

# Performance Evaluation of Cognitive Radio Network Based on 2-D Markov Chain

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**Abstract**—The objective of cognitive radio network is to enhance the wireless network spectrum utilization. In such a network, two types of users are enlisted, namely primary user (PU) and secondary user (SU). The PU can access any channel in case of its availability, but SU users have lower priority and can access a channel only when it is unused by PUs. The performance of such a network solely depends on two traffic parameters: probability of false alarm and probability of misdetection. In this paper the performance of such a network is analyzed based on two dimensional Markov chain including those parameters. The main contribution of this paper is to evaluate blocking probability and PU and SU throughput using the state transition chain instead of existing statistical analysis.

**Keywords**—cognitive radio networks, misdetection, probability of false alarm, throughput and traffic of limited channel.

## 1. Introduction

In a conventional network during low offered traffic period, few traffic channels are idle. Hence network experiences a waste of channel utilization. A cognitive radio network (CRN) overcomes the situation, since the unutilized channels are used by unlicensed users called secondary users (SUs) in case of their availability. The main job of a secondary user is to sense the presence of a primary user (PU), which is done by managing the traffic channel. The detail statistical analysis of CRN spectrum sensing technique is presented in [1]–[4]. To enhance the success rate of sensing a PU by SU, several detected signals of SUs are further analyzed at a host level. Two important traffic parameters under CRN are probability of false alarm and misdetection. Recent literature deals with a statistical model of spectrum sensing based on absence of PU and presence of PU known as hypothesis  $H_0$  and  $H_1$  respectively. If there is no PU on a physical channel and the received signal strength of SU is over a signal to noise ratio (SNR) threshold, then the phenomenon is called false alarm (FA). Again, under presence of a PU on physical channel, if the received SNR is lower than the threshold value, then the phenomenon is called misdetection. Both the parameters govern the performance of a CRN, which are explained in [5]–[8] with a detailed statistical model. Sometimes the false alarm is categorized based on coverage range of sensing shown in [9]. In this

case the SU has to confirm whether the PU is in paging mode inside the sensing zone or in transmitting mode outside the sensing region but this concept is beyond the analysis of the paper. To combat the false alarm and misdetection under fading wireless channel, the space diversity and different combining schemes are incorporated with the sensing unit of SU shown in [10]. The traffic analysis of cognitive radio under Internet, specially for Voice over IP (VoIP) is discussed in [10]–[11]. When a PU requests for a physical channel, the SU on that channel is removed and the phenomenon is called forced termination (FT). In this case the SU has to move to another available traffic channel. In this paper this idea is modeled by two dimensional state transition chains, which is then solved to evaluate the CRN traffic parameters. The detailed statistical analysis of Markov chain and its solution is presented in [13]–[15] but authors emphasis on two dimensional Markov chain with some modification to cope with the CRN traffic. Finally the CRN traffic performance based on blocking probability and throughput (carried traffic) of both PU and SU varying other traffic parameters is analyzed.

The paper is organized as follows. Section 2 provides the traffic model of CRN based on two dimensional Markov chain and the traffic parameters pertinent to the CRN performance are derived solving the state transition chain. Section 3 depicts the results based on analysis of Section 2 and finally Section 4 concludes the entire analysis.

## 2. System Model

Let us first consider a Markov chain of unlimited channel and user case, which is represented by Kendall's notation as  $M/M/\infty$  shown in Fig. 1. The  $\lambda$  and  $\mu$  are the call arrival and termination rate respectively as discussed in [16]–[18]. The chain states  $P_0, P_1, P_2$  etc. are the probability of arrival

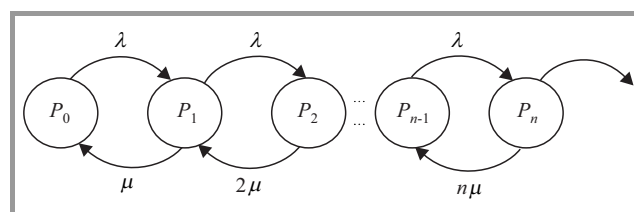


Fig. 1. State transition chain of unlimited trunk network.

of no call, one call, two calls and so on. Basic assumption in Markov chain is that probability of reaching the next state depends on present state but not on previous states. Any new call arrival rate  $\lambda$  will make transition of a probability state  $P_i$  to  $P_{i+1}$  similarly any call termination rate  $i\mu$  will make transition from  $P_i$  to  $P_{i-1}$ .

