Communications in Physics, Vol. 21, No. 3 (2011), pp. 193-198

RADION PRODUCTION IN EXTERNAL ELECTROMAGNETIC FIELD

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Abstract. *A review of Randall-Sundrum model with stressing on radion phenomenology is presented. The radion production in the external electromagnetic field is considered. The total cross sections for the conversions in the presence of the electric field of the flat condenser as well as in the magnetic field of the solenoid are calculated in details. Based on our results a laboratory experiment for production and detection of the light radions may be described.*

I. INTRODUCTION

Much research has been done on understanding possible mechanism for radius stabilization and the phenomenology of the radion field in Randall and Sundrum (RS) model. The motivation for studying the radion is twofold. First, the radion may turn out to be the lightest new particle in the RS-type setup, possibly accessible at the LHC. In addition, the phenomenological similarity and potential mixing of the radion and Higgs boson warrant detailed study in order to facilitate distinction between the radion and Higgs signals at colliders. The aim of this work is to study phenomenology of radion of the RS model and possibility of its conversion in the external electromagnetic field.

II. A REVIEW OF RS MODEL

The RS model is based on a 5D spacetime with non-factorizable geometry [1]. The single extradimension is compactified on a S^1/Z_2 orbifold of which two fixed points accommodate two three-branes (4D hyper-surfaces), the Planck brane at $y = 0$ and TeV brane at $y = 1/2$. The ordinary 4D Poincare invariance is shown to be maintained by the following classical solution to the Einstein equation:

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - b_0^2 dy^2, \qquad \sigma(y) = m_0 b_0 |y|, \tag{1}$$

where x^μ ($\mu = 0, 1, 2, 3$) denote the coordinates on the 4D hyper-surfaces of constant y with metric $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. The m_0 and b_0 are the fundamental mass parameter and compactification radius, respectively.

Gravitational fluctuations about the RS metric,

$$\eta_{\mu\nu} \to g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}(x, y), \qquad b_0 \to b_0 + b(x), \tag{2}$$

yield two kinds of new phenomenological ingredients on the TeV brane: the KK graviton modes $h^{(n)}_{\mu\nu}(x)$ and the canonically normalized radion field $\phi_0(x)$, respectively defined as [2, 3]

$$h_{\mu\nu}(x,y) = \sum_{n=0}^{\infty} h^{(n)}_{\mu\nu}(x)\frac{\chi^{(n)}(y)}{\sqrt{b_0}}, \quad \phi_0(x) = \sqrt{6}M_{\mathrm{Pl}}\Omega_b(x), \tag{3}$$

where $\Omega_b(x) \equiv e^{-m_0[b_0+b(x)]/2}$. The 5D Planck mass M_5 ($\epsilon^2 = 16\pi G_5 = 1/M_5^3$) is related to its 4D one ($M_{\mathrm{Pl}} \equiv 1/\sqrt{8\pi G_{\mathrm{N}}}$) by

$$\frac{M^2_{\mathrm{Pl}}}{2} = \frac{1-\Omega_0^2}{\epsilon^2 m_0}. \tag{4}$$

Here $\Omega_0 \equiv e^{-m_0 b_0/2}$ is known as the warp factor. Because our TeV brane is arranged to be at $y = 1/2$, a canonically normalized scalar field has the mass multiplied by the warp factor, i.e, $m_{\mathrm{phys}} = \Omega_0 m_0$. Since the moderate value of $m_0 b_0/2 \simeq 35$ can generate TeV scale physical mass, the gauge hierarchy problem is explained.

