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ON THE RELATIVELY PSEUDO-COMPLEMENT OPERATION IN FINITE RHAs

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Abstract. Refined hedge algebras were introduced and investigated by Ho & Nam in [6-9]. It is known [9] that every refined hedge algebra (RHA, for short) with a chain of the primary generators is a distributive lattice. In this paper we restrict our consideration to finite version of RHAs (see [7, 9]). It is shown that every finite RHA is a Heyting (pseudo-Boolean) algebra. Furthermore, some computing results for the relatively pseudo-complement operation in these algebras will be exhibited.

1. INTRODUCTION

In the research program initiated by Ho & Wechler in [10], an algebraic approach to the natural structure of domains of linguistic variables was given. The main aim of the investigation is to find out an appropriate algebraic structure for fuzzy linguistic logic and approximate reasoning (Zadeh [18, 19]).

On this approach, every linguistic domain can be interpreted as an algebraic structure called hedge algebra, say $\mathcal{X} = (X, G, H, \leq)$, where $(X, \leq)$ is a poset, G is a set of the primary generators and H is a set of unary operations representing linguistic hedges under consideration.

It is well known that Boolean algebras, Heyting algebras, MV-algebras and so on, are algebraic versions of Boolean logic, Gödel logic, Lukasiewicz logic and other non-classical logic systems (see, e.g., [15-17, 3-5]). In this direction, the idea of connecting abstract algebras with fuzzy logic becomes a natural demand and to play a useful role. This is by no means new (see, e.g., [12-14]). However, previous efforts to develop this idea have concentrated on investigating [0, 1]-valued fuzzy logics, i.e. the algebraic versions of the unit interval [0, 1]. Our motivation is different. We have tried to find a mathematical method for manipulating immediately linguistic terms, which were interpreted by fuzzy sets in the research on fuzzy linguistic logic and approximate reasoning started by Zadeh [18, 19]. Therefore, our focus has been based on natural structure of linguistic domains.

This research project was initiated by Ho & Wechler in [10] and further developed in a series of papers [2, 11, 6-9]. Supporting for this research direction has based on the fact that domains of linguistic variables can be embedded into a relatively well-known algebraic structure: distributive lattice.

Refined hedge algebras were introduced by Ho & Nam in [6]. It is known [8, 9] that every RHA with a chain of the primary generators is a distributive lattice. Further, in [7, 9], symmetrical RHAs were introduced and fundamental properties of these structure were examined. In this work we will restrict ourselves to finite versions of RHAs. It is shown that every finite RHA is a Heyting algebra, i.e. in these algebras we are able to define the relatively pseudo-complement operation satisfying some certain properties. Furthermore, some computing results for the relatively pseudo-complement operation in these algebras will be exhibited.

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2. PRELIMINARIES

Let $\mathcal{A}$ be a lattice. For a, $b \in \mathcal{A}$, an element $c \in \mathcal{A}$ is said to be the pseudo-complement of a relative to b (or: modulo b) if c is the greatest element such that $a \cap c \leq b$, where $\cap$ stands for *meet* in $\mathcal{A}$. The pseudo-complement of a relative to b is denoted, if it exists, by $a \Rightarrow b$. By definition, for any $x \in \mathcal{A}$, $x \leq a \Rightarrow b$ if and only if $a \cap x \leq b$. If for any a, $b \in \mathcal{A}$, there exists the element $a \Rightarrow b$ in $\mathcal{A}$, then $\mathcal{A}$ is called to be a *Brouwerian lattice*.

It is known that every Brouwerian lattice has the unit element (see, e.g., [16]). However, it does not, in general, have the zero element. A Brouwerian lattice with the zero-element is called a Heyting (pseudo-Boolean) algebra. Further, every Brouwerian lattice is distributive (see, e.g., [1, 16]). It is also known [1] that if $\mathcal{A}$ is a finite lattice then

(*) $\mathcal{A}$ is a Brouwerian lattice iff $\mathcal{A}$ is a distributive lattice.

Now, let us consider RHAs. In the paper we attempt to keep our notation and conventions as in the previous papers [6-9]. For more details on RHAs we refer the reader to [6, 7, 9].

