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Simultaneous Decision Making of Optimal Toll Levels and Locations in a Multi-Class Network Equilibrium: Genetic Algorithm Approach

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SIMULTANEOUS DECISION MAKING OF OPTIMAL TOLL LEVELS AND LOCATIONS IN A MULTI-CLASS NETWORK EQUILIBRIUM: GENETIC ALGORITHM APPROACH

By

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Abstract

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The purpose of this thesis is to explore bi-level genetic algorithm (GA) based optimization models to make decisions simultaneously for the second-best optimal toll locations and toll levels. The upper-level subprogram is to minimize the total travel time (system cost). The lower-level subprogram is a user equilibrium problem where all users try to find the route that minimizes their own travel cost (or time). The demand is assumed to be fixed and given a priori. First, two different versions of GA based solution procedures are developed and applied to an example Sioux Falls network assuming homogeneous road users in the network. This kind of problem is referred to as a single-class optimization problem. However, in reality heterogeneous road users exist. As such, the two GA options are compared with one another and the preferred GA option is further applied to the network consisting of multiclass users with different value of times (VOTs). Another heuristic approach is also considered to determine toll rates only on the most congested links for both single-class and multiclass scenarios. Such heuristic toll rates are compared with the combined solution of optimal location and toll rates to demonstrate the most congested links in a network may not be considered as intuitive candidates for optimal toll locations.

Chapter 1 – Introduction

Congestion is continually posing a threat to the economy of many countries and quality of life of millions of people in the world. The impacts of congestion are far-reaching. To name a few, it tends to decrease mobility and indirectly affect accessibility, affects business performance, results in travel time loss and contributes to air pollution. In the U.S., for instance, according to a study by the Texas Transportation Institute (TTI), congestion in 439 urban areas is observed to have caused 4.8 billion hours of travel delay (this is equivalent to the time Americans spend relaxing and thinking in 10 weeks) and 1.9 billion gallons of wasted fuel (which is equivalent to about 2 months of flow in the Alaska Pipeline), for a total cost of $101 billion in the year 2010 (not to mention the cost on the negative effect of uncertain or longer delivery times, missed meetings, business relocations and other congestion-related effects) (TTI, 2011). This figure would be noticeably higher (perhaps almost triple) if it accounted for the significant cost of growing system unreliability and unpredictability to drivers and businesses, the environmental impacts of idle-related auto emissions, or higher gasoline prices.

Undoubtedly, the level of congestion on roads has increased substantially over the past few decades. As a consequence, there is always a need to focus on congestion relief and mitigation strategies. Broadly speaking, there are two strategies that can be considered and applied in order to alleviate the congestion problem; the first being supply-side improvements to improve capacity and the second being demand-side control strategies to make existing transportation facilities work better. The two strategies are briefly discussed in the next two paragraphs.

The first strategy i.e. increasing the supply to enhance capacity includes intersection improvement, increasing the number of lanes, construction of new alternative routes, proposing new infrastructure projects – from roads to bridges to transit facilities, or other road facility expansions. These improvements will increase the capacity of the roadway and therefore will in fact relieve congestion for the time being. However, just adding capacity may lead to opposite effects than those originally anticipated (Triantisa et al., 2011). Through the course of time, once road users realize that their travel time can be greatly reduced by taking the improved roadway, the demand will quickly increase on that roadway because the new facility may attract additional traffic from other routes (Litman, 2011). This can lead to congestion in the improved road because as the demand increases, it may equal or exceed the capacity just as it did before the improvement. Hence, supply improvements are not generally recommended solutions to the congestion problem and are therefore less efficient. Besides, capacity expansions are expensive and time consuming. Therefore, simply expanding all of the roadways is not as such an attractive solution from economy point of view.

The other strategy is to make the existing transportation facilities work better by controlling the travel demand, and it is proving to be a more long-lasting solution as compared to the first strategy. Demand-side strategy can also be relatively easy to implement in a shorter timeframe, within a more constrained budget, than supply side strategy. It includes alternative mode encouragement strategies like Park-and-Ride facilities, High Occupancy Vehicle (HOV) facilities, transit service improvements and transit payment innovations, ridesharing programs, telecommuting, alternative work hours, driving disincentives like increased fuel tax/mile fee, congestion pricing, etc. (Triantisa et al., 2011). In this thesis, congestion pricing is considered and further discussed to address the problem of congestion.

Congestion pricing has long been recognized as a potential way of reducing traffic congestion and air pollution in the past few decades (Yang and Zhang, 2003). In addition, it serves as a source of revenue for federal highways' funding, which could be used to expand and improve transportation infrastructure. It has been efficiently implemented in many metropolises around the world (for example, in London in 2003 and Stockholm in 2006). Congestion pricing works by shifting purely flexible rush hour travel demand to other transportation modes or to off-peak periods. The concept of road pricing comes from the idea that road users are actually paying lower costs than the cost they impose.

To account for this difference, users on each link in a road network are charged a marginal-cost to drive user equilibrium flow pattern to system optimum or simply to make the traffic conditions move closer to an optimal state (Hearn and Ramana, 1998; Yang and Zhang, 2003). This kind of pricing scheme is called the first best pricing.

However, the first-best pricing is not practically appealing because of high operating costs for toll collection and poor public acceptance. For example, introducing a new tax has never been popular. People tend to forget that toll revenues collected by the government become available again to society. Hence, second-best pricing scheme, where only a subset of links is subjected to toll charges, has lately received much attention (Yang and Huang, 2005). Its relative advantage over the first-best pricing scheme has been thoroughly studied and discussed by different researchers (Yang and Lam, 1996; Verhoef et al., 1996; Lindsey and Verhoef, 2001; Verhoef, 2002). A classic example of the second-best pricing problem that concerns a two route network, where an untolled route is available, is presented by different authors elsewhere (Marchand, 1968; Verhoef et al., 1996; Liu and McDonald, 1999). The concepts of user equilibrium, system optimum, first-best pricing and second-best pricing will be discussed in detail in the next chapter.

Designing a congestion pricing scheme is not an easy task because one needs to deal with the complex nature of transportation systems and the network evaluation process. Transport economists and planners normally evaluate the efficiency of a pricing scheme by total system travel time and/or social welfare measures, and these measures can be used as objectives in an optimization framework. The objectives in the framework need to be carefully defined before solving any congestion pricing problem. Once the objective is defined, important principles of transport planning such as traffic assignment i.e. allocating traffic to paths and links can be used along with other heuristic techniques to find satisfactory optimal solutions to even complex congestion pricing problems. The optimal solutions are usually toll levels and/or toll locations for a given network. However, very little attention has been given to the combined determination of toll level and toll locations.

The optimal toll level and toll location problem can be seen as a type of the network design problem. Several methods have been proposed to deal with this challenging problem and a brief review on different methods can be found elsewhere (Shepherd and Sumalee, 2004). It is difficult to solve the network design problem with traditional network optimization methods. There are many combinations of choosing the required number of toll links even from the subset. Therefore, there is a need to find a method which can go through all combinations to search the optimal combination. Genetic algorithm (GA) is capable of doing this, because it has a favorable procedure of natural selection. GA based approach has been applied to address second best optimal toll design problem by a few researchers (Cree et al., 1998; Yang and Zhang, 2003; Shepherd and Sumalee, 2004). However, in these studies the approach was applied to determine either toll locations or toll levels, not both simultaneously. For example, Cree et al. (1998) developed the GA based method to solve the optimal toll problem but not the location problem. Shepherd and Sumalee (2004) developed an alternative GA based approach for finding optimal toll levels for a predefined set of chargeable links and for finding optimal toll locations. The so called "CORDON" method was used to determine the toll rates in the latter case. Yang and Zhang (2003) considered selection of optimal toll levels and optimal toll locations on predetermined links, which are basically the most congested ones, for achieving maximum social welfare using a bi-level programming GA based approach with both discrete and continuous variables. In their study also, GA was used only to determine the optimal toll locations; the optimal toll rates were evaluated using another technique called the simulated annealing method. More explanations as to what GA is and why it is proposed in this thesis are provided in the next chapter.

As such, the first objective of this thesis is to explore bi-level GA based optimization models to solve second-best optimal toll location and toll level (OTLTL) problem simultaneously. The upper-level subprogram is to minimize the total travel time (system cost). The lower-level subprogram is a user equilibrium problem where all users try to find the route to minimize their own travel cost (or time). Two different approaches of GA based solution procedure will be developed to solving the bi-level optimization problem. In this thesis, the travel demand is assumed to be fixed and given a priori. Network experiments will also be conducted to compare the two versions of the GA algorithms and the best one will be considered for further analysis. In this specific task, a single user class is considered i.e. all road users value their time equally.

As has been discussed above, transportation network can be driven from the user equilibrium to a system optimum by imposing tolls on the congested links. This concept has been studied extensively during the past few decades (Button and Verhoef, 1998; Hearn and Ramana, 1998; Cole et al., 2003). In the tolled network, users choose their routes according to total travel time experienced and total monetary travel cost (Han and Yang, 2008). These are sometimes collectively referred to as a generalized cost of travel. In most previous research efforts on the congestion pricing problem, homogeneous users are assumed to have existed in the transportation network. This means that the value of time (VOT) is taken to be identical for each user in the network. However in actual case, road users differ from one another in the values they place on time. That means in reality, heterogeneous groups of people use the network. With such condition, in traffic and transportation analysis with such users in terms of different VOT, either a discrete set of VOT for several distinct user classes or a continuous distributed VOT across the whole group of users can be assumed to develop network equilibrium models (Marcotte and Zhu, 2000; Nagurney, 2000).

As has been indicated earlier, a lot of attention has been devoted on the toll level and location problem for homogenous road users. Very limited research efforts have been done towards the combined optimal toll level and location problems for multiclass users. However, there are still some authors that addressed the problems partially in some way. For example, Zhang (2009) studied the congestion pricing location problem of multi-class network with social and spatial equity constraints when the number of toll links is known. His research is based on the known number of toll links which is usually not given in the realistic traffic network. Han and Yang (2008) also addressed the concept of multiclass and multicriteria traffic equilibrium to evaluate the efficiency loss caused by the models. Therefore, another objective of this thesis is to further investigate the preferred GA using extensive numerical experiment and apply it to a network with multiclass users in order to determine the combined optimal toll rates and locations.

Chapter 2 – Background

The theoretical background of congestion pricing can be traced to the work of Pigou (1920) and Knight (1924). Congestion pricing, in principle, is designed to incur the marginal social cost of a trip to the driver so that road users become aware of the costs that they are imposing upon one another while using the roadways, and that they should be charged for any additional congestion they create, thus encouraging the redistribution of demand in space or in time. It is implemented in several cities today (e.g. Singapore, London, and Stockholm). Congestion pricing makes use of concepts from the fields of traffic engineering, transport economy and optimization theory. Some of these concepts are presented in the subsequent sections.

2.1 Economic Theory of Congestion Pricing

This section presents some of the basic economic principles that provide a foundation for understanding the economic rationale for congestion pricing. Lindsey and Verhoef (2001) discussed that the understanding for congestion pricing comes from the observation that people tend to make socially efficient decisions when they are faced with all the social benefits and costs of their actions. In their study, they illustrated the basic principles of congestion pricing using a simple example. The example considered one origin and one destination connected by a single road and made assumptions that individuals make trips alone in identical vehicles. Also traffic flows, speeds and densities were considered to have been uniform along the road and independent of time of day. An equilibrium condition for their example is shown in Figure 1. The horizontal axis depicted traffic flow or volume and the vertical axis depicted the price or 'generalized cost' of a trip which included vehicle operating costs, distance cost of travel, the time costs of travel, and any toll. It is intuitive that at low volumes, vehicles can travel at the highest free-flow speed, and the trip cost curve, $C(q)$, is constant at the beginning with free flow cost C^{ff}. This is because vehicles have minimal impact on one another. As time goes by, the volume/flow is expected to increase. As a result, congestion develops, speed falls, and $C(q)$ slopes upwards. The flow was interpreted to be the quantity of trips "demanded" per unit of time and a demand curve $p(q)$ was added to Figure 1 to obtain a demand-supply diagram. The demand curve was assumed to slope downwards to reflect the fact that the number of trips people want to make decreases when the price increases. This is a fair assumption because from economics we understand that as the price of a good or service falls, the quantity demanded increases, considering other factors to remain constant. When supply and demand are in balance, a market is said to be in equilibrium. The unregulated 'no-toll' equilibrium occurs at the intersection of $C(q)$ and $p(q)$, resulting in an equilibrium flow of q^n and an equilibrium price of C^n. Since 'external benefits' of road use are not likely to be significant (benefits are normally either purely internal or monetary in nature), $p(q)$ specifies both the private and the marginal social benefit of travel. The total social benefits can thus be measured by the area under $p(q)$ and the cost to the traveler of taking a trip is measured by $C(q)$. If travel costs by environmental externalities other than congestion, such as accidents and air pollution, are ignored, then $C(q)$ measures the average social cost of a trip. The total social cost of q trips is then $TC(q) = qC(q)$, and the marginal social cost of an additional trip is given as: $MC(q) = \partial TC(q) / \partial q = C(q) + q. \partial C(q) / \partial q$.

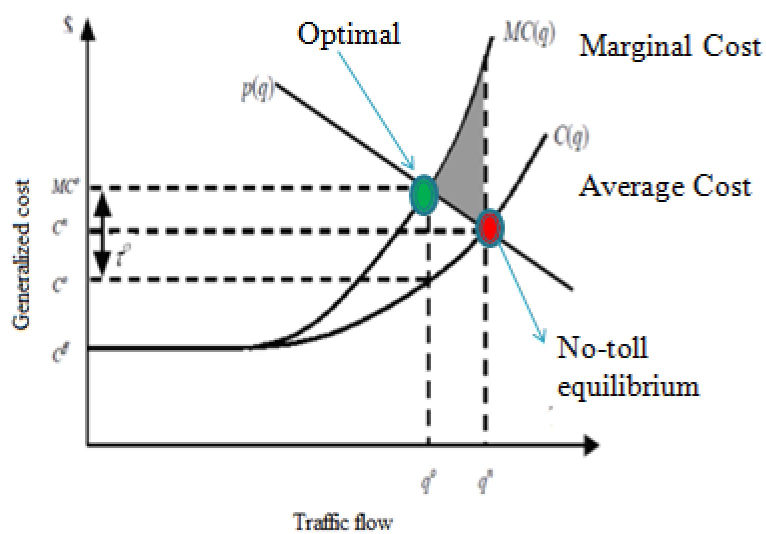


Figure 1. Cost vs. Traffic flow (Lendsey and Verhoef, 2001)

As a summary, from the point of view of society or road users, the efficient or ideal traffic volume would occur at the intersection point where marginal social cost (MC) meets the demand curve at q^o as depicted on Figure 1. This is perhaps because at this level road users somehow value their trips as much as the incremental cost to society of adding more users. At this point, a large number of drivers who are using the facility could be observed, because the value they place on travel is greater than the cost that they face, but on the contrary, the cost to society is greater than the value they receive. Thus, this process leads to an economic efficiency loss due to excessive traffic volumes. To account for this loss, discretionary trips that are valued less than their social cost should be eliminated. One way to do this might be to adjust the price signals that potential users of the roadway facility receive. This may be done by imposing a toll on all users of the facility during peak hours corresponding to the magnitude of the congestion externality; thus, the price that users face is equivalent to the marginal cost to society. In this example, the optimum toll level is indicated as 'τo' in Figure 1. Knowing this level helps in shifting lower valued trips to other routes or time periods (or not made at all), such that the new equilibrium traffic condition is driven to the socially or system optimum level. This is the main connection between congestion pricing and economics. The concept of optimal pricing is discussed in the next section.

2.2 Optimal Congestion Pricing Schemes

In solving optimal congestion pricing problems, knowing information on what links in the traffic network to locate toll facilities and how much to charge at each such facility is very important. The two commonly used optimal pricing schemes are discussed in the next subsections.

2.2.1 First-best

The total system cost can be minimized by letting the road users pay for their external effects (Beckmann et al., 1956). This pricing principle is usually referred to as marginal social cost pricing. Every link in a transportation network is subjected to toll charge in this scheme. From the economic theory discussion in the previous section, the optimal toll level is presented as 'τ_0' in Figure 1. It is equal to the marginal change of travel cost multiplied by the current flow, and this is the increase in travel cost that the users currently traveling on the link would experience if the flow was increased.

As has been discussed in Sheffi (1984), this type of pricing scheme will result in a system optimal travel demand and traffic flow pattern. It should, however, be noted that different toll patterns may result in system optimal flow. For example, if we have elastic travel demand, those toll patterns which produce system optimal flow, will charge the road users the same amount (Yin and Lawphongpanich, 2008). When the demand is fixed, the case may be different. Hearn and Ramana (1998) discussed that there can be pricing schemes which result in system optimal flow with different total toll revenues.

Different researchers have tried to make further experimentation on this pricing scheme. For example, Yildirim and Hearn (2005) made use of alternative objective functions to investigate alternative toll patterns, which give a system optimal flow while minimizing the number of toll facilities in the network. By considering operator cost, a first-best pricing scheme which optimizes the number of toll facilities will give the highest net social surplus.

2.2.2 Second-best

As has been highlighted earlier in the first chapter, unlike the previous scheme not every link in a transport network can be tolled. In this case, prices are not equal to marginal costs, because users can make decisions as to whether to take tolled links or to drive on the untolled alternatives. Examples of the second-best congestion pricing schemes include ‘pay-lanes’, such as used at various locations in the US, and ‘toll cordons’ around central business district areas. More information on second-best pricing can be found elsewhere (for example, Verhoef et al., 1996; Verhoef, 2002). This thesis focuses on this kind of pricing scheme.

2.2.3 Pricing in Practice

In practice, congestion pricing will not match theory. This is because it is not governmentally feasible to identify the appropriate externality tax at every point in time for every road, although mathematical models and technological improvements have made it possible to get close. A good congestion pricing scheme is about getting as close to the optimal price, with as little administrative cost as possible.