The 4D effective Lagrangian is then [4]

$$\mathcal{L} = -\frac{\phi_0}{\Lambda_\phi}T^\mu_\mu - \frac{1}{\hat{\Lambda}_W}T^{\mu\nu}(x)\sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}(x), \tag{5}$$

where $\Lambda_\phi \equiv \sqrt{6}M_{\mathrm{Pl}}\Omega_0$ is the VEV of the radion field, and $\hat{\Lambda}_W \equiv \sqrt{2}M_{\mathrm{Pl}}\Omega_0$. The $T^{\mu\nu}$ is the energy-momentum tensor of the TeV brane localized SM fields. The T^μ_μ is the trace of the energy-momentum tensor, which is given at the tree level as [5]

$$T^\mu_\mu = \sum_f m_f \bar{f} f - 2m_W^2 W^+_\mu W^{-\mu} - m_Z^2 Z_\mu Z^\mu + (2m^2_{h_0}h_0^2 - \partial_\mu h_0 \partial^\mu h_0) + \cdots \tag{6}$$

The gravity-scalar mixing arises at the TeV-brane by

$$S_\xi = -\xi \int d^4x \sqrt{-g_{\mathrm{vis}}} R(g_{\mathrm{vis}})\hat{H}^\dagger \hat{H}, \tag{7}$$

where $R(g_{\mathrm{vis}})$ is the Ricci scalar for the induced metric on the visible brane or TeV brane, $g^{\mu\nu}_{\mathrm{vis}} = \Omega_b^2(x)(\eta^{\mu\nu} + \epsilon h^{\mu\nu})$. $\hat{H}$ is the Higgs field before re-scaling. The parameter ξ denotes the size of the mixing term.

III. PHOTON-TO-RADION CONVERSIONS

Referring the reader for details of the radion-photon coupling to Ref. [2], we lay out the necessary radion-photon coupling

$$\mathcal{L}_{\gamma\gamma\phi} = \frac{1}{2}c_{\phi\gamma\gamma}\phi F_{\mu\nu}F^{\mu\nu}, \tag{8}$$

with

$$c_{\phi\gamma\gamma} = -\frac{\alpha}{4\pi\Lambda_\phi}\left\{a(b_2+b_Y) - a_{12}[F_1(\tau_W) + 4/3F_{1/2}(\tau_t)]\right\}, \tag{9}$$

Let us consider the conversion of the photon γ with momentum q into radion ϕ with momentum p in external EM field. Using the Feynman rules we get the following expression for the matrix element

$$< p|M_\phi|q >= \frac{c_{\phi\gamma\gamma}}{(2\pi)^2\sqrt{p_0 q_0}}\varepsilon^\mu(\mathbf{q},\lambda)q^\nu \int_V e^{i\mathbf{kr}} F^{\text{class}}_{\nu\mu} d\mathbf{r}, \tag{10}$$

where $\mathbf{k} \equiv \mathbf{p} - \mathbf{q}$ and $\varepsilon^\mu(\mathbf{q},\lambda)$ represents the polarization vector of the photon. Expression (10) is valid for an arbitrary external EM field. In the following we shall use it for the cases, namely in the electric field of a flat condenser and in the static magnetic field of a solenoid with the TE_{10} mode. Here we use the following notations: $q \equiv |\mathbf{q}|$, $p \equiv |\mathbf{p}| = (q^2 - m_\phi^2)^{1/2}$ and θ is the angle between $\mathbf{p}$ and $\mathbf{q}$.

III.1. Conversion in electric field

Let us take the EM field as a homogeneous electric field of a flat condenser of size $l_x \times l_y \times l_z$. We shall use the coordinate system with the x axis parallel to the direction of the field, i.e., $F^{01} = -F^{10} = E$. Then the matrix element is given by

$$< p|M_\phi|q >= \frac{c_{\phi\gamma\gamma}}{(2\pi)^2\sqrt{p_0 q_0}}\varepsilon^1(\mathbf{q},\lambda)q^0 F_e(\mathbf{k}), \tag{11}$$

where

$$F_e(\mathbf{k}) = \int_V e^{i\mathbf{kr}} E(\mathbf{r}) d\mathbf{r} \tag{12}$$

For a homogeneous electric field of intensity E we have

$$F_e(\mathbf{k}) = 8E\sin(l_x k_x/2)\sin(l_y k_y/2)\sin(l_z k_z/2)(k_x k_y k_z)^{-1}. \tag{13}$$

