A Comparative Study of Enhanced Nonlinear PI to Multivariable Nonlinear Plant

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Abstract— **The static gains of PI controller have limitation to handle the process nonlinearities of the system. This suggests the development of enhanced nonlinear PI controller where a nonlinear gain function is cascaded to PI parameters. Two nonlinear gain functions are developed and the effectiveness of the control performance is investigated for two different control structures for a multivariable nonlinear wastewater treatment plant (WWTP). It was proved that superior output with the lower mean error was obtained by developed nonlinear PI specifically in the multivariable control structure. The developed nonlinear PI offers simpler control structure and easy implementation hence offering alternative control strategy for the multivariable system.**

Index Terms— **Activated Sludge; Nonlinear Gain Function; Nonlinear Pi.**

I. INTRODUCTION

The study of multivariable control strategies for multiple-input-multiple-output (MIMO) system has become a great interest in literature such as [1-3]. MIMO system is a system with more than one control loops that generally can be controlled by a centralized multivariable controller or block-diagonal controllers. A centralized multivariable control technique is expected to yield better control performance due to the coupled nature of the system. However, the design of the controller may ask for more complex structure in decoupling task, excessive engineering manpower and robustness issues as discussed in [4]. However, the design of diagonal controller with certain input and output effects is also explored. It is the simplest control structure for MIMO control where the system is decomposed into several subsystems and the designs of controllers are based on single-input-single-output (SISO) controller.

Multivariable nonlinear wastewater treatment plants (WWTP) are subject to the large disturbances inflows and loads together with uncertainties concerning the composition of the influent wastewater. As referred to [5], the activated sludge process (ASP) is one of the most popular bioprocesses used for wastewater treatment where the organic matter in the wastewater is oxidized by microorganisms. A basic configuration of ASP with an aerated tank and a clarifier settler are used in the simulation.

Meanwhile, PID controller is frequently used in process industries due to its simplicity, robustness and near to optimal control performances. The controller attempts to minimize the error by adjusting the process control inputs. Each part of PID controller is highly contributed to achieving the control target [6]. However, the PID controller is still faced with a great challenge to control a complex nonlinear system;

specifically with the randomness of the external disturbances. The classical PID controller is regularly adequate to control of a nominal physical process. Difficulties may come to the classical PID to perform well in high-performance control with changes in operating conditions [7]. Besides, the fixed control parameters in the classical PID controller lead to poor performance of transient response. This was supported by the limitation in the operating range of the controller specifically when it deals with the complex nonlinear system [8]. But, the design and analysis of the nonlinear PID controller are strongly complicated and difficult to be implemented [9] while the question to design simple architecture of effective PID controller was appointed [10]. In conjunction with these issues, modification of a linear PI controller using special nonlinear functions is claimed to be more attractive in engineering applications [9].

Therefore, the work is proposed where the performance of nonlinear PI controller that is capable of improving the performance of linear static-gain PI controller is investigated. A sector-bounded nonlinear gain is cascaded to classical PI control architecture that is used in stabilizing the nominal system. Greater correction is generated when the error is large whilst the control action is automatically reduced in avoiding large overshoots in the response when the error diminishes. The automatic gain adjustment thus leads the nonlinear PI controller to operate in high initial gain for a fast response and lower gain in preventing an oscillatory behavior.

The effectiveness of developed nonlinear PI controller is tested to the ASP of the wastewater treatment plant that is highly known with the nonlinearity of the control parameters. The work highlights the importance to control the substrate and dissolve oxygen (DO) concentrations of the process. The comparative work of the control strategies to the wastewater plant has been studied in [11] where tuning algorithm based on adaptive interaction theory has been applied in diagonal control while Davison method is applied in the multivariable controller. However, an enhancement control strategies of the nonlinear PI controller is proposed in this paper.

The paper is organized as follows. In Section II the ASP is briefly explained. Section III discusses on the development of nonlinear PI controller. Meanwhile, the performance of the controlled plant and the conclusion drawn are presented in Section VI and Section V, respectively.