Applying cut equation between nodes in Fig. 1 the generalized probability state  $x$  known as Poisson's probability density function (PDF) is:

$$P_x = \frac{A^x}{x!} e^{-A}. \quad (1)$$

Poisson's distribution is valid for infinite number of trunks or channels but in real conditions number of channels are limited therefore traffic should be analyzed for limited channel case. A limited trunk traffic model is shown in Fig. 2, where number of users  $N$  is infinite, users offer an average arrival rate of  $\lambda$  and average holding time of  $t_h = 1/\mu$ . If the number of trunk is  $n$  then any arrival beyond the state  $P_n$  will not get the service and the call will be lost shown in Fig. 3 using Markov chain.

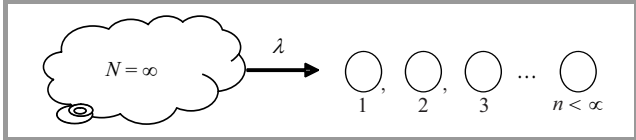


Fig. 2. Limited server or channel network.

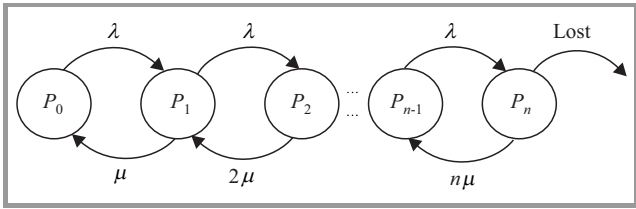


Fig. 3. State transition of limited trunk network.

Again applying cut equations, the probability state  $x$  in generalized form from Fig. 3 becomes:

$$P_x = \frac{A^x}{x!} \frac{A_1}{\sum_{i=0}^n \frac{A_i}{i!}}, \quad (2)$$

which is known as Erlang's PDF.

In teletraffic engineering probability of occupancy of all channels is called call blocking probability:

$$B_n = P_n = \frac{\frac{A^n}{n!}}{\sum_{i=0}^n \frac{A_i}{i!}}. \quad (3)$$

Let us now focus on 2-D Markov chain having two types of Poisson's offered traffic  $A_1$  and  $A_2$  [19]. The arrival and termination rates are:  $\lambda_1, \lambda_2, \mu_1, \mu_2$  and  $n = \infty$  servers or channels shown in Fig. 4. It is convenient to arrange

states  $x_1$  and  $x_2$  along X and Y direction, where any probability state  $P_{x_1, x_2}$  indicates probability of  $x_1$  and  $x_2$  calls of type 1 and type 2 traffic occupancy, respectively. Because the system is reversible it is convenient to apply cut equation between nodes.

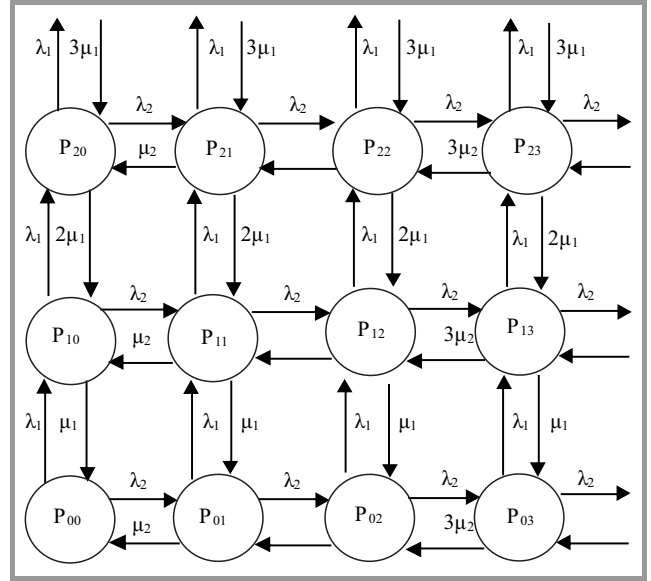


Fig. 4. 2-D Markov chain of Poisson traffic.