Generally, policy makers decide upon where to apply toll in the transportation network (Yang and Zhang, 2003). There is not much documentation available as to how optimal toll is set. Tsekeris and Vos (2009) stated that most of the work on designing road pricing schemes still kept on theoretical ground and validation of their practical applicability is required in realistically complex situations.

2.3 Genetic Algorithms

Genetic Algorithms (GA) were invented by John Holland in the 1960s to study the phenomenon of adaptation as it occurs in nature and to develop ways in which mechanisms of natural adaptation might be imported into computer systems. GAs are the heuristic search and optimization techniques that mimic the process of natural evolution (Holland, 1975, Goldberg, 1989). They represent an intelligent exploitation of a random search used to solve optimization problems. Although randomized, GAs are by no means random, instead they exploit historical information to direct the search into the region of better performance within the search space (Michalewicz, 1999). The basic techniques of the GAs are designed to simulate processes in natural systems necessary for evolution, especially those following the principles first laid down by Charles Darwin of "survival of the fittest". This is because in nature, competition among individuals for scanty resources results in the fittest individuals dominating over the weaker ones.

According to Mitchell (1996), many computational problems require searching through a huge number of possibilities for solutions. One example is the problem of computational protein engineering, in which an algorithm is sought that will search among the vast number of possible amino acid sequences for a protein with specified properties. Another example is searching for a set of rules or equations that will predict the ups and downs of a financial market, such as that for foreign currency. Such search problems can often benefit from an effective use of parallelism, in which many different possibilities are explored simultaneously in an efficient way.

The GA is started with a set of solutions (represented by chromosomes) called population. Solutions from one population are taken and used to form a new population. This is motivated by a hope, that the new population will be better than the old one. Solutions which are selected to form new solutions (offspring) are selected according to their fitness - the more suitable they are the more chances they have to reproduce. Detailed discussion on genetic algorithms can be found elsewhere (Holland, 1975, Goldberg, 1989, Mitchell, 1996).

2.4 Types of Congestion Pricing

Congestion pricing – sometimes called value pricing – is a way to bring shifting rush-hour travels to other transportation modes or to off-peak periods. There exist different categories of congestion pricing that have been considered by many researchers and applied to transportation networks. For instance, travel-distance based charging, travel time or travel-delay based charging, link-based charging and cordon-based charging are discussed in May and Milne (2000). These road pricing categories are of practical interest. As has been discussed earlier, for example, Verhoef (2002) proposed a link-based pricing method to find the second-best toll levels in which only a subset of links can be charged. Mun et al. (2003) presented a simple spatial model of traffic congestion for a monocentric city to investigate the effects of cordon pricing on trip-making and congestion level in each location. According to a study by Ecola and Lights (2009), the primary forms of congestion pricing that have been implemented or have received notable attention generally fall into one of the following five categories:

i. Time-, distance-, and/or place-based pricing: This approach adjusts road-user charges based on how many miles a vehicle is driven, location, time of day, and vehicle type. This is used mainly to raise revenues and reduce various traffic problems. The main advantage of this approach is that it requires no infrastructure on the ground other than installation of an onboard unit in each vehicle, which would typically consist of a Global Positioning System (GPS) receiver and a mobile communication device. The method has been efficiently used in the German heavy vehicles. In the US also there are quite a few studies regarding this pricing scheme. The disadvantage of this approach is that people do not like to have their movements tracked.

ii. Cordon pricing: In this category, a fee is charged every time a vehicle crosses a boundary (usually known as a cordon) into and out of a charged zone or area. The area could be a central business district, or just a heavily traveled portion of the transportation network. Generally, the charge may vary between weekdays and weekends and peak and off-peak hours. In this scheme, drivers are not charged as long as they make intra-zonal trips. Although cordon pricing has been considered to have significantly reduced congestion, it can perhaps be viewed as unfair to some travelers who must travel in and out of charging zones many times each day (e.g., taxis). Furthermore, because drivers who travel entirely within the cordon area (without crossing its boundaries) are not charged, those subject to the charge may perceive it as unfair. Cordon tolls provide a few advantages to other tolls, these include; they are easy for drivers to understand and use, ease of implementation, and the technology needed is available. On the other hand, cordon tolls may simply divert traffic to outside the cordon line and cause congestion to increase in other areas. The London and Stockholm congestion pricings are good examples.

iii. Area-license systems (ALS): This is basically similar to cordon pricing except that it allows drivers to make an unlimited number of trips into and within a particular zone of interest like central business districts during certain hours for a fixed fee. Residents who live within the zone and therefore require a license may receive a discount. Though the system may be perceived to be relatively fair as compared to the cordon pricing, it may be less effective at reducing congestion, since the charge is fixed. Singapore’s ALS was the first urban traffic congestion pricing scheme to be successfully implemented in the world. London had also adopted an area-license system in its downtown core.

iv. High occupancy toll (HOT) lanes: This scheme is described as a high-occupant-vehicle lane that accommodates a limited number of lower-occupant vehicles for a fee. HOT lanes are like that of high-occupancy vehicle (HOV) lanes, which have been introduced on highways in the United States to encourage carpooling during peak periods. Under this pricing scheme, qualified carpool vehicles can use HOT lanes for free or at a discount while vehicles having fewer occupants may also access the lanes by paying a toll. The objective here is to minimize congestion within the lanes. There are alternative parallel general-purpose lanes for vehicles that choose not to pay. The tolls may change dynamically throughout the day according to real-time traffic conditions, which is intended to manage the number of cars in the lanes to keep them less congested. In the United States, HOT lanes are popular and have been implemented in many highways (e.g. San Diego (I-15); Minneapolis (I-394); Houston (I-10, on a stretch commonly known as the Katy Freeway); Denver (I-25)).

v. Toll roads, bridges, and tunnels: This is also referred to as a link-based pricing scheme where tolls are applied to bottlenecks in the transportation network. These are considered as a form of user tax that is usually paid for the cost of road construction and maintenance without raising taxes on non-users. The tolls can be collected manually at tollbooths or electronically using transponder technology. Examples of tolled facilities with time-varying tolls include a number of bridges and tunnels into New York City; the Dulles Greenway outside Washington, D.C.; the 407 express toll route (ETR) in Toronto; and so on. This thesis focuses on this type of pricing scheme because it is an important tolling way in the second-best pricing policy to deal with bottlenecks in the traffic network. It was discussed earlier that the determination of both toll locations and toll levels simultaneously is crucial to the success of the pricing policy. If these parameters are set or determined unreasonably, the congestion pricing policy will increase the revenue only, but may not reduce congestion and because of that the social welfare is anticipated to be significantly reduced. In some cases, the traffic condition with pricing may even be worse than the un-tolled condition. In principle, the optimal congestion charge should make up for the difference between the average cost paid by the driver and the marginal cost imposed on other drivers (such as extra delay) and on society as a whole (such as environmental externalities). However, there are always some practical challenges while setting optimal link-based tolls and determining the toll locations at the same time, especially when one considers the travel pattern in complex networks may not be known precisely.

The preference of different forms of congestion pricing may vary between different regions of the world. In Europe, for example, cordon pricing and area-licensing systems have been the most commonly used schemes; in the United States, HOT-lane pricing has received greater attention lately. A number of factors may be considered to adopt for one type of system over another, for example, existing land-use patterns, available data and so on. In the next section, different cases, for which one or more of the above schemes were applied, have been studied.

2.5 Case Studies

Congestion pricing has been successfully implemented in many countries such as Singapore, Britain, and the US as an effective approach to mitigate traffic congestion. This section will discuss some of the examples of congestion pricing success stories in different countries.

i. Singapore

The first operational congestion pricing scheme in the world was implemented in Singapore in 1975. The scheme is categorized under the Area License Scheme (ALS). All drivers were supposed to purchase a license and display it at the windscreen before entering the so called central 'Restricted Zone' (RZ) during peak periods. The vehicles were monitored manually at control points. Throughout the implementation process, peak periods and toll rates were adjusted a number of times. After observing the traffic pattern, an afternoon peak charge was introduced in 1989, and in 1994 an access fee for inter-peak day-time passages was implemented. As a result, it was observed that the number of vehicles entering the RZ reduced remarkably by 44%. In effect, the speeds increased profoundly in the zone itself and the average commuting times for work trips inside the zone even increased. However, the displaced traffic caused increased traffic outside the charging zone.

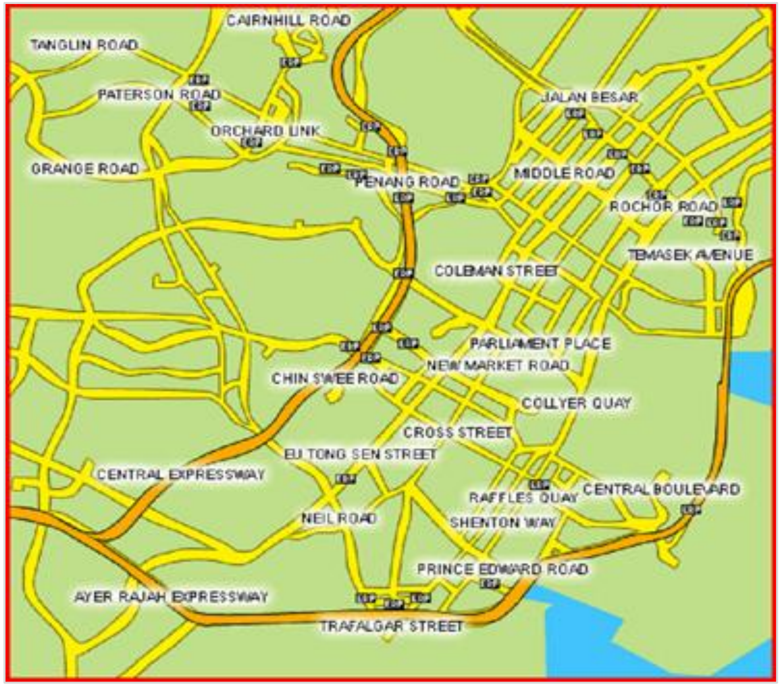


Figure 2. Singapore CBD Priced Zone, 2005 (K.T. Analytics, 2008)

Through the advancement of technology, Singapore was able to switch to a scheme known as Electronic Road Pricing (ERP) in 1998. In this pricing scheme, charges are deducted from a smart card prepared for toll collection purposes when passing through an Electronic Road Pricing framework using microwave technology. The scheme consists of 28 frameworks that are used to collect tolls throughout the day times around the central area. In addition to that, there are 14 tolled expressways and arterial roads during morning peak periods only. However, there were no tolls on weekends. The charges vary by time of day in 30-minute steps and are adjusted quarterly, depending on average speeds measured in the previous quarter. Even though the average charge for ERP is lower than it was for ALS, traffic into the central business district amazingly decreased by another 10% –15% compared to the ALS scheme. One reason might be that every entry by a given car is charged under ERP, whereas ALS allowed for unlimited access throughout the day with a single permit. Another reason might be that vehicles are not monitored manually anymore in the latter case.

ii. London

In 2003, London implemented congestion pricing where drivers have been charged £5 for entering the cordon zone (central business district in this case) or driving within its perimeter between certain periods of times (from 7am to 6:30pm) during weekdays. A map of the cordon zone in London is presented in Figure 3. The perimeter, shown as a white dash line, was carefully designed and modeled to balance the need to reduce the number of vehicles entering the city, while keeping the number of cordon crossing points to a minimum and trying to minimize and discourage the development of cut-through traffic.

Clear signs have been installed at all access points to the cordon zone in order to help users to know that they are entering a charging zone. Here also with the help of technology, the scheme is monitored and enforced using a network of Automatic Number Plate Recognition cameras (ANPR) within the zone. The data collected using the cameras is processed with the help of optical character recognition (OCR) software which can basically translate the images into a database of recognized vehicle number plates. If the OCR software cannot interpret the number plate automatically, the automated system is supported by staff to enter information on vehicles manually. Users have been presented with several options that they can pay to enter the central zone, including the internet, retail outlets and SMS text messaging.

As a result of its implementation, a number of benefits have been observed. For example, 21% reduction in congestion within the original charging zone comparing to pre-charge levels (around 70,000 fewer cars a day), 14% reduction in traffic entering the Western Extension (30,000 fewer cars a day), 6% increase in bus passengers during charging hours, 12% increase in cycle journeys into the Western Extension and £137m being raised, in the financial year 2007/08, to invest back into improving transport in London.

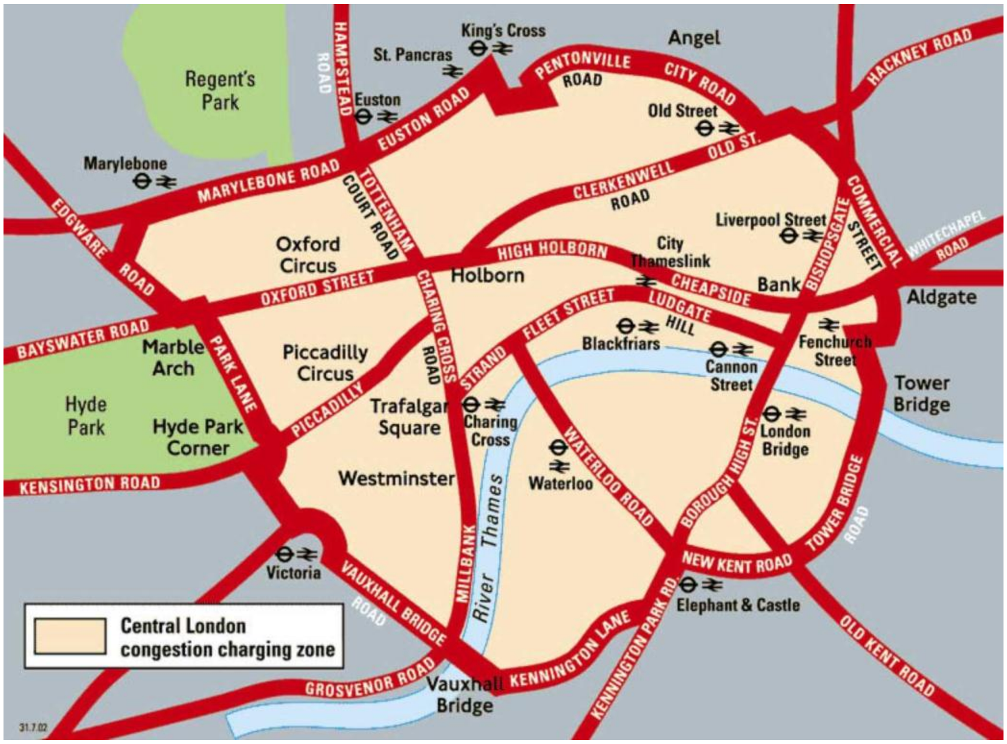


Figure 3. London Congestion Charging Area (Transport for London, 2006)

iii. SR 91 Express Lanes, California, US

State Route 91 is one of the most congested freeways in Southern California in east-west direction. In 1995, four express lanes (two in each direction), shown on Figure 4 bounded by yellow plastic pylons, in the median of the State Route 91 Freeway were added as new capacity to a congested freeway and opened in December 1995. Electronic variable tolling system was proposed and introduced for the first time, even before Singapore. The toll levels are variable depending on the direction of travel, time of day, and day of the week. Charges are collected when a vehicle carrying a transponder, issued by any toll agency in the State of California, passes beneath the toll zone gantry without stopping, and at highway speeds. The toll schedule is revised and adjusted every three months based on traffic observed over the three-month period. Speeds on the express lanes typically move on average at 60-65 mph while congestion on the free lanes causes speeds as low as 15-20 mph. As a result, for example, during a typical Friday afternoon rush hour, the 91 Express Lanes have twice as much vehicle throughput as the free lanes. Toll revenues have been adequate to pay for construction and operating costs. Like the

London congestion pricing scheme, monitoring is done using cameras and OCR. In 2003, the private company that had the contract to build and operate the facility sold the franchise to the Orange County Transportation Authority for a profit.

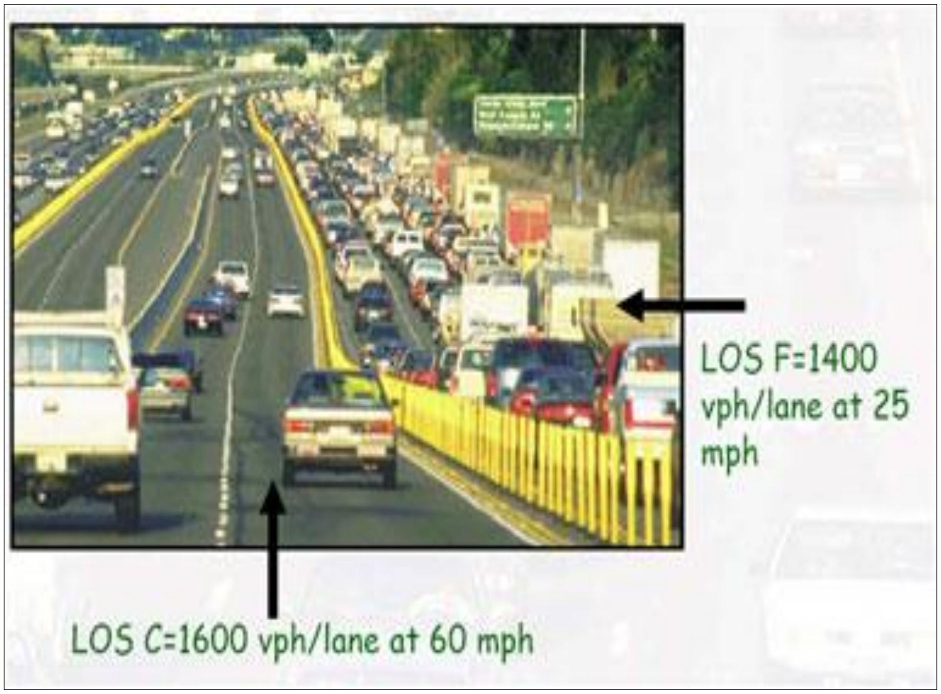


Figure 4. State Route 91 (FHWA, 2011)

iv. Lee County, Florida, US

In 1998, variable pricing was introduced on the Midpoint and Cape Coral toll bridges in Lee County, Florida. Bridge travelers were offered a 50 percent discount on their toll if they traveled during specific off-peak periods and paid their toll electronically. The off-peak periods are 6:30 to 7 am, 9 to 11 am, 2 to 4 pm, and 6:30 to 7 pm. This toll structure was developed to discourage discretionary trips or encourage drivers to shift from peak periods to off-peak/discount periods.

