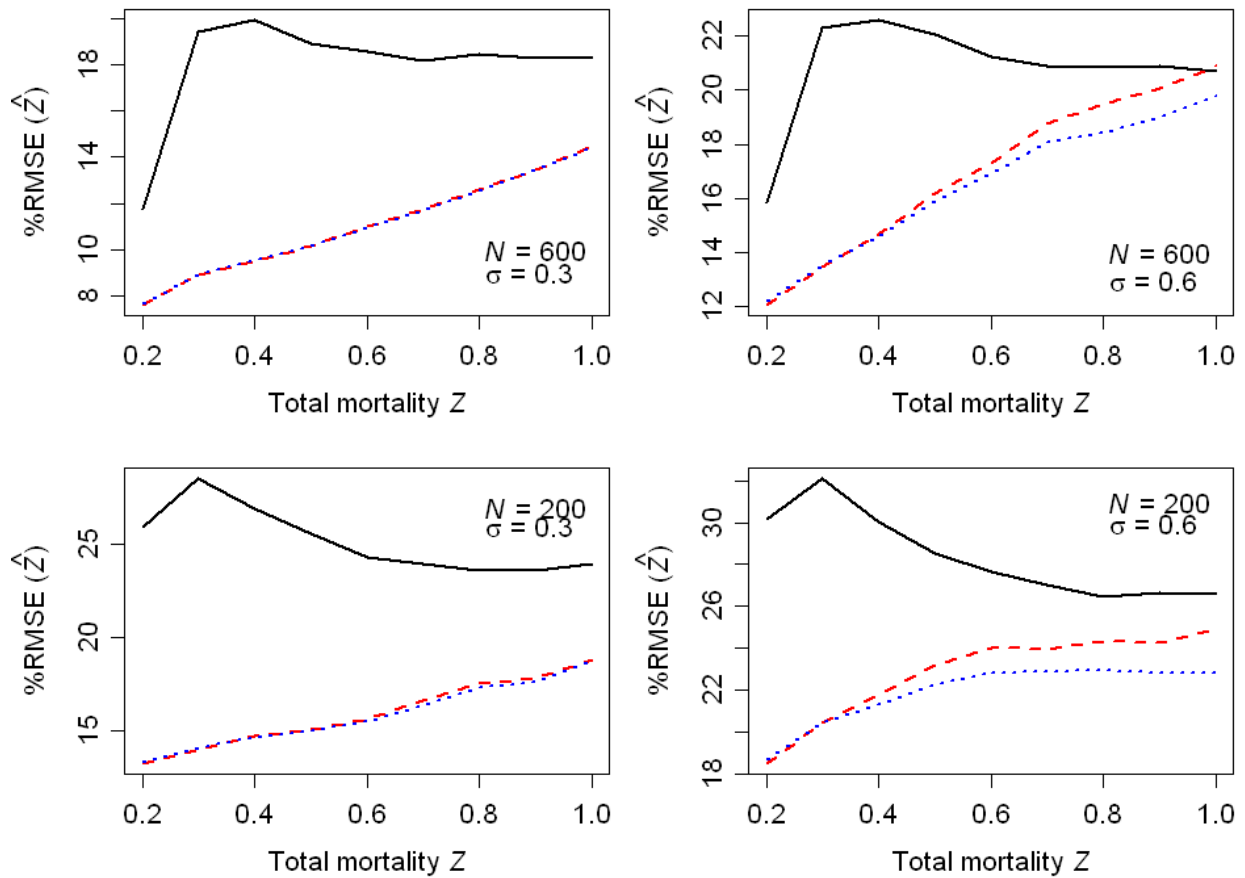


Figure 10. Comparison of %RMSE obtained from unweighted regression (black line) weighted regression (blue dotted line), and iteratively reweighted regression (red dashed line). Results are obtained using the RG right truncation method from the scenario where natural mortality rate ( $M$ ) equaled 0.2 and the selectivity/partial recruitment curve was steep.



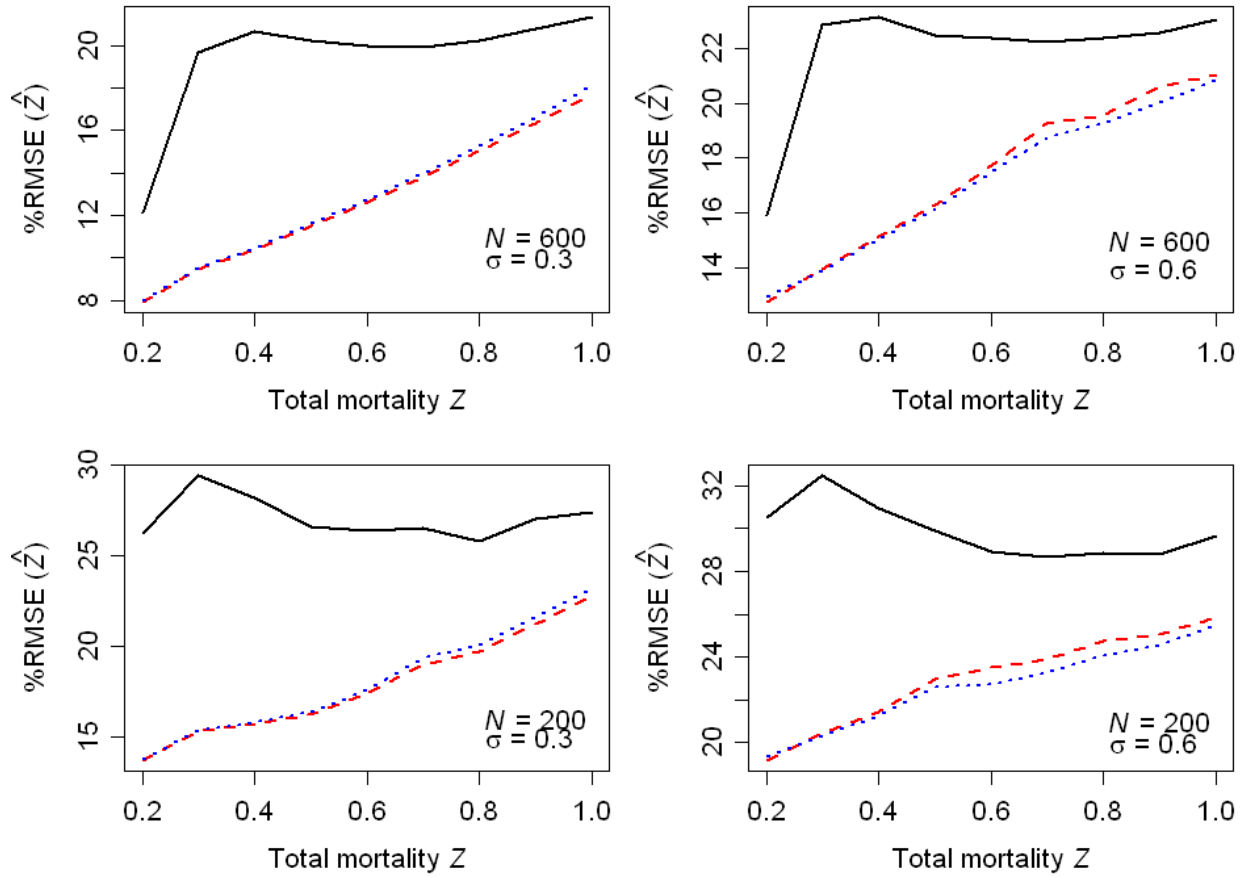


Figure 11. Comparison of %RMSE obtained from unweighted regression (black line) weighted regression (blue dotted line), and iteratively reweighted regression (red dashed line). Results are obtained using the RG right truncation method from the scenario where natural mortality rate ( $M$ ) equaled 0.2 and the selectivity/partial recruitment curve was gradual.

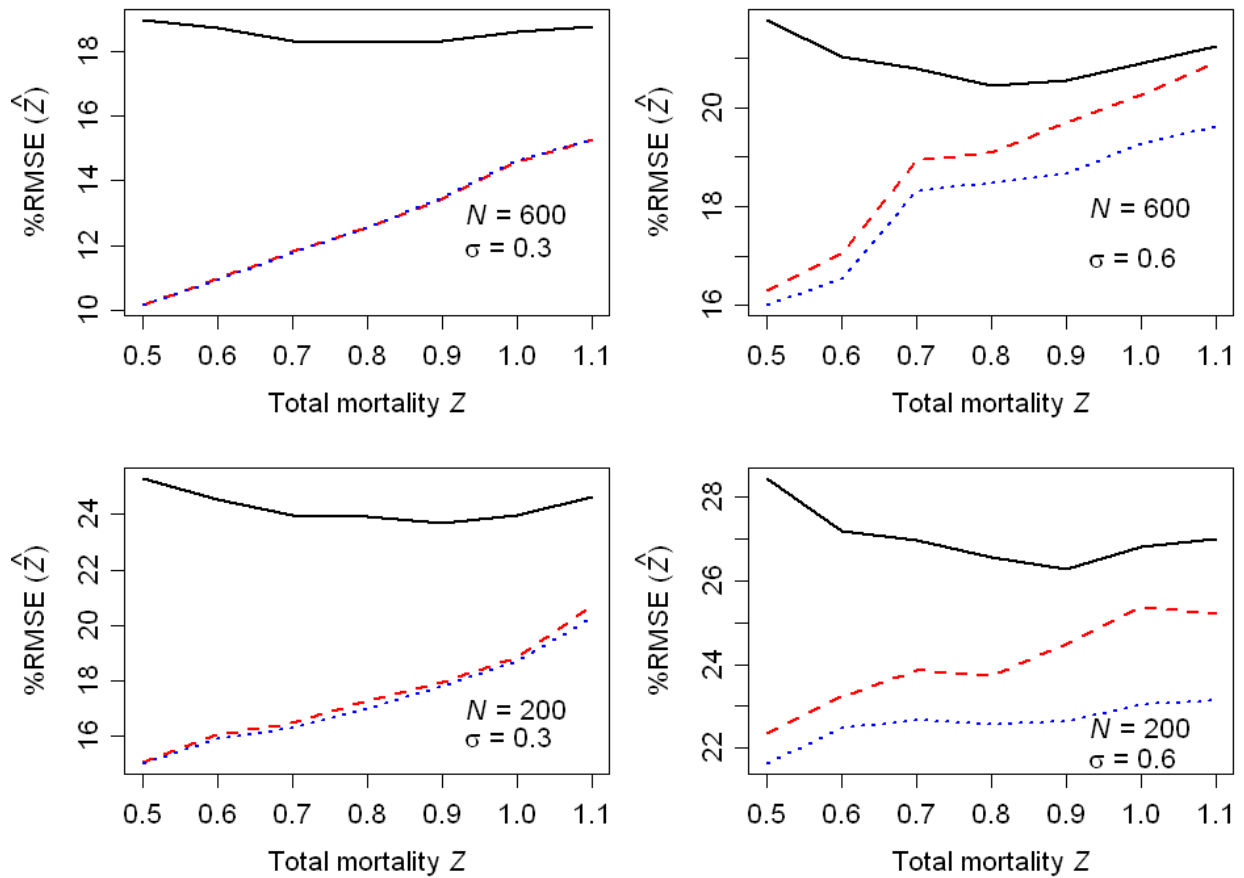


Figure 12. Comparison of %RMSE obtained from unweighted regression (black line) weighted regression (blue dotted line), and iteratively reweighted regression (red dashed line). Results are obtained using the RG right truncation method from the scenario where natural mortality rate ( $M$ ) equaled 0.5 and the selectivity/partial recruitment curve was steep.

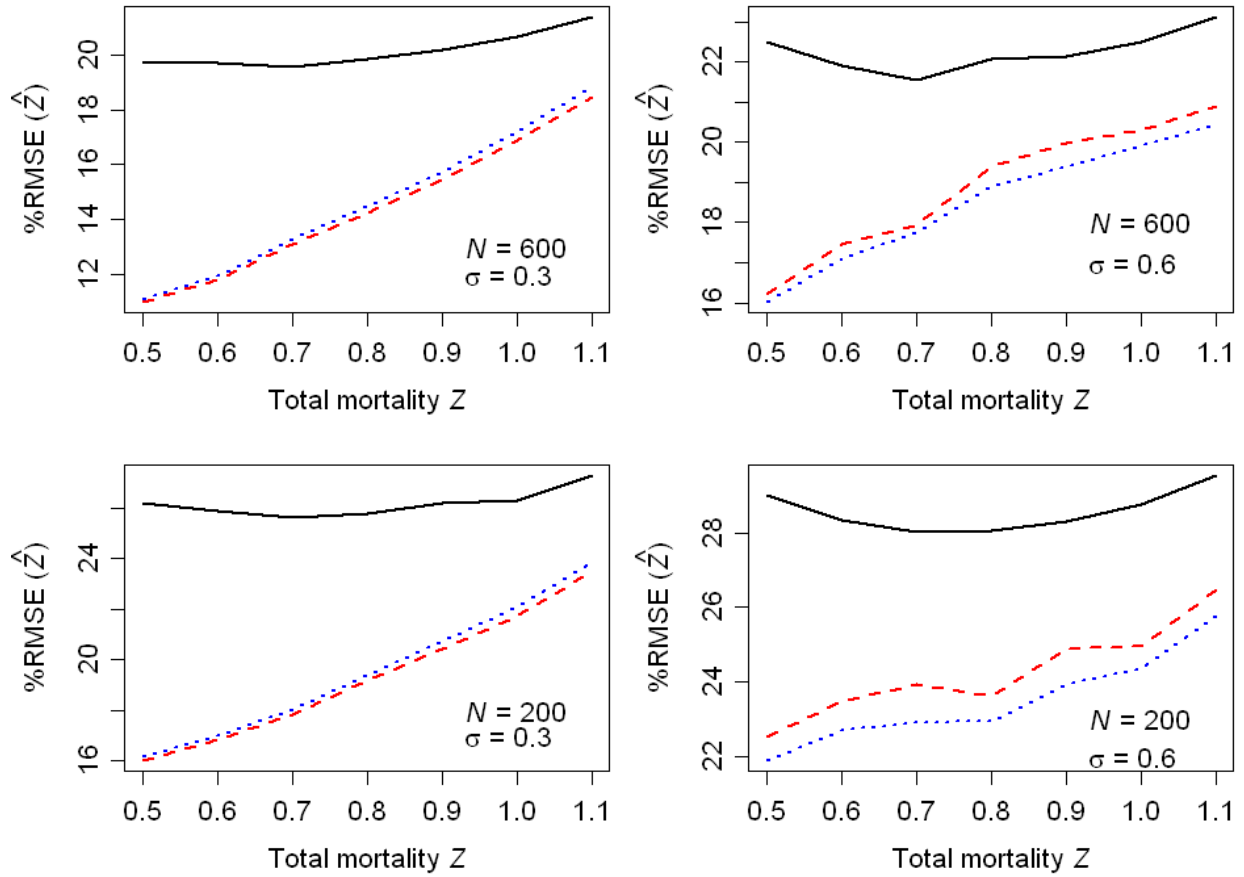


Figure 13. Comparison of %RMSE obtained from unweighted regression (black line) weighted regression (blue dotted line), and iteratively reweighted regression (red dashed line). Results are obtained using the RG right truncation method from the scenario where natural mortality rate ( $M$ ) equaled 0.5 and the selectivity/partial recruitment curve was gradual

### **Performance of weighted and unweighted regression estimators**

The following section shows results comparing the performance, in terms of %BIAS and %RMSE, of the two preferred weighted regression estimators (RG Peak —●— and RG z-test —▲—). For all plots in this section, comparisons between the two preferred weighted regression estimators and all other weighted regression estimators are in the left column and the comparisons of the two preferred weighted regression estimators and all unweighted regression estimators are in the right column. Figures 14 - 17 depict results where  $M = 0.2$  and selectivity/partial recruitment is steep. Figures 18 - 21 depict results where  $M = 0.2$  and selectivity/partial recruitment is gradual. Figures 22 - 25 depict results where  $M = 0.5$  and selectivity/partial recruitment is steep. Figures 26 - 29 depict results where  $M = 0.5$  and selectivity/partial recruitment is gradual.

