An Effective Noise Reduction Algorithm in Signal Edge Detection

Ehsan Ehsaeyan

Faculty of Sirjan University of Technology, Electrical Engineering Department, Sirjan, Iran.

ehsaeyan@sirjantech.ac.ir

***Abstract*—Noise as an unwanted factor always degrades the edge detection performance. Exploiting real edges under noise contaminated condition has been a challenge in the edge detection issue, especially when the noise power is high. This paper presents a robust edge detection method in a noisy condition based on the 1D wavelet transform domain as a solution for the noise problem. First of all, a new group of wavelet basis functions for the edge detection is introduced. Then, a basic ramp function is modeled by a Gaussian approximation, and edge detection according to introduced bases derived from relevant formulas is discussed. We develop the multiscale production method and present a new algorithm to reinforce real peaks and suppress fake edges. Finally, the simulation results of the edge detecting for the noisy uniform step edge are provided to evaluate proposed algorithm efficiency. The results showed that our scheme is more effective than the production of adjacent coefficients method in the low signal to noise ratio condition.**

***Index Terms*—Wavelet Transform; Edge Detection; Gaussian Filter; Multiscale Analysis; Wavelet Basis Function; Step Edge; Multiplying Scales.**

I. INTRODUCTION

A major problem in the edge detection issue is the presence of noise. Real scenes, signals and images are contaminated by noise and distinguishing between correct edges and fake edges is difficult, especially in low SNR cases because they both have high frequency characteristics (signal details and noise). Traditionally, the maximum modulus of points along the gradient direction that is larger than the threshold has been considered as edges. The classical edge detectors, such as Roberts, Prewitt and Sobel use a small mask to cover the image: Although these detectors are simple to use, they are weak in noisy or blurred images. They can perform well when the edge is laid in certain orientation only.

An approach that provides an efficient method of obtaining scaled edge map outputs by directly using linear combinations of edge map outputs obtained at the lowest scale has been presented [1]. A noise-robust color edge detector using gradient matrix and anisotropic Gaussian directional derivative (ANDD) matrix has been proposed and the results have been compared with color Canny edge detector [2]. A novel method based on entropy-driven gradient evaluation (P-Edge) for detecting perceptual edges that represent boundaries of objects has been proposed [3]. In this method, P-Edge is characterized by iteratively employing a shape-changeable mask centered at a target pixel to sample gradient orientations of neighboring pixels for measuring the directivity of the target pixel. In recent years, the edge detection by the wavelet transform is developed in image processing and computer vision fields [4-8].

In this paper, a set of wavelet basis functions based on odd derivations of Gaussian smooth filter is introduced. Gaussian filter has the optimal localization in the spatial and frequency domain. Hence, it is considered as a basic edge detector in this article. A Gaussian family filter, i.e. Gabor filter has been considered [9], and optimal parameters of the step edge detector have been derived [10]. However, this approach does not cover other types of the edge.

Zhang and Bao have proposed a wavelet based edge detection scheme by scale multiplication [11]. The dyadic wavelet transformed at the two adjacent scales is multiplied as a product function to magnify the edge structures and suppress the noise. They determined the edges as the local maxima directly in the scale product after an efficient thresholding. It has shown that the scale multiplication achieves better results than either of the two scales, especially on the localization performance. However, some fake edges remained and were not deleted.

One of the most bolded research is the work of Canny [12]. He introduced Gaussian filter as optimal edge detectors and presented three optimal criteria for designing edge filters based on local maxima, which have been used until now.

To have a detection with less noise or blurred condition, we combine multi-resolution and multi-derivation Gaussian filter to open a new computer vision idea in the edge detection. The proposed algorithm can effectively suppress noise and produce proper edges.

This paper is organized as follows: Section 2 is dedicated to the study of the proposed wavelet bases. In this section, the new wavelet basis functions are introduced. Section 3 discusses the ramp edge detection. A ramp edge is approximated by Gaussian filtering of a step edge function. This modified model helps us to reach an easier relationship. Then, an edge detection algorithm based on proposed bases is introduced; that is the subject of Section 4. In this section, the performance of the proposed scheme on the ideal and on the noisy uniform step edge is described. Three noise conditions are considered: low, medium and high. In each condition, the proposed method is described and the results are shown in relevant figures for comparison. The introduced algorithm can develop across the *scale*. This is the motivation to compare our work with Zhang's method [11] at the end of this section. Finally, the conclusion and discussion of the paper are devoted in Section 5.