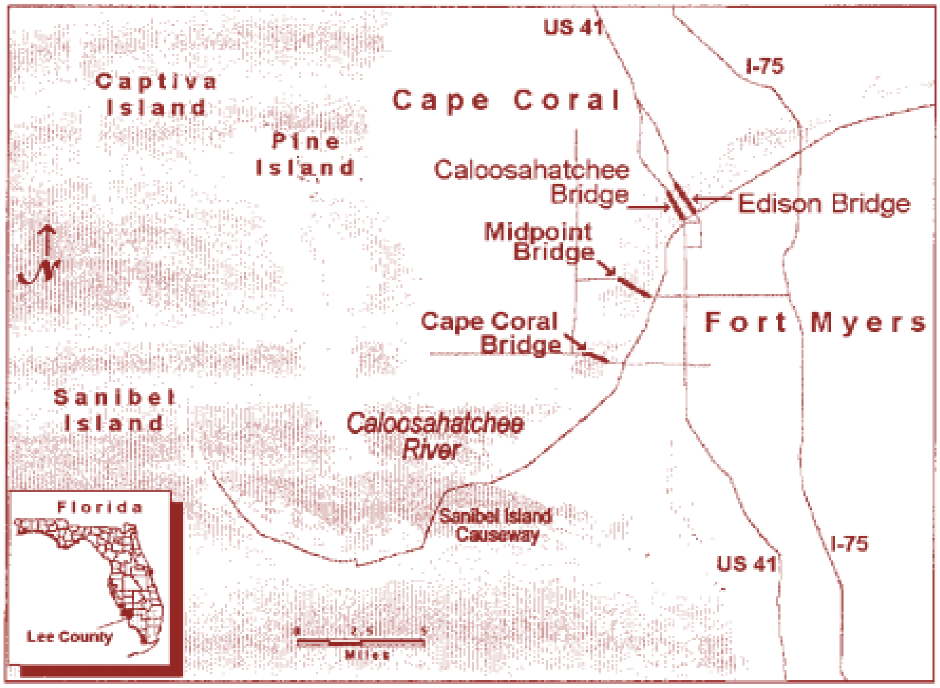


Figure 5. Lee County, Florida (Burris, 2001)

v. HOT Lanes on I-15 in San Diego, US

HOT lanes were introduced on I-15 in 1998. In this case, single-occupant vehicles are required to pay a per-trip fee each time they use the lanes. Tolls vary with the level of traffic demand on the lanes. Figure 6 shows the traffic condition during peak hours of the day. The project generates $2 million in revenue annually, about one-half of which is used to support transit service in the corridor.

Figure 6. I-15 Managed Price Lanes - San Diego, CA (Cronin et.al. 2010)

2.6 Summary

The importance of understanding the economic theory behind road pricing before implementing or designing a pricing scheme was described. It was also discussed that first-best pricing is not appealing to implement and nowadays the second-best pricing is getting much attention. Congestion pricing has been implemented in many countries such as Singapore, England and the United States. The case studies reviewed for these countries showed that congestion pricing has greatly reduced congestion and it was able to collect a good amount of money from the system.

Chapter 3 – Model Formulation

In a network design problem such as setting toll levels, different concepts of traffic assignment such as user equilibrium, system optimal and bi-level optimal model formulation are usually employed. Traffic assignment is the computation of vehicle and travelers flows in a transportation network (Sheffi, 1984). It is based on data on the travelers such as their O-D and car ownership; and on the network characteristics such as link flow capacities. Because drivers make important choices that affect the distribution of flows (such as whether or not to travel, and where to go), all assignment schemes should be based on a behavioral principle. Besides, travelers normally choose routes that they perceive as being the shortest under prevailing traffic conditions. For that reason, planners have been making use of this concept for decades for evaluating projects, optimizing tolls and estimating demands. Many network design problems are solved using Wardrop's principles on human behaviors. These principles have been addressed in this chapter.

3.1 Mathematical Notation

A network design problem can be described in terms of "nodes", "links" and "routes". Consider a connected network with a directed graph $G = \{N, K\}$ consisting of a finite set of N nodes and K links (arcs), such that link $k \in K$, which connect pairs of nodes. In order to formulate the model, the following notations are used.

Sets/Indices:

k= Link

n= Node

w= Origin-Destination (O-D) pair

Data/Parameter:

N= number of nodes

K=Set of links (arcs) such that $k \in K$

W= Set of O-D pairs

P_w = *Set of paths between O-D pair* $w \in W$

f_p^w = *the flow on path* $p \in P_w$ *between O-D pair* $w \in W$

y_k^{max}= *upper bound toll level of link* $k \in K$

y_k^{min}=*lower bound toll level of link* $k \in K$

δ_{kp}^w = *1 if link* k *is used in path* p *or* $\delta_{kp}^w = 0$ *otherwise,* $w \in W$

q_w = *a priori demand between O-D pair w*

VOT^m= *Average value of time*

$t_k(v_k)$=*travel time on link* $k \in K$ *given* v_k

Decision Variables:

$\overline{K}$ = *subset of links to be tolled i.e.* $\overline{K} \subseteq K$

v_k=*the link flow on link* $k \in \overline{K}$

y_k= *toll level on link* $k \in \overline{K}$

3.2 User Equilibrium Assignment (UE)

The user equilibrium assignment is based on Wardrop's first principle, which states that under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by unilaterally switching routes (Wardrop, 1952). It is assumed that drivers have perfect knowledge about traffic conditions on a network and choose the best route according to Wardrop's first principle; this assumption is usually referred to as deterministic user equilibrium. In other words, every trip-maker in the network experiences equal costs, and Wardrop’s principle can be restated as - under equilibrium conditions traffic arranges itself in congested networks in such a way that all routes between any O-D pair have equal and minimum costs while all unused routes have greater or equal costs. Beckmann (1956) had formulated and analyzed the static, deterministic user equilibrium (UE) model as the following nonlinear mathematical optimization program:

$$min\, R = \sum_{k \in K} \int_{0}^{v_k} t_k(\omega) d\omega \tag{1}$$

Subject to:

$$v_k = \sum_{w \in W} \sum_{p \in P_w} f_p^w * \delta_{kp}^w \,, k \in K$$

$$\sum_{p \in P_w} f_p^w = q_w \,, w \in W$$

$$f_p^w \geq 0, p \in P_w \,, w \in W$$

Equation (1) represents flow conservation equation subjected to non-negativity constraints. These constraints naturally consist of points that could possibly minimize the objective function which is designated as *R*. The user equilibrium principle is addressed with the help of the equations above. For example, the path connecting origin-destination pair may or may not carry flow. If the path does not carry any flow, then it indicates that the travel time is greater than (or equal to) the minimum O-D travel time. If the flow pattern satisfies these equations no drivers in the transportation network can be better off by unilaterally changing routes. All other routes have either equal or higher travel times. The user equilibrium criterion is thus met for every origin-destination pair. The UE is a naturally convex optimization problem assuming that the link travel time functions are monotonically increasing function of flow, and the link travel time of a particular link is dependent on the flow. To solve such convex problems the so called Frank Wolfe algorithm is applied (Sheffi, 1984).

3.3 System Optimum assignment (SO)

The system optimum assignment is based on Wardrop's second principle, which states that drivers cooperate with one another in order to minimize total system travel time rather that the individual travel time. In other words, under equilibrium conditions traffic should be arranged in congested networks in such a way that the average (or total) travel cost is minimized. In case all trip makers perceive travel costs in the same way, then for the 2nd equilibrium the following characterization holds: marginal costs for all used routes are minimal and identical. This assignment can be thought of as a model which is developed by assuming that the drivers are informed with which routes to use. Obviously, this is not a realistic assumption, but it can be useful to transport planners and engineers, trying to manage the traffic to minimize costs and therefore achieve an optimum social equilibrium. Once again the optimization of the objective function will be the tool for allocating for each of the links on the paths between each O-D pairs to find the SO travel patterns and travel time of the system. Basically the same algorithm as the UE is used but with different objective function as expressed by Equation (2) (Beckmann, 1956).

$$min\, Z = \sum_{k \in K} v_k t_k(v_k) \tag{2}$$

Subject to:

$$v_k = \sum_{w \in W} \sum_{p \in P_w} f_p^w * \delta_{kp}^w \,, k \in K$$

$$\sum_{p \in P_w} f_p^w = q_w \,, w \in W$$

$$f_p^w \geq 0, p \in P_w \,, w \in W$$

3.4 Bi-level Model Formulation

Basically, bi-level programming formulation involves two players at different levels, the leader and the follower. The two levels have their own decision variables and objectives, and make an attempt to optimize their own objectives in sequence. As has been presented by many authors (Clegg et. al., 2001; Fan, 2004), the general bi-level programming formulation can be formulated as follows:

(UP) $$\min_{x \in X} F[x, z)] \tag{3}$$

Subject to $H[x, z] \leq 0$

(LP) $$\min_{z \in Z} f[x, z] \tag{4}$$

Subject to $h[x, z] \leq 0$

Where *x* and *z* are called leader's and follower's vectors, whereas *F* and *f* expressed by Equations (3) and (4) are called leader's and follower's objective functions, respectively. *H* and *h* are constrain set of the upper and lower level programs, respectively. In bi-level programming, the leader moves first by choosing a vector *x* to optimize *F*. For each fixed *x*, the lower level optimizes its objective function *f* by selecting a vector *z* which is an optimal solution to the upper level programming.

A bi-level programming model for determining optimal toll levels and location for a given subset of links is proposed in this thesis. It exists in the area of engineering and economics extensively. It is important to design effective and efficient algorithms to solve network design problems such as congestion pricing problem. The upper-level program is to minimize the total system travel time, and the lower-level program is the traffic user equilibrium model in terms of generalized travel cost. In other words, at the upper-level, we decide on how the congestion pricing scheme is designed, trying to either maximize the social surplus or minimize the total system travel time and at the lower level, the road users are reacting to the pricing scheme in order to minimize their own, individual, travel cost. The bi-level formulation has been adopted by different authors in the past year to solve different network design problems (Yang 1996; Yang and Bell, 1997; Zhang and Yang, 2004; Clegg et al. 2001). In the next two sub-sections, the proposed bi-level optimization models for single class and multiclass users are presented.

3.4.1 Single Class Bi-level Optimization Model

In single-class optimization model, the value of time for all users in the transportation network is assumed to be the same. The upper-level program which minimizes the total system travel time (cost) is basically similar with the system optimal objective function presented by Beckmann (1956). It is expressed as follows:

$$min(\sum_{k \in K} t_k(v_k(y_k))\, v_k(y_k)) \tag{5}$$

Subject to

$$y_k^{min} \leq y_k \leq y_k^{max}$$

Where $v_k(y)$ in the above equation is the solution for the following lower-level program, which represents the deterministic user equilibrium proposed by Beckmann (1956) with given fixed O-D demand.

$$min\, r = \sum_{k \in K} \int_0^{v_k} C_k(\omega, y_k) d\omega \tag{6}$$

Subject to:

$$v_k = \sum_{w \in W} \sum_{p \in P_w} f_p^w * \delta_{kp}^w \,, k \in K$$

$$\sum_{p \in P_w} f_p^w = q_w, w \in W$$

$$f_p^w \geq 0, p \in P_w \,, w \in W$$

Where,

The expression $C_k(\omega, y_k)$ in the Equation (6) of the lower level program stands for the generalized link cost function, which is equal to $t_k(v_k(y)) + \frac{y_k}{VOT}$ if $k \in \bar{K}$, and $t_k(v_k)$ otherwise. $t_k(v_k)$ is usually expressed by one of the most widely-used functions called the BPR-function (Bureau of Public Roads) (Sheffi, 1984):

$$t_k(v_k) = t_{0k}\left[1 + \alpha(v_k/\mu_k)^{\beta}\right] \tag{7}$$

Where,

t_{ok}= the free flow travel time on link k

μ_k = capacity of link k

α, β = empirically determined coefficients

Common values for the coefficients (to be used in Equation (7)) are α = 0.15 and β = 4. It is normally assumed that capacity at this value of α in the formula above represents the level of traffic intensity whereby the travel time on the link is 15% higher than the travel time at free flow.

3.4.2 Multiclass Bi-level Optimization Model

A second-best multiclass based bi-level programming model is formulated for the optimal toll design problem with fixed demand in this section. The travel demand is subdivided into *M* classes corresponding to groups of users with different values of time. Let $VOT^m > 0$ be the average value of time for users of class *m* and q_w^m be the demand for travel of class *m* between O–D pair $w \in W$. The generalized travel cost (travel time) is transferred into equivalent amount of money. The proposed multiclass network equilibrium model, which is modified from the single-class, is formulated as:

Upper level:

$$\min \sum_{m\in M} \sum_{k\in K} \bar{t}_k(v_k^m(y_k)) * v_k^m\,(y_k) \tag{8}$$

Subject to

$$y_k^{min} \leq y_k \ \leq y_k^{max}$$

Where,

$$\bar{t}_k(v_k, y_k) = t_k(v_k) + \frac{y_k}{VOT}\ if\ k \in \bar{K}$$

$$\tilde{t}_k(v_k, y_k) = t_k(v_k)\ otherwise$$

Here also $t_k(v_k)$ is computed using Equation (7) presented in the previous subsection. It is adopted from Sheffi (1984). The lower level program is formulated as:

Lower level:

$$min \sum_{k\in K} \int_0^{v_k} \tilde{t}_k(\omega, y_k)\, d\omega + \sum_{k\epsilon K} \sum_{m\in M} \frac{1}{VOT^m} v_k^m y_k \tag{9}$$

Subject to

$$\sum_{p\in Pw} f_{p,w}^m = q_w^m, w \in W, m \in M$$

$$v_k = \sum_{w \in W} \sum_{m \in M} \sum_{p \in Pw} f_{p,w}^{m} \delta_{kp}^{w} \, , k \epsilon K \, , m \in M$$

$$v_k^m = \sum_{w \in W} \sum_{p \in Pw} f_{p,w}^{m} \delta_{kp}^{w} \, , m \in M$$

$$f_p^w \; \geq 0, p \in P_w \, , w \in W$$

Here $v_k^m(y_k), k \in K, m \in M$ is the solution of the lower-level multiclass network equilibrium program expressed by Equation (9). It should be noted that here the conventional Frank-Wolf algorithm needs to be modified and applied to solve the above multiclass network equilibrium problem (Sheffi, 1984).

Chapter 4 – Solution Methodology

4.1. GA Implementation

A conventional approach to the combined toll location and rate problem would be to solve the level setting problem for all combination of links, and for each combination compute the total system travel time. The number of ways to combine the links will however even for a network of moderate size be vast. As such, Genetic Algorithms (GA) are adopted in this thesis. GAs are computational models, similar to adaptive heuristic search algorithms, which are inspired by the evolutionary ideas of natural selection and genetics (Goldberg, 1989, Holland, 1975, Michalewicz, 1999). GAs are better than conventional Artificial Intelligence (AI) in that they more robust (Holland, 1975). Unlike older AI systems, they do not break easily even if the inputs changed slightly, or in the presence of reasonable noise. Also, in searching a large state-space, multi-modal state-space, or n-dimensional surface, a genetic algorithm may offer significant benefits over a more typical search of optimization techniques such as linear programming, depth-first, breadth-first , and praxis (Goldberg,1989, Mitchell,1996).GAs have been proven to have provided a robust search as well as a near optimal solution in a reasonable time. The working process of genetic algorithms is simple to understand; it involves nothing more than copying strings or swapping partial strings. The simplicity of the operations and the ability to find good solutions are two characteristics that make this method very suitable for solving network design problems. However, it should be noted that problems with poorly known fitness functions cannot be solved by means of genetic algorithms as bad solutions may appear through generations. Another disadvantage is that there is no guarantee that a genetic algorithm will find a global optimum.

An implementation of a genetic algorithm begins with a population (a set of solutions) of typically random chromosomes. One then evaluates these structures and selects the fittest chromosomes based on how close an individual is from the solution.

The fit individuals are then put together in a group of two and their chromosomes are intermingled to create new two individuals (offspring). The process continues iteratively until the required generation of accuracy is reached. In the next subsections, the whole GA processes consisting of representation, initialization, selection, crossover and mutation are discussed.

4.1.1. Representation

The process of representing a solution in the form of a string that conveys the necessary information is the first basic step in GA implementation. This process is usually referred to as encoding. As in a biological chromosome, each gene controls a particular characteristic of the individual; each bit in a string represents a characteristic of the solution. The most common approach is to encode solutions as binary strings where every chromosome is a string of 0's or 1's. Let us assume that we want to optimize a function *g* of *t* variables, $g(x_1, x_2, ..., x_t): R^t \rightarrow R$ where each decision variable x_i can take values within a domain $D_i = [a_i, b_i] \subseteq R$ and $g(x_1, x_2, ..., x_t) > 0$ for all $x_i \in D_i$. The decision variables are first encoded into binary strings meeting some desirable required precision. For example, Fan (2004) discussed that if the required precision is five places after the decimal point, the domain D_i should be cut into $(b_i - a_i) * 10^5$ equal size ranges. For five decimal places precision, the minimum required bits m_i for the variable or chromosome x_i could be computed using the following relationship:

$$2^{m_i - 1} - 1 < (b_i - a_i)10^5 \leq 2^{m_i} - 1 \quad (10)$$

This means that any value in between can be coded as a binary string using at least m_i bits. Mapping between decimal and binary value for variable x_i can be done using Equation (11) presented below:

$$x_i = a_i + (\text{Decoded value of } m_i)\frac{(b_i - a_i)}{2^{m_i - 1}} \quad (11)$$

where m_i is the binary string representation for the variable x_i. As has been described on Fan (2004), each chromosome could have a *t*-dimensional vector $S_i = (x_1, x_2, ..., x_t)$ as a potential solution. Thus, it is anticipated that the representation of a single solution would have a binary string of length $m = \sum_{i=1}^{k} m_i$. It should be known that the first m_1 bits map into a value within the specific domain range $[a_1, b_1]$ for variable x_1, the next m_2 bits map into a value within the range $[a_2, b_2]$ for the next variable and so on; the last m_t bits map into a value from the range $[a_t, b_t]$ for the last variable.

