Paper

Optical pulse splitting under temporal variations of reflecting medium

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Abstract — A possibility of light pulse transformation by transient reflecting medium is investigated theoretically. After solving 1D problem of such a reflection one estimates such a transformation in a plane optical waveguide with time-dependent conductivity of one of the reflected media. Three types of the conductivity time-dependences are considered: harmonic, Bessel-like and splash-like ones. Obtained results show a possibility of pulse splitting under an influence of time-harmonic conductivity and pulse collapse vy the other considered nonstationarities.

Keywords* — *optical pulse splitting, time dependent conductivity, reflecting boundary.

Motivation and common formulation of the problem

In optics a transformation of electromagnetic field of light has two reasons to be one of the basic problem. The first one is that light sources could not radiate all types of required fields, laser sources radiate fields of a finite number of frequencies. The second one is, as in radio range electromagnetics, a necessity to modulate field for information transmission. Optical waverange field modulation is usually made based on electro-optical Pokker's effect, magneto-optical Faradey's or Kerr's effects and on acousto-optical effect. In all these methods the field is modulated during the transmission through the correspondent modulating media, which is connected with additional losses and additional element of transmission tract. For information transmission it is not convenient to use frequency or phase light modulation, because in the existing light sources for optical communication there is not enough coherence. Besides, such a modulation techniques are well-developed only for harmonic initial fields.

The present work has a goal to find a possibility to transform light pulses by transient reflecting medium and to estimate such a transformation in a plane optical waveguide where one of the reflecting media has time-dependent conductivity.

One has chosen the conductivity as a transient parameter because it can be more easily changed than permittivity whose change besides can leads to destructuring of the full reflection in optical waveguide.

To estimate pulse transformation in a plane optical waveguide with time-dependent conductivity of one of the reflecting media (Fig. 1) one firstly solves rigorously one-space

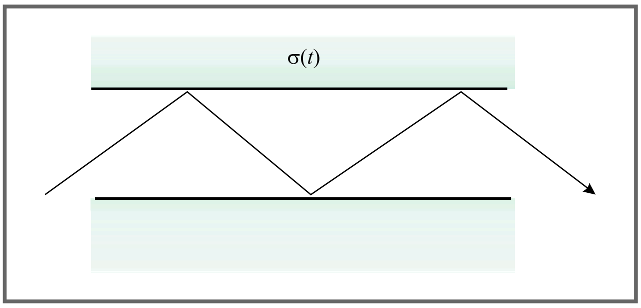

Fig. 1. Plane waveguide with nonstationary reflecting medium

dimensional problem of pulses reflection from a transient conductive half-space for an initial pulse propagating normally to the reflection boarder (Fig. 2).

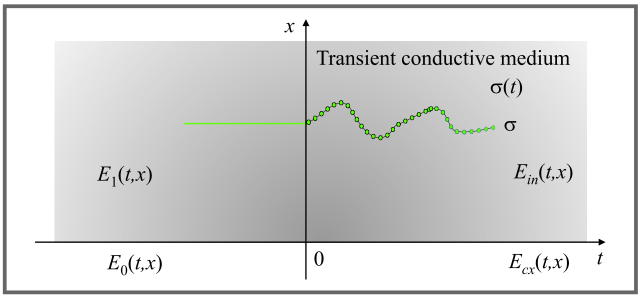

Fig. 2. Formulation of 1D problem

For the conductivity time-dependences considered in 1D case two-space-dimensional problem is investigated. It is solved approximately for the initial pulse falling angle providing a full reflection, in assumption of constant polarisation of the initial and scattered fields (Fig. 9). Then one estimates multiply re-reflection of the transformed pulse from the transient and stationary media forming a plane light waveguide.

One-space-dimensional problem

Problem formulation

The formulation of one-space-dimensional problem is in determination of the reflected field by the reflected field and the initial one and by time-dependence of the relected medium conductivity.

It is considered that the conductivity change starts after the moment $t = 0$ (Fig. 2). Before and after this moment the fields are called the initial and scattered fields, correspondingly. The fields are assumed to have only those components, which are normal to the *x*-axis and independent on the *y*- and *z*-coordinates.