Squaring the matrix element (11) we obtain

$$\frac{d\sigma^e(\gamma\to\phi)}{d\Omega} = \frac{8c^2_{\phi\gamma\gamma}E^2q^2}{\pi^2}\left[\frac{\sin(\frac{1}{2}l_x k_x)\sin(\frac{1}{2}l_y k_y)\sin(\frac{1}{2}l_z k_z)}{k_x k_y k_z}\right]^2\left(1 - \frac{q_x^2}{q^2}\right). \tag{14}$$

We shall explore the following case: The momentum of photon is parallel to the z axis, i.e. $q^\mu = (q, 0, 0, q)$. In the spherical coordinates we then have

$$p_x = p\sin\theta\cos\varphi, \quad p_y = p\sin\theta\sin\varphi, \quad p_z = p\cos\theta, \tag{15}$$

where φ is the angle between the x axis and the projection of $\mathbf{p}$ on the xy plane. Substitution of Eq.(15) into Eq.(14) yields

$$\begin{aligned}\frac{d\sigma^e(\gamma\to\phi)}{d\Omega} &= \frac{8c^2_{\phi\gamma\gamma}E^2q^2}{\pi^2}\left[\sin\frac{l_x p\sin\theta\cos\varphi}{2}\sin\frac{l_y p\sin\theta\sin\varphi}{2}\sin\frac{l_z(q - p\cos\theta)}{2}\right]^2 \\ &\times \quad [p^2(q - p\cos\theta)\sin^2\theta\cos\varphi\sin\varphi]^{-2}.\end{aligned} \tag{16}$$

Because the integrand in the general formula (16) does not simultaneously vanish in the integrated domain, the corresponding total cross-section is always different from zero. On the other hand, the cross-section as given in the range of provided high momenta q (at least larger than the radion mass) is in the rapid oscillation with q. In that case, the relevant quantity should be an average over several oscillations. Also, the resulting cross-section will almost be not depended on the radion mass values if $m_\phi^2/q^2 \ll 1$. To evaluate the average total cross-section for Eq.(16), the parameters are chosen as follows: $\Lambda_\phi =$ 5 TeV, $\xi = 0, \pm 1/6, \alpha = 1/128$, $l_x = l_y = l_z = 1$ m $= 5.07 \times 10^6$ eV^{-1}, $E = 100$ KV/m $=$ 6.517×10^{-2} eV^2 [6], and the radion mass can be taken in the limit $m_\phi = 10$ GeV [5]. The average cross-section value $\overline{\sigma}$ on the ranges of momenta q for the radion production are given in Table 1. Here the different values $\xi = 0, \pm 1/6$ approximately yield the same contribution to the cross-section. We can see from Table 1 that the cross-section is quite small to be measurable because of the current experimental limits.

Table 1. Average cross-section for conversion in electric field.

q[GeV]	100–200	200–300	300–400	400–500	500–600
$\overline{\sigma}$[cm^2]	1.308×10^{-47}	9.717×10^{-47}	2.569×10^{-45}	4.672×10^{-45}	6.700×10^{-45}

III.2. Conversion in magnetic field

Next, we consider the conversion of photon into radion in a homogeneous magnetic field of the solenoid with radius R and a length l. Without loss of generality we suppose that the direction of the magnetic field is parallel to the z-axis, i.e. $F^{12} = -F^{21} = B$. The matrix element is given then

$$< p \mid M \mid q >= \frac{c_{\phi\gamma\gamma}}{(2\pi)^2\sqrt{p_0 q_0}}(\varepsilon^2(\mathbf{q},\sigma)q^1 - \varepsilon^1(\mathbf{q},\sigma)q^2)F_m(\mathbf{k}), \tag{17}$$

where

$$F_m(\mathbf{k}) = \int_V e^{i\mathbf{kr}} B(\mathbf{r}) d\mathbf{r}. \tag{18}$$

In the cylindrical coordinates, the integral (17) becomes

$$F_m(\mathbf{k}) = B\int_0^R \varrho d\varrho \int_0^{2\pi} \exp\{i[k_x\cos\varphi + k_y\sin\varphi]\}d\varphi \int_{-l/2}^{l/2} \exp\{ik_z z\}dz. \tag{19}$$

After some manipulations we get

$$F_m(\mathbf{k}) = \frac{4\pi BR}{k_z\sqrt{k_x^2+k_y^2}} j_1\left(R\sqrt{k_x^2+k_y^2}\right)\sin\left(\frac{lk_z}{2}\right), \tag{20}$$

where j_1 is the spherical Bessel function of the first kind.