4.1.2. Population Size

Once encoding has been done, GAs start with some initial solutions, called initial populations, and try to improve them toward some optimal solution(s). In other words, population size represents how many chromosomes are in a population in one generation. It is possible to set the initialize population with a pre-specified number (*pop_size*) of chromosomes randomly or use available knowledge to arrange for sets of initial solutions (Fan, 2004). On one hand, if there are only a few chromosomes in a population, the GA would have a few opportunities to perform crossover and only a small part of search space is considered. On the other hand, if there are many chromosomes, the algorithm slows down. Once *(pop_size)* is set, each chromosome is evaluated using the objective function g based on the decoded sequences of variables in each generation Fan (2004). Then a new population is selected with respect to the probability distribution based on the fitness values.

4.1.3. Selection

Suppose we want to minimize a certain objective function, selection allocates more copies of those solutions with lower fitness values and thus imposes the survival-of-the-fittest mechanism on the candidate solutions (Goldberg, 1989). The main objective of selection is to prefer better solutions to worse ones. During the past years, many selection schemes, each with different characteristics, have been proposed, including roulette-wheel selection, stochastic universal selection, ranking selection and tournament selection. Even though a roulette wheel is commonly adopted by many authors for the selection process, tournament selection is increasingly being used as a GA selection scheme as it has a number of advantages (Miller and Goldberg, 1995). For example, it is simple to code and needs only a preference ordering between pairs or groups of strings and it can thus adapt in situations where there is no formal objective function. This selection scheme has been adopted in this thesis. Goldberg (1989) described that *s* chromosomes are chosen at random in tournament selection (either with or without replacement) and entered into a competition against each other. The most widely used value of s is 2 because at least two parents are required to compete in a tournament. The winner of the tournament (the fittest individual) is selected for parenthood (reproduction). Using this selection procedure, n tournaments are required to choose n individuals.

4.1.4. Crossover

The main purpose of crossover is to combine two chromosomes (parents) to produce a new chromosome (offspring). The notion behind this is that the new chromosome may be better than both of the parents if it takes best characteristics from each of the parents. The crossover operators are of many types. One-point and two-point crossovers are the simplest and most widely applied crossover methods so far (Goldberg, 1989). In this scheme, two individuals are randomly selected and recombined with probability of crossover p_C. Fan (2004) described the one-point crossover procedure as follows: 1) Set $i = 1$; 2) Generate a random number r within the range [0,1]; 3) If $r \leq p_C$, then select chromosome S_i for crossover; 4) $i = i + 1$; and 5) Repeat the above steps until $i > pop\_size$.It should however be noted that if $r > pc$, the two offspring are simply copies of their parents. The value of p_c can either be set experimentally, or can be set based on schema-theorem principles (Goldberg, 1989). One-point, two-point and uniform crossover methods are illustrated using simple examples in the following figure 7.

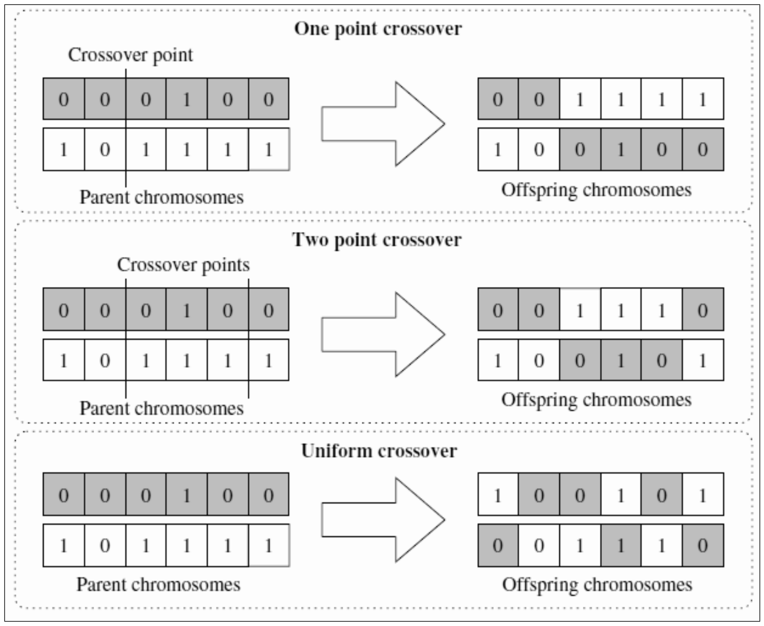


Figure 7. Crossover Examples

4.1.5. Mutation

Mutation represents random change of the values of a gene in a population. Like crossover, mutation operators are of many types. One of the most common mutations is the bit-flip mutation. In bitwise mutation, each bit in a binary string is changed (a 0 is converted to 1, and vice versa) with a mutation probability, *pm* (Goldberg, 1989). Fan (2004) presented the whole mutation procedure as follows: 1) Set $i = 1$; 2) Generate a random number $r \in [0,1)$ for each bit; 3) If $r \le p_m$, mutate the bit; 4) $i = i + 1$; and 5) Repeat the above steps until $i > m \cdot pop\_size$.

If p_m is chosen to be small, many bits (genes) that might be usefully chosen for further improvements will be rarely studied. On the other hand, if p_m is chosen to be too big, the offspring will lose their similarity to their parents due to random perturbations, and as a result, the GA will lose the ability to learn from the search history (Fan, 2004). Therefore, this probability parameter needs to be carefully chosen.

4.2 GA Solution Procedure

First, two different GA approaches (termed OPTION1 and OPTION2) were developed to determine the optimal toll rates and the corresponding locations for the single class user scenario. The basic difference between the approaches is the way their chromosomes are set up during the population initialization step. In OPTION1, populations of toll locations and toll levels are initiated separately at first but used in combination when evaluating the objective function to optimize the toll rates and determine the corresponding locations to be tolled. In the case of OPTION2, both the toll rates and locations are changed simultaneously throughout the GA procedure i.e. the population is initiated after the chromosomes for both toll locations and toll rates are combined. For example, suppose a toll rate in a single link varies between $2 and $10 and assume a network consisting of 76 links, the toll locations and toll levels are represented by 7 bits and 10 bits respectively. Figure 8 shows chromosome structures for both options in this example for the first six populations.

OPTION1

Toll Locations

1	0	1	0	1	0	0
1	1	1	0	1	0	0
1	1	0	1	0	1	1
0	0	1	0	1	0	0
1	0	1	1	0	1	1
0	1	0	0	1	0	1

Toll Rates

0	0	1	0	1	0	0	1	0	1
0	1	0	0	1	1	0	0	1	0
1	0	0	0	1	0	1	1	0	1
1	0	0	1	0	1	0	0	1	0
0	1	1	0	0	0	1	1	0	0
1	1	0	1	0	0	1	0	1	1

OPTION2

Combined Toll Locations and Rates

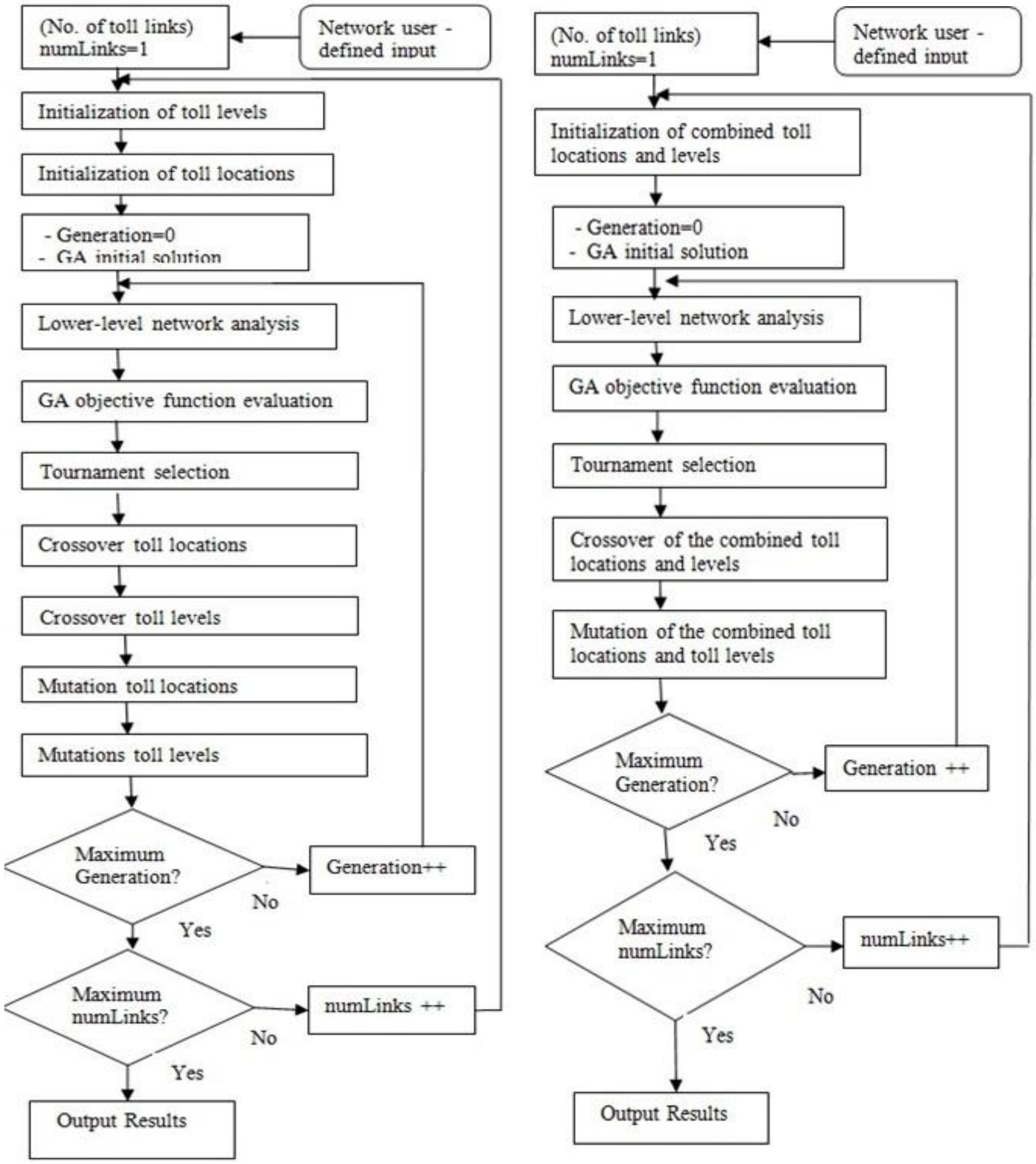
1	0	1	0	1	0	0	0	0	1	0	1	0	0	1	0	1
1	1	1	0	1	0	0	0	1	0	0	1	1	0	0	1	0
1	1	0	1	0	1	1	1	0	0	0	1	0	1	1	0	1
0	0	1	0	1	0	0	1	0	0	1	0	1	0	0	1	0
1	0	1	1	0	1	1	0	1	1	0	0	0	1	1	0	0
0	1	0	0	1	0	1	1	1	0	1	0	0	1	0	1	1

Figure 8. Chromosome Structure for the Two GA Options

After the required number of population is initialized, each chromosome in the population is evaluated using the objective function based on the decoded sequences of variables in each generation. Then a new population is selected with respect to the probability distribution based on the fitness values, and the chromosomes are altered in the new population by using mutation and crossover operators. After a certain number of generations, the current best chromosome is considered as an optimal solution.

Figure 9 presents flow charts showing the whole GA process for both options for a single user class. The process starts by setting the number of links to be tolled as one. A possible set of solutions are initialized randomly. Each solution is evaluated based on the fitness values (the objective function values). The fittest solutions will then be selected for parenthood to perform crossover and mutation operations. Once the convergence criterion (the maximum number of generation in this case) is met, the number of links to be tolled will be set to two and the whole steps are repeated. It can be seen from figure 9 that there are also intermediate steps within the genetic algorithm process. For example, a lower-level network analysis has to be performed in order to evaluate the fitness values discussed earlier. Lower-level network analysis involves determining the flows which are going to be used as inputs in the upper level program using the most commonly applied Frank-Wolfe (FW) algorithm. The algorithm works iteratively by using an adaptive step size to calculate the right amount of flow to shift to get as close to equilibrium condition as possible. The detail algorithm is available elsewhere (Sheffi, 1984).

Once the two GA options are compared, the preferred option will further be modified and applied to a network with multiclass users. The whole GA process in this scenario is somehow very similar to the previous one. If we assume OPTION2 is preferred, a flow chart as shown in Figure 10 can be presented for multiclass network equilibrium case. As can be seen from Figure 10, the lower level network analysis procedure is not anymore a Frank-Wolf algorithm for single class user equilibrium; rather the conventional Frank-Wolf algorithm needs to be modified so that it can be applied to any required number of user classes. Different discrete values of time also need to be set before the iteration process. It should be noted that for a traffic network with fixed demand of either homogenous or heterogeneous users in terms of their different values of time (VOT), the system performance can be measured by the total system travel time.



(a) OPTION1 (b) OPTION2

Figure 9. Flow Charts Showing the Whole GA Process for Single Class Users

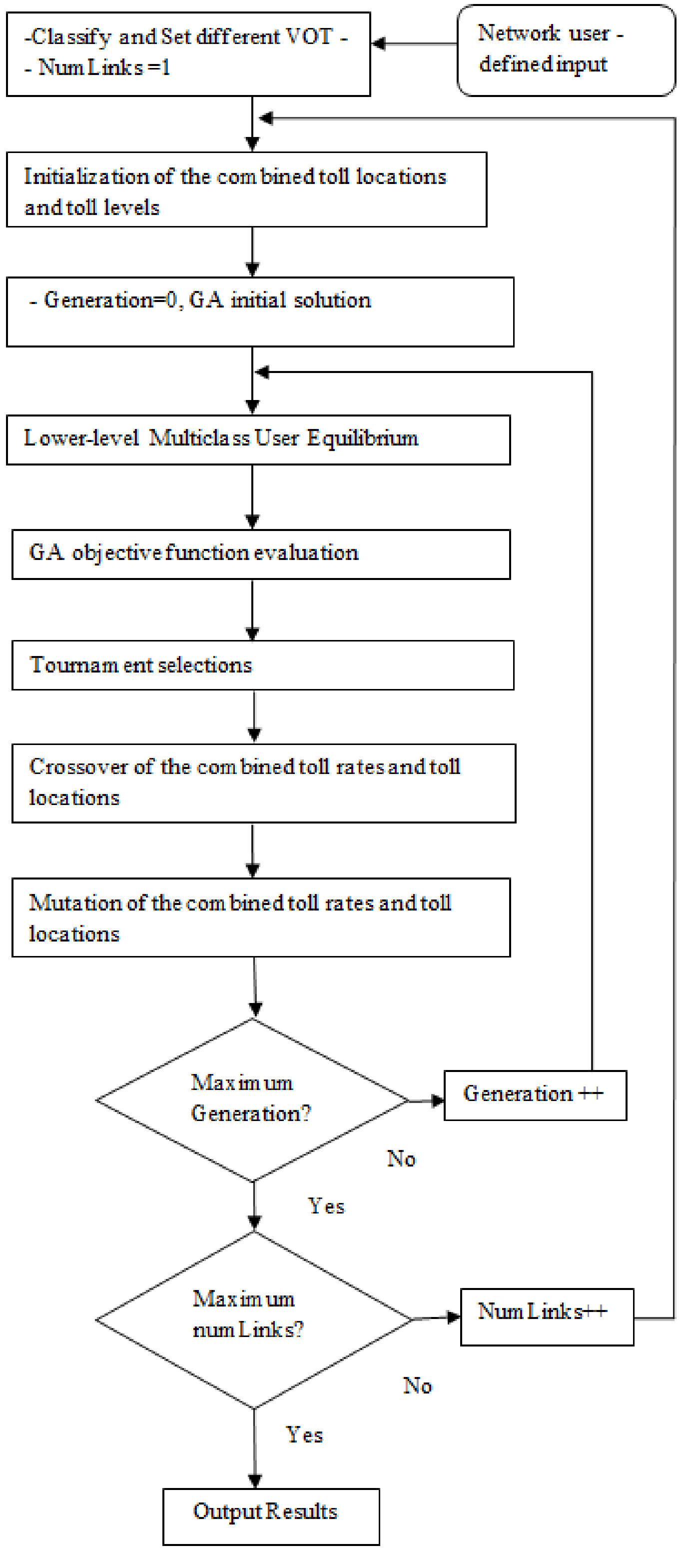


Figure 10. GA Procedure Multiclass Network Equilibrium

Chapter 5 – Numerical Experiments

5.1. Example Network Description

To solve and analyze the optimal toll rate and location problem, an example Sioux Falls network shown in Figure 11 is considered and the proposed algorithms are implemented and tested using a computer program in MATLAB software package. This example network contains 24 travel demand zones, 76 links and 576 O-D pairs (out of which 24 intra zonal and 24 inter zonal zero-demand O-D pairs are assumed to be zero). This network has been used in many publications as it is good for code debugging. Besides, Bar-Gera (2010) found the Sioux Falls user equilibrium solution, using the quadratic BPR cost functions, which could be used for cross checking results.