II. ACTIVATED SLUDGE PROCESS

In general, a WWTP is aiming to remove suspended substances and organic material before releasing to the recipients. The best technology available used to control the

discharge of pollutants emphasized in the biological process; namely as activated sludge process (ASP). In ASP, the organic materials are oxidized by microorganisms. There are several models describing the biological processes in the bioreactor. Figure 1 shows a basic nonlinear activated sludge WWTP with an aerated tank and a clarifier settler. Oxygen supplied in the aerated tank is used by microorganisms to oxidize the organic matter. The biomass in the system is retained by the secondary clarifier for producing a high-quality effluent. Meanwhile, to ensure the right concentration of microorganisms, the settled biomass is recycled back to the aerated tank. The mass balance components used in representing the dynamic behaviors' of the ASP are described as in Equation (1) to (4).
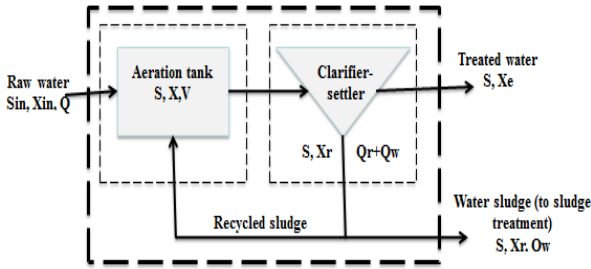

Figure 1: A non-linear activated sludge wastewater treatment plant

$$\dot{X}(t) = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \quad (1)$$

$$\dot{S}(t) = -\frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \quad (2)$$

$$C(t) = \frac{K_o\mu(t)X(t)}{Y} - D(t)(1+r)C(t) + \quad (3)$$
$$K_{La}(C_s - C(t) + D(t)C_{in})$$

$$\dot{X}_r(t) = D(t)(1+r)X(t) - D(t)(\beta+r)X_r(t) \quad (4)$$

where the state variables, X(t), S(t), C(t) and Xr(t) represent the concentrations of biomass, substrate, dissolved oxygen (DO) and recycled biomass, respectively. D(t) is the dilution rate, while Sin and Cin correspond to the substrate and DO concentrations of the influent stream. The parameters r and b represent the ratio of recycled and waste flow to the influent flow rate, respectively. Details on ASP can be referred in [12].

III. Nonlinear PI Control Strategy

The PI control remains a very popular controller choice for many industrial processes. For a PI controller, the error signal, e(t) is used to generate the proportional (P) and integral (I) control actions with the resulting signals weighted and summed to form the control signal, u(t) that applied to the plant model. The mathematical description of the PI controller is briefly expressed in Equation (5) where Kp and Ki are the proportional and integral gains of the controller.

$$u(t) = K_p\,e(t) + K_i\int_0^t e(t)dt \quad (5)$$

However, the static gains of PI controller have limitation to handle the process nonlinearities of the system. This suggests the development of enhanced nonlinear PI controller where the gains of the PI parameters are not fixed but are adjusted

according to the dynamic changes of the output measured [13].

$$u(t) = \left[k_p + k_i\int_0^t dt\right]\cdot f(e)$$
$$\quad (6)$$
$$= k_p\,[k_{non}(e)\cdot e(t)] + k_i\int_0^t[k_{non}(e)\cdot e(t)dt]$$

Referring to Equation (6), the nonlinear gain function, f(e) is connected in cascade with a linear static-gain PI controller. f(e) can be represented by any general nonlinear function of e(t) which is bounded in the sector 0<Knon<Kmax. The knon(e) is a sector-bounded function that acts on the error in resulting the scaled error f(e)=knon(e).e(t). The nonlinear gain function presented in [12] has been developed and tested in cascading to the PI controller,

$$k_{non}(e) = \frac{\exp^{(k_n\,e)} + \exp^{(-k_n\,e)}}{2}$$

$$e = \begin{cases} e & |e| \le e_{max} \\ sign(e)\cdot\sqrt{|e_{max}|} & else \end{cases} \quad (7)$$

with emax is the user-defined positive constants, the knon is functional of error that depends on the selection of kn under the limitation of 1≤ knon(e)≤ knon(emax). The knon(e) is in lower bounded gain when e=0 and in upper bounded gain when e=emax. Therefore, emax indicates the range of variation while kn denotes the rate of variation of knon(e).