Considering  $x_2$ -th column in Fig. 4 and applying cut equation between first and second node we get

$$\lambda_1 P_{0, x_2} = P_{1, x_2} \mu_1 \Rightarrow P_{1, x_2} = \frac{\lambda_1}{\mu_1} P_{0, x_2} = A_1 P_{0, x_2}.$$

Similarly between second and third node:

$$\lambda_1 P_{1, x_2} = P_{2, x_2} 2\mu_1 \Rightarrow P_{2, x_2} = \frac{\lambda_1}{2\mu_2} P_{1, x_2} = \frac{A_1^2}{2!} P_{0, x_2}.$$

In generalized form

$$P_{x_1, x_2} = \frac{A_1^{x_1}}{x_1!} P_{0, x_2}. \quad (4)$$

Again considering  $x_1$ -th row in Fig. 4:

$$P_{x_1, x_2} = \frac{A_2^{x_2}}{x_2!} P_{x_1, 0}. \quad (5)$$

Putting  $x_2 = 0$  in Eq. (4)

$$P_{x_1, 0} = \frac{A_1^{x_1}}{x_1!} P_{0, 0}. \quad (6)$$

From Eqs. (5) and (6)

$$P_{x_1, x_2} = \frac{A_2^{x_2}}{x_2!} \frac{A_1^{x_1}}{x_1!} P_{0, 0}. \quad (7)$$

Considering entire sample space

$$\sum_{x_1=0}^{\infty} \sum_{x_2=0}^{\infty} P_{x_1, x_2} = 1 \Rightarrow \sum_{x_1=0}^{\infty} \sum_{x_2=0}^{\infty} \frac{A_2^{x_2}}{x_2!} \frac{A_1^{x_1}}{x_1!} P_{0, 0} = 1 \Rightarrow e^{A_1} e^{A_2} P_{0, 0} = 1. \quad (8)$$

From Eqs. (7) and (8) the probability state  $(x_1, x_2)$  in normalized form becomes

$$P_{x_1 x_2} = \frac{A_1^{x_1} A_2^{x_2}}{x_1! x_2!} e^{-(A_1 + A_2)}. \quad (9)$$

When multidimensional Poisson traffic is applied to a limited number of servers ( $n < \infty$ ) fully available to all traffic components (complete sharing), loss probabilities are calculated using multidimensional Erlang's loss formula. Using the concept of Eqs. (4) to (9) the sample space for  $n$  channels case becomes

$$\sum_{x_1=0}^n \frac{A_1^{x_1}}{x_1!} \sum_{x_2=0}^{n-x_1} \frac{A_2^{x_2}}{x_2!} P_{00} = 1 \therefore P_{00} = \frac{1}{\sum_{x_1=0}^n \frac{A_1^{x_1}}{x_1!} \sum_{x_2=0}^{n-x_1} \frac{A_2^{x_2}}{x_2!}}. \quad (10)$$

The probability state in normalized form is:

$$P_{x_1 x_2} = \frac{\frac{A_1^{x_1} A_2^{x_2}}{x_1! x_2!}}{\sum_{x_1=0}^n \frac{A_1^{x_1}}{x_1!} \sum_{x_2=0}^{n-x_1} \frac{A_2^{x_2}}{x_2!}}. \quad (11)$$

Blocking probability is the sum of all state  $P_{x_1 x_2}$  where  $x_1 + x_2 = n$ ,

$$B_n = \sum_{x_1=0}^n P_{x_1} P_{n-x_1}. \quad (12)$$

Figure 5 shows the probability states of a limited channel system ( $n = 3$ ), where sum of the complete occupied states is the blocking probability expressed as  $B = P_{30} + P_{21} + P_{12} + P_{03}$ .