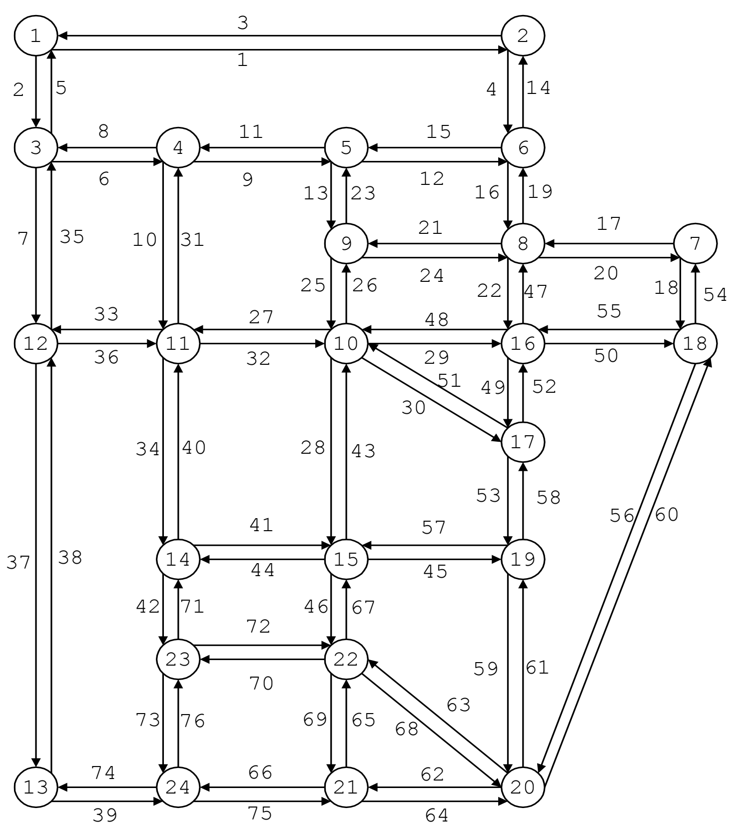


Figure 11. Sioux Falls Test Network (Bar-Gera, 2010)

Bar- Gera (2010) took all the network data, which are also used in this thesis, from LeBlanc et.al. (1975). Table 1 and 2 present hourly O-D demand and network data such as free flow time, capacity, coefficient for BPR function and so on respectively.

Table 1 Peak Hour O-D Demand for Sioux Falls Network (Bar-Gera 2010)

O/D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	100	100	500	200	300	500	800	500	1300	500	200	500	300	500	500	400	100	300	300	100	400	300	100
2	100	0	100	200	100	400	200	400	200	600	200	100	300	100	100	400	200	0	100	100	0	100	0	0
3	100	100	0	200	100	300	100	200	100	300	300	200	100	100	100	200	100	0	0	0	0	100	100	0
4	500	200	200	0	500	400	400	700	700	1200	1400	600	600	500	500	800	500	100	200	300	200	400	500	200
5	200	100	100	500	0	200	200	500	800	1000	500	200	200	100	200	500	200	0	100	100	100	200	100	0
6	300	400	300	400	200	0	400	800	400	800	400	200	200	100	200	900	500	100	200	300	100	200	100	100
7	500	200	100	400	200	400	0	1000	600	1900	500	700	400	200	500	1400	1000	200	400	500	200	500	200	100
8	800	400	200	700	500	800	1000	0	800	1600	800	600	600	400	600	2200	1400	300	700	900	400	500	300	200
9	500	200	100	700	800	400	600	800	0	2800	1400	600	600	600	900	1400	900	200	400	600	300	700	500	200
10	1300	600	300	1200	1000	800	1900	1600	2800	0	4000	2000	1900	2100	4000	4400	3900	700	1800	2500	1200	2600	1800	800
11	500	200	300	1500	500	400	500	800	1400	3900	0	1400	1000	1600	1400	1400	1000	100	400	600	400	1100	1300	600
12	200	100	200	600	200	200	700	600	600	2000	1400	0	1300	700	700	700	600	200	300	400	300	700	700	500
13	500	300	100	600	200	200	400	600	600	1900	1000	1300	0	600	700	600	500	100	300	600	600	1300	800	800
14	300	100	100	500	100	100	200	400	600	2100	1600	700	600	0	1300	700	700	100	300	500	400	1200	1100	400
15	500	100	100	500	200	200	500	600	1000	4000	1400	700	700	1300	0	1200	1500	200	800	1100	800	2600	1000	400
16	500	400	200	800	500	900	1400	2200	1400	4400	1400	700	600	700	1200	0	2800	500	1300	1600	600	1200	500	300
17	400	200	100	500	200	500	1000	1400	900	3900	1000	600	500	700	1500	2800	0	600	1700	1700	600	1700	600	300
18	100	0	0	100	0	100	200	300	200	700	200	200	100	100	200	500	600	0	300	400	100	300	100	0
19	300	100	0	200	100	200	400	700	400	1800	400	300	300	300	800	1300	1700	300	0	1200	400	1200	300	100
20	300	100	0	300	100	300	500	900	600	2500	600	500	600	500	1100	1600	1700	400	1200	0	1200	2400	700	400
21	100	0	0	200	100	100	200	400	300	1200	400	300	600	400	800	600	600	100	400	1200	0	1800	700	500
22	400	100	100	400	200	200	500	500	700	2600	1100	700	1300	1200	2600	1200	1700	300	1200	2400	1800	0	2100	1100
23	300	0	100	500	100	100	200	300	500	1800	1300	700	800	1100	1000	500	600	100	300	700	700	2100	0	700
24	100	0	0	200	0	100	100	200	200	800	600	500	700	400	400	300	300	0	100	400	500	1100	700	0

The coloring in Table 1 helps us understand different patterns in the travel demands between the O-D pairs in the study network. For example, it can be clearly seen that large number of trips are generated from or attracted to zone 10 (as has been highlighted in red color). The intra-zonal trips were not considered and the first 6 zones contribute to the least amount of demand generation. Table 2 presents all the required information in order to develop the BPR link performance function which will be used to calculate flows on each link.

Table 2 Link Performance Information for Sioux Falls Network (Bar-Gera 2010)

Origin	Destination	Capacity	FreeflowTime	α	β
1	2	25900.2	6	0.15	4
1	3	23403.47	4	0.15	4
2	1	25900.2	6	0.15	4
2	6	4958.181	5	0.15	4
3	1	23403.47	4	0.15	4
3	4	17110.52	4	0.15	4
3	12	23403.47	4	0.15	4
4	3	17110.52	4	0.15	4
4	5	17782.79	2	0.15	4
4	11	4908.827	6	0.15	4
5	4	17782.79	2	0.15	4
5	6	4947.995	4	0.15	4
5	9	10000	5	0.15	4
6	2	4958.181	5	0.15	4
6	5	4947.995	4	0.15	4
6	8	4898.588	2	0.15	4
7	8	7841.811	3	0.15	4
7	18	23403.47	2	0.15	4
8	6	4898.588	2	0.15	4
8	7	7841.811	3	0.15	4
8	9	5050.193	10	0.15	4
8	16	5045.823	5	0.15	4
9	5	10000	5	0.15	4
9	8	5050.193	10	0.15	4
9	10	13915.79	3	0.15	4
10	9	13915.79	3	0.15	4
10	11	10000	5	0.15	4
10	15	13512	6	0.15	4
10	16	4854.918	4	0.15	4
10	17	4993.511	8	0.15	4
11	4	4908.827	6	0.15	4
11	10	10000	5	0.15	4
11	12	4908.827	6	0.15	4
11	14	4876.508	4	0.15	4
12	3	23403.47	4	0.15	4
12	11	4908.827	6	0.15	4
12	13	25900.2	3	0.15	4
13	12	25900.2	3	0.15	4
13	24	5091.256	4	0.15	4
14	11	4876.508	4	0.15	4
14	15	5127.526	5	0.15	4
14	23	4924.791	4	0.15	4
15	10	13512	6	0.15	4
15	14	5127.526	5	0.15	4
15	19	14564.75	3	0.15	4
15	22	9599.181	3	0.15	4
16	8	5045.823	5	0.15	4
16	10	4854.918	4	0.15	4
16	17	5229.91	2	0.15	4
16	18	19679.9	3	0.15	4
17	10	4993.511	8	0.15	4
17	16	5229.91	2	0.15	4
17	19	4823.951	2	0.15	4
18	7	23403.47	2	0.15	4
18	16	19679.9	3	0.15	4
18	20	23403.47	4	0.15	4
19	15	14564.75	3	0.15	4
19	17	4823.951	2	0.15	4
19	20	5002.608	4	0.15	4
20	18	23403.47	4	0.15	4
20	19	5002.608	4	0.15	4
20	21	5059.912	6	0.15	4
20	22	5075.697	5	0.15	4
21	20	5059.912	6	0.15	4
21	22	5229.91	2	0.15	4
21	24	4885.358	3	0.15	4
22	15	9599.181	3	0.15	4
22	20	5075.697	5	0.15	4
22	21	5229.91	2	0.15	4
22	23	5000	4	0.15	4
23	14	4924.791	4	0.15	4
23	22	5000	4	0.15	4
23	24	5078.508	2	0.15	4
24	13	5091.256	4	0.15	4
24	21	4885.358	3	0.15	4
24	23	5078.508	2	0.15	4

5.2. Preliminary Analysis

The analyses of user equilibrium (UE) and system optimum (SO) are the basic steps in any network design problem in order to gain some sort of perspective about optimal solutions. For example, in second-best optimal congestion pricing problem in a network of single-class users, we expect the total system time to be between the systems cost of UE and SO. As has been discussed in the previous chapters, in the case of UE, users are aware of the traffic conditions and try to optimize their travel time by taking less congested routes. In the case of SO on the other hand, the total travel time in a system is optimized, rather than the individual travel times. Even though UE models traffic behavior more accurately than the theoretical SO, the study of a SO of a network can help us understand how to design networks efficiently so that the natural tendency of the network leans toward optimizing the total system time.

Using the Sioux Falls network information, Matlab routines were developed to implement the UE and SO traffic assignment models. A traffic assignment function contains Dijkstra's shortest path algorithm which was called within a loop in the convex combination algorithm. Separate Matlab functions for the line search algorithm were also developed to find the step size which will also be called within a loop in the convex combination algorithm. The objective function within the convex combination algorithm is modified in order to achieve UE or SO total travel times.

Table 3 presents the results of the analyses for both cases. As can be seen from the table, major difference exists in the flows on each link. As one might expect, the total system travel time (cost) for the SO traffic assignment 7246945.8 is less than that of the system cost for the UE assignment 7480481.8. The average v/c ratio and link cost in the network in the SO case are also less than the UE case. However, assuming that a link is congested if v/c ratio is greater than 1, it can be observed that the number of congested links increases by 3%, as one switches from UE to SO assignment. This may be because some of the drivers tend to shift to routes, which are at the verge of congestion, in order to cooperate with other drivers to minimize the total system cost in the SO scenario.

Table 3 User Equilibrium and System Optimal Analysis Result for Single-class Users

Link #	UE Volume	UE Cost	UE System cost	UE v/c	SO Volume	SO Cost	SO System Cost	SO v/c	Link#	UE Volume	UE Cost	UE System cost	v/c	SO Volume	SO Cost	SO System Cost	SO v/c
1	4495	6.00	26973.67	0.17	7557	6.01	45391.29	0.29	39	11120	17.66	196390.41	2.18	10573	15.16	160281.31	2.08
2	8120	4.01	32550.57	0.35	11241	4.03	45322.97	0.48	40	9815	13.84	135865.56	2.01	9495	12.62	119862.36	1.95
3	4521	6.00	27129.77	0.17	7623	6.01	45789.48	0.29	41	9035	12.23	110537.25	1.76	8958	11.99	107377.13	1.75
4	5967	6.57	39224.66	1.20	6582	7.33	48240.67	1.33	42	8399	9.08	76257.41	1.71	7717	7.62	58783.32	1.57
5	8095	4.01	32449.51	0.35	11175	4.03	45048.55	0.48	43	23194	13.81	320345.33	1.72	23516	14.26	335265.59	1.74
6	14015	4.27	59835.67	0.82	17555	4.66	81890.85	1.03	44	9078	12.37	112336.66	1.77	8961	12.00	107497.01	1.75
7	10031	4.02	40326.42	0.43	14577	4.09	59624.35	0.62	45	19085	4.33	82565.76	1.31	18428	4.15	76535.59	1.27
8	14038	4.27	59960.06	0.82	17524	4.66	81664.17	1.02	46	18406	9.09	167276.37	1.92	16338	6.78	110711.88	1.70
9	18012	2.32	41704.52	1.01	18977	2.39	45337.45	1.07	47	8403	10.78	90574.35	1.67	7976	9.68	77227.28	1.58
10	5207	7.13	37143.10	1.06	6252	8.37	52317.61	1.27	48	11072	20.24	224056.04	2.28	10737	18.35	197060.70	2.21
11	18037	2.32	41793.03	1.01	18959	2.39	45266.50	1.07	49	11691	9.50	111081.55	2.24	10590	7.04	74590.20	2.02
12	8797	10.00	87954.39	1.78	7018	6.43	45113.23	1.42	50	15286	3.16	48356.72	0.78	18944	3.39	64151.47	0.96
13	15784	9.65	152336.29	1.58	17003	11.27	191598.27	1.70	51	8100	16.31	132094.94	1.62	8335	17.31	144319.95	1.67
14	5993	6.60	39550.91	1.21	6648	7.42	49354.87	1.34	52	11680	9.47	110642.94	2.23	10587	7.04	74508.59	2.02
15	8804	10.02	88222.27	1.78	6992	6.39	44695.90	1.41	53	9953	7.44	74016.75	2.06	8219	4.53	37216.04	1.70
16	12492	14.69	183519.41	2.55	12561	14.97	188036.10	2.56	54	15861	2.06	32724.20	0.68	16637	2.08	34548.60	0.71
17	12106	5.55	67214.45	1.54	13310	6.73	89639.26	1.70	55	15342	3.17	48570.24	0.78	18982	3.39	64339.17	0.96
18	15801	2.06	32585.23	0.68	16596	2.08	34450.98	0.71	56	18983	4.26	80855.64	0.81	20837	4.38	91204.09	0.89
19	12525	14.82	185672.60	2.56	12600	15.13	190658.97	2.57	57	19118	4.34	82886.68	1.31	18460	4.16	76816.75	1.27
20	12045	5.50	66264.52	1.54	13268	6.69	88733.90	1.69	58	9942	7.41	73692.83	2.06	8222	4.53	37259.99	1.70
21	6884	15.17	104462.69	1.36	7544	17.47	131786.61	1.49	59	8686	9.46	82161.43	1.74	8685	9.45	82078.50	1.74
22	8384	10.73	89955.91	1.66	7951	9.62	76520.63	1.58	60	19000	4.26	80944.37	0.81	20817	4.38	91086.46	0.89
23	15802	9.67	152807.78	1.58	17012	11.28	191925.65	1.70	61	8707	9.52	82849.28	1.74	8720	9.54	83180.07	1.74
24	6838	15.04	102828.95	1.35	7516	17.36	130468.67	1.49	62	6303	8.17	51468.16	1.25	6446	8.37	53955.96	1.27
25	21746	5.68	123572.36	1.56	21764	5.69	123888.90	1.56	63	7001	7.71	53999.62	1.38	7227	8.08	58412.62	1.42
26	21817	5.72	124733.10	1.57	21845	5.73	125230.46	1.57	64	6242	8.08	50445.57	1.23	6426	8.34	53600.38	1.27
27	17728	12.41	219928.06	1.77	17523	12.07	211524.20	1.75	65	8617	4.21	36307.78	1.65	8090	3.72	30075.91	1.55
28	23127	13.72	317357.26	1.71	23412	14.11	330385.81	1.73	66	10307	11.92	122901.28	2.11	9756	10.16	99088.97	2.00
29	11046	20.08	221856.81	2.28	10726	18.29	196228.87	2.21	67	18382	9.06	166488.85	1.91	16313	6.75	110166.28	1.70
30	8100	16.31	132094.94	1.62	8329	17.29	143992.90	1.67	68	7001	7.71	53999.62	1.38	7161	7.97	57083.79	1.41
31	5305	7.22	38318.15	1.08	6339	8.50	53898.83	1.29	69	8606	4.20	36154.24	1.65	8127	3.75	30470.60	1.55
32	17606	12.20	214850.50	1.76	17384	11.85	205992.05	1.74	70	9661	12.37	119466.05	1.93	9307	11.20	104265.81	1.86
33	8366	13.59	113695.84	1.70	7355	10.54	77491.46	1.50	71	8393	9.07	76090.66	1.70	7736	7.65	59204.66	1.57
34	9778	13.69	133873.39	2.01	9473	12.54	118830.13	1.94	72	9626	12.24	117852.45	1.93	9252	11.03	102088.27	1.85
35	9983	4.02	40129.57	0.43	14542	4.09	59468.62	0.62	73	7903	3.76	29709.78	1.56	7765	3.64	28261.63	1.53
36	8405	13.74	115443.98	1.71	7380	10.60	78212.31	1.50	74	11111	17.62	195742.72	2.18	10564	15.12	159743.99	2.07
37	12298	3.02	37174.35	0.47	15597	3.06	47714.01	0.60	75	10257	11.75	120546.21	2.10	9701	10.00	96977.94	1.99
38	12389	3.02	37457.89	0.48	15688	3.06	48014.25	0.61	76	7863	3.72	29273.53	1.55	7729	3.61	27897.22	1.52
									Sum			**7480481.8**				**7246945.8**	

5.3. Single-class Users - Numerical Analyses

In this section, numerical experiments on the optimal congestion charging have been discussed using the Sioux Falls network with homogenous road users. The main objective here is to find the optimal toll rates and locations to collect the fees simultaneously. As such, the two GA options discussed on section 4.2 have been explored and compared with each other. In practice, policy makers first may decide upon the desired number of links where toll is to be charged. Normally, the most congested links could be chosen as candidate set of toll links. This set, however, may not be the optimal toll link set. Therefore, further effort will be put into this thesis to compare the results of the optimal toll locations with that of the most congested links.