Let $\mathcal{X} = (X,\ G,\ LH,\ \leq)$ be an RHA constructed from PN-homogeneous hedge algebra $(H(G),$ $G,\ H,\ \leq)$, where G is a set of the primary generators, H is a set of unary operations representing linguistic hedges under consideration, and $H(G)$ is the set of all elements generated from G by means of hedge operations in H.

It is known that H can be decomposed into two disjoint subsets H^+ and H^- such that $H^+ + I$ and $H^- + I$ are finite modular lattices satisfying the chain condition on their grades defined by the height function (see [6]), where I is the identity, i.e. $Ix = x$ for every x in X, and is their zero-element.

As constructed in [6], $LH^+ + I$ and $LH^- + I$ are distributive lattices generated from $H^+ + I$ and $H^- + I$, respectively, and $LH = LH^+ \cup LH^- \cup \{I\}$. For simplicity of notation, in the sequel by "c" we mean either "+" or "−". With this notation we have

$$LH^c + I = \bigcup_{i=0}^{l(H^c+I)} LH_i^c\,,$$

where $l(H^c + I)$ denotes the length of $H^c + I$.

Recall that for $i = 1, \ldots, l(H^c + I) - 1$, if $o(H_i^c) > 1$ then $o(H_{i-1}^c) = o(H_{i+1}^c) = 1$, where $o(H_i^c)$ denotes the number of elements of H_i^c. Furthermore, for any $h \in LH_i^c$, $k \in LH_j^c$ and $i < j$, then $h < k$. Let

$$I^c = \{0,\ 1, \ldots,\ l(H^c + I)\} \quad \text{and} \quad SI^c = \{i \in I^c \mid o(H_i^c) > 1\}.$$

It is known that, for any $i \in SI^c$, LH_i^c is the free distributive lattice generated by incomparable elements of the grade H_i^c in $H^c + I$, and, is also a sublattice of $LH^c + I$.

For any h, $k \in LH$, if $x \leq hx$ iff $x \geq kx$ for every x in X then h and k are said to be *converse*, or h is converse to k and vice-versa. If $x \leq hx$ iff $x \leq kx$ for every x in X then h and k are said to be *compatible*.

For the sake of convenience we repeat the relevant material from [7, 9] without proofs.

For any $x \in X$, let $LH[x] = \{hx \mid h \in LH\}$, $LH(x) = \{\delta x \mid \delta \in LH^*\}$, where LH^* denotes the set of all strings of elements in LH.

Theorem 2.1. *Let $\mathcal{X} = (X,\ G,\ LH,\ \leq)$ be an RHA. If G is a chain then $\mathcal{X}$ is a distributive lattice. Moreover, for any two incomparable elements x and y in X, then there exist two compatible hedge operations h and k in LH and an element w in $LH(a)$, for an element $a \in G$, such that both h and k together belong to LH_i^c for an index $i \in SI^c$ and $x = \delta hw$, $y = \delta' kw$, where δ, $\delta' \in LH^*$, and*

$$x \cup y = \begin{cases} \delta(h \vee k)w \cup \delta'(h \vee k)w, & \text{if } hw > w \\ \delta(h \wedge k)w \cup \delta'(h \wedge k)w, & \text{if } hw < w \end{cases}$$

$$x \cap y = \begin{cases} \delta(h \wedge k)w \cap \delta'(h \wedge k)w, & \text{if } hw > w \\ \delta(h \vee k)w \cap \delta'(h \vee k)w, & \text{if } hw < w \end{cases}$$

where $\cup$ *and* $\cap$ *stand for join and meet, respectively, in* $\mathcal{X}$ *while* $\vee$ *and* $\wedge$ *stand for join and meet, respectively, in* $LH^c + I$.

Proposition 2.1. $\forall x \in X$, *$LH[x]$ and $LH(x)$ are distributive sublattices of* $\mathcal{X}$.

Proposition 2.2. $\forall h,\ k \in LH_i^c$ *for some* $i \in SI^c$, $\forall x \in X$ *such that* $hx \neq kx$. *Then $LH(hx)$ is lattice-isomorphic with $LH(kx)$.*

3. RESULTS

Here and subsequently, $\mathcal{X}$ stands for the RHA $\mathcal{X} = (X,\ G,\ LH,\ \leq)$ considered as in the previous section, in which G is a finite chain. Furthermore, the underlying set X is defined as follows.