From Eqs.(17,20) we obtain the differential cross-section as follows

$$\frac{d\sigma^m(\gamma\to\phi)}{d\Omega'} = \frac{2c^2_{\phi\gamma\gamma}R^2B^2}{k_z^2(k_x^2+k_y^2)} j_1^2\left(R\sqrt{k_x^2+k_y^2}\right)\sin^2\left(\frac{lk_z}{2}\right)(q_x-q_y)^2. \tag{21}$$

Eq.(21) shows that when the momentum of the photon is parallel to the z-axis (the direction of the magnetic field), the differential cross-section vanishes. This result is the same as the previous section. It implies that if the momentum of the photon is parallel to the

EM field, then there is no conversion. If the momentum of the photon is parallel to the x-axis, i.e. $q^\mu = (q, q, 0, 0)$, then Eq.(21) gets the form

$$\frac{d\sigma^m(\gamma \to \phi)}{d\Omega'} = \frac{2c^2_{\phi\gamma\gamma} R^2 B^2 q^2 j_1^2 \left(R\sqrt{(q - p\cos\theta)^2 + (p\sin\theta\cos\varphi')^2}\right)}{(p\sin\theta\sin\varphi')^2[(q - p\cos\theta)^2 + (p\sin\theta\cos\varphi')^2]} \times \sin^2\left(\frac{lp}{2}\sin\theta\sin\varphi'\right), \tag{22}$$

where φ' is the angle between the y-axis and the projection of $\mathbf{p}$ on the yz-plane.

To evaluate the average total cross-section from the general formula (22), the parameter values for Λ_ϕ, α and m_ϕ are given as before. The remaining ones are chosen as follows: $R = l = 1$ m $= 5.07 \times 10^6$ eV^{-1} and $B = 9$ Tesla $= 9 \times 195.35$ eV^2 [7]. The average cross-section on the ranges of momenta q by Eq.(22) for three cases $\xi = 0, \pm\frac{1}{6}$ yield the same value which is presented as in Table 2.

From Table 2 we see that the cross-sections for the radion production in the magnetic field are much bigger than that of the electric field, this is due to $B \gg E$. It is worth mentioning here if the radion mass is much smaller than the provided photon momentum, the cross-sections are much larger.

Table 2. Average cross-section for conversion in magnetic field.

q[GeV]	100–200	200–300	300–400	400–500	500–600
$\overline{\sigma}$[cm^2]	4.740×10^{-38}	6.625×10^{-38}	7.662×10^{-38}	2.732×10^{-37}	4.734×10^{-37}

IV. CONCLUSION

We have given a brief review of the RS model with stressing on radion phenomenology. With the help of the coupling of radion to photons, we have obtained the cross-sections of conversions of photon into radion in the presence of several external fields such as the static electric field of the condenser and the static magnetic field of the solenoid. The numerical evaluations of the total cross-sections are also given.

Let us mention that since the Randall-Sundrum model radion is quite heavy with masses at least in the GeV order, the experiments are only available if the provided photon sources are in high energies, as we often take some hundreds of GeV. Also, the light radions in the model if they really exist are favored in these experiments.

In this work we have considered only a theoretical basis for the experiments, other techniques concerning construction and particle detection can be found in Ref. [7]. It is emphasized that our study can be applied for searching the possible light radions in other models such as the large extradimensions.

ACKNOWLEDGMENTS

The work was supported in part by National Foundation for Science and Technology Development (NAFOSTED) of Vietnam under Grant No. 103.01.15.09.

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Received 29 September 2010.