The Embedded MATLAB function is used as the platform to write the nonlinear gain function. This block can be found in Simulink > User Defined Function > Embedded MATLAB Function. The effectiveness of developed nonlinear PI controller is tested in both diagonal and centralized multivariable control structures of the WWTP.

IV. Development Of The Controller

The main objective of the control algorithm is to regulate the substrate and DO concentrations of the ASP. However, it is of great interest to investigate how effective the controller in the diagonal and multivariable control structure of the WWTP.

The referred WWTP model is two-input two-output (TITO) model structure. As in diagonal platform, the WWTP is partitioned into two subsystems which contribute to two diagonal PI controllers. Both controllers are arranged in diagonal so that the output y1, the substrate is controlled by manipulating the input signals u1, dilution rate while the output y2, dissolved oxygen is controlled by u2 air flow rate; as shown in Figure 2. Despite that, a centralized nonlinear PI controller is just demanded centralized multivariable control as shown in Figure 3.
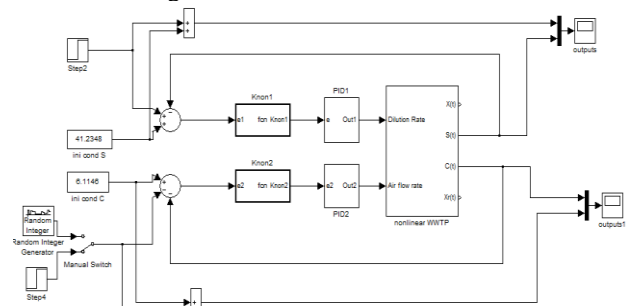

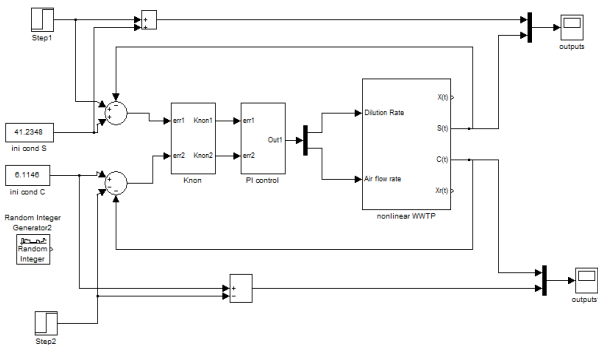
Figure 2: Diagonal Control Structure

Figure 3: Multivariable Control Structure

V. RESULTS AND DISCUSSION

As previously discussed, the effectiveness of developed nonlinear PI controller is tested for both diagonal and centralized multivariable control structure of the WWTP. Figure 4 shows the output of substrate and DO for decentralized and multivariable controls. The WWTP is simulated in 1400 hours with step inputs and 1 hours sampling time. According to Equation (7), e_{max} which indicate the range of variation is set to 1 while k_n that denote the rate of variation of k_{non} is best tuned to 0.5 for both control studies.

Overall, both output variables show good performance in tracking the reference input with superior performance in DO rather than substrate control as presented in Figure 4(a). It was observed that, the larger peak is obtained by diagonal controller compared to multivariable control for substrate control before it tracked back to the set point hence ask for longer settling time as shown in Figure 4(b) and Figure 4(c) for the output resulted at t=400 and t=900 simulation time, respectively.

(a)