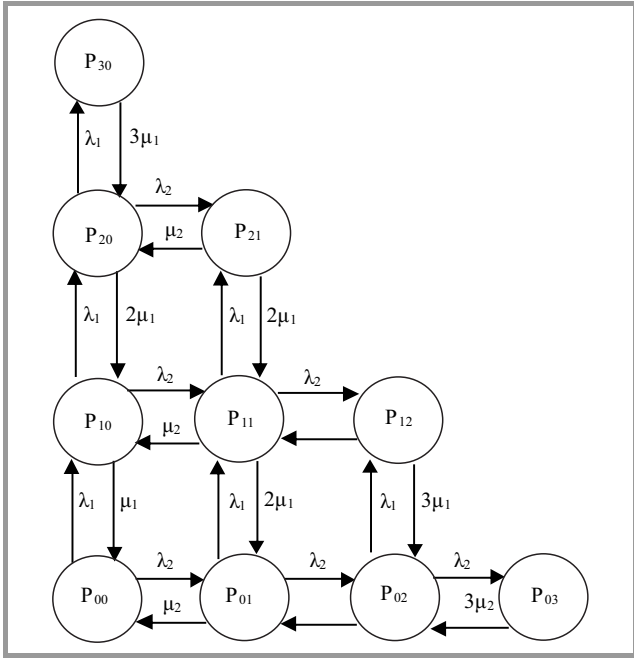


Fig. 5. 2-D Markov chain of Erlang's traffic ( $n = 3$ ).

In cognitive radio network the Markov chain from Fig. 5 has to be modified like it is presented in Fig. 6 (the only  $n = 3$  channels are considered). Since any arrival of PU under a complete occupied state, a SU has to be terminated

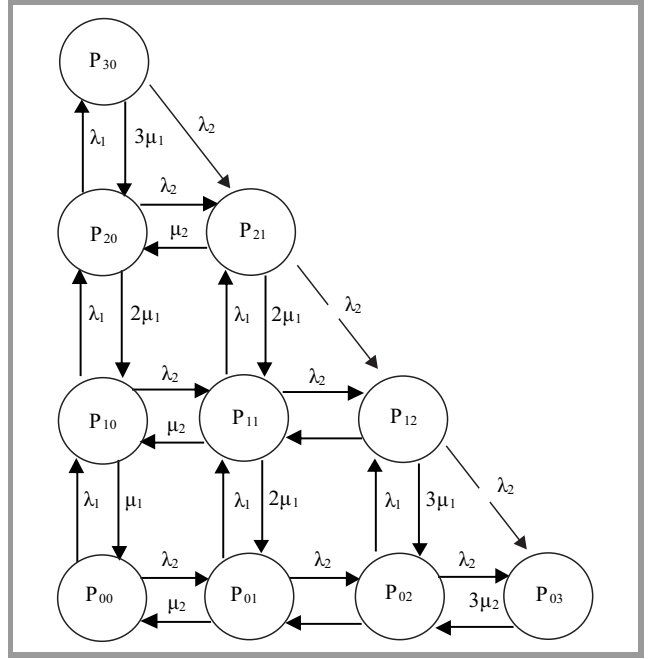


Fig. 6. 2-D Markov chain of Erlang's traffic ( $n = 3$ ) for performance evaluation of cognitive radio network under unlimited user.

forcibly as discussed in [20], [21]. The annotation is as follows:  $\lambda_1 = \lambda_2 =$  arrival rate of SU and  $\lambda_2 = \lambda_p =$  arrival rate of PU,  $\mu_1 = \mu_s =$  termination rate of SU and  $\mu_2 = \mu_p =$  termination rate of PU. The offered PU traffic and SU are  $A_p = \lambda_p / \mu_p$  and  $A_s = \lambda_s / \mu_s$  respectively. For example a CRN of  $n$  channel, the complete occupied states will be  $(x_1, x_2) = (i, n - i)$ ;  $i = 0, 1, 2, 3, \dots, n$ , where  $x_1$  is the number of SU and  $x_2$  is the number of PU. Any arrival of PU at state  $(i, n - i)$  will change the state to  $(i - 1, n - i + 1)$  forcefully.