II. Proposed Wavelet Basis Functions

Let $\theta(x)$ be a differentiable smooth function which its integral over $(-\infty,+\infty)$ is 1 and converges to 0 at $\pm\infty$ (fast reduces at $\pm\infty$). Assuming that $g(x)$ be a Gaussian filter with variance σ^2 and zero, it means that:

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

The Gaussian filter is a smooth or primitive function because its integral over $[-\infty,+\infty]$ is 1 and reaches to zero in infinity. Canny used the first derivative of the Gaussian filter for edge detecting and introduced it as the optimal edge detector [12]. We develop this idea to the higher order derivation of the Gaussian filter. Let wavelet $\psi^n(x)$ be the nth order derivative of $g(x)$. With this assumption, we have:

$$\psi^n(x) = \frac{d^n g(x)}{dx^n} \qquad n = 1,2,\ldots \tag{2}$$

We introduce odd *nth* order derivative of Gaussian smooth filter as wavelet bases. Assuming that $g_s(x)$ is a Gaussian filter at the scale *s*:

$$g_s(x) = \frac{1}{\sqrt{2\pi}s} e^{-\frac{x^2}{2s^2}} \tag{3}$$

The scaled function of $\psi(x)$ is denoted by $\psi_s(x)$ and would be defined as:

$$\psi_s^n(x) = s^n \frac{d^n g_s(x)}{dx^n} \tag{4}$$

We use these wavelet functions to reach the ramp edge detection and step edge detection as described in the following subsection.

A. Ramp Edge Detection

In the noisy condition, the step shape is more similar to a ramp rather a step. So, it is essential to study the performance of the ramp edge as a real shape in signals. It can be assumed that a ramp is an output of the step function through a Gaussian smooth filter. In this situation, the slope of the ramp edge can be modeled by the slope of the output function in the break point [13]. Let $g_\sigma(x)$ be a smoothing Gaussian filter with variance σ^2 as:

$$g_\sigma(x) = \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \tag{5}$$

Therefore, the output of the unit step function through the Gaussian filter would be:

$$u_{-1}(x) * g_\sigma(x) = \frac{\operatorname{erf}\left(\frac{x}{\sqrt{2\pi}\sigma}\right) + 1}{2} \tag{6}$$

where $\operatorname{erf}(x)$ is the error function [14]. Figure 1 illustrates this approximation for the different *sigma*.

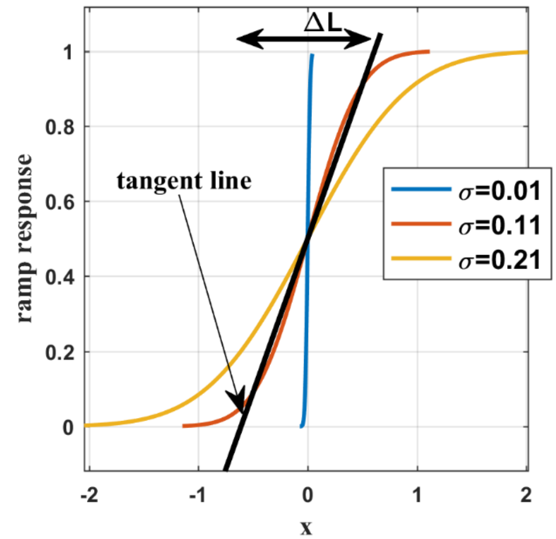


Figure 1: An approximation of the ramp by the convolution of the step signal and Gaussian filter

The slope of the tangent line is the derivation of Equation (6) at $x = 0$.

$$m = \frac{d}{dx} u_{-1}(x) * g_\sigma(x)\Big|_{x=0} = \frac{1}{\sqrt{2\pi}\sigma} \tag{7}$$