It is noted that the parameter set chosen will have some effects on the optimal solutions. As a result, such parameters inherent in the GA algorithms need to be carefully chosen. Based on some previous research efforts such as (Chen &Yang, 2004, Recker et al., 2005), the following parameters are assumed in this numerical experiment: Population size,64; Maximum number of generation,1000; Crossover probability,0.6; Mutation probability,0.05; Maximum toll rate (y_k^{max}),10$; Minimum toll rate $\left(y_k^{min}\right)$, 2$; and Value of Time (*VOT)*, 10$/hr.

In addition, it is assumed that a maximum of 10 links are to be tolled to determine where to collect the tolls, and what toll levels to charge on. For both GA approaches , the travel time function for each link follows the BPR- form discussed previously on model formulation section: $t_k(v_k) = t_{0k}\left[1 + \alpha(v_k/\mu_k)^\beta\right]$.

Commonly, GA terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. In this study, the number of genetic algorithm iterations is used as stopping criteria. Therefore, it is important to do sensitivity analysis to check if the total system travel time is decreasing through a number of iterations as more surviving offspring is produced for every new generation. Figures 12 and 13 present the sensitivity of travel time over different generation for OPTION1 and OPTION2 respectively for different number of toll links.

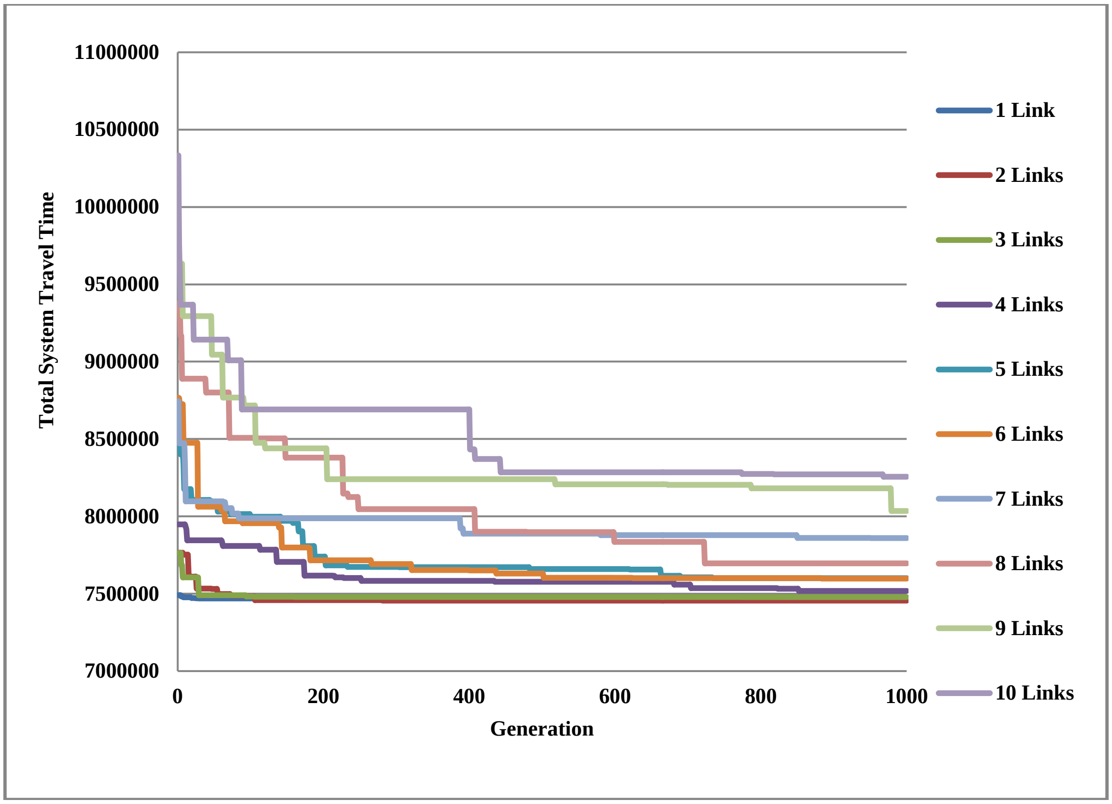


Figure 12. Sensitivity of Total System Travel Time over Generation (OPTION1)

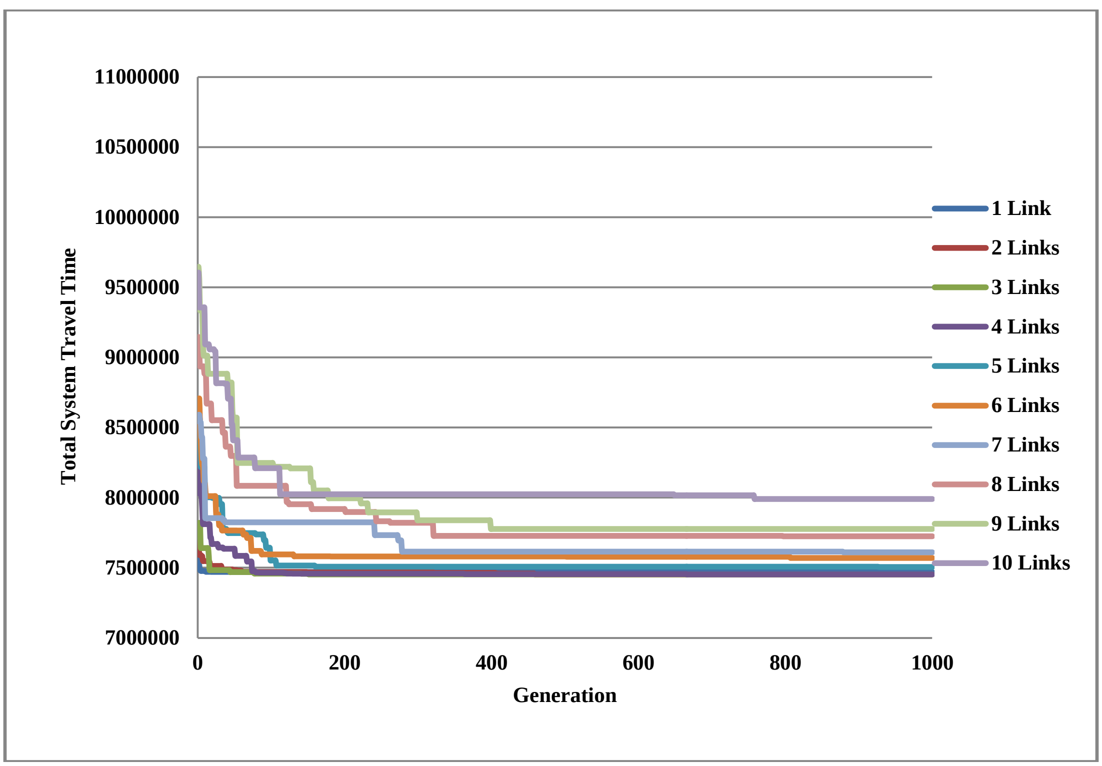


Figure 13. Sensitivity of Total System Travel Time over Generation (OPTION2)

As can be seen from the above figures, the value of total system travel time declined sharply at the beginning of the first few iterations and further declined until the convergence criteria is met for all number of toll links to be charged. This illustrates that every new generation gives a better result than the previous one as one might expect. In addition to the above observation, the figures also show that OPTION2 seems to have converged faster than OPTION1.

The final system costs (fitness values) for the proposed GA options at the 1000th iteration are presented in Figures 14 and 15. As can be seen from the figures, the optimal number of toll links is two for OPTION1 and four for OPTION2. It has been shown in Table 3 that, the system cost for non-tolling equilibrium case (when no link is charged) is nearly equal to 7480481.8. The total travel times at optimal solutions are 7456064.815 and 7452548.414 for OPTION1 and OPTION2 respectively. These values are somewhat between the systems costs of UE and SO traffic assignments as one may anticipate. By simply looking at these values, we may have an indication that the second GA option (OPTION2) gives a better result, since the objective is to minimize the total system cost. Even though levying all the links in the network (first-best) may drive the system from UE to SO, it is not always advantageous to consider large numbers of toll links when implementing road pricing. For example, Figure 14 and 15 reveal that charging larger numbers of links may even be worse than not charging at all. In other words, it can be seen from the figures that charging more than four links in a network results in higher total travel time value than the case in the absence of toll charge. Therefore, the value of system cost does not necessarily decrease as the number of toll links increases.

Another important observation is that, the fitness curve for OPTION2 (as shown in Figure 15) is smoother than the curve for OPTION1 (as shown in Figure 14). This shows that OPTION2 relatively avoids local optima values as compared to the OPTION1. This can also be considered as another advantage of the second GA option over the first one.

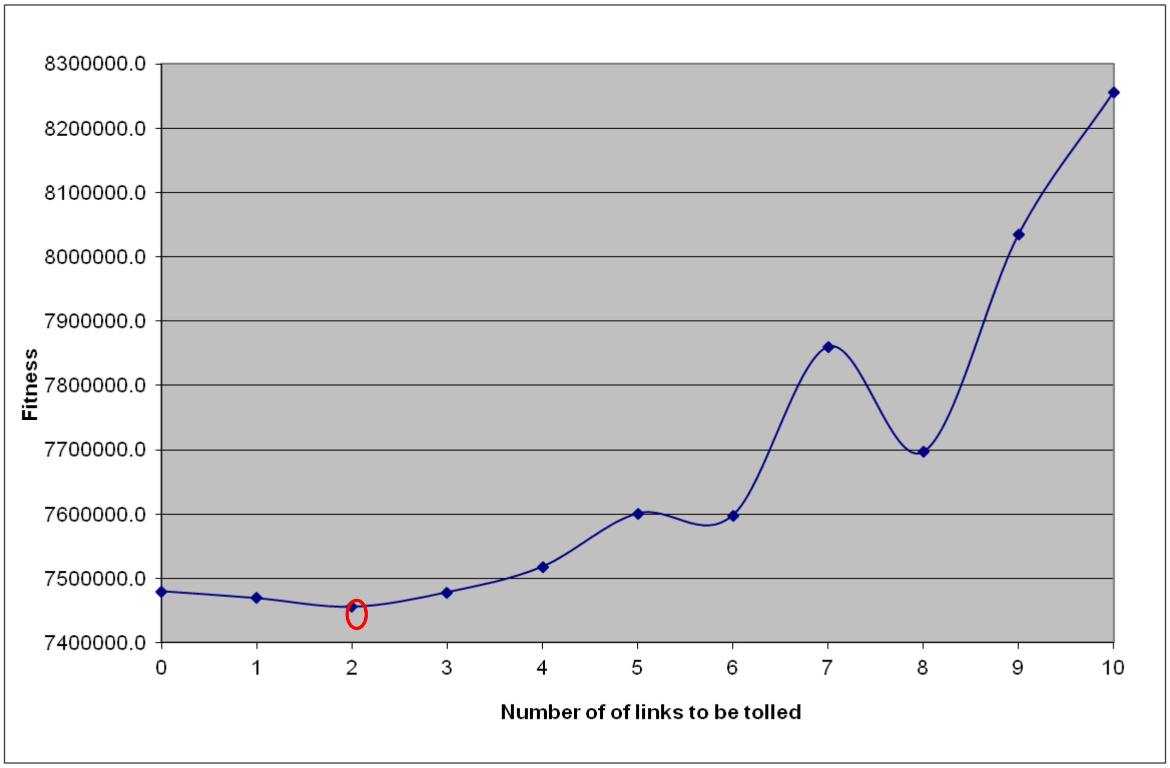


Figure 14. Number of Toll Links vs. Fitness (OPTION1)

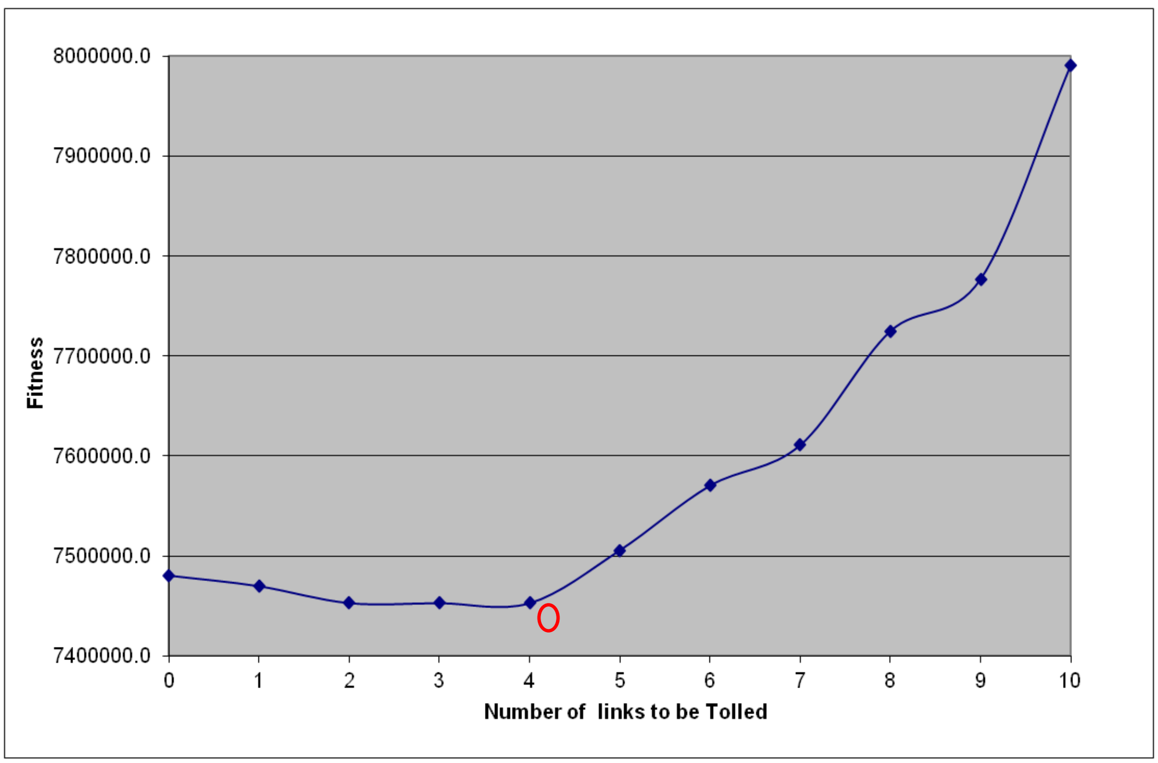


Figure 15. Number of Toll Links Vs. Fitness (OPTION2)

The results of the optimal toll levels and locations evaluated using the two GA options are presented in Tables 4 and 5. As has been indicated previously, charging only two links (Link numbers 49 and 58) gives an optimal result in the case of OPTION1 whereas four links (Links numbers (63, 64, 65, and 76) need to be tolled to get an optimal result in the second case i.e.OPTION2. The corresponding toll rates indicate that the links are charged a little over than the minimum toll level. This tells us charging a higher rate does not necessarily guarantee an optimal solution.

Table 4 Number of Toll Links, Optimal Toll Locations, Toll Rates and System Cost (OPTION1)

Number of Toll Links	*Optimal Toll Locations*	*Corresponding Optimal Toll Rates ($)*	*System Cost*
1	39	2.03	7469567.161
2	58,49	2.05,2.00	7456064.815
3	58,49,30	5.67,5.80,4.74	7478733.750
4	29,67,71,74	2.10,2.35,2.05,2.30	7518480.333
5	31,36,39,35,74	3.31,3.54,3.69,3.60,2.18	7601101.652
6	12,68,49,74,58,64	2.05,2.92,3.02,2.56,3.93,2.84	7598440.337
7	44,42,74,27,71,58,49	3.94,2.02,3.52,2.02,3.85,3.96,3.61	7860020.844
8	16,39,53,13,36,52,14,10	3.62,3.64,2.34,2.13,3.28,2.56,4.82,4.14	7696801.687
9	5,14,71,74,73,72,49,19,58	5.40,4.97,5.9,2.41,6.11,5.64,3.56,3.38,4.10	8035190.264
10	22,7,20,16,36,10,75,65,76,25	4.74,4.54,5.3,3.61,6.24,3.66,2.27,2.57,5.08,6.96	8256472.052

Table 5 Number of Toll Links, Optimal Toll Locations, Toll Rates and System Cost (OPTION2)

Number of Toll Links	*Optimal Toll Locations*	*Corresponding Optimal Toll Rates ($)*	*System Cost*
1	39	2.03	7469567.161
2	58,49	2.21,2.18	7452677.604
3	64,76,65	2.26,2.00,2.21	7452613.474
4	76,65,63,64	2.15,2.55,2.15,2.25	7452548.414
5	53,39,62,63,46	2.63,2.33,2.48,2.82,2.45	7505391.536
6	36,14,35,5,10,39	4.06,7.87,2.75,6.38,2.91,4.39	7570033.239
7	64,68,58,15,5,14,74	2.4,2.88,2.20,2.58,5.10,7.80,2.38	7610519.152
8	32,74,71,41,72,48,75,67	2.28,2.97,3.31,2.25,3.66,3.45,3.77,2.31	7724486.124
9	74,67,8,53,19,40,73,33,69	2.74,2.62,3.29,2.05,3.00,2.95,5.01,4.07,2.66	7776557.413
10	59,75,52,42,8,14,62,76,46,35	3.92,2.34,2.64,2.42,4.31,3.07,2.00,2.63,2.80,2.63	7990413.178

Based on the above discussions, at this point, the second GA option (OPTION2) gives a better optimal solution as compared to the first one. Therefore, the second GA option is further considered and applied to the four most congested links to make additional comparisons. Here also 1000 iterations were used to determine the toll rates and total travel times by varying the number of congested links to be tolled from 1 to 4. Table 6 presents the results of toll rates and system costs for the four most congested links. It can be seen from the table that charging even a single most congested link is worse than not charging at all. The system cost even increases as the number of congested links to be tolled increases. Thus, this result reinforces our idea of not using the most congested links as candidate toll links.