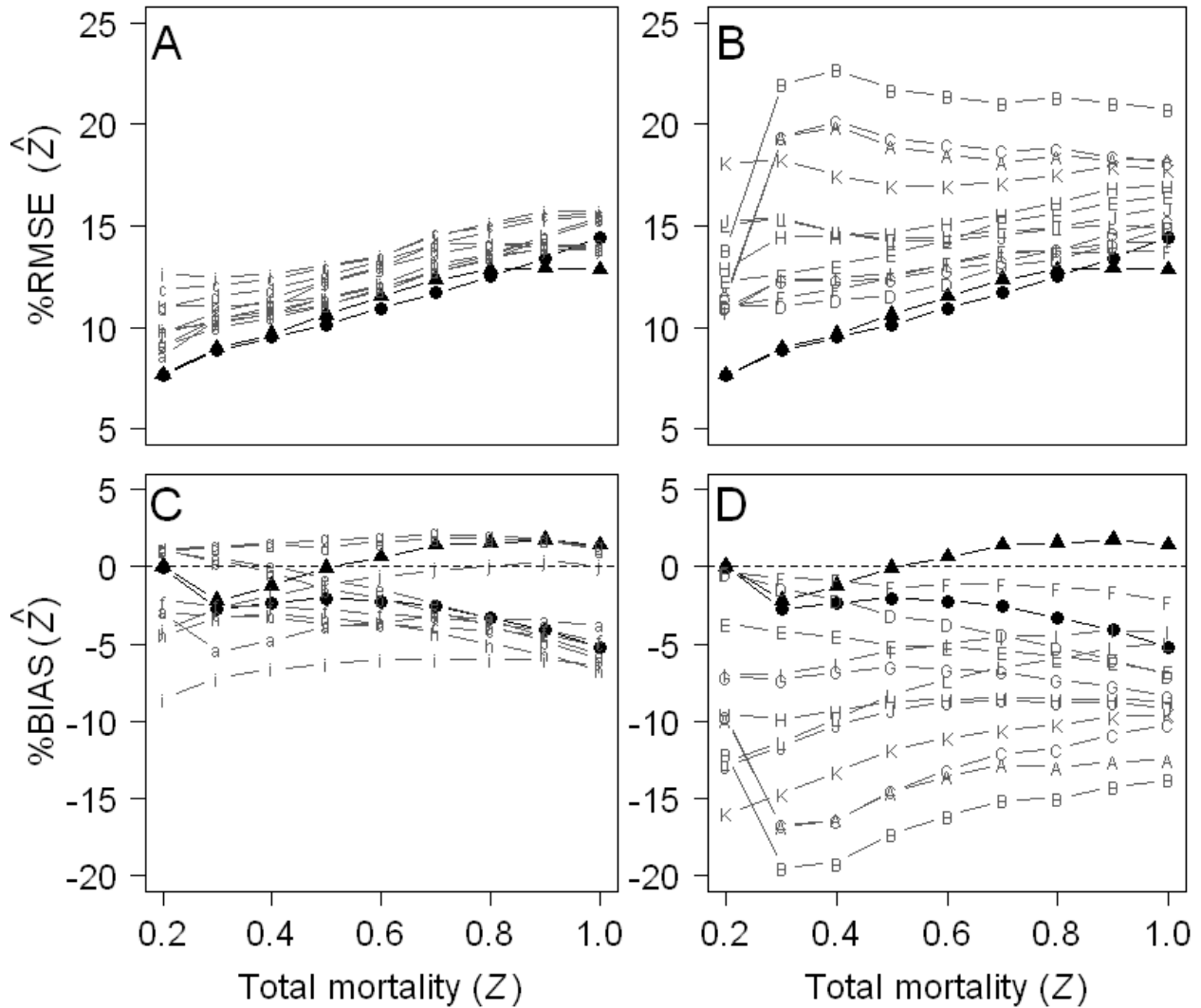


Figure 14. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.2$  year<sup>-1</sup>, steep selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

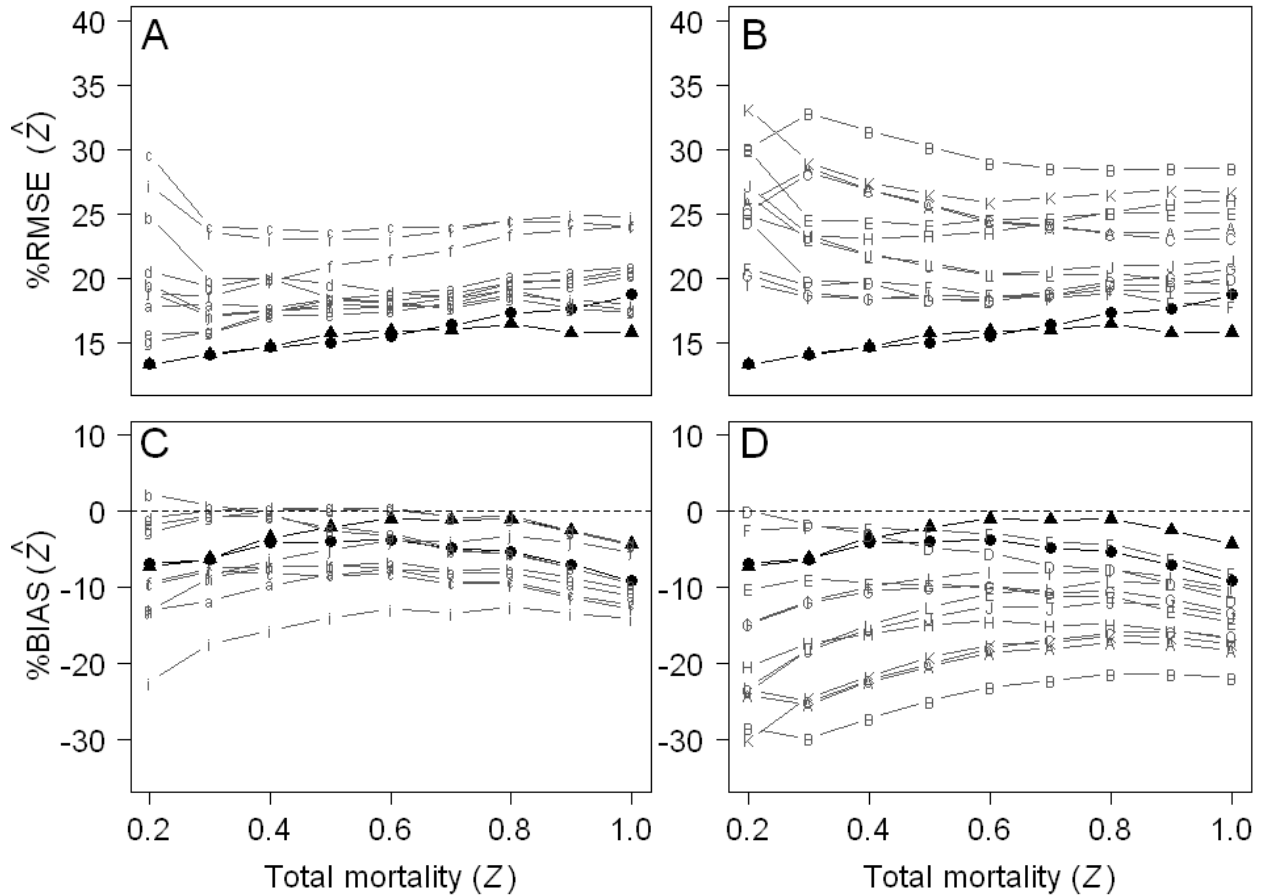


Figure 15. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.2$  year<sup>-1</sup>, steep selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

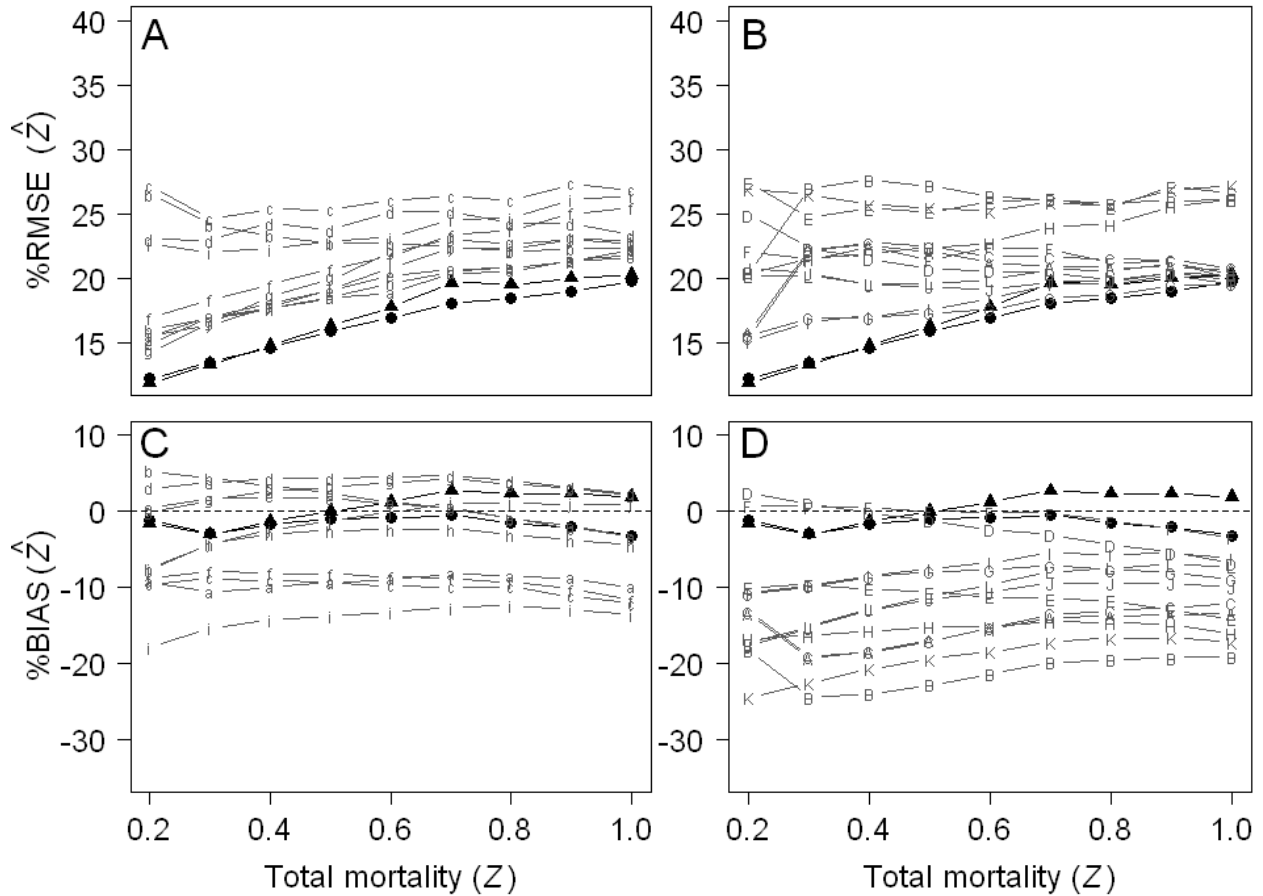


Figure 16. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.2$  year<sup>-1</sup>, steep selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

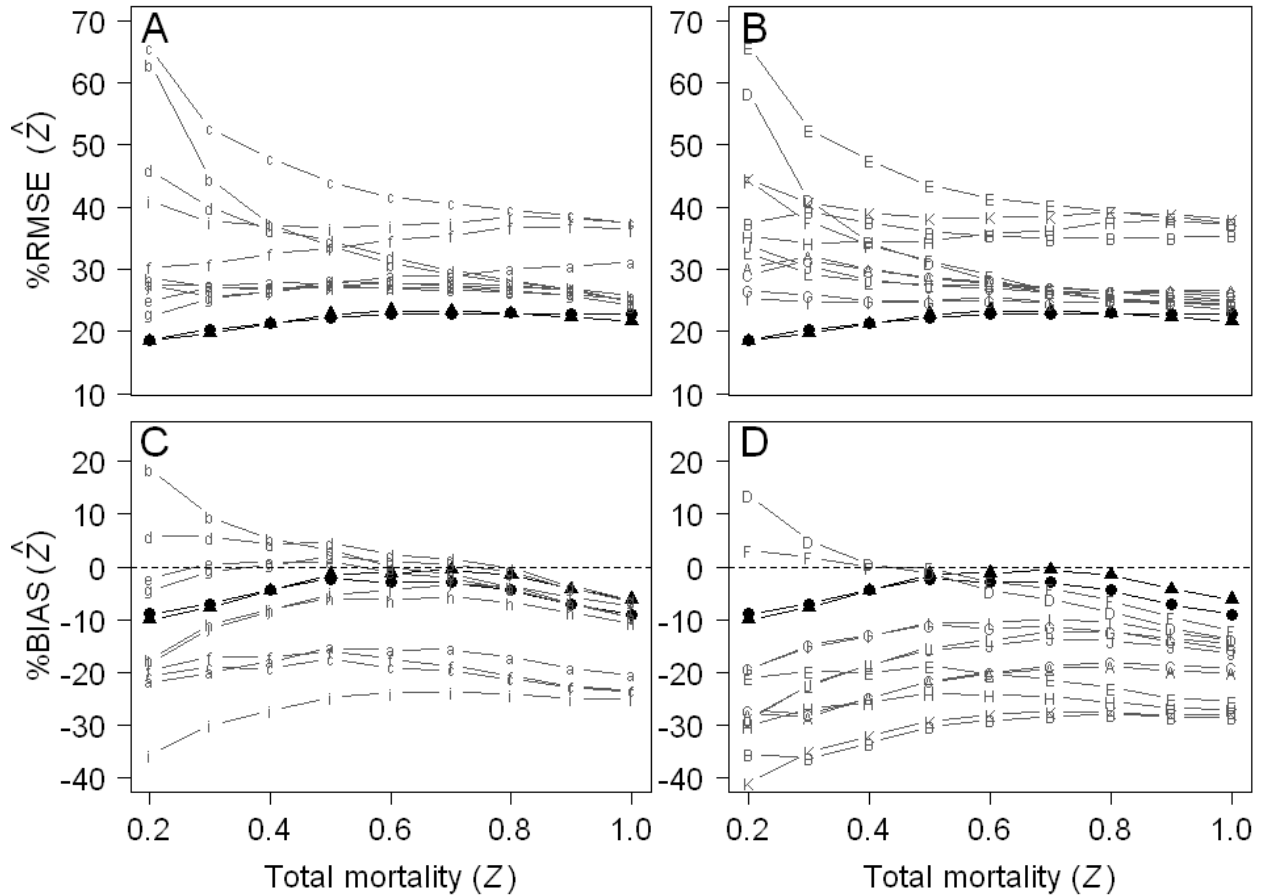


Figure 17. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.2$  year<sup>-1</sup>, steep selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.



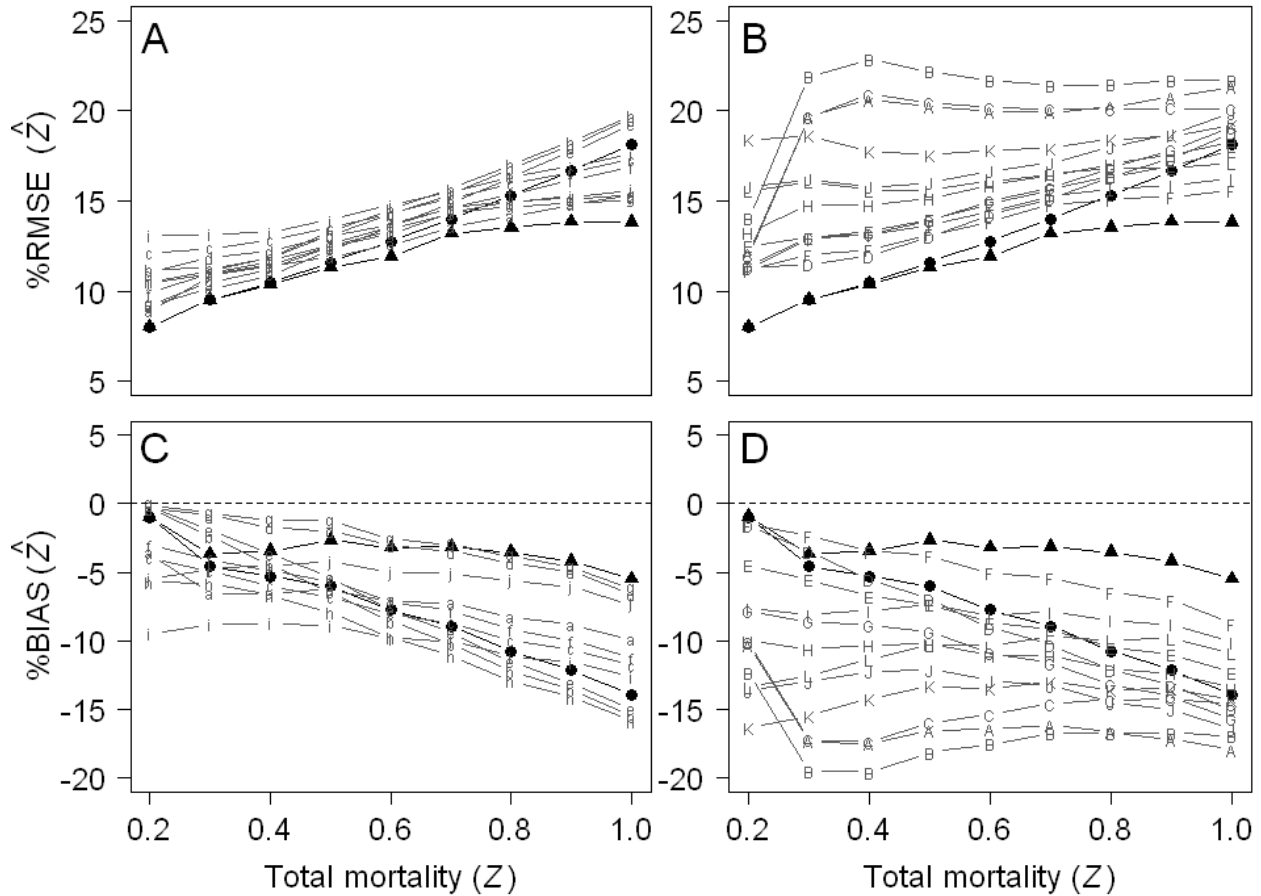


Figure 18. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.2$  year<sup>-1</sup>, gradual selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