Mathematically, the problem is formulated in terms of the Volterra integral equation for the electrical component of electromagnetic field [6], which has the following form for the internal field (inside the transient region) for $t > 0$, $x > 0$:

$$\begin{aligned}
E_{in}(t,x) &= A(t,x) + \\
&-\frac{2\pi}{\varepsilon v}\,\Theta(vt-x)\int_{t-x/v}^{t} \mathrm{d}t'\sigma(t')E_{in}\Big(t',x-v(t-t')\Big) + \\
&-\frac{2\pi}{\varepsilon v}\,\Theta(x-vt)\int_{0}^{t} \mathrm{d}t'\sigma(t')E_{in}\Big(t',x-v(t-t')\Big) + \\
&-\frac{2\pi}{\varepsilon v}\int_{0}^{t} dt'\sigma(t')E_{in}\Big(t',x+v(t-t')\Big)
\end{aligned} \qquad (1)$$

where $A(t,x)$ is known because it is determined by the initial field and prehistory of its interaction with the medium:

$$\begin{aligned}
A(t,x) &= E_0(t,x) + \\
&-\Theta(x-vt)\,\frac{2\pi}{\varepsilon v}\int_{t-x/v}^{0} \mathrm{d}t'\sigma(t')E_1\Big(t',x-v(t-t')\Big) + \\
&-\frac{2\pi}{\varepsilon v}\int_{-\infty}^{0} \mathrm{d}t'\sigma(t')E_1\Big(t',x+v(t-t')\Big)
\end{aligned}$$

and for the external field for $t > 0$, $x < 0$:

$$\begin{aligned}
E_{ex}(t,x) &= B(t,x) + \\
&-\frac{2\pi}{\varepsilon v}\,\Theta(vt+x)\int_{0}^{t+x/v} \mathrm{d}t'\sigma(t')E_{in}\Big(t',v(t-t')+x\Big)
\end{aligned} \qquad (2)$$

where

$$\begin{aligned}
B(t,x) &= E_0(t,x) + \\
&-\frac{2\pi}{\varepsilon v}\,\Theta(vt+x)\int_{-\infty}^{0} \mathrm{d}t'\sigma(t')E_1\Big(t',v(t-t')+x\Big) + \\
&-\frac{2\pi}{\varepsilon v}\,\Theta(x+vt)\int_{-\infty}^{t+x/v} \mathrm{d}t'\sigma(t')E_1\Big(t',v(t-t')+x\Big)
\end{aligned}$$

where ε is the dielectric permittivity, $v = c/\sqrt{\varepsilon}$ is the light velocity in considered medium and the conductivity time-dependence (or time-spatial dependence) $E_{ex}(t,x)$ is a function to be found.

Analytical solution of the problem

To obtain an equation for the external field, we firstly solve the equations (1) and (2) jointly to express the initial field after the external one.

As one can see from (2), the external field is determined by the sum of the known function B and a function of one variable $t - x/v$. This fact, which is due to the assumed homogeneity and losslessness of the external half-space, makes impossible to express the internal field through the external one directly from this expression. However, it allows to obtain a non-integral formula for the external field determined by the internal field on the boundary $x = 0$.

For this purpose we introduce a new function F of one variable as:

$$F(t) = -\frac{2\pi}{\varepsilon v}\int_{0}^{t} \mathrm{d}t'\sigma\Big(t',v(t-t')\Big)E_{in}\Big(t',v(t-t')\Big), \qquad (3)$$

which determines the external field in the external region $-vt < x < 0$ by the expression

$$E_{ex}(t,x) - B(t,x) = F(t + x/v).$$

From (1) we can obtain that the internal field on the boundary is determined by the same function F:

$$E_{in}(t,0) - A(t,0) = F(t).$$

Considering this expression for shifted time moment $t + y/v$, $-vt < y < 0$, we have

$$E_{in}(t+y/v,0) - A(t+y/v,0) = F(t+y/v).$$

After comparing this formula with (2) we obtain the following expression for the external field:

$$\begin{aligned}
E_{ex}(t,x) = B(t,x) + E_{in}(t+x/v,0) - A(t+x/v,0),\\
-vt < x < 0.
\end{aligned} \qquad (4)$$