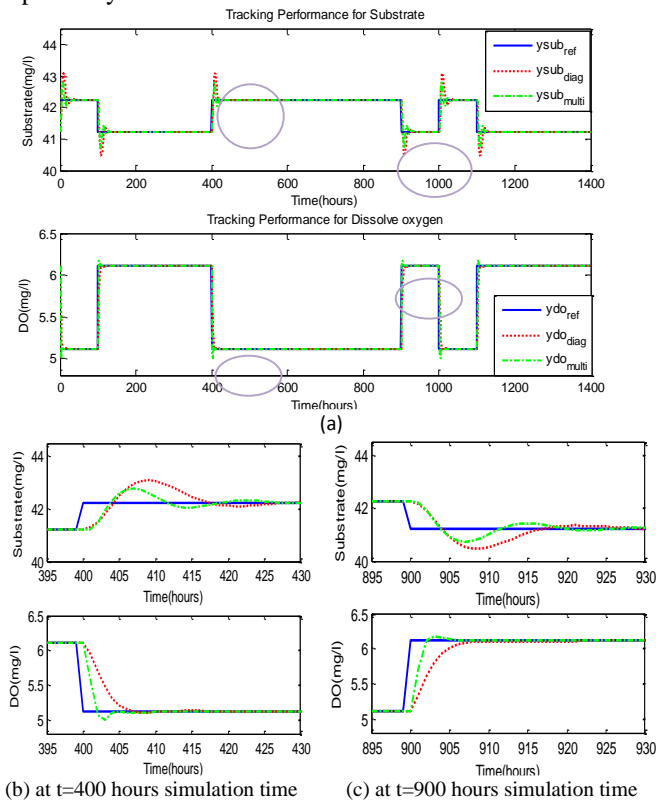(b) at t=400 hours simulation time   (c) at t=900 hours simulation time

Figure 4: The output performances by diagonal and multivariable control

The performance of error resulted by diagonal and multivariable control is presented in Table 1. mean error1 and mean error2 are referring to substrate's error and DO's error, respectively. As observed, the developed nonlinear PI controller with proposed nonlinear gain is best applied in centralized multivariable control with lower mean error resulted.

Table 1
Comparison between diagonal and centralized multivariable control

	Diagonal Control	Multivariable Control
mean error$_1$	0.0000086	0.0000029
mean error$_2$	0.0000058	0.0000023

In order to investigate the effectiveness of the nonlinear PI controller with an applied gain in Equation (7), a nonlinear gain function expressed in Equation (8); as referred in [9] are tested to the WWTP.

$$k_{non} = k_o + k_1 \left\{ 1 - \frac{2}{\exp^{(k_{2e})} + \exp^{(-k_{2e})}} \right\} \qquad (8)$$

Here, k0, k1, and k2 are user-defined positive constants. The gain knon is now upper-bounded by kmax= k0+k1 when error,e = +∞ and lower bounded by kmin= k0 when error,e = 0. Thus, k0 defines the minimum value, k1 defines the range of variations while k2 specifies the rate of variations knon. The best tuned of k0 is achieved at 0.05, k1 =0.25 and k2 =1000 of the nonlinear gain function

The output performances of the nonlinear PI controller with two nonlinear gain functions are compared in Figure 5 for diagonal control and Figure 6 for centralized multivariable control. knon1 is referring to the nonlinear gain function described in Equation (8) while knon2 is referring to the proposed nonlinear function; previously described in Equation (7).
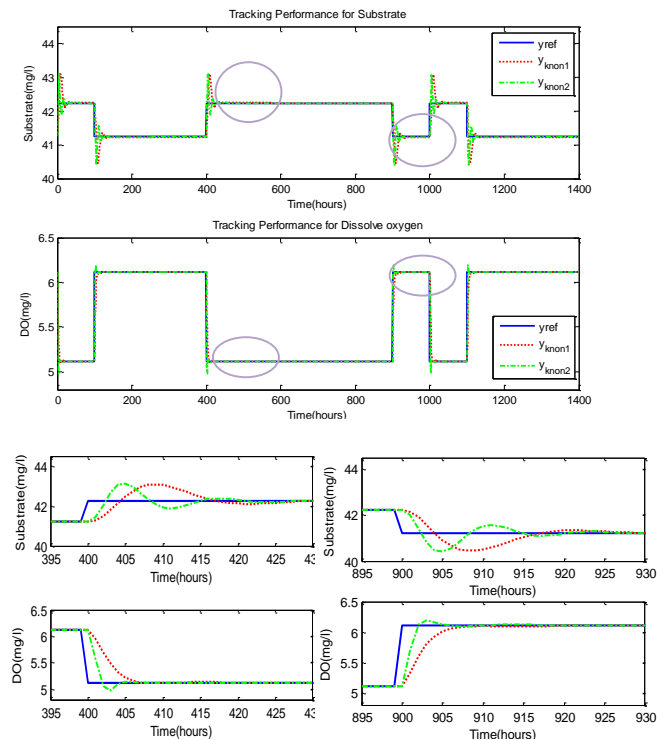