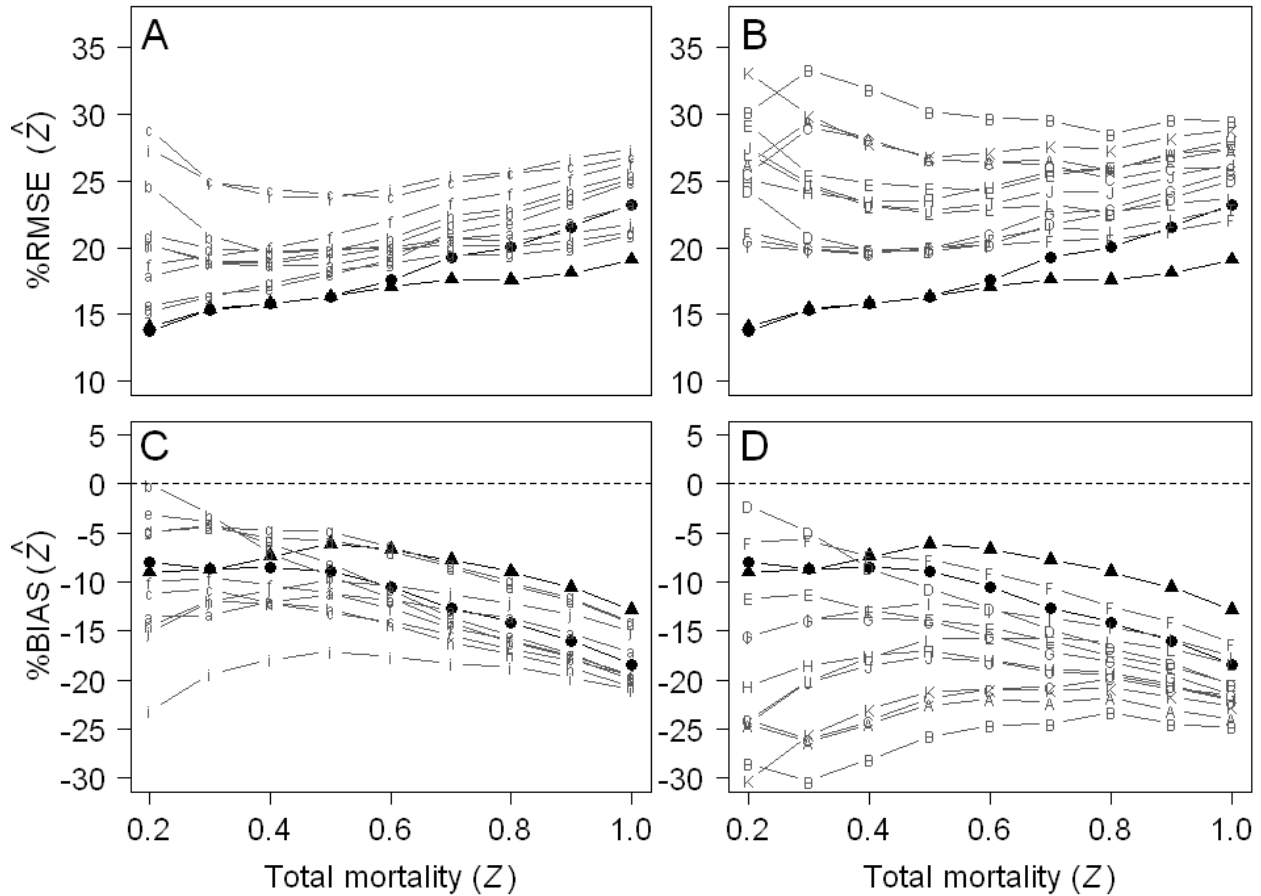


Figure 19. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.2$  year<sup>-1</sup>, gradual selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

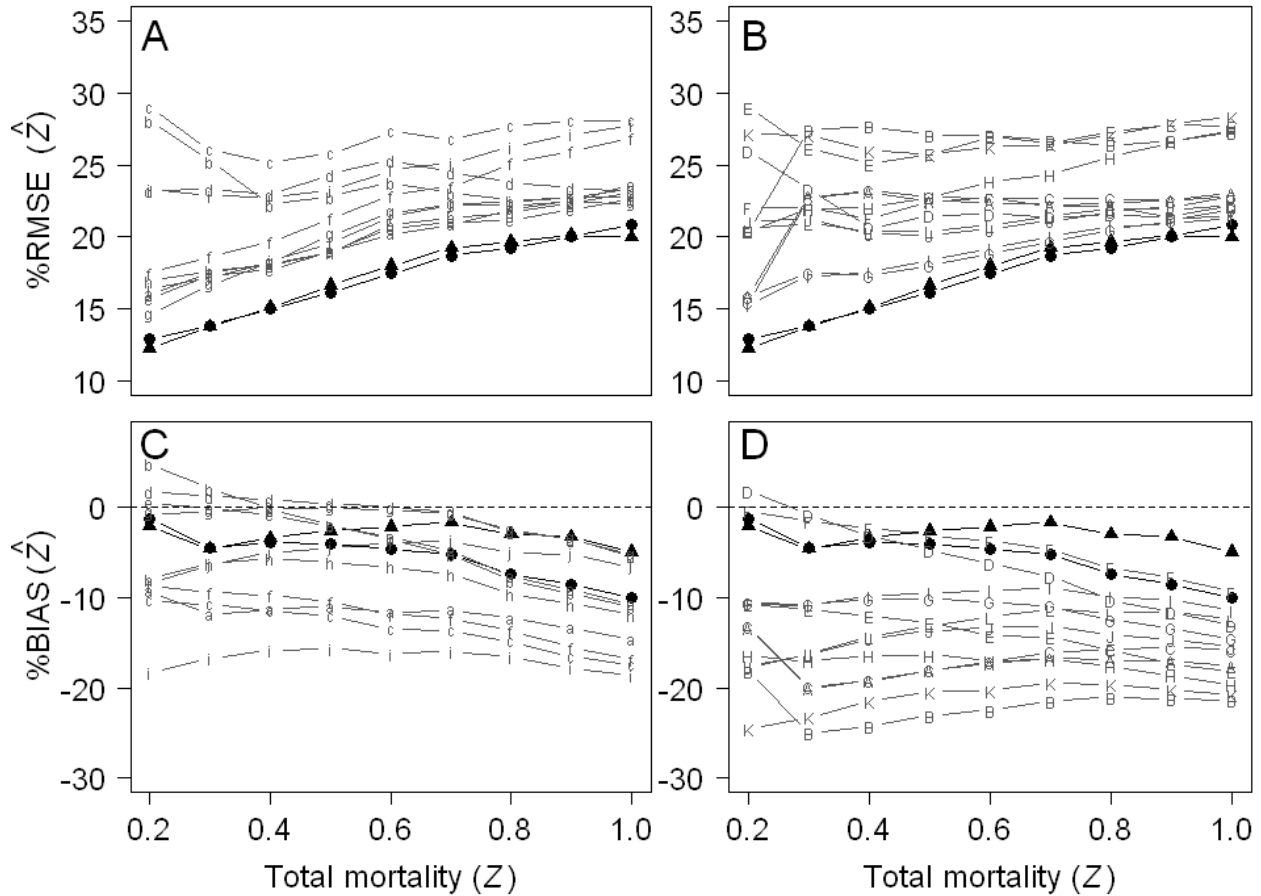


Figure 20. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak —●— and RG z-test —▲—) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.2$  year<sup>-1</sup>, gradual selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

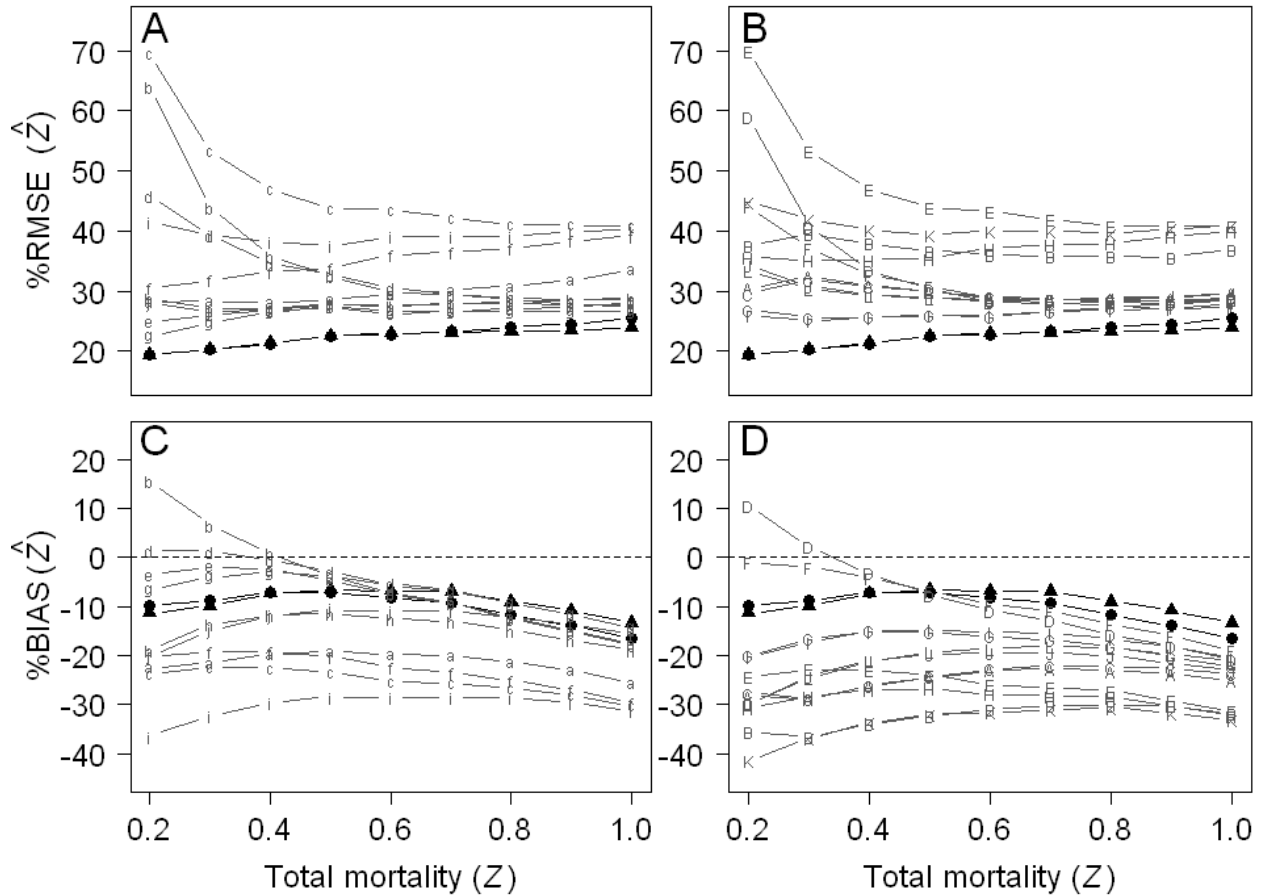


Figure 21. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak —●— and RG z-test —▲—) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.2$  year<sup>-1</sup>, gradual selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

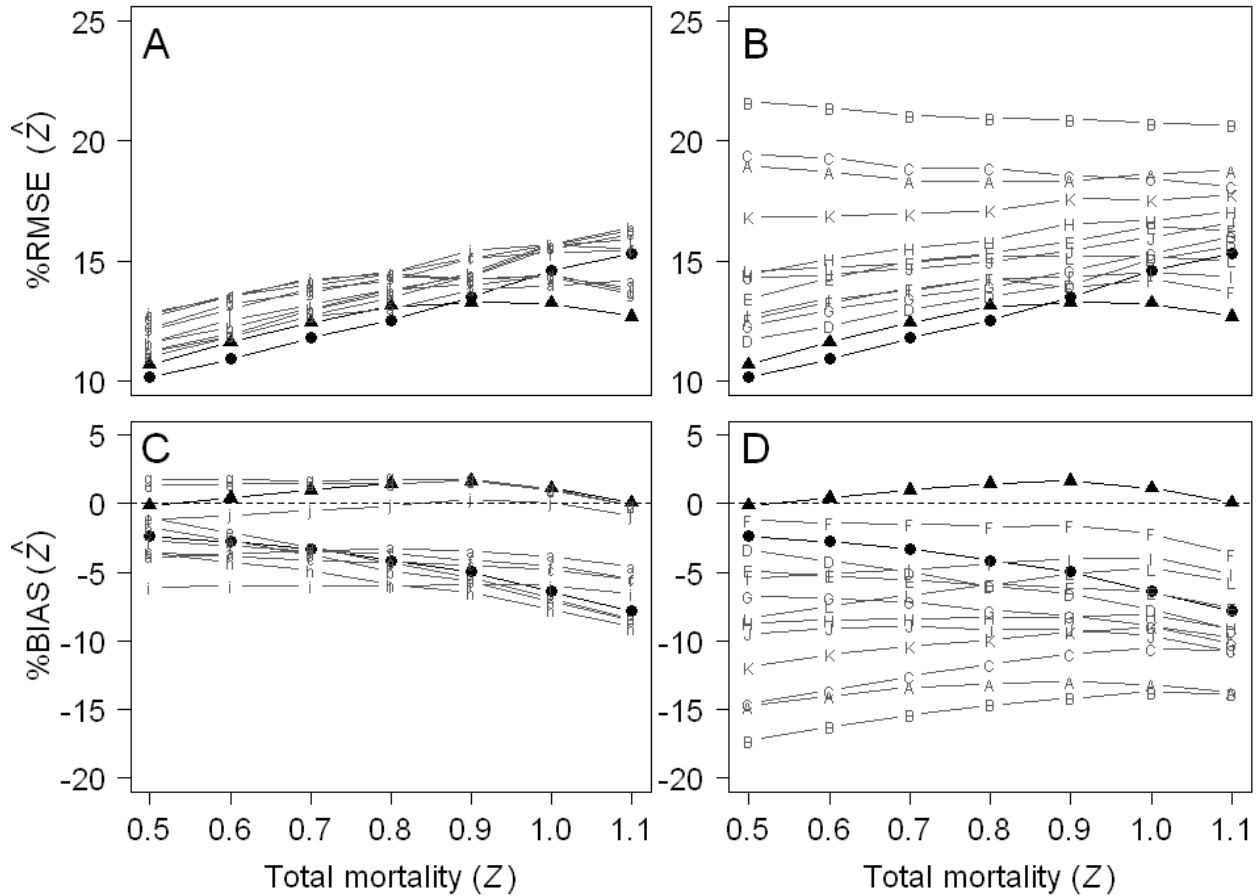


Figure 22. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.5$  year<sup>-1</sup>, steep selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

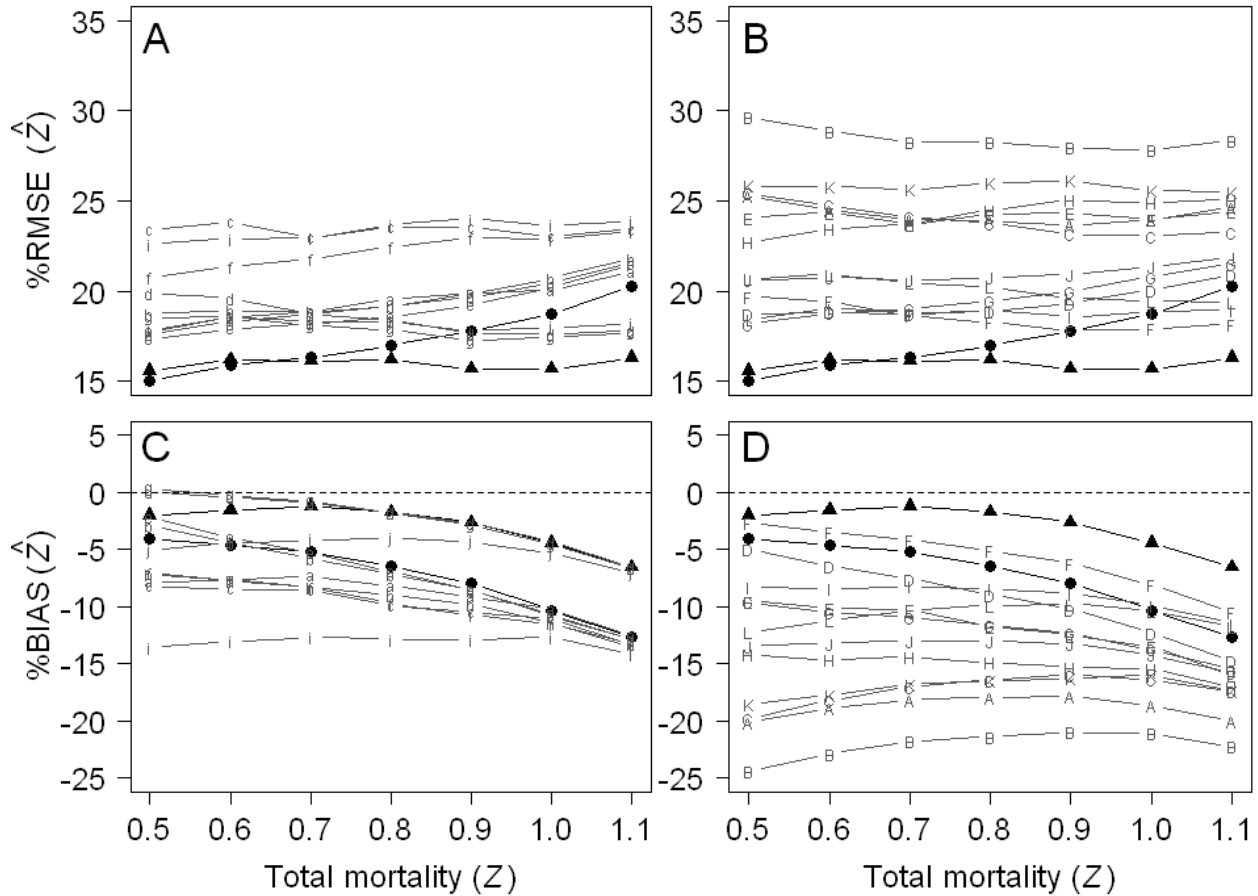


Figure 23. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.5 \text{ year}^{-1}$ , steep selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