Table 6 Number of Toll Links, Optimal Toll Locations, Corresponding Toll Rates and System Cost for the First Four Most Congested Links (OPTION2)

Number of Toll Links	*Optimal Toll Locations*	*Corresponding Optimal Toll Rates ($)*	*System Cost*
1	19	2.06	7535873.999
2	19,16	2.35,2.05	7572913.574
3	19,16,48	2.10,2.47,2.17	7604861.749
4	19,16,48,29	2.11,2.14,2.1,2.04	7616903.149

Another interesting observation out of the above three tables is that the most congested links may not normally give an optimal solution to the toll design problem and cannot be taken as intuitive candidates of toll links. Table 7 presents volume-capacity (v/c) ratios for each link for the Sioux Falls Network. The four most congested links, based on their v/c ratios, are link 19, link 16, link 48 and link 29. As has been discussed a bit earlier, in the first GA option, charging links 58 and 49 and in the second GA option, charging links 76, 65, 63 and 64 gave the optimal solutions. It can be noted that these optimal toll locations do not belong to the most congested links in the network.

Table 7 Volume-Capacity (v/c) Ratio for Sioux Falls Network

Link Number	*v/c ratio*	*Link Number*	*v/c ratio*	*Link Number*	*v/c ratio*	*Link Number*	*v/c ratio*
1	0.17	20	1.54	39	2.18	58	2.06
2	0.35	21	1.36	40	2.01	59	1.74
3	0.17	22	1.66	41	1.76	60	0.81
4	1.20	23	1.58	42	1.71	61	1.74
5	0.35	24	1.35	43	1.72	62	1.25
6	0.82	25	1.56	44	1.77	63	1.38
7	0.43	26	1.57	45	1.31	64	1.23
8	0.82	27	1.77	46	1.92	65	1.65
9	1.01	28	1.71	47	1.67	66	2.11
10	1.06	29	2.28	48	2.28	67	1.92
11	1.01	30	1.62	49	2.24	68	1.38
12	1.78	31	1.08	50	0.78	69	1.65
13	1.58	32	1.76	51	1.62	70	1.93
14	1.21	33	1.70	52	2.23	71	1.7
15	1.78	34	2.00	53	2.06	72	1.93
16	2.55	35	0.43	54	0.68	73	1.56
17	1.54	36	1.71	55	0.78	74	2.18
18	0.67	37	0.47	56	0.81	75	2.10
19	2.56	38	0.48	57	1.31	76	1.55

As a summary, in this section two GA approaches were investigated and compared with each other in terms of total system travel times assuming homogenous network users. The second option (OPTION2) was found to be better than the first one. The optimal number of locations to be tolled was determined to be four in the section option. OPTION2 was then applied to the four most congested links to make further comparisons. It was found that charging the most congested links makes the traffic congestion worse. In the next section, the preferred GA options i.e. OPTION2 will be applied to solve optimal toll level and location problem for the same network with heterogeneous users having different values of time.

5.4. Multiclass Users - Numerical Analyses

The preferred GA approach from the previous analysis is further modified and applied to the Sioux Falls network. Value of time (VOT) is a very important concept in multiclass network equilibrium design in determining the optimal toll rates and locations as it helps us introduce heterogeneity to the network. Here also fixed demand is considered. The whole O-D demand is divided into some classes, and each class is assumed to have an average VOT belonging to some interval.

In this thesis, the total number of users is divided into 3 classes according to their respective VOTs which may be related to their income level. For the numerical experimentation, the potential market shares of the three classes are assumed to be 75%, 4% and 21% and their VOTs are 5$/hr, 10$/h and 15$/h, respectively. The VOTs are assumed so that their average gives 10$/h which is equal to the VOT for single-class user equilibrium case.

In addition to the above assumption, the following parameters have also been considered: Population size,64; Maximum number of generation, 250; Crossover probability,0.6; Mutation probability,0.05 ; Maximum toll rate (y_k^{max}),10$; Minimum toll rate (y_k^{min}), 2$. Like the previous case, 10 links are assumed to be tolled. As has been discussed earlier, this value is usually set by policy makers.

Figure 16 presents the fitness values computed for 10 different numbers of toll links. The fitness value is decreasing at first, but when the number of toll links exceeds 3 it begins to increase. It can be seen from the figure that the optimal number of toll links is two. In fact, it is the only number of toll links that gives a better result than the no-tolling scenario. The value of system travel time at the optimal solution is 7445565. Even though this value is less than the total system travel time of the single-class, it may not always hold true because GA may not guarantee global optimal solution. However, this still gives a good indication of the importance of modeling traffic behavior with assumption close to reality. It should be noted that this result is obtained in only 250 iterations.

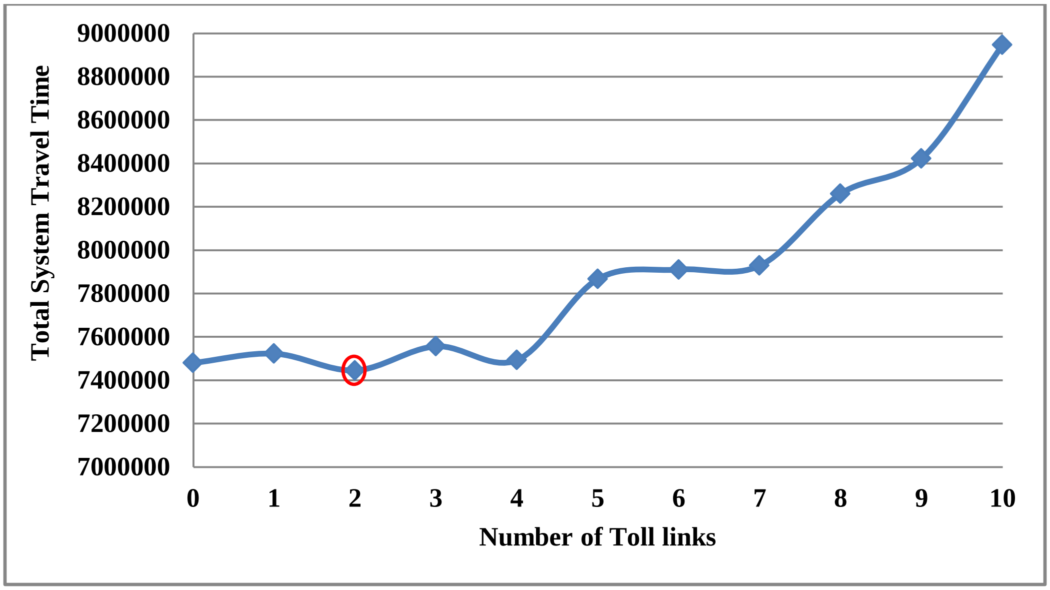


Figure 16. Number of toll links Vs. Fitness (Multiclass Users)

Figure 17 presents the sensitivity of system cost over generations. It can be seen that irrespective of the number of toll links, the total travel time decreases over generations.

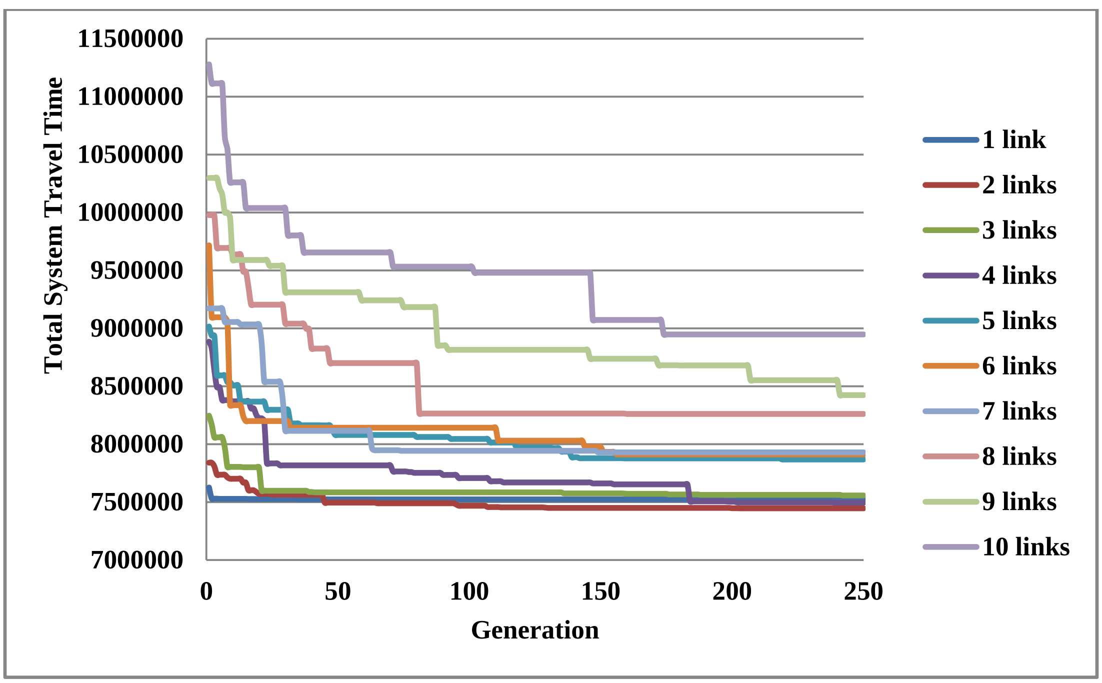


Figure 17. Sensitivity of Total System Travel Time Over Generations (Multiclass Users)

The optimal toll rates and locations are presented in Table 8. As can be seen from the table, the optimal toll levels are very close to the lower boundary of toll rates. Neither of the toll locations is also part of the four most congested links.

Table 8 Number of Toll Links, Optimal Toll Locations, Corresponding Toll Rates and System Cost (Multiclass-Users)

Number of Toll Links	*Optimal Toll Locations*	*Corresponding Optimal Toll Rates ($)*	*System Cost*
1	58	2.01	7522729
2	58,49	2.02,2.05	7445565
3	42,5,14	2.02,7.10,7.29	7556428
4	14,2,4,5	2.72,3.03,3.87,2.42	7493236
5	5,74,19,14,42	9.60,2.09,2.63,8.32,2.05	7867167
6	74,40,2,31,4,42	2.06,2.27,3.52,3.74,4.51,2.06	7910711
7	33,4,14,44,5,2,65	3.05,9.55,5.86,2.06,6.04,9.27,2.93	7928852
8	4,42,31,74,33,76,22,32	3.54,2.22,3.67,2.47,3.83,2.05,2.13,2.1	8260229
9	15,23,49,19,33,4,74,44,31	8.26,3.30,2.28,2.92,6.54,2.12,4.10,2.42,2.88	8423011
10	31,44,66,19,23,33,51,70,34,4	9.18,6.72,5.07,3.87,3.95,5.74,3.32,3.94,4.21,3.51	8946598

In the multiclass users case too, it is important to study if charging the most congested links could lead to an optimal solution. Using the same assumptions presented earlier in this section, the preferred GA option is employed to solve the multiclass user equilibrium problem on the four most congested links i.e. link number 19, 16, 48, 29, to be consistent with the single-class case. Table 9 shows the toll rates, locations and the fitness values for 4 different number of toll links.

Table 9 Number of Toll Links, Optimal Toll Locations, Corresponding Toll Rates and System Cost for the First Four Most Congested Links (Multiclass Users)

Number of Toll Links	*Optimal Toll Locations*	*Corresponding Optimal Toll Rates ($)*	*System Cost*
1	19	2.00	7635151
2	19,16	2.03,2.00	7817865
3	19,16,48	2.05,2.04,2.10	7945653
4	19,16,48,29	2.04,2.01,2.03,2.02	8075845

Table 9 reveals that as the number of congested toll links increases, the total system travel time also increases. Charging even one link is worse than the UE condition.

Once again, this indicates that the most congested links in a network cannot be taken as candidate toll links by default.

5.5. Sensitivity Analysis

It is shown that assuming heterogeneous users in a network gives a better result than assuming homogenous users. As a future extension of this research, conducting sensitivity analysis of the p_c (crossover rate) and p_m (mutation rate) might be important. These values are usually determined through numerical experimentation. Generally a crossover rate value between 0.5 to 0.7 and a mutation rate of less than 10% are typically recommended by many authors. Table 10 presents the sensitivity analysis for different mutation and crossover rates using the multiclass user GA approach.

Table 10 Sensitivity Analysis for Crossover and Mutation Rates

Scenario	p_c	p_m	Optimal System Cost
I	0.5	0.05	7488868
II	0.6	0.05	7445565
III	0.7	0.05	7439439
IV	0.7	0.01	7474381
V	0.7	0.001	7525329

The sensitivity analysis was done as follows. First, the mutation rate was set to be fixed while allowing the crossover rate to vary. Then the crossover rate for the scenario with the lowest travel time was taken and let to be fixed while changing the mutation rate. As can be seen from the above analysis, scenario III gives the best optimal solution. Hence, those parameters can be used if further investigation is needed for future research purposes. The corresponding results of the five scenarios such as fitness values, system cost over generation and optimal toll rates and locations can be found from Appendix 1 through 5.

Chapter 6 – Conclusions

Two different bi-level optimization approaches (OPTION1 and OPTION2), where only genetic algorithm was employed, were investigated to determine the optimal toll location and level simultaneously in which the demand is assumed to be fixed and given a priori. In both GA options, it was first assumed that the network consists of single-class (homogenous road users) in the sense that all drivers value their time equally. The two approaches were compared with each other in terms of the total system travel time (cost) and convergence criteria. Network experiments are conducted and numerical results are described to make the comparisons. The study showed that setting combined chromosome structure for toll locations and toll rates (OPTION2) gives a better optimal solution than setting separate chromosomes for each (OPTION1). Another important point is that it is not a good idea to consider the most congested links in a network as candidate toll links for toll design problem. Sensitivity analysis of the two GA algorithms showed that each new generation gives better or at least the same results compared to the previous one.

Next, the preferred GA based options were further studied and modified so that they could be implemented to a network with heterogeneous users. Here, value of time plays the key role. Three different user classes were considered. It was found that the optimal number of toll links is two and four in the multiclass and single class respectively. It was also found that the value of system travel time at the optimal solution (i.e.74.4E+05) is less than the total system travel time of the single-class users. Irrespective of the number of toll links, the total travel time decreases over generation in both cases.

In the multiclass users case also, the effect of charging the most congested links was investigated. For that, preferred GA option was employed to solve the multiclass user equilibrium problem on the most congested links i.e. link number 19, 16, 48, 29, 49, 52, 39, 74, 66 and 75. It was revealed that as the number of congested toll links increases, the total system travel time also increases and levying a toll on even one link is worse than the UE condition. Here also, it was learned that the most congested links in a network should not be taken as candidate toll links by default.

In order to achieve further improvements of the results reported here, sensitivity analysis of the p_c (crossover rate) and p_m (mutation rate) were conducted. It was discovered that a crossover rate of 0.7 and a mutation rate of 0.05 could give the best optimal solution.

In addition, the variation of demand across time of the day was not considered. Another future research may be conducted toward this end with further insight provided for solving combined toll levels and toll locations problem by considering stochastic demand as it is a very important issue in both design (of a new) and redesign of an existing road networks.