Introduce another new function

$$\Phi(t,x) \equiv E_{in}(t-x/v,x) - A(t-x/v,x) \qquad (5)$$

for $0 < x < vt$, satisfying the following equation obtained from (1):

$$\begin{aligned}
&\Phi'_x(t,x) + \frac{2}{v}\,\Phi'_t(t,x) = \\
&-\frac{4\pi}{\varepsilon v}\frac{\partial}{\partial t}\int_{0}^{t-x/v} \mathrm{d}t'\sigma\Big(t',v(t-t')\Big)E_{in}\Big(t',v(t-t')\Big)
\end{aligned} \qquad (6)$$

with the boundary and initial conditions:

$$\Phi(t,0) = F(t), \text{ and } \Phi(x/v,x) = E_{in}(0,x) - A(0,x) = 0\,.$$

Knowing the external field at any point, the field in the whole external region can be determined, including the region close to the boundary:

$$\begin{aligned}
&E_{ex}(t,x) - B(t,x) = E_{ex}(t-x_1/v,x+x_1) + \\
&-B(t-x_1/v,x+x_1) = F(t+x/v)
\end{aligned} \qquad (7)$$

So it would be enough to obtain the solution for external field close to the boundary at the points where $|x| \ll vt$.

Under this approximation, we can solve Eq. (6), hence expressing the internal field through the external one, because the integral at the right-hand part of (6) will be equal to $F(t)$:

$$\Phi'_x(t,x) + \frac{2}{v}\,\Phi'_t(t,x) \approx 2F'(t)\,. \qquad (8)$$

After substitution of this equation solution into (3) we obtain the conductivity time-dependence in the half-space determined by the scattered field:

$$\sigma(t)\Big(E_{ex}(t-x/v,x)-E_0(t-x/v,x)+E_0(t,0)\Big)= \\ =\sigma_0 A(0,0)\,, \tag{9}$$

where $\sigma_0=\sigma(0)$ is the known value of initial conductivity, and x means an arbitrary point coordinate (not only $|x|\ll vt$) inside the external region $-vt<x<0$.

Pulse splitting with amplification by time-harmonic conductivity of the reflecting half-space

For the rectangular pulse $E_0=\Theta(t-x/v)-\Theta(t-x/v-t_0)$ scattering on the homogeneous half-space with time-harmonic conductivity, computer analysis revealed the scattered pulse features dependence on the conductivity frequency. When it is comparable with reverse incident pulse duration then the pulse of scattered field just changes a little in its shape under the same duration.

When the conductivity frequency is more then four times as much as reverse pulse duration, the scattered pulse has deep valleys (Fig. 2). Their number grows with the frequency increasing. Thus, it becomes a consequence of pulses with joint duration less then that of the initial pulse. These pulses amplitude can be more then ten times as much as that of the initial pulse. The more is the conductivity frequency (Fig. 3(a)), the more is the number of reflected field pulses (Fig. 3(b)).

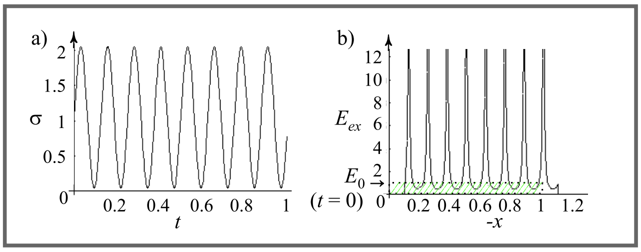

Fig. 3. Time-harmonic conductivity of the reflecting half-space leading to (b) pulse splitting with amplification by

Pulse transformation by reflecting half-space with conductivity changing with time as Bessel function

Bessel-type conductivity time-dependence provides the initial rectangular pulse transformation into a pulse with duration 10^{-3} as much as the initial one, following by reducing oscillating tail (Fig. 4).

Such a pulse collapse can be also followed by the reflected pulse amplification, as Fig. 5 shows.

Consider the same pulse scattering on the half-space with time-splashing conductivity (Fig. 6(a)) with its time dependence described by a difference of reducing exponents.