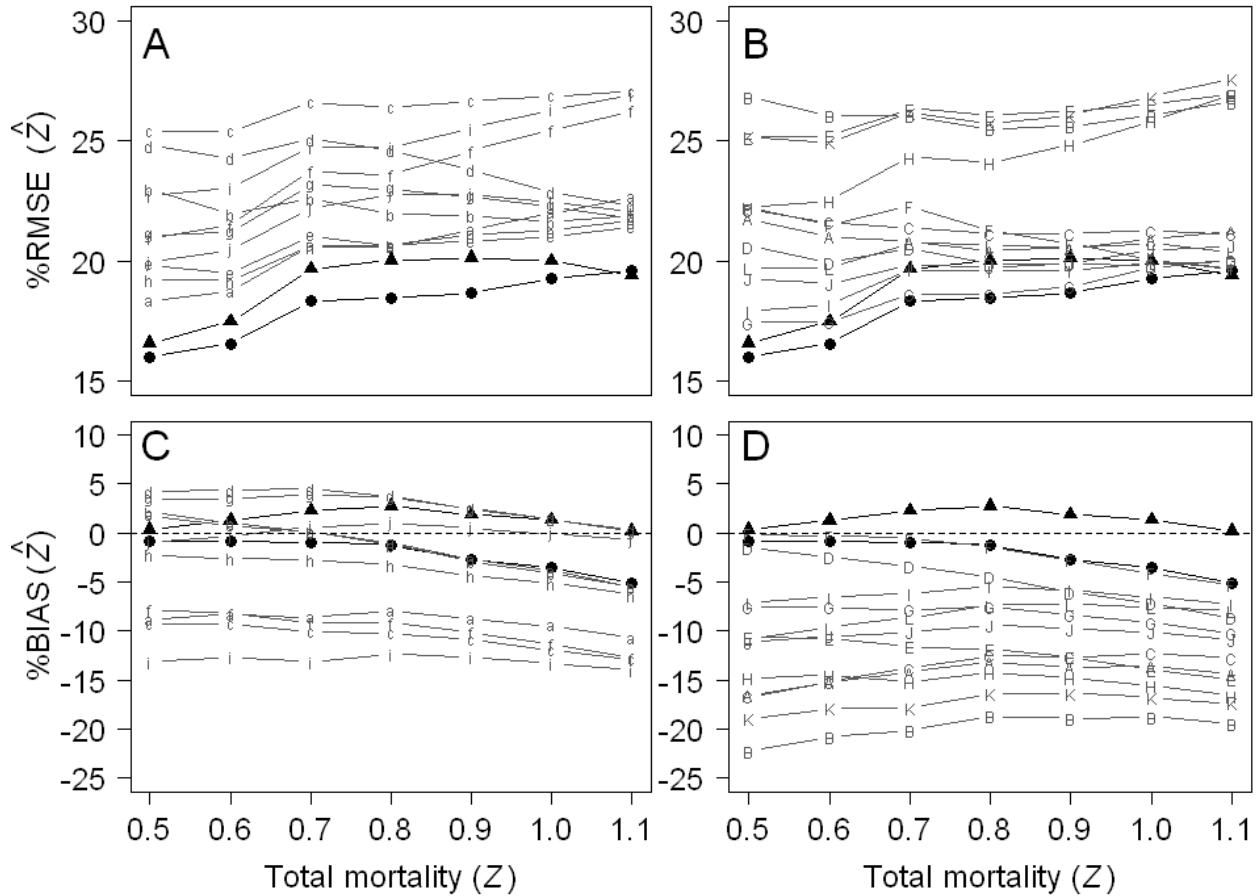


Figure 24. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.5$  year<sup>-1</sup>, steep selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

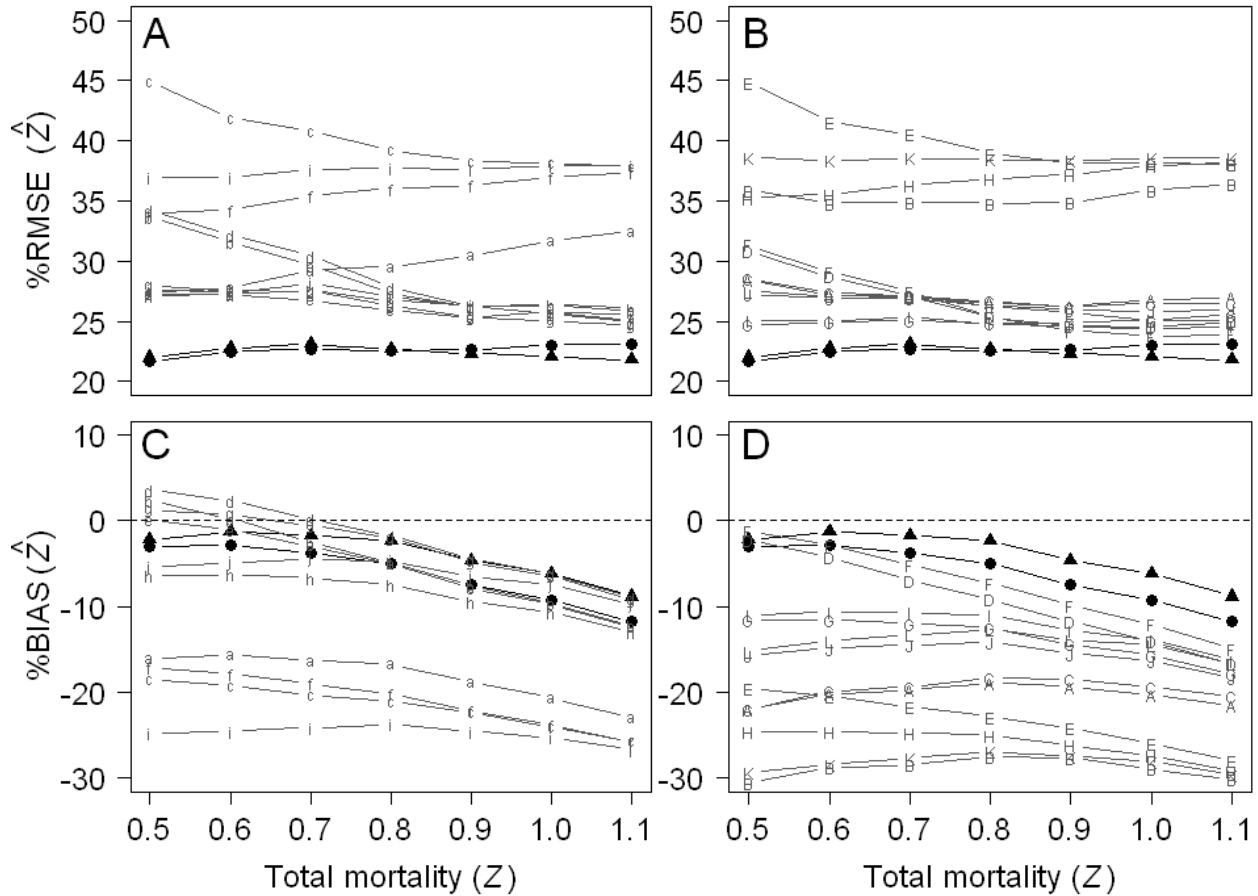


Figure 25. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.5$  year<sup>-1</sup>, steep selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.



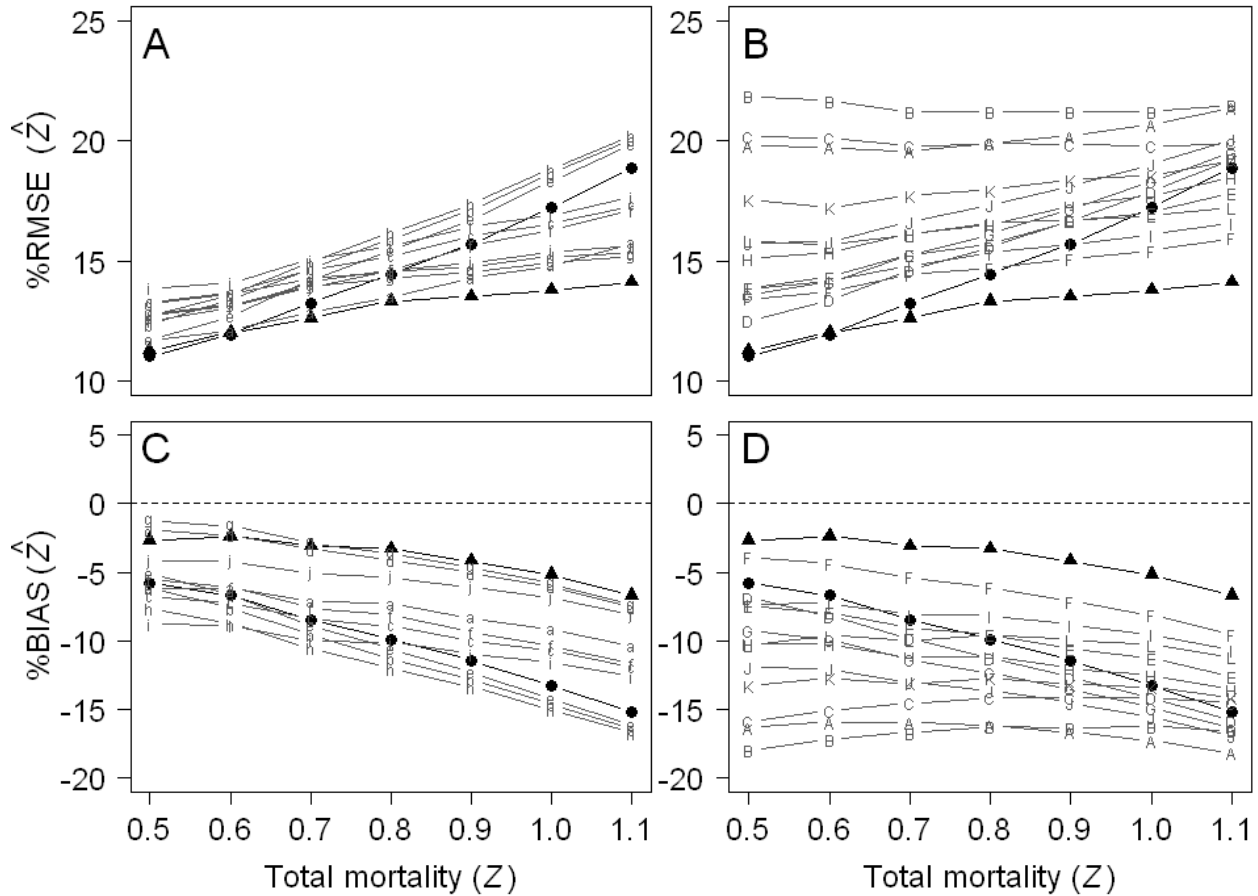


Figure 26. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.5$  year<sup>-1</sup>, gradual selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

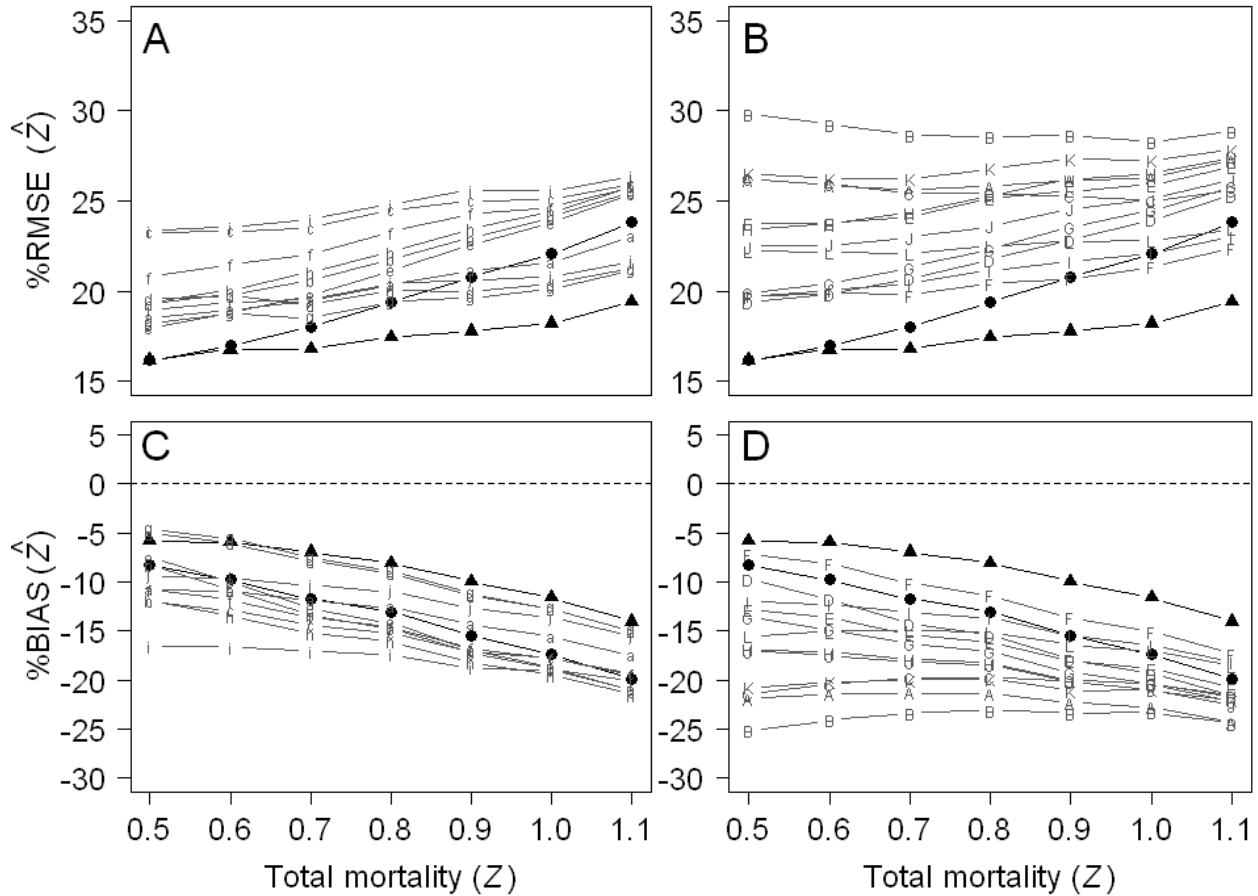


Figure 27. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak —●— and RG z-test —▲—) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.5$  year<sup>-1</sup>, gradual selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

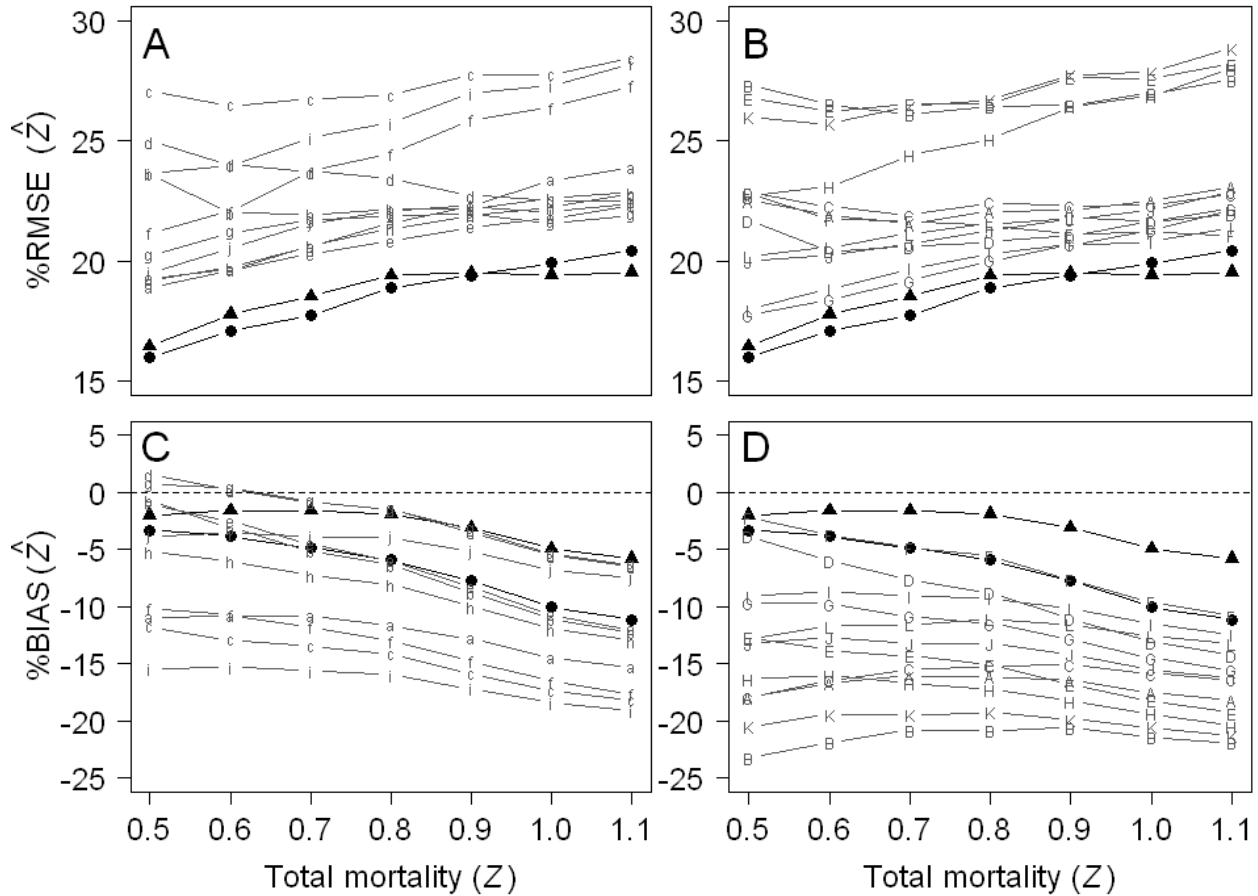


Figure 28. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.5$  year<sup>-1</sup>, gradual selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