(a) at t=400 hours of simulation time

(b) at t=900 hours of simulation time

Figure 5: The control performances by diagonal control
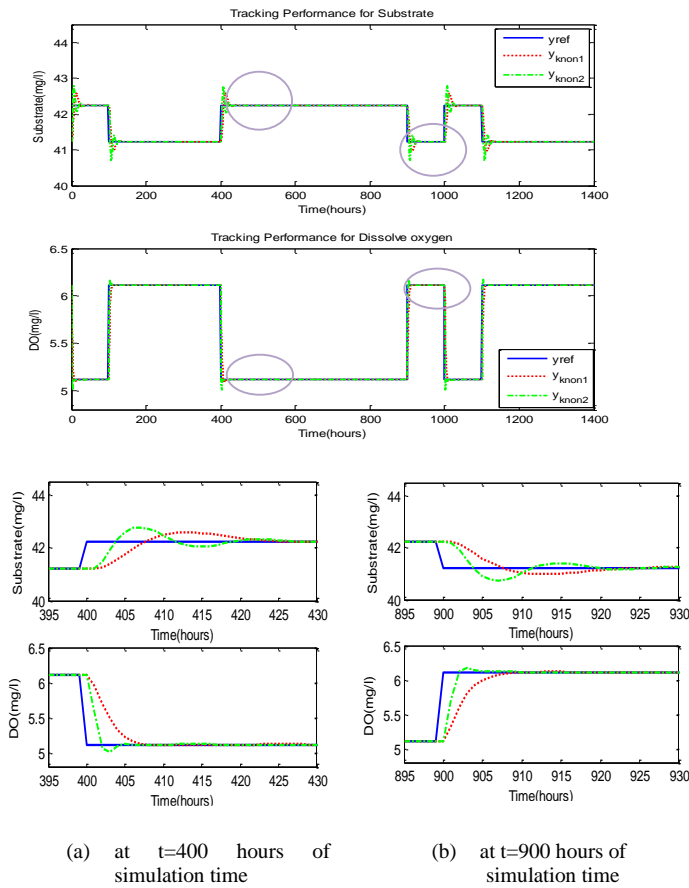
Figure 6: The control performances by centralized multivariable control

| | (a) | at t=400 hours of simulation time | (b) | at t=900 hours of simulation time |

In graphical output performance, it can be seen that no significant different resulted by both nonlinear PI controller as referred to Figure 5 and Figure 6. Both knon1 and knon2 are performed well to regulate the substrate and DO with even slight peak triggered at the changing time; specifically at 400 and 900 simulation time. However, with respect to the performance of error (see Table 2 and Table 3), superior output performances are obtained by the PI controller with the proposed nonlinear gain, knon2 for both diagonal and multivariable control structures.

Table 2
Comparison of error by decentralized control

	Substrate		DO	
	k_{non1}	k_{non2}	k_{non1}	k_{non2}
mean error	0.000019	0.0000029	0.000031	0.0000023

Table 3
Comparison of error by centralized multivariable control

	Substrate		DO	
	k_{non1}	k_{non2}	k_{non1}	k_{non2}
mean error	0.000036	0.0000086	0.000039	0.0000058

VI. CONCLUSION

Limitation of the static gain PI controller to handle the nonlinearities of the system is undeniable thus motivates the development of enhanced nonlinear PI controller. Two case studies are emphasized in this work. Firstly, the control performance of the nonlinear PI controller with the proposed nonlinear gain is investigated in two different control

structures; diagonal and multivariable. It was observed that both output variables show good performance in tracking the reference input with superior performance in DO rather than substrate control. Compared to centralized multivariable control, higher peak in diagonal substrate control results in every changing simulation time but it immediately tracked back to the set point. The output performance of the nonlinear PI controller with the proposed nonlinear gain function is next compared to other established nonlinear function. The PI controller with the proposed nonlinear gain function (knon2) is performing well with a lower mean error in both diagonal and multivariable control structures. To conclude, the developed nonlinear PI with the proposed nonlinear gain function offers simpler control structure and easy implementation hence offering alternative control strategy for a multivariable system. The nonlinear PI controller will then be tested for more complex multivariable WWTP for optimum output in the future study.

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