If the equivalent edge width is denoted by ΔL as shown in Figure 1, we will have:

$$m = \frac{1}{\Delta L} \tag{8}$$

Equation (8) indicates that there is a reverse relationship between width ΔL and slope m. The ramp model $y(x)$ as a piecewise function is formulated as:

$$y(x) = \begin{cases} 1 & \frac{\pi\sigma}{\sqrt{2}} \le x \\ \frac{x}{\sqrt{2\pi}\sigma} + \frac{1}{2} & -\frac{\pi\sigma}{\sqrt{2}} \le x \le \frac{\pi\sigma}{\sqrt{2}} \\ 0 & x \le -\frac{\pi\sigma}{\sqrt{2}} \end{cases} \tag{9}$$

We use the function $y(x)$ instead of the ramp function. This approximation makes the problem easier to be solved. So:

$$r(x) = u_{-1}(x) * g_\sigma(x) \cong y(x) \tag{10}$$

where, $r(x)$ is an approximation model of $y(x)$ in this equation. Near to the zero amount of ΔL, the approximation model equals to the step edge i.e.

$$\Delta L \to 0 \ : \ m \to \arctan(\infty) \ : \ y(x) \to u_{-1}(x) \qquad (11)$$

It is possible to measure root square error of this approximation. This criterion helps us to find the deviation of the approximate model from the real signal. It is defined and calculated as:

$$RS = \sqrt{\int_{-\infty}^{+\infty} (u_{-1}(x) * g_\sigma(x) - y)^2 dx}$$
$$= \sqrt{\begin{array}{c} \int_{-\infty}^{-\frac{\pi\sigma}{\sqrt{2}}} \left(\frac{\operatorname{erf}\left(\frac{x}{\sqrt{2\pi}\sigma}\right) + 1}{2} \right)^2 dx + \\ \int_{-\frac{\pi\sigma}{\sqrt{2}}}^{\frac{\pi\sigma}{\sqrt{2}}} \left(\frac{\operatorname{erf}\left(\frac{x}{\sqrt{2\pi}\sigma}\right)}{2} - \frac{x}{\sqrt{2}\pi\sigma} \right)^2 dx + \int_{\frac{\pi\sigma}{\sqrt{2}}}^{\infty} \left(\frac{\operatorname{erf}\left(\frac{x}{\sqrt{2\pi}\sigma}\right)}{2} - \right. \end{array}} \qquad (12)$$

After simplifying the formula, we reach to:

$$RS = \sqrt{0.0178\sigma} \qquad (13) \qquad (13)$$

Root Square Error of this approximation versus *sigma* is traced in Figure 2. With reference to this figure, it is clear that by increasing the *sigma*, the line slope will be reduced, and the error approximation increases.

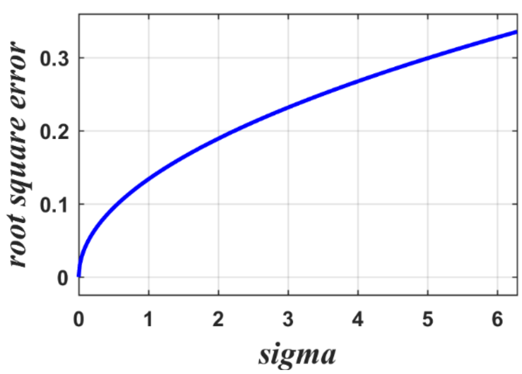


Figure 2: Root square error of the ramp approximation versus *sigma*

For the introduced wavelet basis functions, we have:

$$\begin{aligned} W_s^n f(x) &= r(x) * \psi_s^n(x) \\ &= s^n u_{-1}(x) * g_\sigma(x) * \frac{d^n g_s(x)}{dx^n} \\ &= s^n \frac{d^{n-1}}{dx^{n-1}} g_{\sqrt{s^2+\sigma^2}}(x) \end{aligned} \qquad (14)$$

Assume $\psi_s^n(x)$ is defined at the finest resolution *s=1*. Figure 3 shows wavelet coefficients of the approximated ramp in this case.