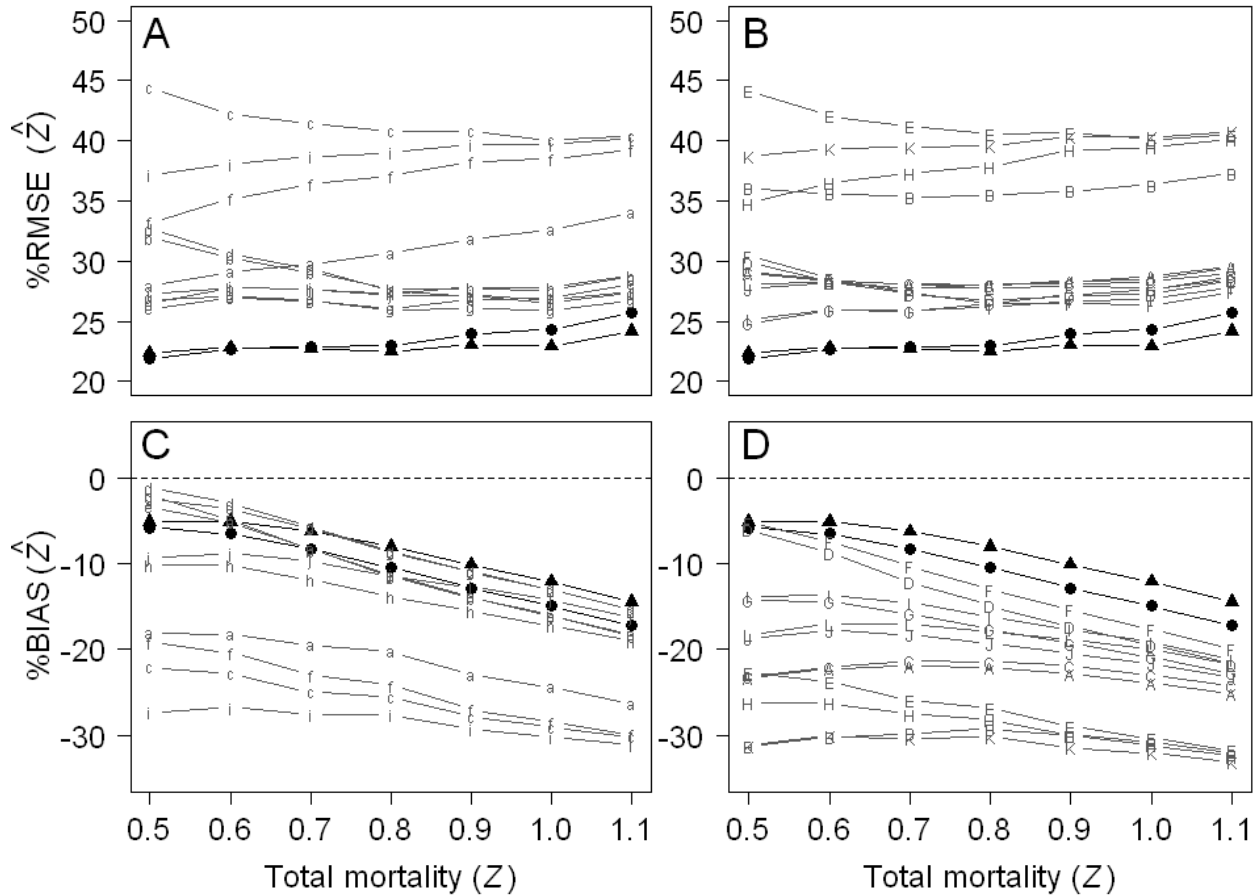


Figure 29. %BIAS (**bottom**) and %RMSE (**top**) of various mortality estimators as a function of total mortality rate ( $Z$ ). **Panels A and C:** the preferred weighted regression methods (RG Peak  $\bullet$  and RG z-test  $\blacktriangle$ ) compared to all other simulated weighted regressions (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). Simulation conditions were  $M = 0.5$  year<sup>-1</sup>, gradual selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error. **Panels B and D:** the preferred weighted regressions (as indicated above) compared to all simulated unweighted regressions (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were the same as in Panels A and C.

## **Performance of Chapman and Robson estimator with different age of full recruitment selection criteria**

The following plots show the relative performance of the Chapman and Robson total mortality rate estimator with the Peak, Peak Plus, and Z-test age of full recruitment selection criteria. Figures 30 - 33 show the %RMSE of the estimated  $Z$  for a given combination of natural mortality rate ( $M = 0.2 \text{ yr}^{-1}$  or  $M = 0.5 \text{ yr}^{-1}$ ), partial recruitment/selectivity curve (steep or gradual), sample size ( $N = 200$  or  $N = 600$ ) and non-sampling error ( $\sigma = 0.3$  or  $\sigma = 0.6$ ). Figures 34 - 37 show the %BIAS of the estimated  $Z$  for the same combinations as the %RMSE

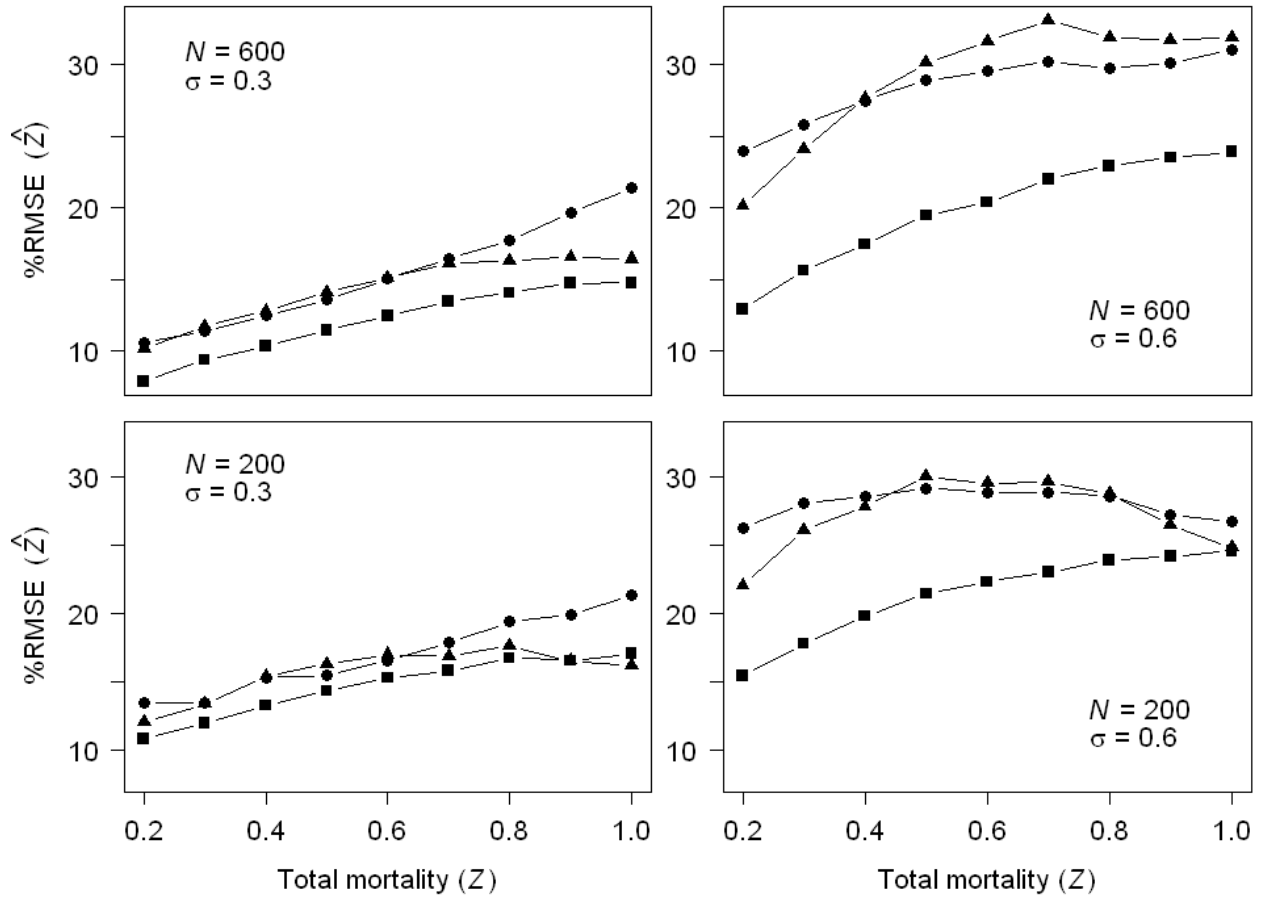


Figure 30. Percent root mean square error (%RMSE) of the Chapman-Robson total mortality rate ( $Z$ ) estimator, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.2 \text{ year}^{-1}$  and steep selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate.

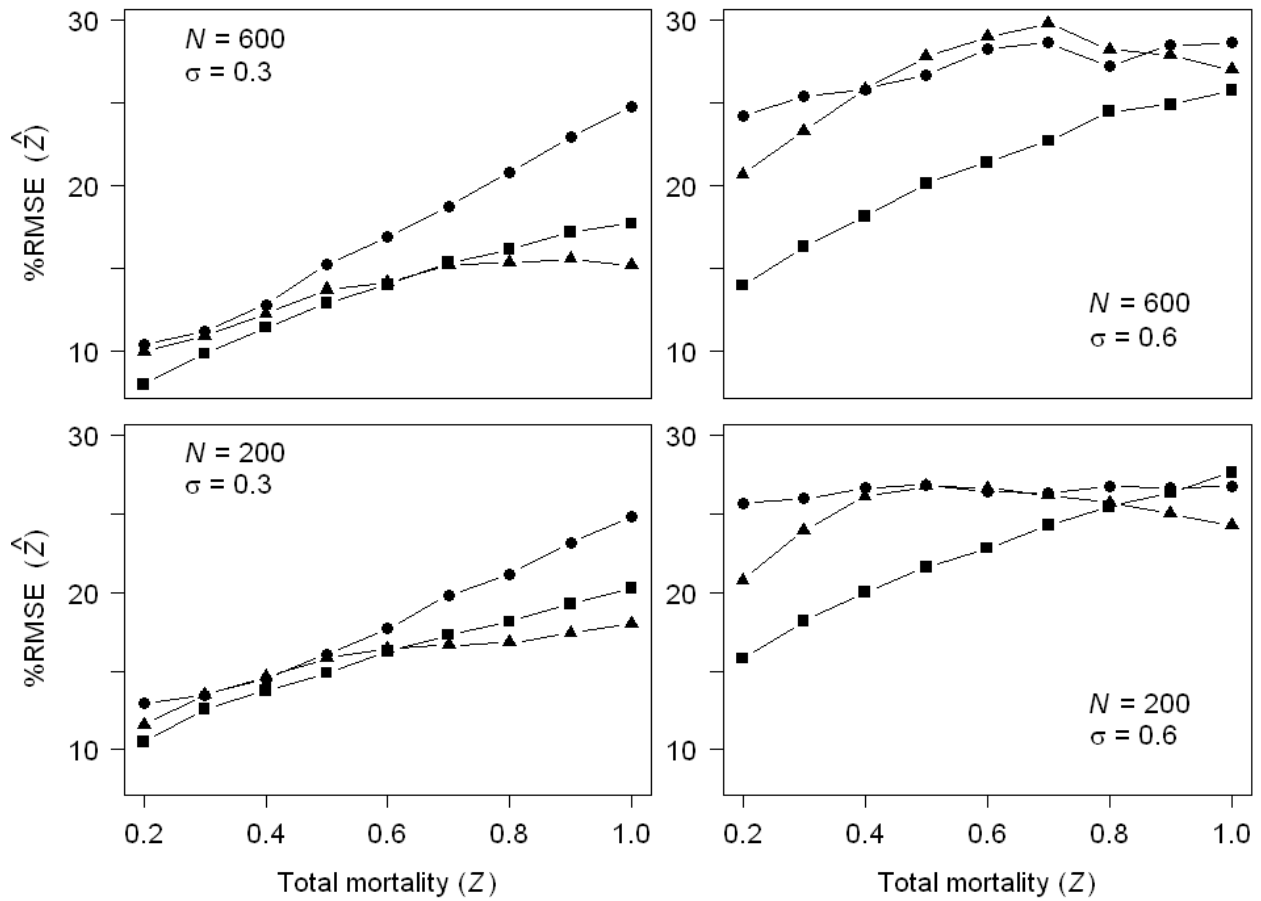


Figure 31. Percent root mean square error (%RMSE) of the Chapman-Robson total mortality rate ( $Z$ ) estimator, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.2 \text{ year}^{-1}$  and gradual selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate.

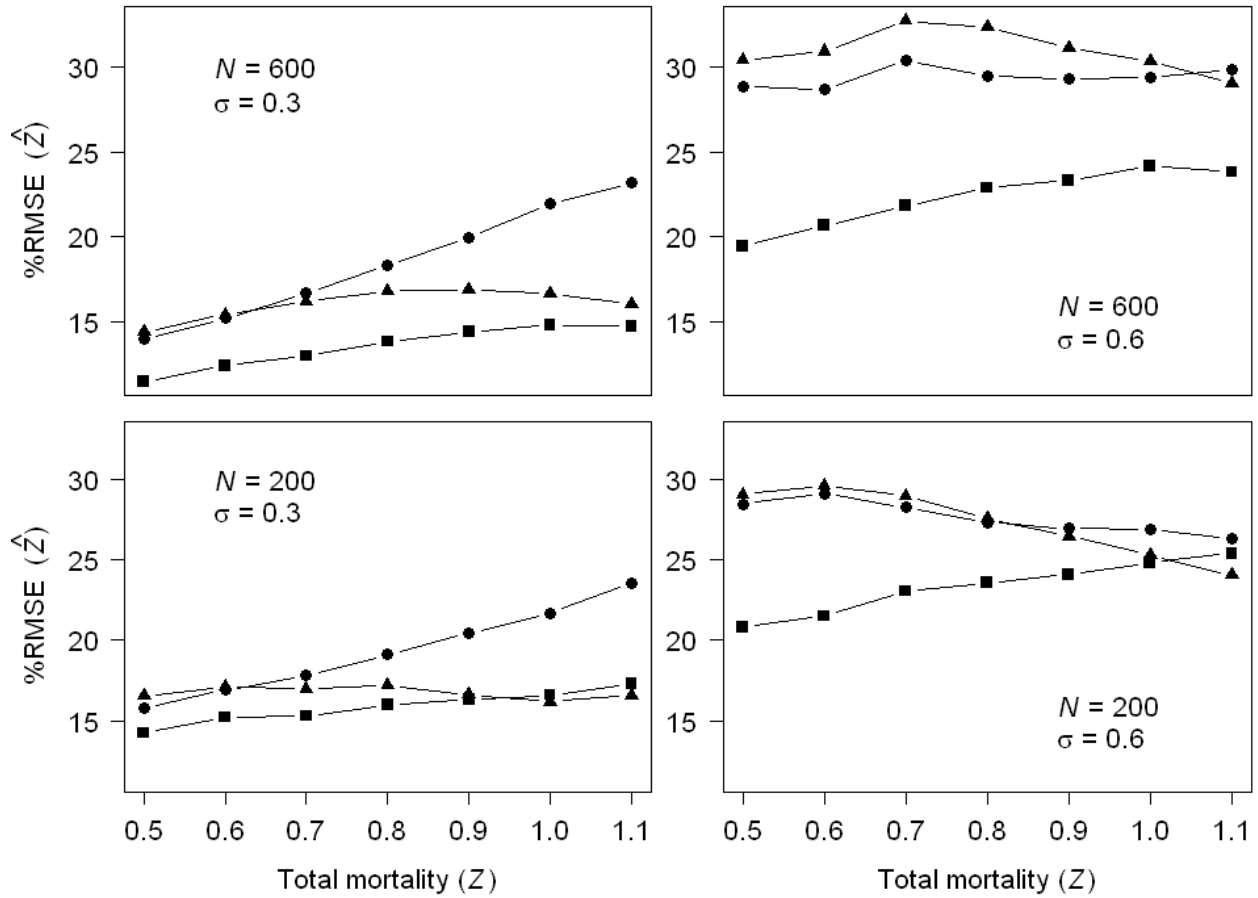


Figure 32. Percent root mean square error (%RMSE) of the Chapman-Robson total mortality rate ( $Z$ ) estimator, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.5 \text{ year}^{-1}$  and steep selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate.