First, define $LH[G] = \bigcup\limits_{a \in G} LH[a]$. Then, define $LH_n[G]$ for $n \geq 0$ by

$$LH_0[G] = G\,, \quad LH_{n+1}[G] = LH[LH_n[G]]\,.$$

Notice that by convention made upon the identity I (see [9]), it follows that

$$G \subseteq LH_1[G] \subseteq LH_2[G] \subseteq \cdots \subseteq LH_n[G] \subseteq \cdots$$

Let p be an arbitrary but fixed positive integer. For any $x \in LH_p[G]$ and $x \notin LH_{p-1}[G]$, we define $hx = x$ for every $h \in LH$. Let $X = LH_p[G]$. Clearly, $\mathcal{X}$ is well-defined and, is a complete distributive lattice. Furthermore, it is known [16] that $\mathcal{X}$ is a Heyting (pseudo-Boolean) algebra.

To simplify notation, we write $\varphi_{\mathcal{A}}$ instead of the relatively pseudo-complement operation defined on Browerian lattice $\mathcal{A}$. That is, for any $x,\ y \in \mathcal{A}$, $\varphi_{\mathcal{A}}(x,y) = \max\{z \in \mathcal{A} \mid x \wedge z \leq y\}$. In addition, if $\mathcal{A}$ is a complete lattice then we denote by $1_{\mathcal{A}}$ and $0_{\mathcal{A}}$ the unit and zero element, respectively, in $\mathcal{A}$. Similarly as in [1], we shall denote by $\breve{\mathcal{A}}$ the dual of $\mathcal{A}$ in the category of posets, and $\breve{\leq}$ the converse of the ordering relation $\leq$. Then $\breve{\mathcal{A}}$ is also a lattice with the ordering relation $\breve{\leq}$.

By definition, it is easily seen that the following holds.

Proposition 3.1. *Let $\mathcal{A}$ be a Brouwerian lattice. For any $x, y \in \mathcal{A}$, we have*

(i) $x \leq y$ *iff* $\varphi_{\mathcal{A}}(x,y) = 1_{\mathcal{A}}$.

(ii) $\varphi_{\mathcal{A}}(x,y) \geq y$.

(iii) If $x > y$ and $\varphi_{\mathcal{A}}(x,y) > y$ then x and $\varphi_{\mathcal{A}}(x,y)$ are incomparable and $x \cap \varphi_{\mathcal{A}}(x,y) = y$.

(iv) If x and y are incomparable then so are x and $\varphi_{\mathcal{A}}(x,y)$ and $x \cap \varphi_{\mathcal{A}}(x,y) < y$.

We are now ready to establish some fundamental results for the relatively pseudo-complement operation in $\mathcal{X}$.

Proposition 3.2. *Let $x = h_n \ldots h_1 a$, $y = k_m \ldots k_1 b$ be two canonical representations of x and y with respect to a and b, respectively, in $\mathcal{X}$, where $a,\ b \in G$ and $a \neq b$. Then*

(i) If $a > b$ then $x > y$ and $\varphi_{\mathcal{X}}(x,y) = y$.

(ii) If $a > b$ then $x < y$ and $\varphi_{\mathcal{X}}(x,y) = 1_{\mathcal{X}}$.

Proof: The proof is straightforward.

The following theorem gives us a recursive formular for computing of the pseudo-complement operation in a sublattice $LH(x_j)$ of $\mathcal{X}$, for some x_j of $\mathcal{X}$. Note that, according to (*) in Section 2, it is legitimate to consider the operations $\varphi_{LH(x)}$, $\varphi_{LH_i^c}$, $\varphi_{\breve{LH}_i^c}$ defined on $LH(x)$, LH_i^c, $\breve{LH}_i^c$ respectively, for any x of $\mathcal{X}$ and $i \in SI^c$.