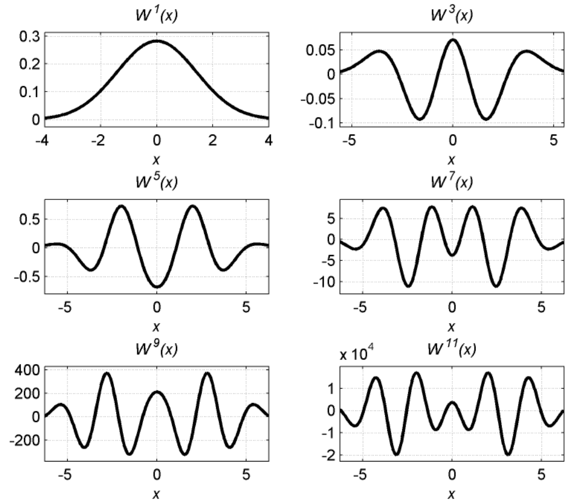


Figure 3: Wavelet ramp responses in scale=1 and *sigma*=1

B. Step Edge Detection

In this section, we demonstrate the performance of the introduced wavelet basis functions in the step edge detection and improved the algorithm with the use of multiplication of responses.

The technique of scales multiplication was first introduced by Rosenfeld [15] and developed by Bao [16], Zhang [11] and Zhu [9]. Multiscale product decreases the input noise correlation. We used this procedure to reinforce the edge amplitude and suppress the noise power, not only across the scale but also across the derivations *n*. Figure 4 illustrates the step edge responses by introducing the bases and multiplying them without noise at the *scale=1*.

If we consider a noiseless step signal, single W^n or ΠW^n with small n (i.e. $W^1 \times W^3$ or $W^3 \times W^5$) is suitable for a breakpoint detection, with less amplitude and location error in these selections. However, low SNR W^n and ΠW^n with greater n is suitable in noisy conditions. Further, in higher scales, amplitudes are better distinguished because of the suppressing noise. The use of higher multiplication across the derivation leads to sharper peaks. Three cases are discussed to clarify the matter.

i. SNR= 17 dB

First, we study the high Signal to Noise Ratio case. The results of the noisy step function are traced in Figure 5 at the finest *scale=1*. The signal is corrupted with AWGN.

In high SNR mode, every basis can be solely used to detect the edge. Because the power of the noise is too weak and cannot influence the decision significantly, using the product of bases in this case is not necessary. In this case, the edge exploiting from W^1 is a better choice.

ii. SNR=10 dB

Figure 6 illustrates the responses of the noisy step edge with SNR=10 dB and scale s=1. Focus on this Figure, we understand that the second column, the multiplication of adjacent responses, is more suitable for the edge detection ($W^1 \times W^3$ or $W^1 \times W^3 \times W^5$ or $W^1 \times W^3 \times W^5 \times W^7$).

iii. SNR=0.5 dB

In this case, the step edge is contaminated crucially by noise. As shown in Figure 7, the product of higher responses ($W^1 \times W^3 \times W^5$ or $W^1 \times W^3 \times W^5 \times W^7$) is more effective to detect the edge and suppress noise.