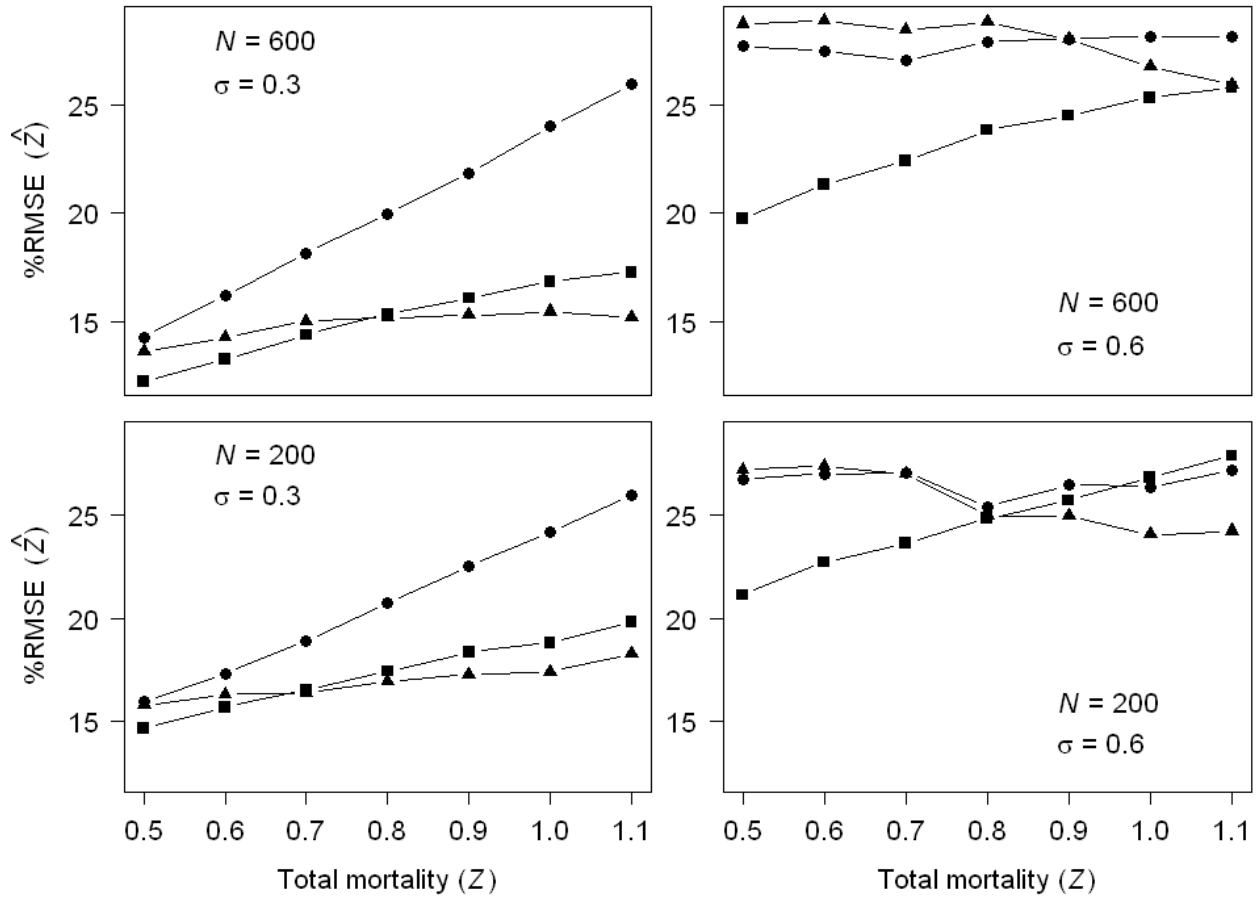


Figure 33. Percent root mean square error (%RMSE) of the Chapman-Robson total mortality rate ( $Z$ ) estimator, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.5 \text{ year}^{-1}$  and gradual selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate.

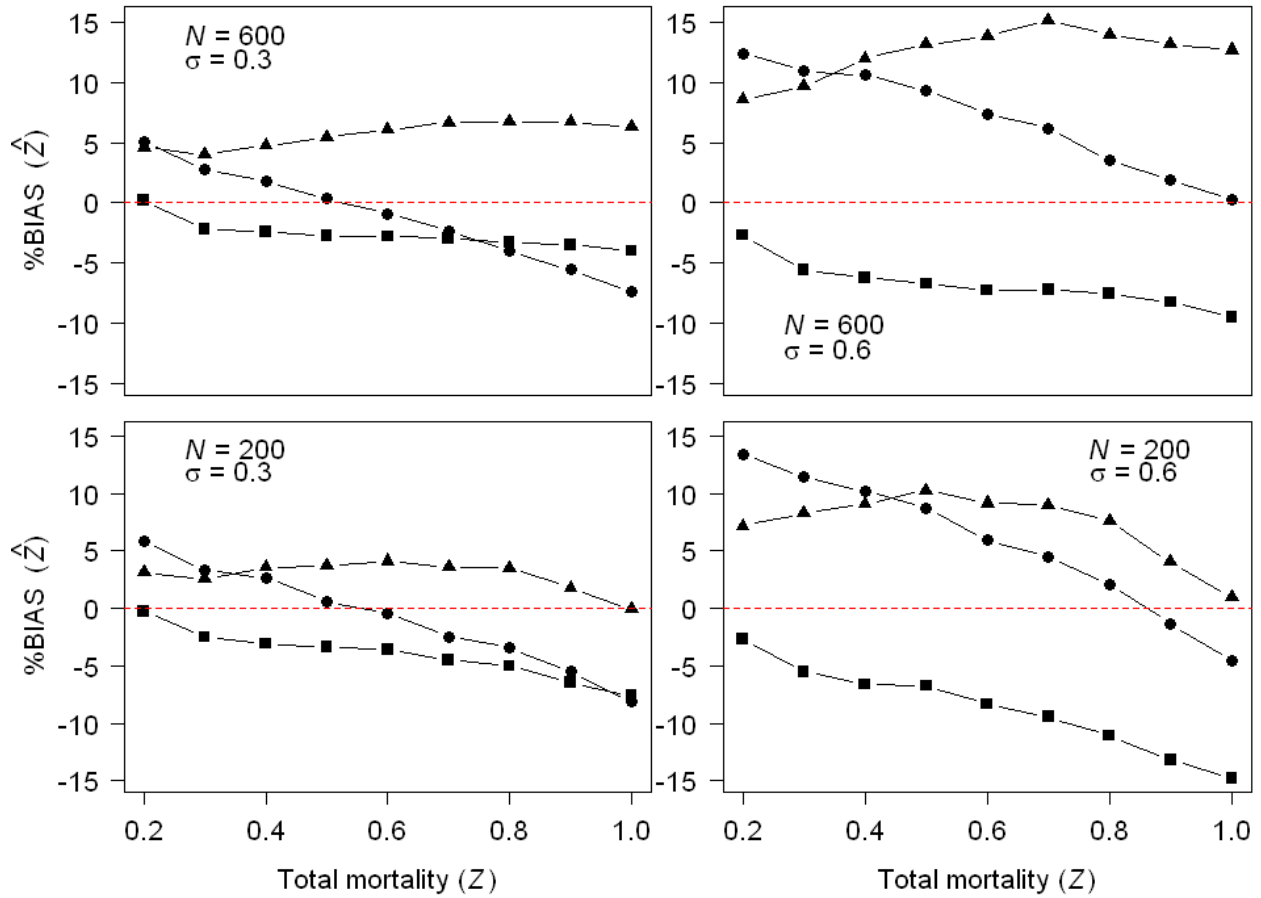


Figure 34. Percent bias (%BIAS) of the Chapman-Robson total mortality rate ( $Z$ ) estimator, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.2 \text{ year}^{-1}$  and steep selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate.

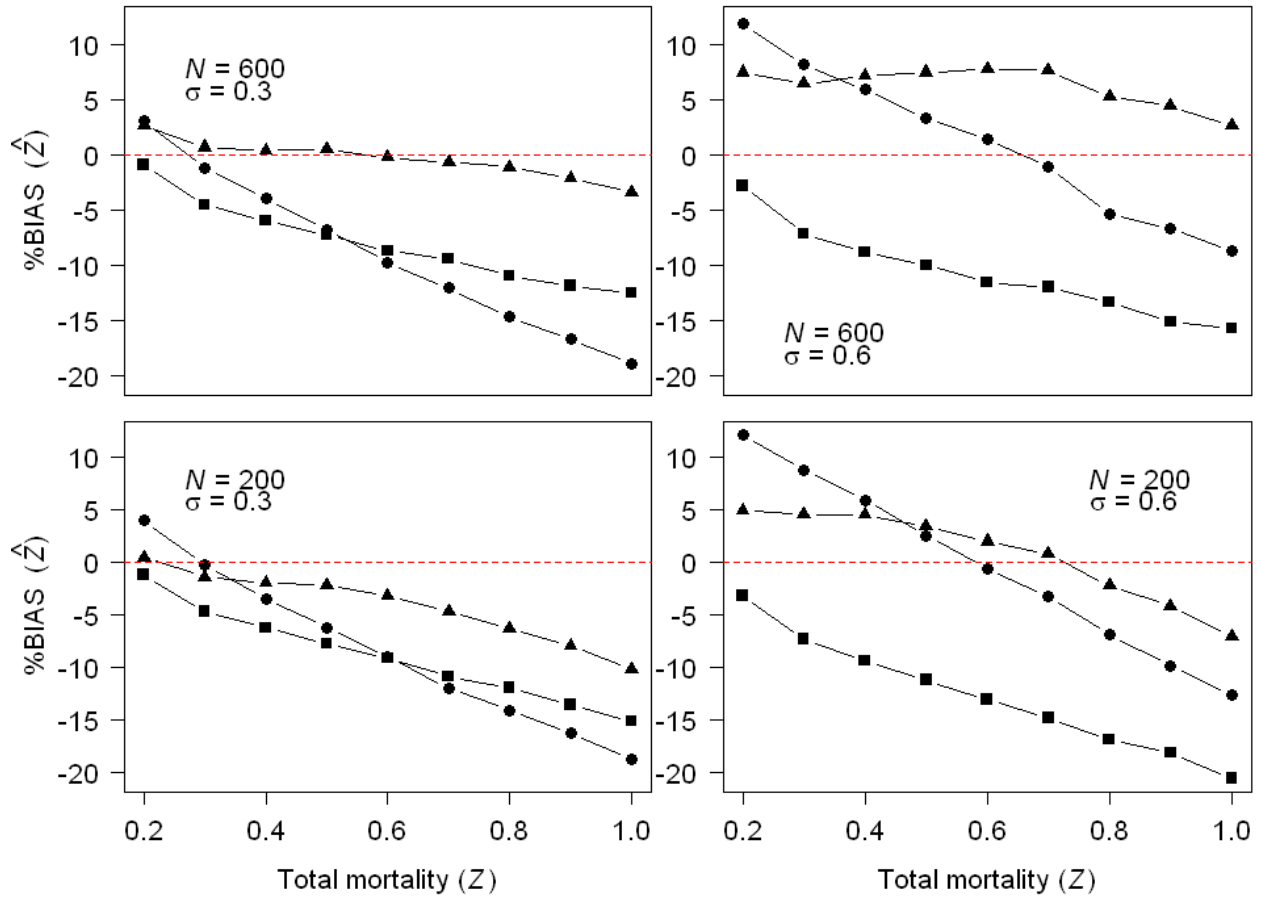


Figure 35. Percent bias (%BIAS) of the Chapman-Robson total mortality rate ( $Z$ ) estimator, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.2 \text{ year}^{-1}$  and gradual selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate.

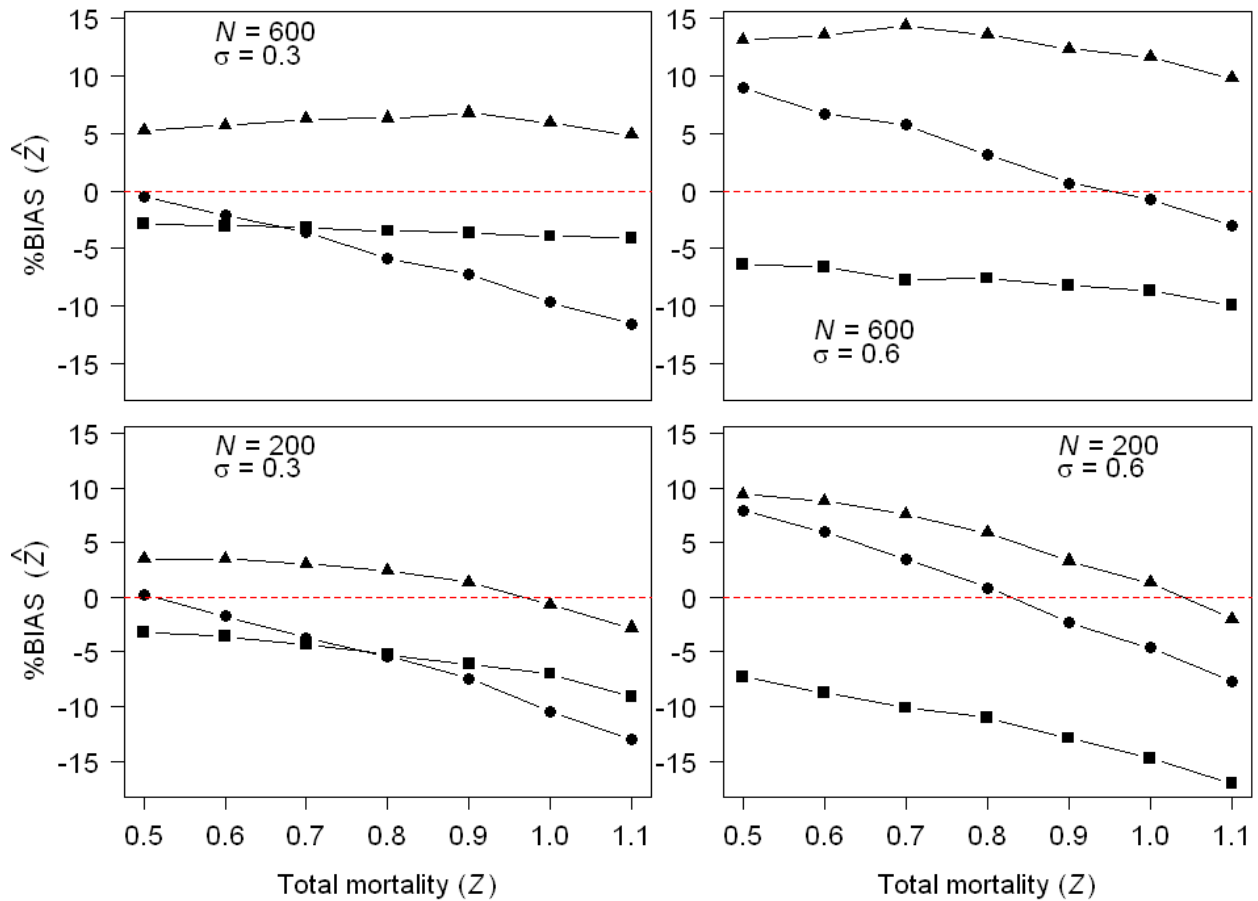


Figure 36. Percent bias (%BIAS) of the Chapman-Robson total mortality rate ( $Z$ ) estimator, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.5 \text{ year}^{-1}$  and steep selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate.

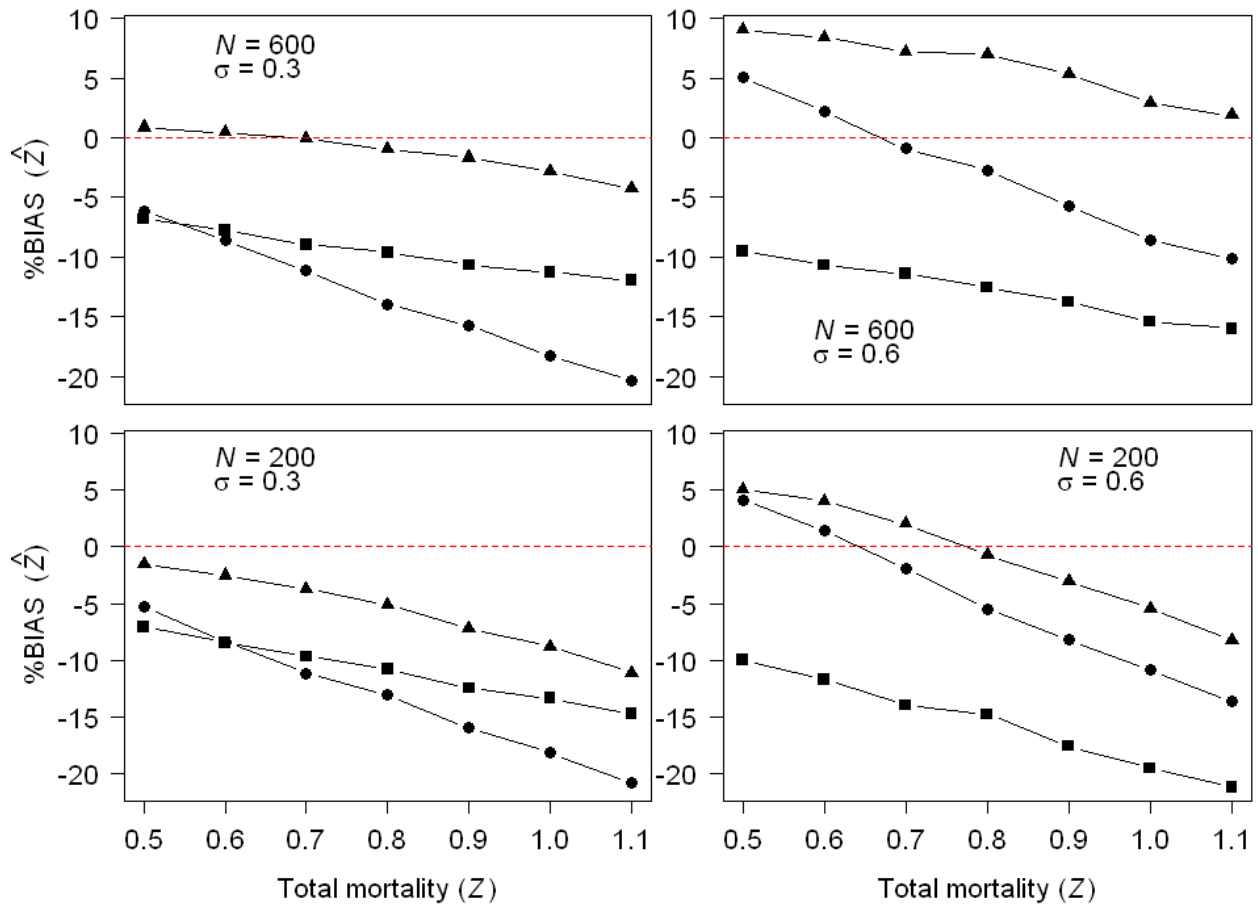


Figure 37. Percent bias (%BIAS) of the Chapman-Robson total mortality rate ( $Z$ ) estimator, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.5 \text{ year}^{-1}$  and gradual selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate.