Theorem 3.1. *Let $x = h_n \dots h_1 a$, $y = k_m \dots k_1 a$ be two canonical representations of x and y with respect to a, where $a \in G$, and $x \not\leq y$. Then, there exists an index $j \leq \min\{n, m\} + 1$ such that $h_i = k_i$ for all $i < j$ and*

$$\varphi_{LH(x_j)}(x, y) = \begin{cases} y\,, & \text{if } \nexists i \in SI^c \text{ such that } h_j, k_j \in LH_i^c\,, \\ \varphi_{LH(hx_j)}(h_n ... h_{j+1} h x_j,\ k_m ... k_{j+1} h x_j)\,, & \text{if } \exists i \in SI^c \text{ such that } h_j,\ k_j \in LH_i^c \\ & \text{and } h_j x_j > x_j\,, \\ \varphi_{LH(\check{h}x_j)}(h_n ... h_{j+1} \check{h} x_j,\ k_m ... k_{j+1} \check{h} x_j)\,, & \text{if } \exists i \in SI^c \text{ such that } h_j, k_j \in LH_i^c \\ & \text{and } h_j x_j < x_j\,, \end{cases}$$

where $x_j = h_{j-1} \dots h_1 a$, and

$$h = \begin{cases} \varphi_{LH_i^+}(h_j, k_j), & \text{if } i \in SI^+ \\ \varphi_{\breve{LH}_i^-}(h_j, k_j), & \text{if } i \in SI^- \end{cases}$$

and

$$\check{h} = \begin{cases} \varphi_{\breve{LH}_i^+}(h_j, k_j), & \text{if } i \in SI^+ \\ \varphi_{LH_i^-}(h_j, k_j), & \text{if } i \in SI^- \end{cases}$$

Notice that LH_i^c is a sublattice of $LH^c + I$, while $\breve{LH}_i^c$ is a sublattice of $\breve{LH^c} + I$.

Proof: Let j be the least index such that $h_j \neq k_j$. If $\nexists i \in SI^c$ such that $h_j, k_j \in LH_i^c$ then it implies that x and y are comparable, and hence $x > y$ and $h_j x_j > k_j x_j$, since $x \not\leq y$. Assume that $h_j \in LH_{i_0}^c$ for some $i_0 \in I^c$. If $\varphi_{LH(x_j)}(x, y) > y$ then it follows from (iii) of Proposition 3.1 that x and $\varphi_{LH(x_j)}(x, y)$ are incomparable. Thus, there exists $h' \in LH_{i_0}^c$ such that $\varphi_{LH(x_j)}(x, y) \in LH(h' x_j)$. Hence, there exists $h'' \in LH_{i_0}^c$ such that $(x \cap \varphi_{LH(x_j)}(x, y)) \in LH(h'' x_j)$. Since $k_j \notin LH_{i_0}^c$ and $h_j x_j > k_j x_j$, it follows that $x \cap \varphi_{LH(x_j)}(x, y) > y$, which is impossible. So we infer $\varphi_{LH(x_j)}(x, y) = y$.

Now suppose that there exists $i \in SI^c$ such that $h_j, k_j \in LH_i^c$ and $h_j x_j > x_j$. By the properties of $\mathcal{X}$, it is easily seen that $\varphi_{LH(x_j)}(x, y) \in LH(h' x_j)$ for some $h' \in LH_i^c$. Assume that $\varphi_{LH(x_j)}(x, y) = \delta h' x_j$ for some $\delta \in LH^*$. Then we have $\delta h' x_j \cap x \leq y$, i.e. $\delta h' x_j \cap h_n \dots h_j x_j \leq k_m \dots k_j x_j$. It follows from Theorem 2.1. that

$$\delta (h' \wedge h_j) x_j \cap h_n ... h_{j+1} (h' \wedge h_j) x_j \leq k_m ... k_j x_j\,. \tag{1}$$

Hence, it implies that $(h' \wedge h_j) x_j \leq k_j x_j$. Again by Theorem 2.1 we obtain $h' x_j \cap h_j x_j \leq k_j x_j$. Let

$$h = \begin{cases} \varphi_{LH_i^+}(h_j, k_j), & \text{if } i \in SI^+ \\ \varphi_{\breve{LH}_i^-}(h_j, k_j), & \text{if } i \in SI^- \end{cases}$$

then by the definition of h and the last inequatily we infer

$$h' x_j \leq h x_j\,. \tag{2}$$

On the other hand, we have

$$\delta h x_j \cap x = \delta (h \wedge h_j) x_j \cap h_n \dots h_{j+1} (h \wedge h_j) x_j\,. \tag{3}$$