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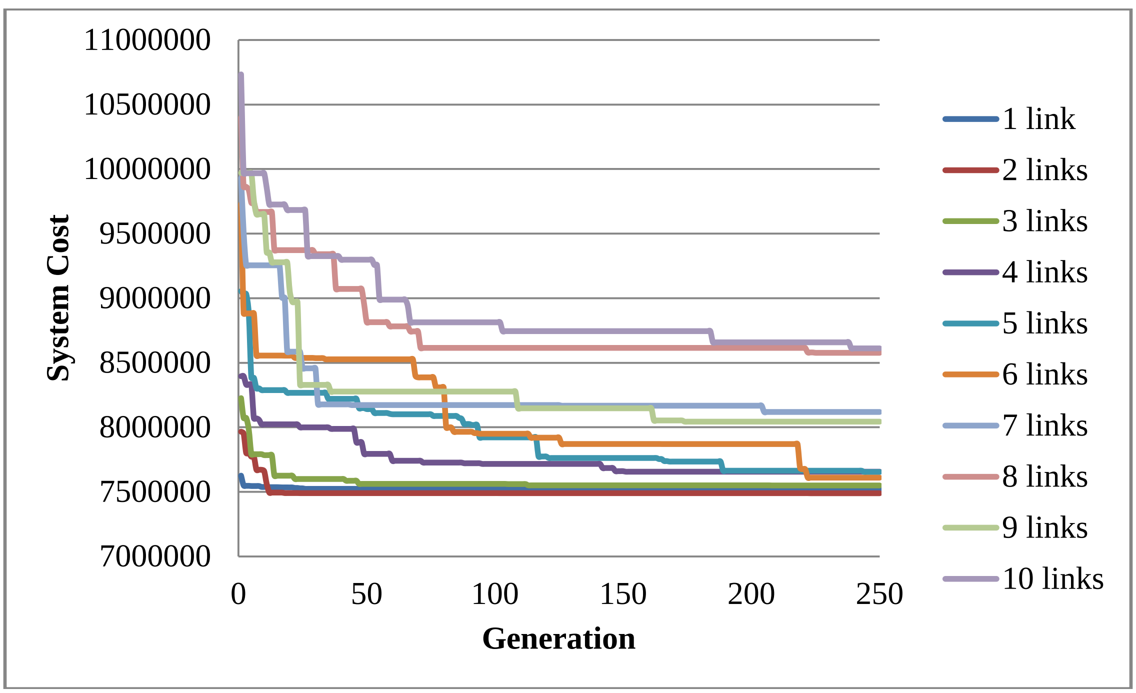
Appendix 1: Sensitivity Analysis for Crossover and Mutation Rates - Scenario I

Fitness

# of links	Fitness
1	7522729
2	7488868
3	7549353
4	7655778
5	7653902
6	7609826
7	8118670
8	8576781
9	8043513
10	8611196



Toll level

1 link	2.01									
2 links	3.72	3.18								
3 links	2.04	4.46	4.09							
4 links	2.61	2.19	2.03	2.19						
5 links	2.52	5.43	2.36	2.22	2.86					
6 links	2.44	2.26	3.31	2.7	3.24	2.3				
7 links	2.01	8.4	2.48	2.28	7.74	2.4	8.89			
8 links	2.18	4.56	4.68	5.64	5.25	5.97	6.77	2.35		
9 links	2.27	3.68	3.02	3.03	3.05	2.06	3.92	3.06	3.72	
10 links	2.63	3.1	2.36	2.94	3.7	2.06	3.84	4.25	2.74	4.51

Toll locations

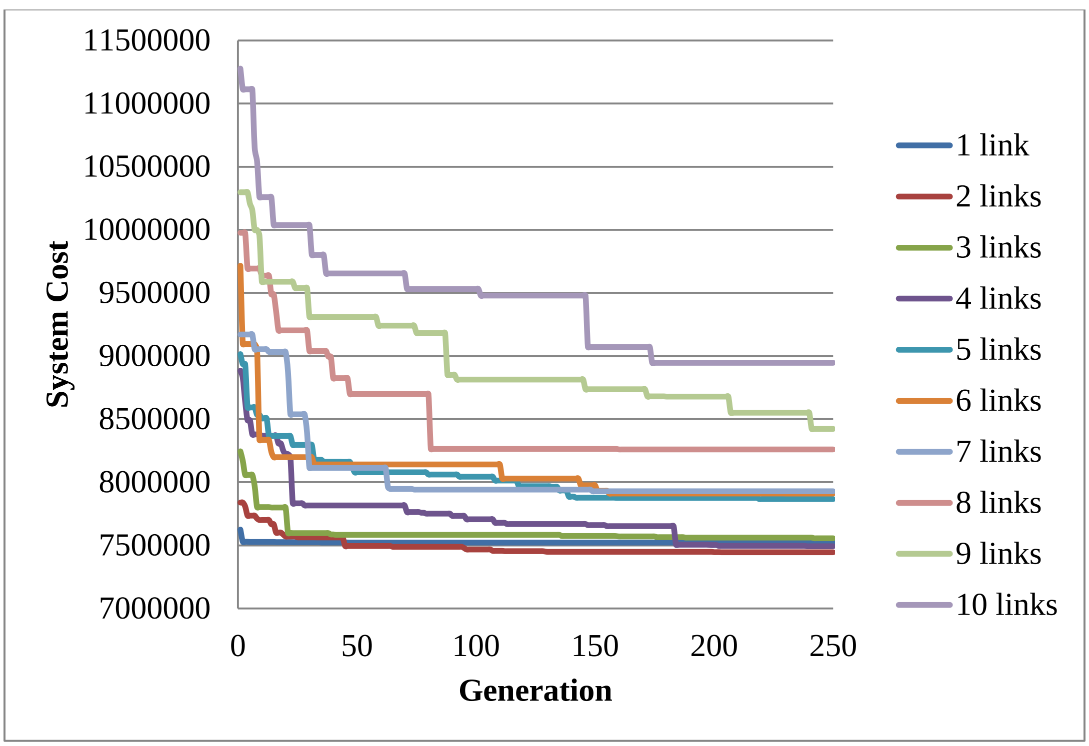
1 link	58									
2 links	14	5								
3 links	42	4	2							
4 links	4	2	44	74						
5 links	38	14	33	5	8					
6 links	58	30	33	49	74	7				
7 links	31	10	33	44	2	32	4			
8 links	75	53	50	47	51	3	48	22		
9 links	16	26	21	15	33	42	31	74	14	
10 links	23	51	16	27	63	39	53	52	3	15

Appendix 2: Sensitivity Analysis for Crossover and Mutation Rates - Scenario II

Fitness

# of links	Fitness
1	7522729
2	7445565
3	7556428
4	7493236
5	7867167
6	7910711
7	7928852
8	8260229
9	8423011
10	8946598

Generation



Toll Level

1 link	2.01									
2 links	2.02	2.05								
3 links	2.02	7.1	7.29							
4 links	2.72	3.03	3.87	2.42						
5 links	9.6	2.09	2.63	8.32	2.05					
6 links	2.06	2.27	3.52	3.74	4.51	2.06				
7 links	3.05	9.55	5.86	2.06	6.04	9.27	2.93			
8 links	3.54	2.22	3.67	2.47	3.83	2.05	2.13	2.1		
9 links	8.26	3.3	2.28	2.92	6.54	2.12	4.1	2.42	2.88	
10 links	9.18	6.72	5.07	3.87	3.95	5.74	3.32	3.94	4.21	3.51

Toll Location

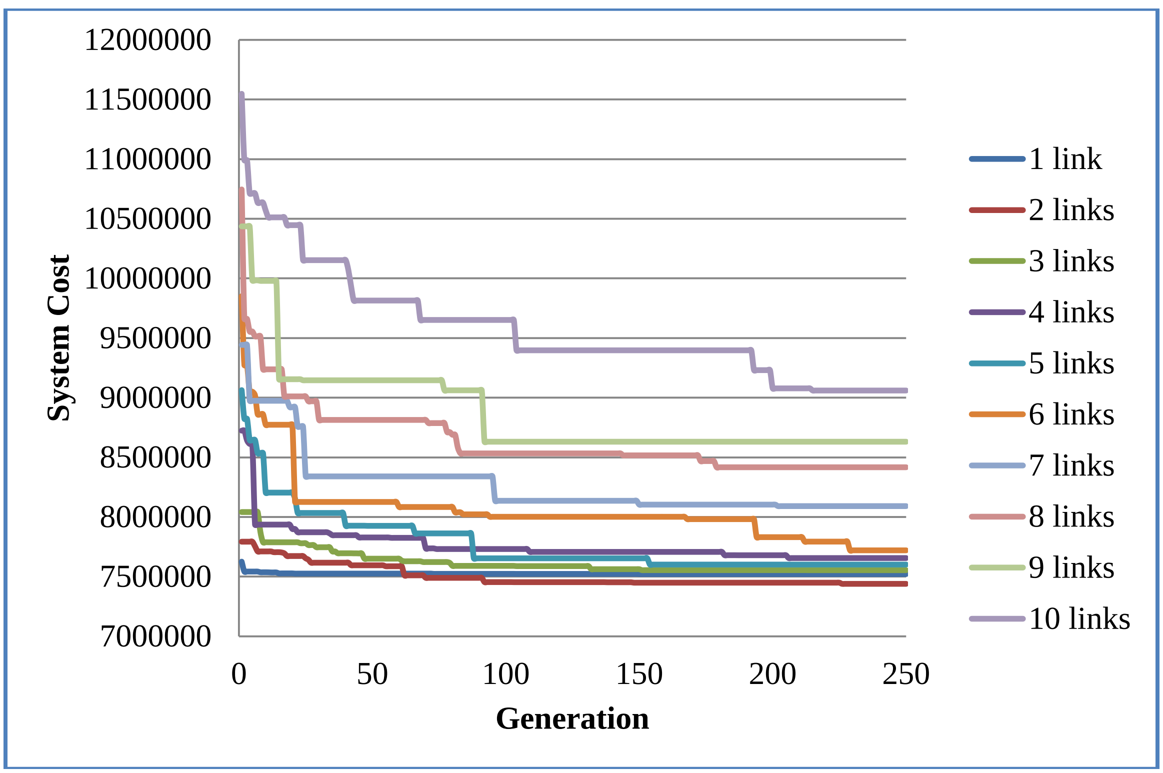
1 link	58									
2 links	58	49								
3 links	42	5	14							
4 links	14	2	4	5						
5 links	5	74	19	14	42					
6 links	74	40	2	31	4	42				
7 links	33	4	14	44	5	2	65			
8 links	4	42	31	74	33	76	22	32		
9 links	15	23	49	19	33	4	74	44	31	
10 links	31	44	66	19	23	33	51	70	34	4

Appendix 3: Sensitivity Analysis for Crossover and Mutation Rates - Scenario III

Fitness

# of Links	Fitness
1	7521405
2	7439439
3	7553854
4	7655502
5	7600989
6	7720298
7	8091016
8	8416739
9	8630941
10	9060260

Generation



Toll level

1 link	2.02									
2 links	2	2.02								
3 links	2.99	2.21	2.07							
4 links	2.06	2.2	2.34	2.03						
5 links	3.42	3.41	3.02	2.85	2.02					
6 links	6.5	5.88	2.07	2.01	2.56	2.05				
7 links	2.02	4.98	2.16	3.51	2.19	2.19	2.26			
8 links	3.38	3.34	3.76	5.35	2.09	2.16	5.22	2.02		
9 links	3.4	3.29	5.69	3.78	4.76	7.33	2.91	3.85	4.42	
10 links	7.3	7	2.71	2.48	2.18	6.74	5.57	2.78	5.08	6.98

Toll Location

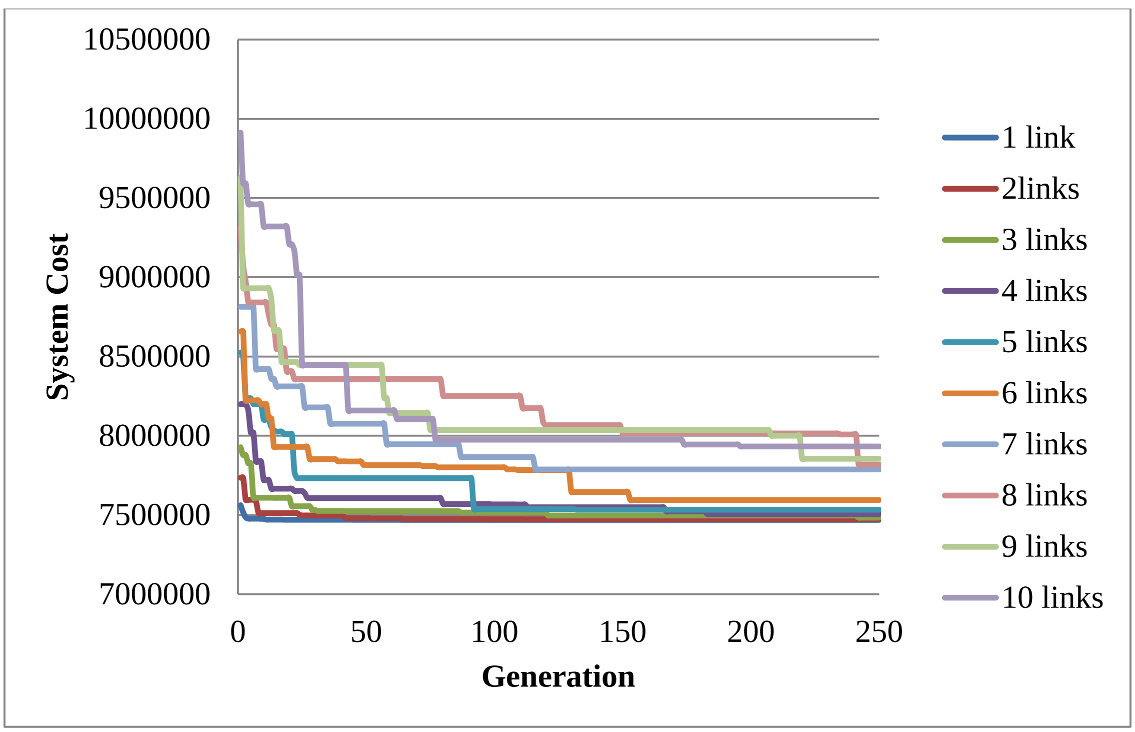
1 link	53									
2 links	58	49								
3 links	14	5	74							
4 links	76	71	34	75						
5 links	74	33	3	8	40					
6 links	14	5	76	49	64	65				
7 links	44	1	58	2	60	34	74			
8 links	49	30	47	58	75	74	31	36		
9 links	32	42	41	9	4	76	14	3	1	
10 links	58	14	44	75	65	76	64	72	30	49

Appendix 4: Sensitivity Analysis for Crossover and Mutation Rates - Scenario IV

Fitness

# of links	Fitness
1	7499567
2	7474381
3	7483522
4	7506462
5	7533790
6	7595024
7	7787820
8	7818825
9	7855245
10	7932662

Generation



Toll level

1 link	2.03									
2links	2.01	2.02								
3 links	4.17	2.05	3.05							
4 links	2.01	2.5	2.19	2.54						
5 links	2.53	2.34	8.41	7.07	2.27					
6 links	3.62	4.22	3.01	2.02	3.54	2.19				
7 links	6.76	6.02	3.42	2.59	2.69	7.23	2.86			
8 links	2.03	3.05	6.18	9.21	5.95	3.34	5.99	4.78		
9 links	3.63	2.18	4.22	2.15	3.51	5.24	3.67	2.66	4.65	
10 links	2.99	6.12	6.92	5.44	2.67	2.81	2.32	2.1	5.39	2.16

Toll Location

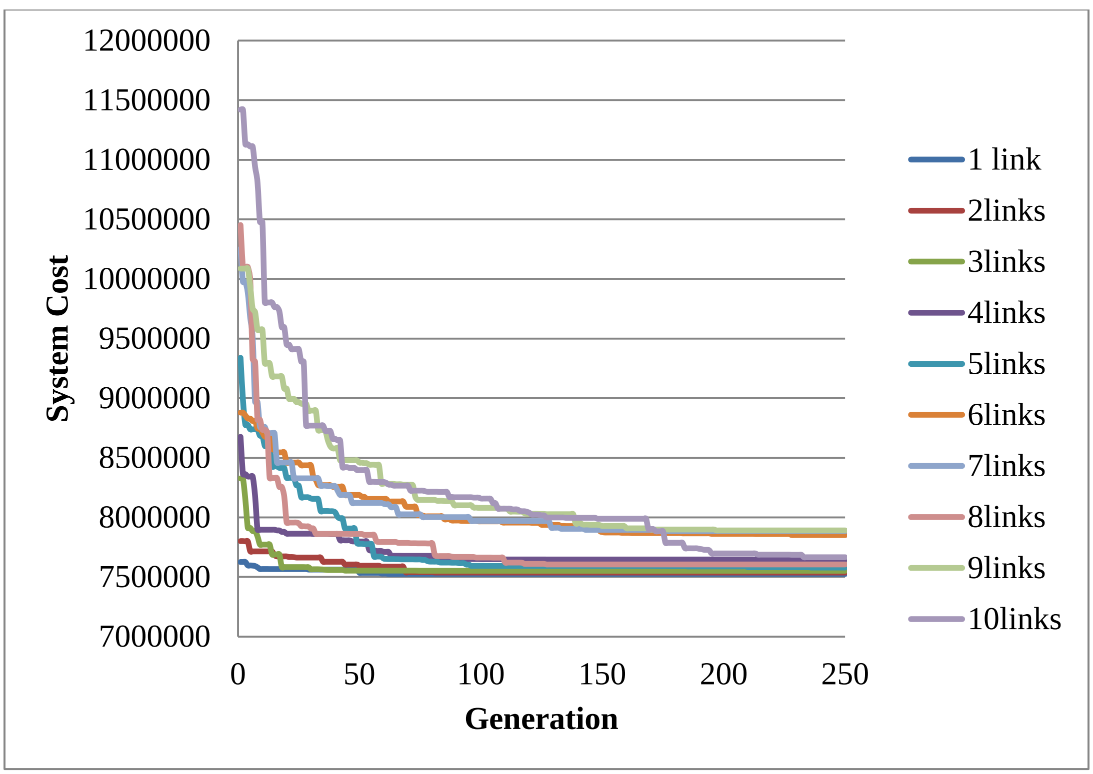
1 link	39									
2links	65	76								
3 links	14	58	5							
4 links	64	65	72	71						
5 links	69	31	4	2	73					
6 links	53	49	64	52	58	65				
7 links	4	5	14	64	68	2	62			
8 links	42	69	10	4	36	13	2	39		
9 links	48	19	63	12	51	52	62	10	53	
10 links	59	53	39	42	36	52	10	51	46	29

Appendix 5: Sensitivity Analysis for Crossover and Mutation Rates - Scenario V

Fitness

#ofLinks	Fitness
1	7525329
2	7537227
3	7551824
4	7646905
5	7579106
6	7851431
7	7884586
8	7605266
9	7890676
10	7665574

Generation



Toll Level

1 link	2.02									
2 links	2.04	2.01								
3 links	2.01	2.04	2.91							
4 links	2.01	2.01	2.06	2.11						
5 links	2.32	2.02	2.01	2.48	3.01					
6 links	2.04	2.15	2.18	2.2	2.12	2.01				
7 links	3.02	2.32	2.25	2.3	2.47	2	2.09			
8 links	2.04	2.24	4.24	2.09	2.02	3.78	4.49	3.54		
9 links	4.55	2.01	3.76	2.31	2.24	2.21	2.03	2.12	2.5	
10 links	3.28	3.92	2.4	2	2.13	3.35	2.15	4.09	3.56	2.69

Toll Location

1 link	71									
2 links	40	74								
3 links	74	5	14							
4 links	34	31	71	33						
5 links	3	8	40	33	74					
6 links	18	42	40	41	21	22				
7 links	69	36	62	10	70	3	8			
8 links	44	38	64	10	36	68	71	67		
9 links	5	74	3	71	14	33	15	16	2	
10 links	36	72	1	31	33	64	34	71	65	6