## **Comparison of Chapman-Robson Peak Plus and un-truncated (RG) weighted regression total mortality rate estimators**

Percent bias (%Bias) and percent root mean square error (%RMSE) of the two best performing total mortality rate estimators for all natural mortality rates ( $M$ ), sample sizes ( $N$ ), non-sampling errors ( $\sigma$ ), and partial recruitment/selectivity curves (steep and gradual) simulated. For each scenario (separated by horizontal dotted and dashed lines), total mortality rate ( $Z$ ) increases from bottom to top. When  $M$  equals 0.2  $Z$  ranges from 0 to 0.8 and when  $M$  equals 0.5  $Z$  ranges from 0 to 0.6.

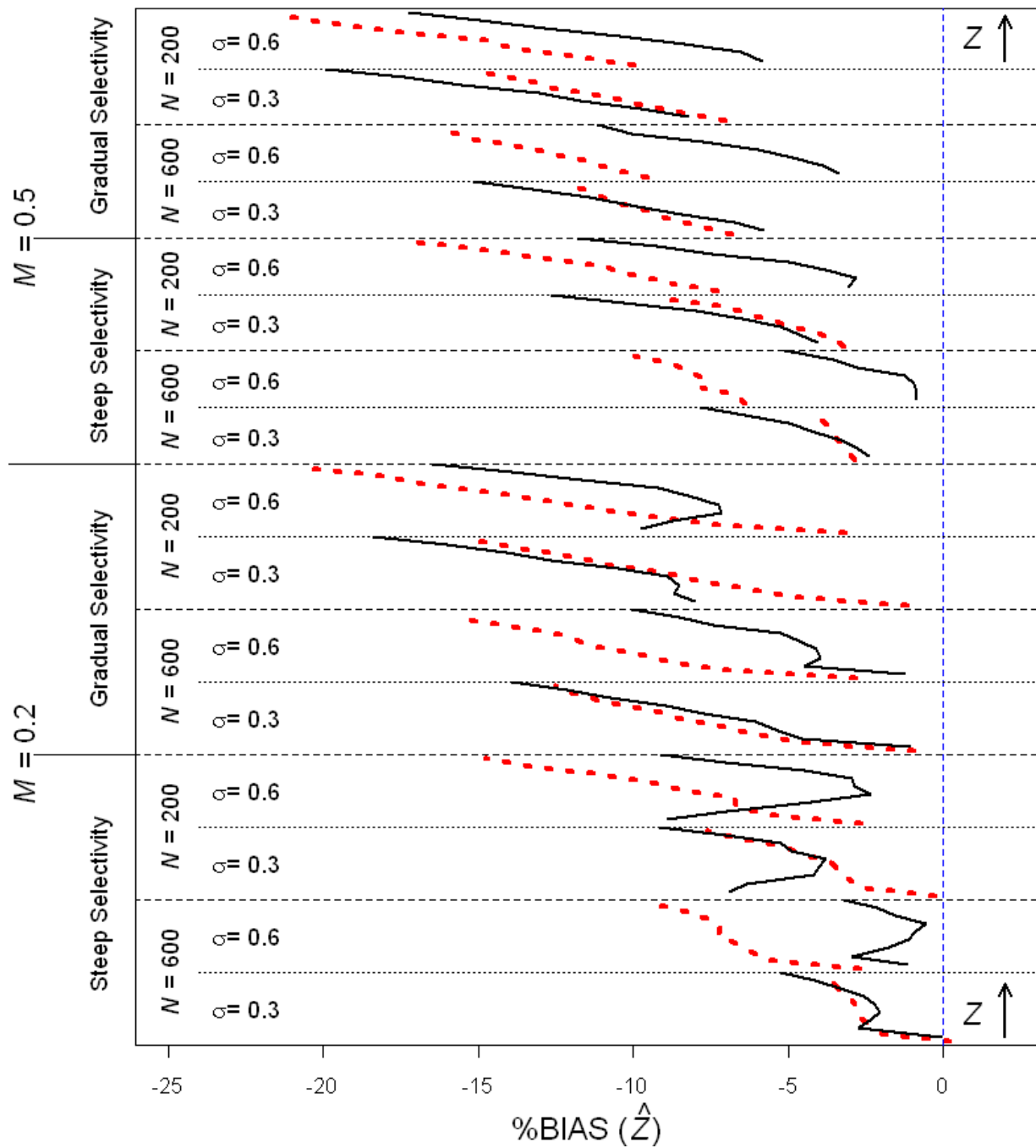


Figure 38. Percent bias (%BIAS) of the Chapman-Robson Peak Plus total mortality rate estimator (dotted red line) and the weighted regression estimator with Peak age of full recruitment (solid black line) as a function of natural mortality rate ( $M$ ), selectivity/partial recruitment pattern, sample size ( $N$ ), nonsampling error ( $\sigma$ ), and total mortality rate ( $Z$ ). In each strip,  $Z$  increases from bottom to top. Results of all 128 scenarios are shown.

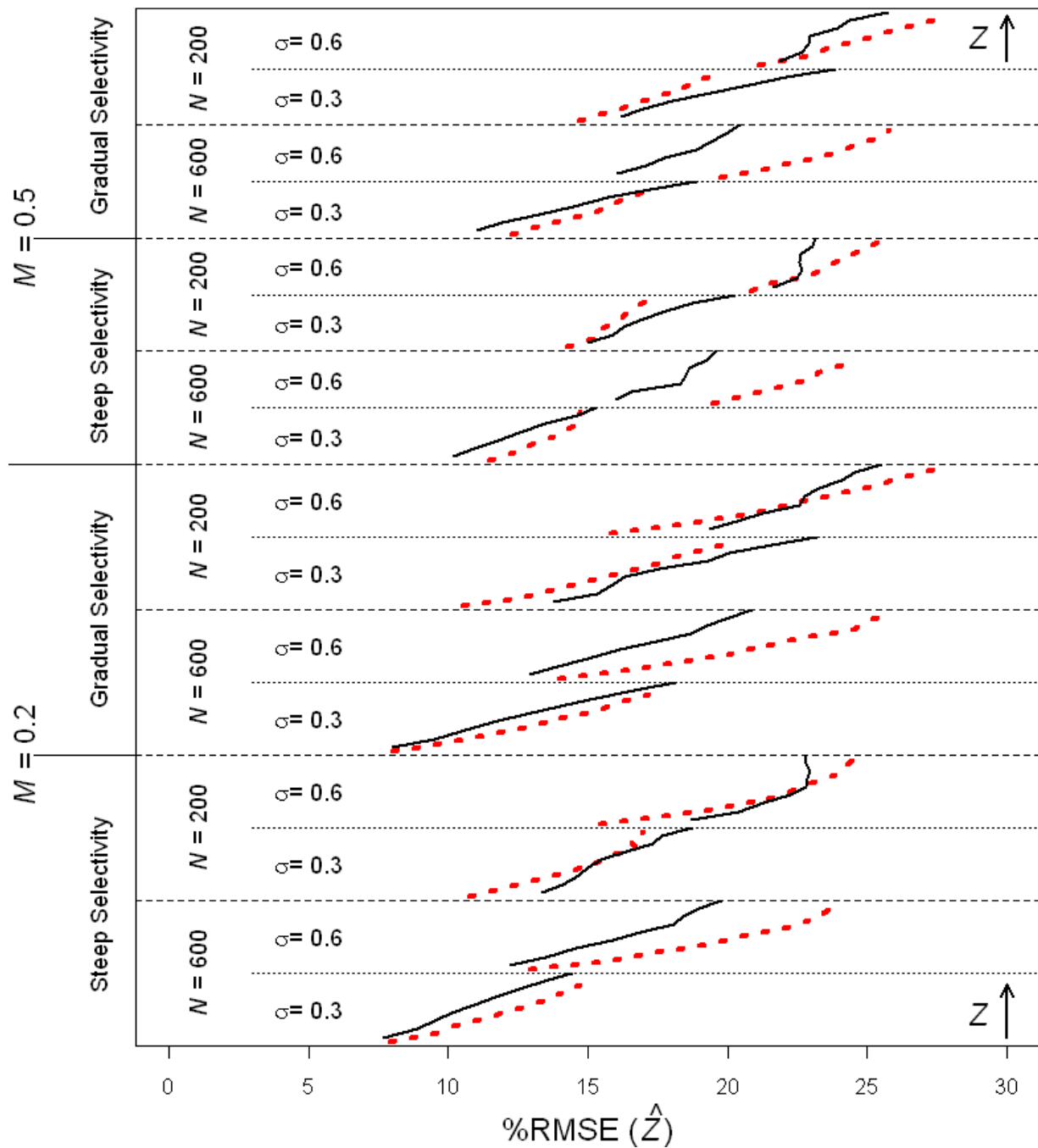


Figure 39. Percent root mean square error (%RMSE) of the Chapman-Robson Peak Plus total mortality rate estimator (dotted red line) and the weighted regression estimator with Peak age of full recruitment (solid black line) as a function of natural mortality rate ( $M$ ), selectivity/partial recruitment pattern, sample size ( $N$ ), nonsampling error ( $\sigma$ ), and total mortality rate ( $Z$ ). In each strip,  $Z$  increases from bottom to top. Results of all 128 scenarios are shown.



## **Comparison of the uncorrected and corrected Chapman-Robson total mortality rate variance estimators.**

Percent root mean square error (%RMSE) and percent bias (%BIAS) of the uncorrected and corrected Chapman-Robson variance estimator. A quasi-likelihood approach was used to correct the variance estimator presented by Chapman and Robson (1960) for a substantial negative bias which occurred as a result of overdispersion in the simulated data (see Smith et al. 2012). Simulated results are presented for three age of full recruitment selection criteria, at various sample sizes ( $N$ ), and levels of non-sampling error ( $\sigma$ ). Only results for the scenario where natural mortality rate ( $M$ ) is equal to 0.2 and selectivity/partial recruitment was steep are presented.

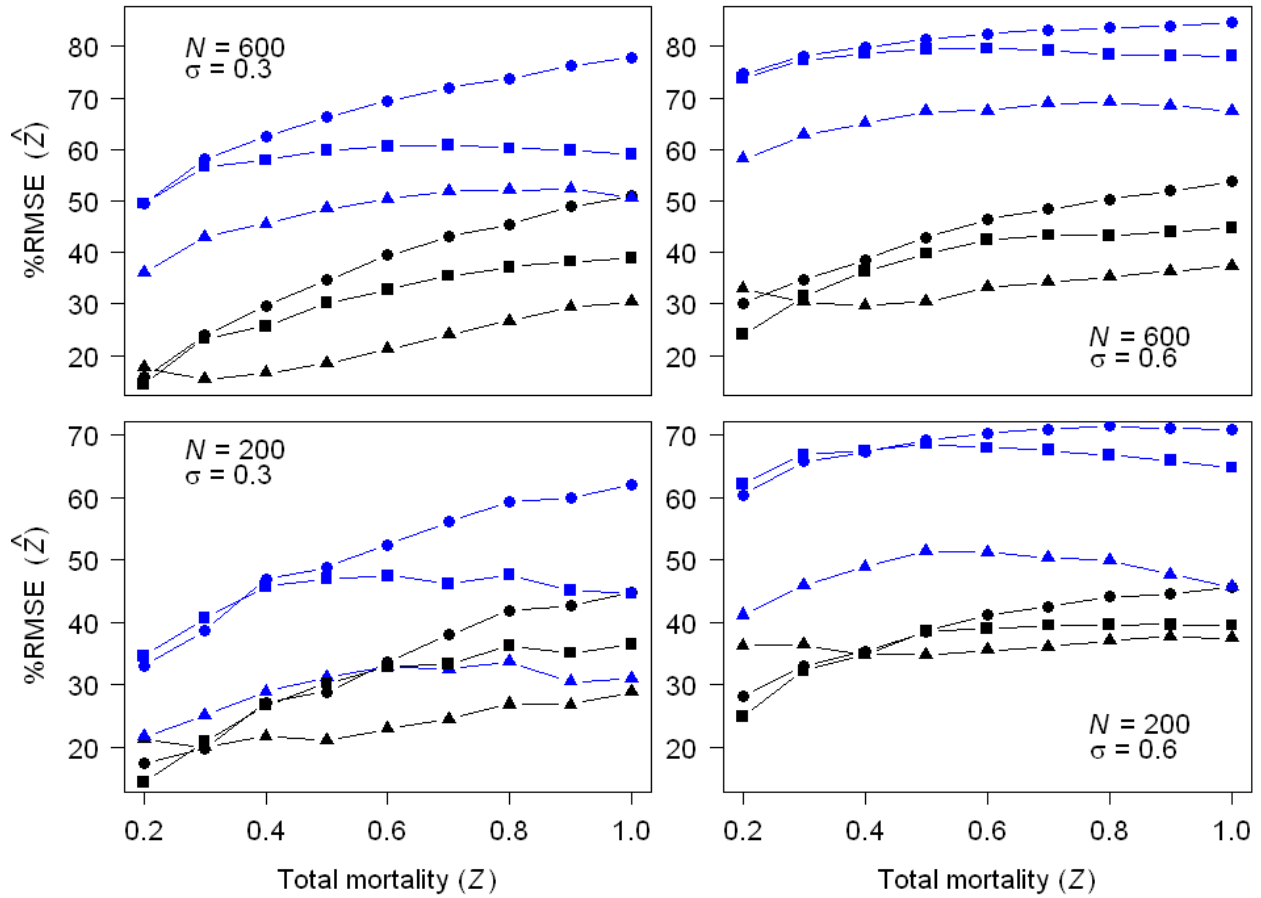


Figure 40. %RMSE for uncorrected (blue) and corrected (black) Chapman-Robson variance estimators, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.2 \text{ year}^{-1}$  and steep selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size,  $\sigma$  is the level of non-sampling error, and  $Z$  is the instantaneous total mortality rate.

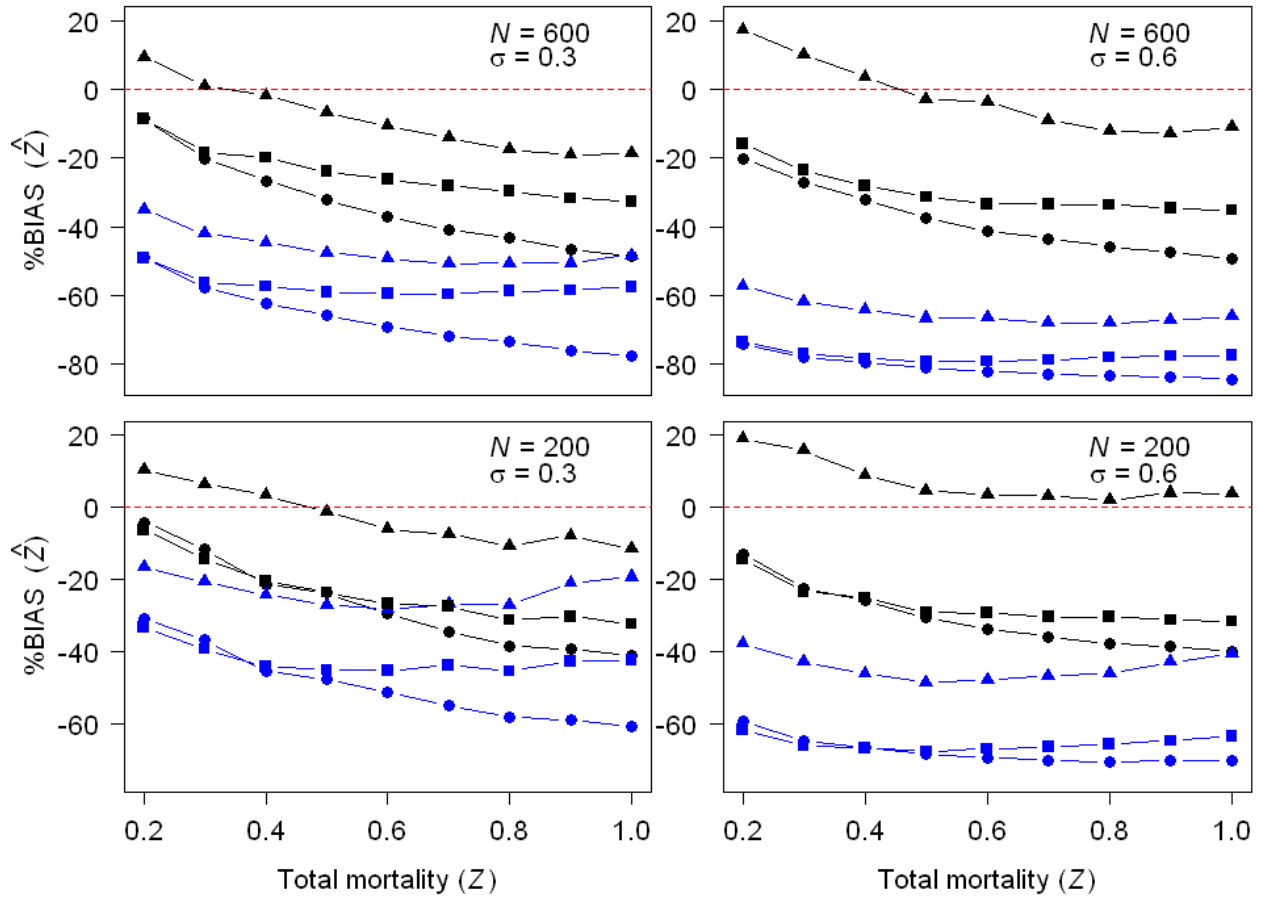


Figure 41. %BIAS for uncorrected (blue) and corrected (black) Chapman-Robson variance estimators, with three methods of selecting the first fully recruited age, (—●—) Peak, (—■—) Peak Plus and (—▲—) z-test. Conditions were:  $M = 0.2 \text{ year}^{-1}$  and steep selectivity/partial recruitment pattern, where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size,  $\sigma$  is the level of non-sampling error, and  $Z$  is the instantaneous total mortality rate.