From Proposition 2.2 and (1) it is easy to check that

$$\delta k_j x_j \cap h_n \dots h_{j+1} k_j x_j \leq k_m \dots k_j x_j\,. \tag{4}$$

Once again, since $(h \wedge h_j)x_j \leq k_j x_j$, it follows from Proposition 2.2 and (4) that

$$\delta(h \wedge h_j)x_j \cap h_n \dots h_{j+1}(h \wedge h_j)x_j \leq k_m \dots k_{j+1}(h \wedge h_j)x_j \leq k_m \dots k_j x_j .$$

From the last inequalities and (3), we obtain $\delta h x_j \cap x \leq y$, and hence $\delta h x_j \leq \delta h' x_j$. Thus, we have $hx_j \leq h'x_j$. Since x_j does not be a fixed point and by (2) it implies $h' = h$. Further, it follows again by Proposition 2.2 and (4) that

$$\delta h x_j \cap h_n \dots h_{j+1} h x_j \leq k_m \dots k_{j+1} h x_j .$$

Therefore, we have

$$\varphi_{LH(x_j)}(x, y) = \delta h x_j \leq \varphi_{LH(hx_j)}(h_n \dots h_{j+1} h x_j, k_m \dots k_{j+1} h x_j) . \tag{5}$$

Now assume that $\varphi_{LH(hx_j)}(h_n \dots h_{j+1} h x_j, k_m \dots k_{j+1} h x_j) = \delta' h x_j$, for some $\delta' \in LH^*$. Since $\delta' h x_j \cap h_n \dots h_{j+1} h x_j \leq k_m \dots k_{j+1} h x_j$, we have

$$\delta'(h \wedge h_j)x_j \cap h_n \dots h_{j+1}(h \wedge h_j)x_j \leq k_m \dots k_{j+1}(h \wedge h_j)x_j ,$$

or

$$\delta' h x_j \cap h_n \dots h_j x_j \leq k_m \dots k_{j+1}(h \wedge h_j)x_j \leq k_m \dots k_j x_j ,$$

and hence $\delta' h x_j \cap x \leq y$. We thus get

$$\delta' h x_j \leq \delta h x_j = \varphi_{LH(x_j)}(x, y) . \tag{6}$$

From (5) and (6), we obtain the desired equality

$$\varphi_{LH(x_j)}(x, y) = \varphi_{LH(hx_j)}(h_n \dots h_{j+1} h x_j, k_m \dots k_{j+1} h x_j) .$$

The proof for the remain case can be obtained by a similar argument. Consequently, the proof is complete.

Now the main result is this. The following theorem establishes a recursive formula for computing the pseudo-complement operation in $\mathcal{X}$.

Theorem 3.2. *Under the same hypothesises and notation as in Theorem 3.1, we have*

$$\varphi_{\mathcal{X}}(x, y) = \begin{cases} \varphi_{LH(x_j)}(x, y), & \text{if } h_i \in \bigcup\limits_{s \in (I^c \setminus SI^c), c \in \{+,-\}} LH_s^c \text{ for any } i \leq j-1, \\ \varphi_{LH(hx_t)}(h_n \dots h_j h_{j-1} \dots h_{t+1} h x_t, k_m \dots k_j h_{j-1} \dots h_{t+1} h x_t), & \text{if there exists the} \\ \qquad \text{least index } t \text{ such that } h_t \in LH_{i_t}^c \text{ for some } i_t \in SI^c \text{ and } h_t x_t > x_t , \\ \varphi_{LH(\check{h}x_t)}(h_n \dots h_j h_{j-1} \dots h_{t+1} \check{h} x_t, k_m \dots k_j h_{j-1} \dots h_{t+1} \check{h} x_t), & \text{if there exists the} \\ \qquad \text{least index } t \text{ such that } h_t \in LH_{i_t}^c \text{ for some } i_t \in SI^c \text{ and } h_t x_t < x_t , \end{cases}$$

where $x_t = h_{t-1} \dots h_1 a$, *and*

$$h = \begin{cases} 1_{LH_{i_t}^+}, & \text{if } i_t \in SH^+ \\ 0_{LH_{i_t}^-}, & \text{if } i_t \in SI^- \end{cases}$$

and

$$\check{h} = \begin{cases} 0_{LH_{i_t}^+}, & \text{if } i_t \in SH^+ \\ 1_{LH_{i_t}^-}, & \text{if } i_t \in SI^- \end{cases}$$