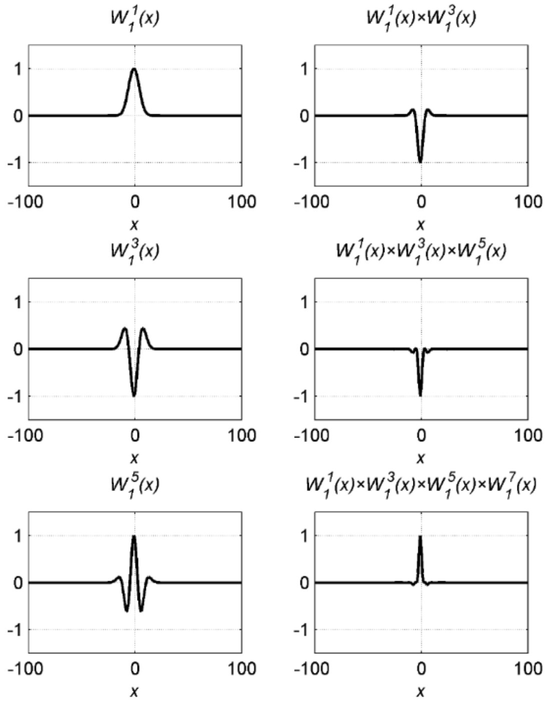


Figure 4: Step responses, single and multiplication at *scale=1*

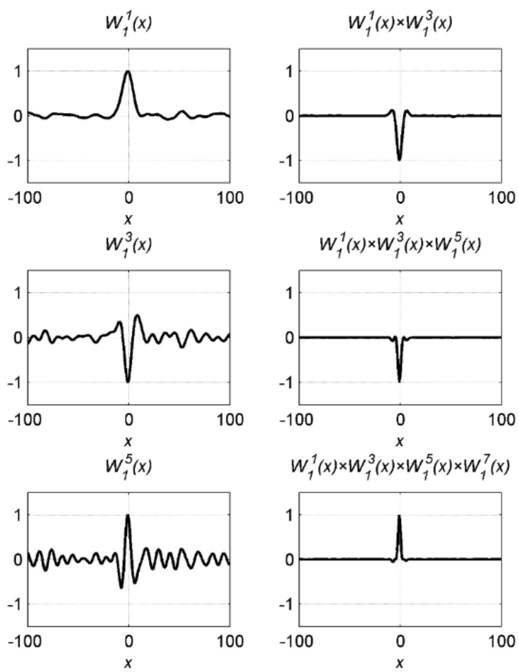


Figure 5: Noisy step responses in SNR=17 dB and s=1

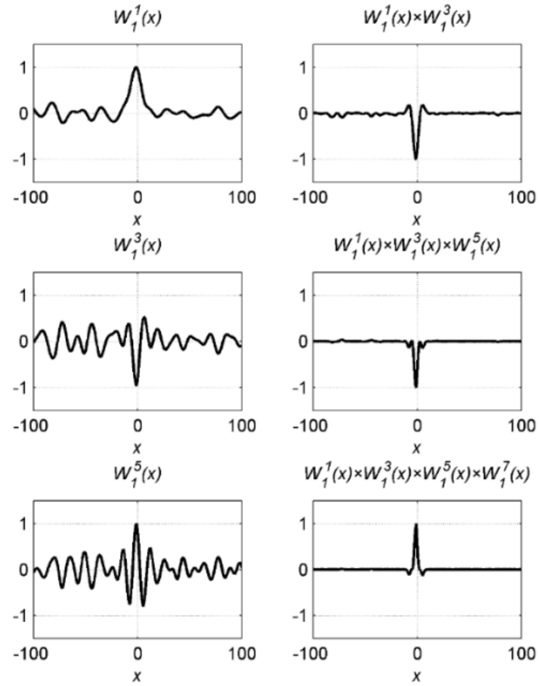


Figure 6: Noisy step responses in SNR=10 dB and s=1

These diagrams were traced in the resolution of *s=1*. It is possible to proceed across the *scale* and reach similar responses. A comparison between our method and Zhang method [11] is illustrated in Figure 8. Zhang's algorithm has been based on the multiplication of adjacent wavelet coefficients to reinforce the real peaks, but it is poor in the noise blocking. As shown in this Figure, our method is more powerful to eliminate pseudo edges created by noise. In fact, our response has a sharper peak in the break point.

Figure 9(a) shows a step edge corrupted with Gaussian white noise. Figure 9(b) and 9(c) are the edge detection results of the Zhang's and our method. Pratt [17] introduced a criterion that shows the quantity performance of edge detection. This parameter is called *figure of merit* and it is defined as:

$$F = \frac{1}{\max\{N_I, N_A\}} \sum_{i=1}^{N_A} \frac{1}{1 + \alpha d^2(i)} \qquad (13)$$

where N_I is the number of true edges, and N_A represents the number of marked edges by the detector algorithm. α is a penalty scaling number that controls false edges, and similar to Pratt's work, it is set at 1/9. d means the Euclidian distance between the point detected by the algorithm procedure and marked as the edge point and its actual edge in the reference map.