For the time less then a pulse duration after it began its interaction with the half-space. Figure 6(b) shows the reflected pulse front which then will save its shape and size,

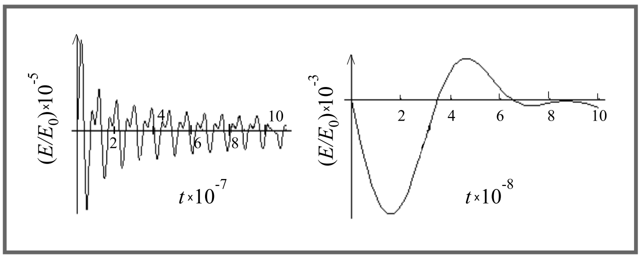

Fig. 4. Reflected pulse amplitude time-dependence for different amplitudes of Bessel-type conductivity change(initial pulse length =1 cm)

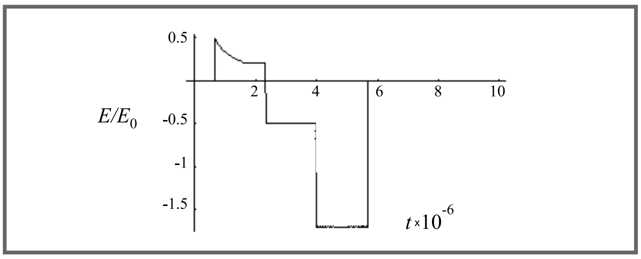

Fig. 5. Reflected pulse amplitude time-dependence for Bessel-type conductivity change (initial pulse length = 0.5 cm)

moving with the correspondent to the medium light velocity (circle incision in Fig. 7). With time this pulse of a small amplitude moving away from the boundary leaves a field trace of a high amplitude (Fig.7). After the end of the conductivity splash the trace amplitude decreases forming so a splash-like pulse. The front of this pulse moves with a velocity lower then that of light for the considered medium. This trace evolution with time leads to its transformation

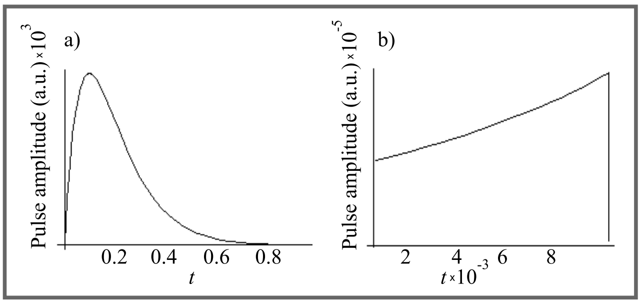

Fig. 6. Conductivity splash (a) and initial stage of the pulse reflection (b)

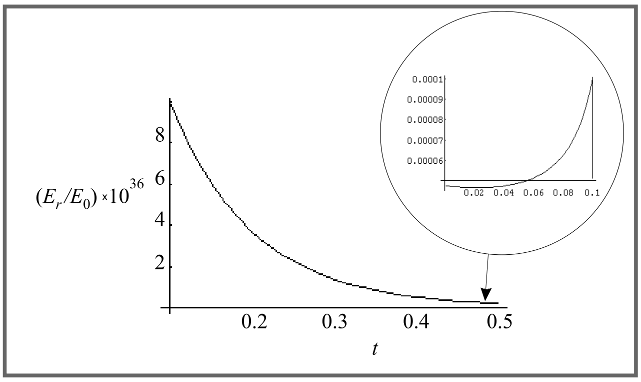

Fig. 7. Reflected field front and trace

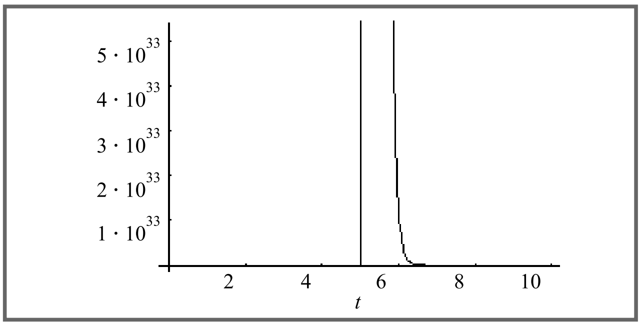

Fig. 8. Short pulse formed from the reflected field trace

into a shot pulse of a very high amplitude which is like to Dirac delta function as Fig. 8 shows.

Obtained solutions enable one to analyse a large number of scattering on the transient conductive half-space problems. Results for the special cases can be useful, for example, for creation short pulses of high amplitude.

The obtained results demonstrate different possibilities of initial pulse splitting or time-compression by time variation of the reflecting medium.

They also show possibilities of the reflected pulse amplification.

For the case of time-harmonic conductivity in the reflecting half-space one can obtain an easy read information when the modulated signal amplitude and frequency are translated into amplitudes and a number of pulses which are the result of the initial pulse subdivision.