Proof: For the case where $h_i \in \bigcup_{s\in(I^c\setminus SI^c),\, c\in\{+,-\}} LH_s^c$ for any $i \leq j-1$, it is easy to check that $\varphi_X(x,y) = \varphi_{LH(x_j)}(x,y)$, by the assumptions made upon elements x and y and the definition of RHA.

In the opposite case, let t be the least index satisfying $t \leq j-1$ such that $h_t \in LH_{i_t}^c$, for some $i_t \in SI^c$. Assume first that $h_t x_t > x_t$, then we have $hx_t \geq h_t x_t$. By the assumption that $x \not\leq y$ and Proposition 2.2 we get

$$h_n ... h_{t+1} h x_t \not\leq k_m ... k_{t+1} h x_t \,,$$

with a notice that $h_i = k_i$ for any i such that $t+1 \leq i \leq j-1$.

Let $\varphi_{LH(hx_t)}(h_n \ldots h_{t+1} hx_t, k_m \ldots k_{t+1} hx_t) = \delta hx_t$. It follows immediately from Theorem 2.1 that

$$x \cap \delta hx_t = x \cap \delta h_t x_t \,, \tag{7}$$

since $h_t x_t > x_t$. In addition, since

$$h_n ... h_{t+1} hx_t \cap \delta hx_t \leq k_m ... k_{t+1} hx_t \,,$$

it implies by Proposition 2.2 that

$$x \cap \delta h_t x_t \leq y \,. \tag{8}$$

We infer from (7) and (8) that $x \cap \delta hx_t \leq y$. This shows that

$$\varphi_X(x,y) \geq \delta hx_t \,. \tag{9}$$

On the other hand, by the definition of RHA and Proposition 3.1, it can easily be seen that $\varphi_X(x,y) \in LH(kx_t)$ for some $k \in LH_{i_t}^c$. Let $\varphi_X(x,y) = \delta' kx_t$. By (9) and the definition of h, it is easy to check that $h = k$. Further, by definition we have $x \cap \delta' hx_t \leq y$. This shows that

$$x \cap \delta' h_t x_t \leq y \,, \tag{10}$$

by Theorem 2.1. From (10) and Proposition 2.2 we infer

$$h_n ... h_{t+1} hx_t \cap \delta' hx_t \leq k_m ... k_{t+1} hx_t \,,$$

or

$$\delta' hx_t \leq \delta hx_t = \varphi_{LH(hx_t)}(h_n ... h_{t+1} hx_t, k_m ... k_{t+1} hx_t) \,. \tag{11}$$

Thus $\varphi_X(x,y) = \varphi_{LH(hx_t)}(h_n \ldots h_{t+1} hx_t, k_m \ldots k_{t+1} hx_t)$, by (9) and (11). For the case where $h_t x_t < x_t$, the proof is similar. Consequently, the theorem is completely proved.

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Tóm tắt. Để xây dựng một cơ sở đại số cho logic mờ theo nghĩa của Zadeh, đại số gia tử mịn hóa đã được giới thiệu và nghiên cứu trong [6-9]. Trong [9], các tác giả đã chứng minh rằng đại số gia tử mịn hóa với dây chuyền các phần tử sinh nguyên thủy là một dàn phân phối. Trong bài báo này chúng tôi hạn chế xem xét trên các đại số gia tử mịn hóa hữu hạn. Khi đó đại số gia tử mịn hóa trở thành một đại số Heyting, tức là trong các đại số gia tử mịn hóa hữu hạn chúng ta có thể định nghĩa toán tử tựa phần bù tương đối thỏa mãn một số tính chất xác định (xem [1, 16]). Chúng tôi cũng đưa ra các kết quả tính toán cho toán tử đó trong các đại số gia tử mịn hóa hữu hạn.

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