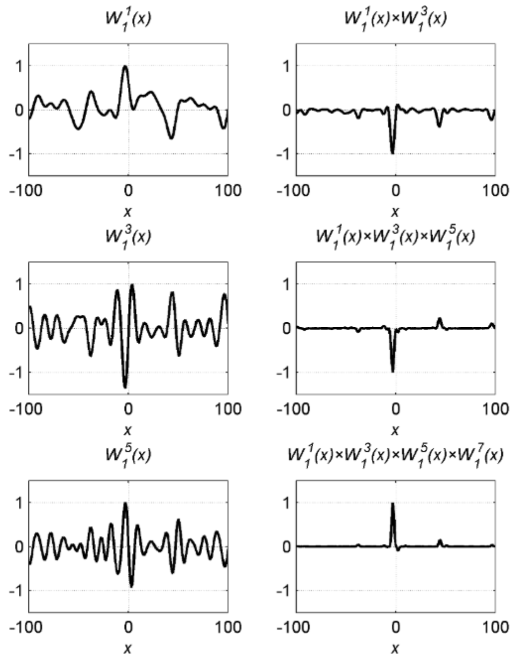


Figure 7: Noisy step responses in SNR=0.5 dB and s=1

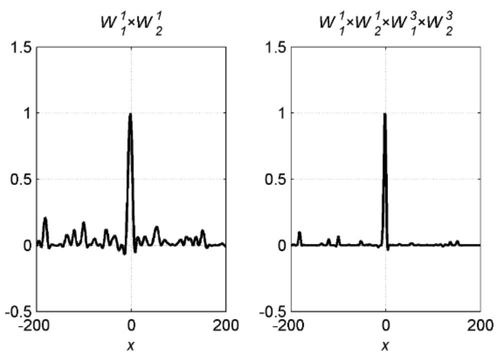


Figure 8: Step edge detection. A comparison between the production of adjacent coefficients which has been proposed by Zhang (left diagram) and our work, the production of adjacent coefficients and adjacent derivations (right diagram) in SNR= 0.1 dB

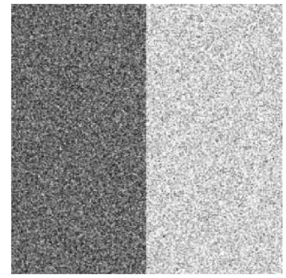
These values are calculated and showed in Table 1. The results show that our method has higher *figure of merit* than Zhang's method.

Table 1
The *figure of merit* values of the Zhang method and our method for the isolated step edge

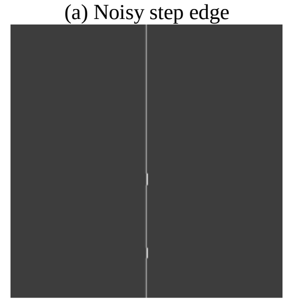
	Zhang method	Our method
Figure of merit	0.9929	0.9974

The number of the multiplication of the coefficients (s and n) depends on the degree of the noise refining. In higher scales, the edges are blurred and diluted, and this is a drawback of the edge detecting in high scales. We introduce the multiplication of wavelet coefficients across the derivations to achieve sharper peaks.

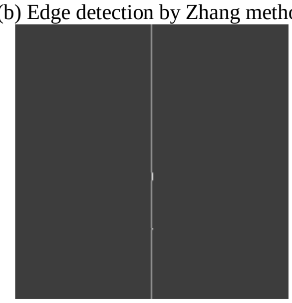
The degree of freedom in our method is two (s and n); but degree of freedom in the multiscale wavelet is one (only *scale*). Thus, our method is more powerful than the multiscale analysis in the edge detection because our algorithm has a better fine-to-coarse performance and can adjust details with these two parameters.



(a) Noisy step edge



(b) Edge detection by Zhang method



(c) Edge detection by our method

Figure 9: Noisy step and edge maps

III. CONCLUSION

In this paper, we develop Canny edge detector and define new wavelet basis functions based on derivations of the Gaussian function. A ramp function is modeled and the edge detection by introduced bases is discussed. An accomplished algorithm is introduced and applied to the uniform step edge in the presence and absence of noise. Edge and noise can be better distinguished in the case of low SNR in the product of wavelet coefficients. It is also shown that the wavelet multiplication across the derivation achieves better results in the noise dilution and yields sharper peaks in comparison to either of the single wavelet responses. Moreover, our scheme has two freedom degrees (scale and the number of derivations) to compare multiresolution analysis with one freedom degree (only scale) to focus on details, control noise ability and solve edge detection problems. From the simulation results, it is obvious that our proposed scheme certainly has a better performance of inhibiting false responses wherever other edge detections have problems in low SNR condition.