2D Problem

In 3D nonstationary media an integral equation for electromagnetic field is [5]:

$$\boldsymbol{E}(t,\boldsymbol{r}) = \boldsymbol{E}_1(t,\boldsymbol{r}) - \frac{2\pi\sigma}{\varepsilon v}\left(\text{graddiv} - \frac{\partial^2}{\partial t^2}\right)\times \times\int_0^t \mathrm{d}t' \int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{r}'\sigma(t',\boldsymbol{r}')\frac{\Theta\big(t-t'-|\boldsymbol{r}-\boldsymbol{r}'|/v\big)}{|\boldsymbol{r}-\boldsymbol{r}'|}\boldsymbol{E}(t',\boldsymbol{r}'), \tag{10}$$

where $\boldsymbol{r} \equiv (x,y,z)$ and $|r-\boldsymbol{r}'| = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$.

For the considered 2D problem we assume $\sigma(t,r) \equiv \sigma(t)\Theta(x)$ and $\boldsymbol{E} = E_z\boldsymbol{k}$. In this case we have the following scalar integral equation inside the transient region:

$x \geq 0$

$$\begin{aligned} E_{in}(t,x,y) &= E_1(t,x,y) + \\ &-\Theta(vt-x)\int_{t-x/v}^{t}\mathrm{d}t'\sigma(t')\int_{x-v(t-t')}^{x+v(t-t')}\mathrm{d}x'G + \\ &-\Theta(x-vt)\int_0^{\infty}\mathrm{d}t'\sigma(t')\int_{x-v(t-t')}^{x+v(t-t')}\mathrm{d}x'G + \\ &-\Theta(vt-x)\int_0^{t-x/v}\mathrm{d}t'\sigma(t')\int_0^{x+v(t-t')}\mathrm{d}x'G \end{aligned} \tag{11}$$

where

$$G \equiv \frac{2\pi}{\varepsilon v}\int_{y-\sqrt{v^2(t-t')-(x-x')^2}}^{y+\sqrt{v^2(t-t')-(x-x')^2}}\mathrm{d}y' \ln\frac{v(t-t')+\sqrt{v^2(t-t')^2-(x-x')^2-(y-y')^2}}{v(t-t')-\sqrt{v^2(t-t')^2-(x-x')^2-(y-y')^2}}E_{in}(t',x',y')$$

and the following formula which expresses the external field after the internal one:

$x < 0$

$$\begin{aligned} E_{in}(t,x,y) &= E_1(t,x,y) + \\ &-\Theta(vt+x)\int_0^{t+x/v}\mathrm{d}t'\sigma(t')\int_0^{v(t-t')-x}\mathrm{d}x'\,G. \end{aligned} \tag{12}$$

We are interested in the external field for which we also can obtain a usual wave equation with constant coefficients directly from Maxwell's equations or from (12):

$$\left(\Delta - \frac{1}{v_0^2}\frac{\partial^2}{\partial t^2}\right)E_{ex}(t,x,y) = 0. \tag{13}$$

However, boundary conditions here will contain the unknown internal field.

Carrying out the procedures analogous with those in 1D problem solution, the following approximate integral equation for the function F determining the field on the boundary as

$$E(t,0,y) = E_1(t,0,y) + F(t,y)$$

can be obtained:

$$F(t,y) = \frac{2\pi}{\varepsilon v}\int_0^t \mathrm{d}t' \, \sigma(t')\int_{y-v(t-t')}^{y+v(t-t')}\mathrm{d}y'\,\frac{F(t',y')(y-y')}{v^2(t-t')^2-(y-y')^2}. \tag{14}$$

This equation can be solved numerically for concrete types of the conductivity time-dependence using standard procedures. After obtaining the field on the boundary, we can estimate the reflected field by solving the wave equation (13).

Estimating solution of this 2D problem has shown that there will be the same effects of pulse splitting, compression and amplification as in 1D problem.

These effects are not influenced later by multiply re-reflection, because the obtained pulses of short duration already does not feel the conductivity nonstationarity of correspondingly low frequency. It means that for pulse transformation by nonstationarity of the reflecting medium in optical waveguide a homogeneous simultaneous change of the conductivity is not necessary, but it is enough to create local nonstationarity.

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