**Performance of the Chapman-Robson Peak plus variance estimator with correction for overdispersion and the RG Peak weighted regression variance estimator**

The following plots highlight the performance of the Chapman-Robson Peak Plus variance estimator corrected for overdispersion and the RG Peak weighted regression variance estimator. These variance estimators are compared against all other weighted (left column) and unweighted (right column) variance estimators tested in the simulation. Performance is measured as the %RMSE and %BIAS of the estimated  $se(Z)$  with the true standard error set equal to the standard deviation of the 5000 estimates of  $Z$  obtain from the simulation. In each plot, %RMSE results are displayed in the top row, and %BIAS results are displayed in the bottom row. Results are displayed for the scenarios where natural mortality rate ( $M$ ) is equal to  $0.2 \text{ year}^{-1}$  and partial recruitment/selectivity curve is steep.

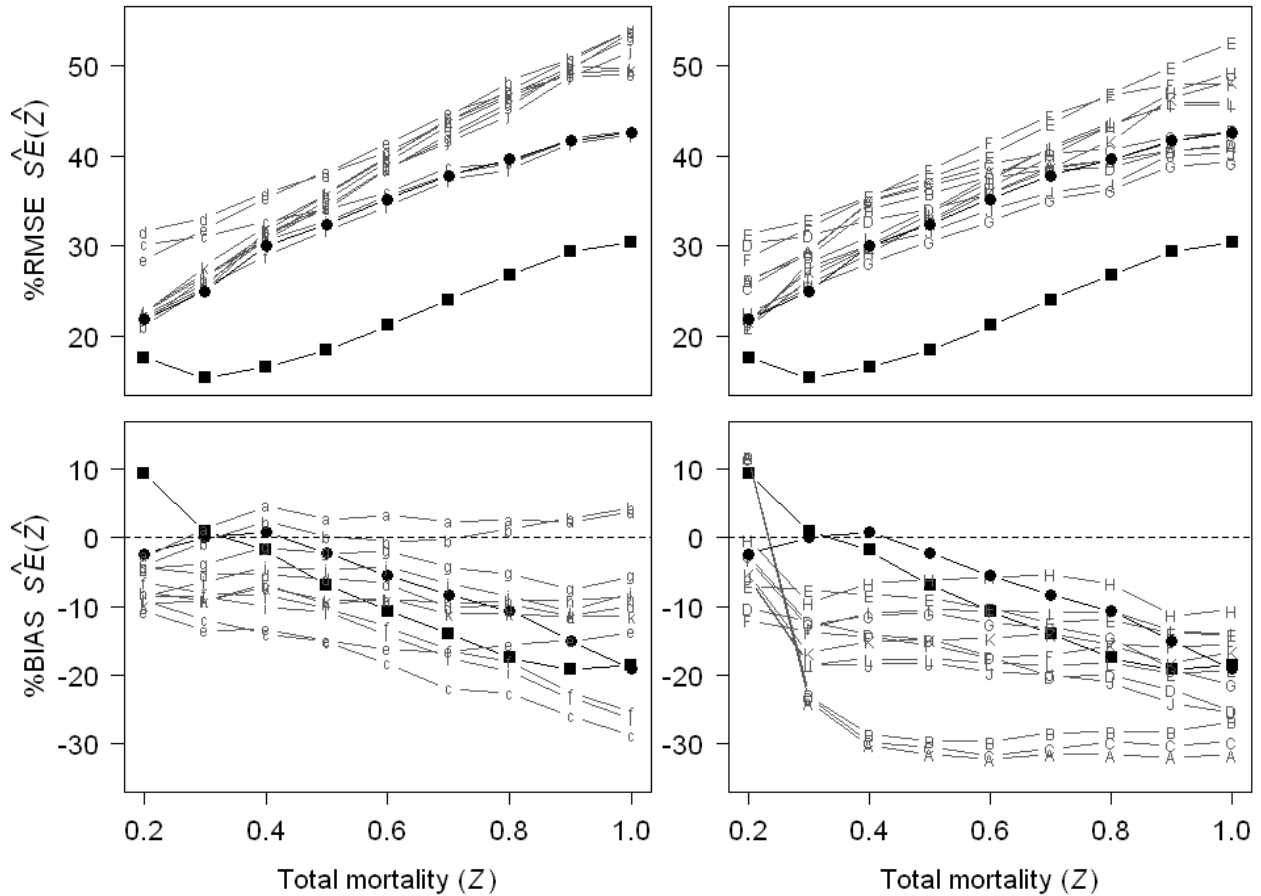


Figure 42. Performance of the Chapman-Robson Peak Plus variance estimator with correction for overdispersion (—■—), and the RG Peak weighted regression variance estimator (—●—) in terms of %RMSE (**top**) and %BIAS (**bottom**). In the left column, highlighted variance estimators are plotted against all other simulated weighted regression variance estimators (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). In the right column, the preferred highlighted variance estimators are plotted against all other simulated unweighted regression variance estimators (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were  $M = 0.2 \text{ year}^{-1}$ , steep selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error.

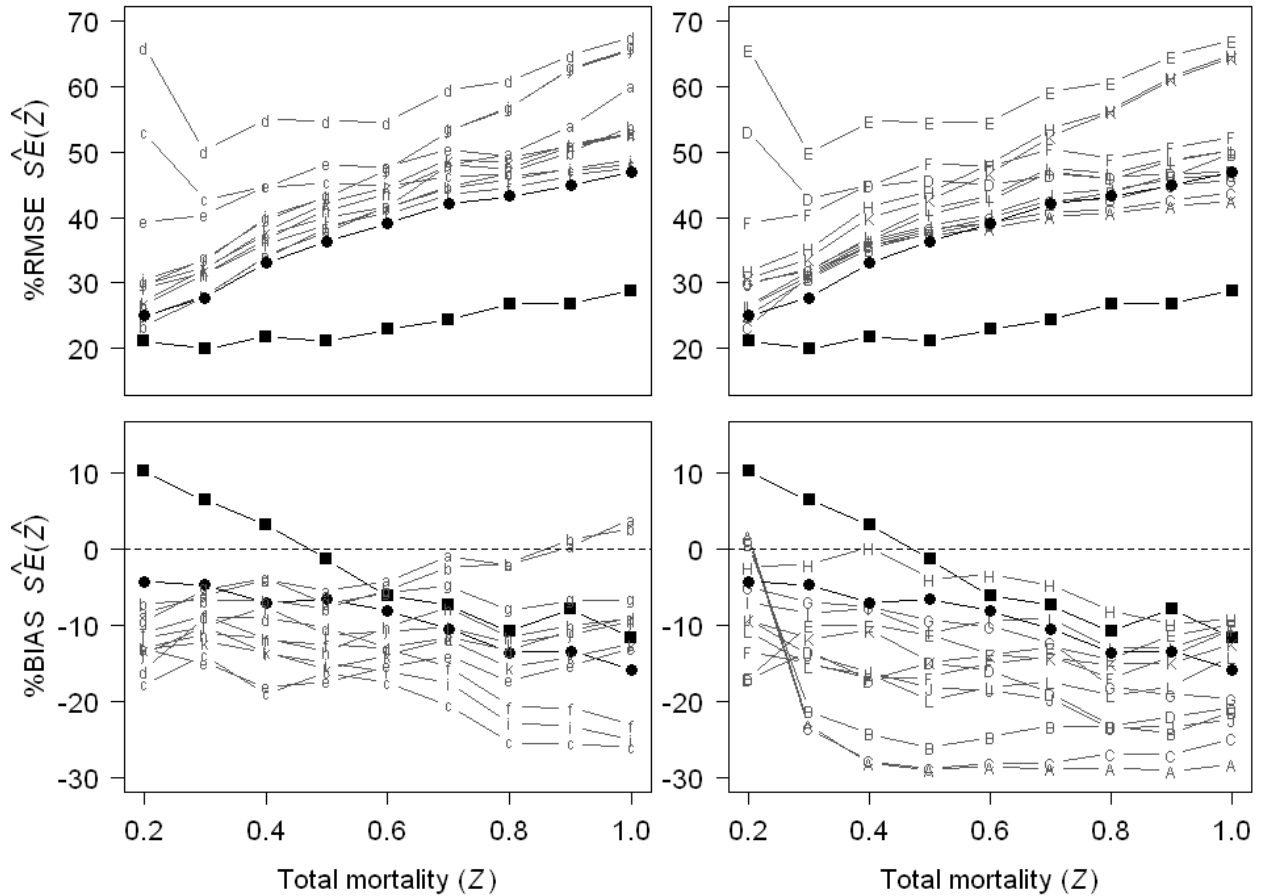


Figure 43. Performance of the Chapman-Robson Peak Plus variance estimator with correction for overdispersion ( $\blacksquare$ ), and the RG Peak weighted regression variance estimator ( $\bullet$ ) in terms of %RMSE (**top**) and %BIAS (**bottom**). In the left column, highlighted variance estimators are plotted against all other simulated weighted regression variance estimators (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). In the right column, the preferred highlighted variance estimators are plotted against all other simulated unweighted regression variance estimators (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were  $M = 0.2 \text{ year}^{-1}$ , steep selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.3$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error.

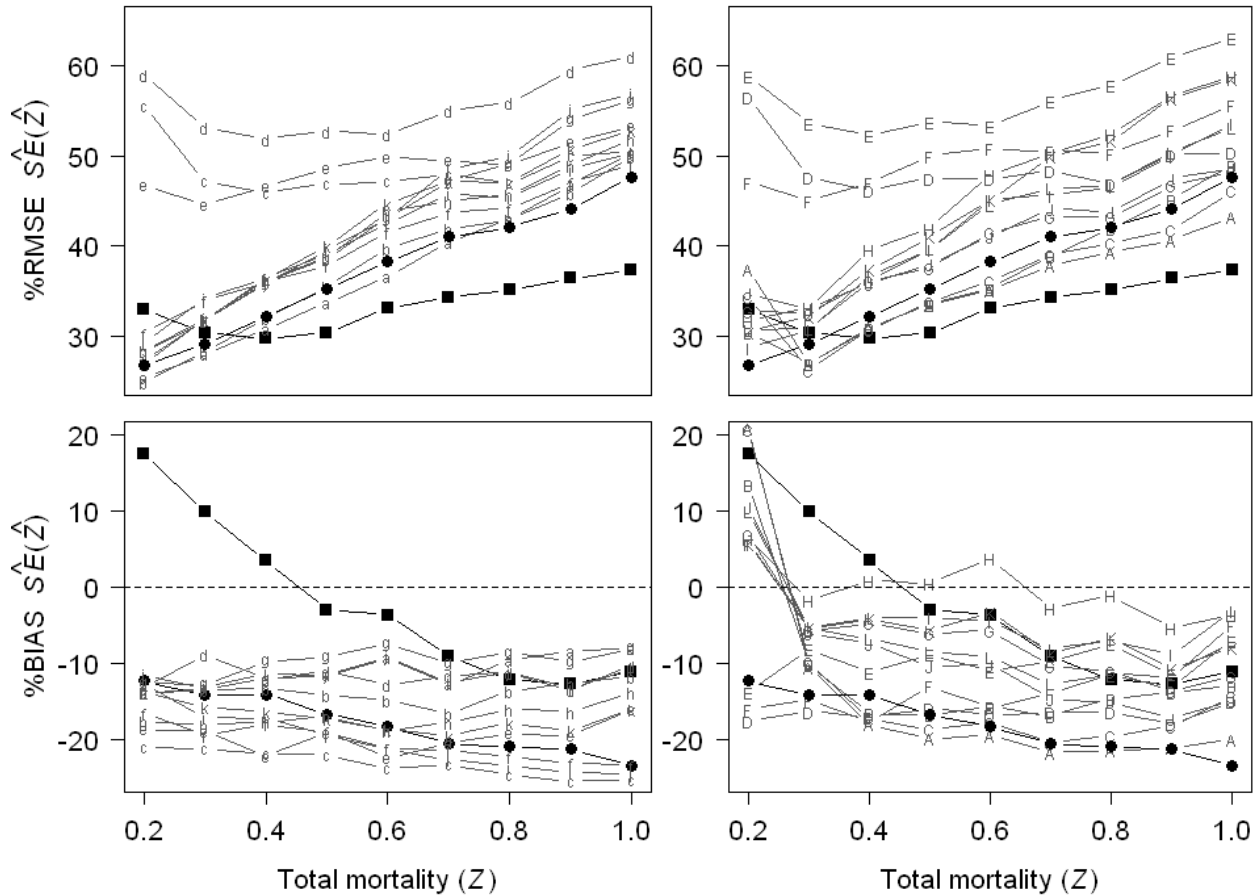


Figure 44. Performance of the Chapman-Robson Peak Plus variance estimator with correction for overdispersion (—■—), and the RG Peak weighted regression variance estimator (—●—) in terms of %RMSE (**top**) and %BIAS (**bottom**). In the left column, highlighted variance estimators are plotted against all other simulated weighted regression variance estimators (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). In the right column, the preferred highlighted variance estimators are plotted against all other simulated unweighted regression variance estimators (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were  $M = 0.2 \text{ year}^{-1}$ , steep selectivity/partial recruitment curves,  $N = 600$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error.

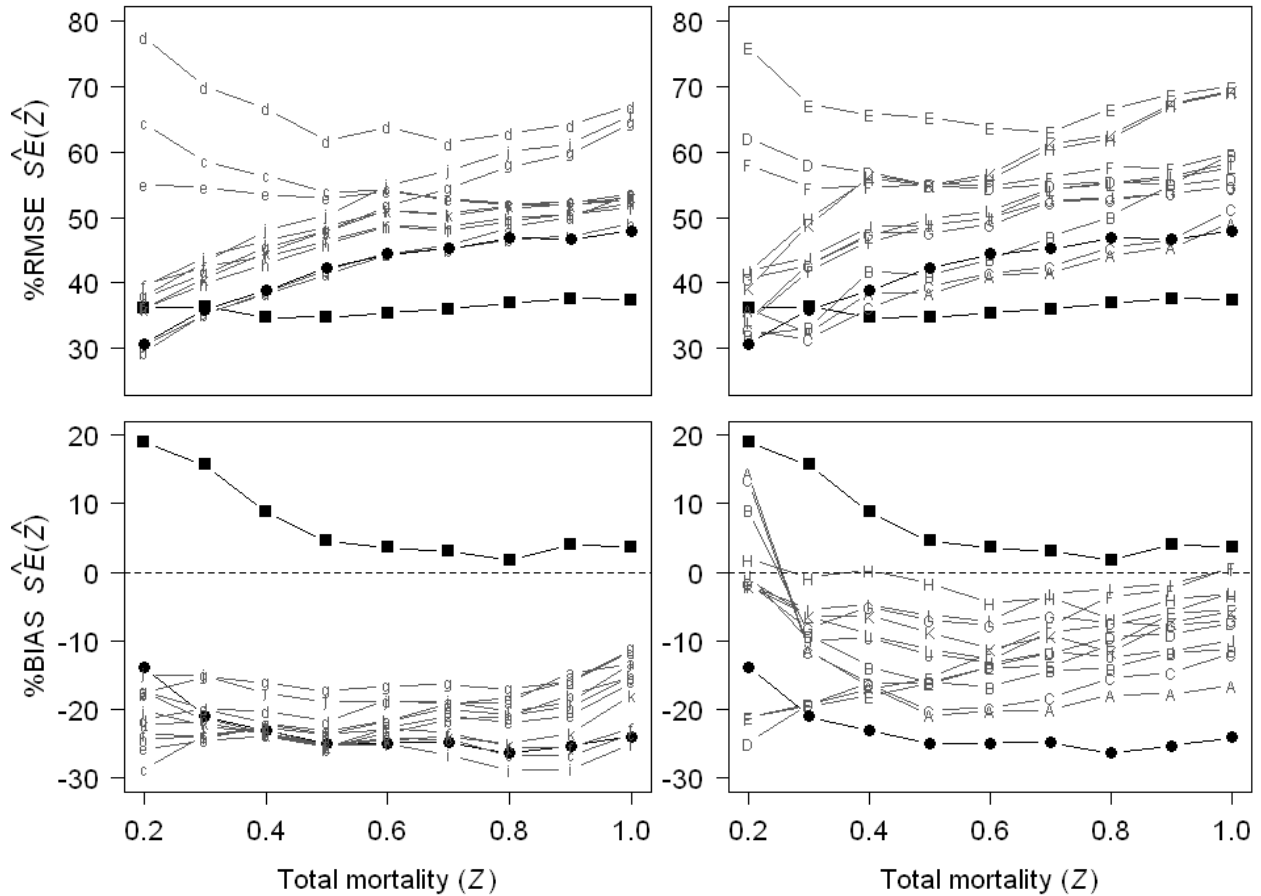


Figure 45. Performance of the Chapman-Robson Peak Plus variance estimator with correction for overdispersion (—■—), and the RG Peak weighted regression variance estimator (—●—) in terms of %RMSE (**top**) and %BIAS (**bottom**). In the left column, highlighted variance estimators are plotted against all other simulated weighted regression variance estimators (a = RG Peak plus, b = R1 Peak, c = R1 Peak plus, d = R1 z-test, e = R1' Peak, f = R1' Peak plus, g = R1' z-test, h = R1'' Peak, i = R1'' Peak plus, j = R1'' z-test). In the right column, the preferred highlighted variance estimators are plotted against all other simulated unweighted regression variance estimators (A = RG Peak, B = RG Peak plus, C = RG z-test, D = R1 Peak, E = R1 Peak plus, F = R1 z-test, G = R1' Peak, H = R1' Peak plus, I = R1' z-test, J = R1'' Peak, K = R1'' Peak plus, and L = R1'' z-test). Simulation conditions were  $M = 0.2 \text{ year}^{-1}$ , steep selectivity/partial recruitment curves,  $N = 200$ ,  $\sigma = 0.6$  where  $M$  is the instantaneous natural mortality rate,  $N$  is the sample size, and  $\sigma$  is the total error.