REFERENCES

[1] B. Gardiner, S. Coleman, and B. Scotney, "Multiscale edge detection using a finite element framework for hexagonal Pixel-based images," *IEEE Transactions on Image Processing*, pp. 1–1, 2016.

[2] F.-P. Wang and P.-L. Shui, "Noise-robust color edge detector using gradient matrix and anisotropic Gaussian directional derivative matrix," *Pattern Recognition*, vol. 52, pp. 346–357, Apr. 2016.

[3] C.-S. Tseng and J.-H. Wang, "Perceptual edge detection via entropy-driven gradient evaluation," *IET Computer Vision*, vol. 10, no. 2, pp. 163–171, Mar. 2016.

[4] C. Hu, X. Qu, D. Guo, L. Bao, and Z. Chen, "Wavelet-based edge correlation incorporated iterative reconstruction for undersampled MRI," *Magnetic Resonance Imaging*, vol. 29, no. 7, pp. 907–915, Sep. 2011.

[5] F. Guo, Y. Yang, B. Chen, and L. Guo, "A novel multi-scale edge detection technique based on wavelet analysis with application in multiphase flows," *Powder Technology*, vol. 202, no. 1-3, pp. 171–177, Aug. 2010.

[6] Z. Zhang, S. Ma, H. Liu, and Y. Gong, "An edge detection approach based on directional wavelet transform," *Computers & Mathematics with Applications*, vol. 57, no. 8, pp. 1265–1271, Apr. 2009.

[7] W. Sun, R. Mukherjee, P. Stroeve, A. Palazoglu, and J. A. Romagnoli, "A multi-resolution approach for line-edge roughness detection," *Microelectronic Engineering*, vol. 86, no. 3, pp. 340–351, Mar. 2009.

[8] W. Cao, R. Che, and D. Ye, "An illumination-independent edge detection and fuzzy enhancement algorithm based on wavelet transform for non-uniform weak illumination images," *Pattern Recognition Letters*, vol. 29, no. 3, pp. 192–199, Feb. 2008.

[9] Z. Zhu, H. Lu, and Y. Zhao, "Scale multiplication in odd Gabor transform domain for edge detection," *Journal of Visual Communication and Image Representation*, vol. 18, no. 1, pp. 68–80, Feb. 2007.

[10] S. Liu, Z. Niu, G. Sun, and Z. Chen, "Gabor filter-based edge detection: A note," *Optik - International Journal for Light and Electron Optics*, vol. 125, no. 15, pp. 4120–4123, Aug. 2014.

[11] L. Zhang and P. Bao, "Edge detection by scale multiplication in wavelet domain," *Pattern Recognition Letters*, vol. 23, no. 14, pp. 1771–1784, Dec. 2002.

[12] J. Canny, "A computational approach to edge detection," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. PAMI-8, no. 6, pp. 679–698, Nov. 1986.

[13] L. Fan, F. Song, and S. Jutamulia, "Edge detection with large depth of focus using differential Haar–Gaussian wavelet transform," *Optics Communications*, vol. 270, no. 2, pp. 169–175, Feb. 2007.

[14] Andrews LC. *Special functions of mathematics for engineers*. 2nd ed. SPIE Society of Photo-Optical Instrumentation Engi; May 15, 2005.

[15] A. Rosenfeld, "A nonlinear edge detection technique," *Proceedings of the IEEE*, vol. 58, no. 5, pp. 814–816, 1970.

[16] P. Bao, L. Zhang, and X. Wu, "Canny edge detection enhancement by scale multiplication," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 27, no. 9, pp. 1485–1490, Sep. 2005.

[17] W.K. Pratt, Digital Image Processing, John Wiley & Sons, New York, Usa, 2001.