Let us introduce the traffic parameters of CRN like:  $P_{md}$  – probability of misdetection,  $P_f$  – probability of false alarm,  $A_{s\_original}$  – offered traffic of SU,  $A_p$  – offered traffic for primary user. Channel will experience the traffic from SU side as

$$A_s = A_{s\_original} \cdot (1 - P_{md})P(H_1) + A_{s\_original} \cdot (1 - P_f)P(H_0),$$

where  $P(H_1)$  and  $P(H_0)$  are the probability of presence and absence of an PU on a traffic channel. The probabilities are widely used in test symbols statistical analysis of detected energy, and are known as two hypothesis model.

Solving the Markov chain from Fig. 6 for  $n$  channels case, the entire sample space is

$$S(n) = A + B + C + D, \quad (13)$$

where

$$A = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1-i} \frac{A_p^i}{i!} \frac{A_s^j}{j!},$$

$$B = \sum_{x=1}^{n-1} A_p \left\{ \frac{A_p^{n-x-1}}{(n-x-1)!} \frac{A_s^x}{x!} + \frac{A_p^{n-x-1}}{(n-x-1)!} \frac{A_s^{x+1}}{(x+1)!} \right\} + A_s \frac{A_p^{n-x}}{(n-x)!} \frac{A_s^{x-1}}{(x-1)!},$$

$$C = \frac{A_p \left\{ \frac{A_p^0}{0!} \frac{A_s^{n-1}}{(n-1)!} \right\}}{n + A_p} \quad \text{and} \quad D = \frac{A_p \left\{ \frac{A_s^{n-1}}{(n-1)!} \frac{A_p^0}{0!} \right\} + A_p \frac{A_p^{n-x}}{(n-x)!} \frac{A_s^1}{1!}}{n}$$

The blocking probability of SU is:

$$B_{SU}(n) = \frac{B + C + C}{S(n)}, \quad (14)$$

and the blocking probability of PU:

$$B_{PU}(n) = \frac{D}{S(n)}. \quad (15)$$

The throughput of PU could be expressed as

$$X_{p\_bar}(n) := (1 - B_{pu}(n)) \cdot A_p \quad (16)$$

and the throughput of SU

$$X_{s\_bar}(n) := (1 - B_{su}(n)) \cdot A_{s\_original}. \quad (17)$$

The next section provides the profile of throughput and blocking probability to evaluate the CRN performance.

### 3. Results

Figure 7 shows that the variation of throughput against number of channels taking. The offered traffic of PU is  $A_p = 15$  Erl, offered traffic of SU  $A_{s\_original} = 10$  Erl, the probability of misdetection  $P_{md} = 0.12$ , the probability of false alarm  $P_f = 0.04$  and the probability of hypothesis  $P(H_1)$  and  $P(H_0)$  are both 0.5. The picture shows that the throughput gradually increases with the number of channels of both the PU and SU. The throughput of PU is found much larger than the SU because of its priority of getting a channel.

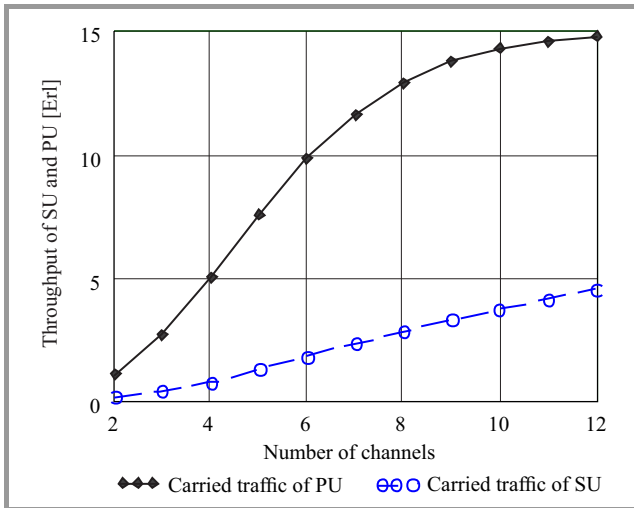


Fig. 7. Throughput of SU and PU with the variation of channel.

The variation of blocking probability (PU and SU) against the number of channel is shown in Fig. 8. The blocking probability of PU is much lower than SU at the same

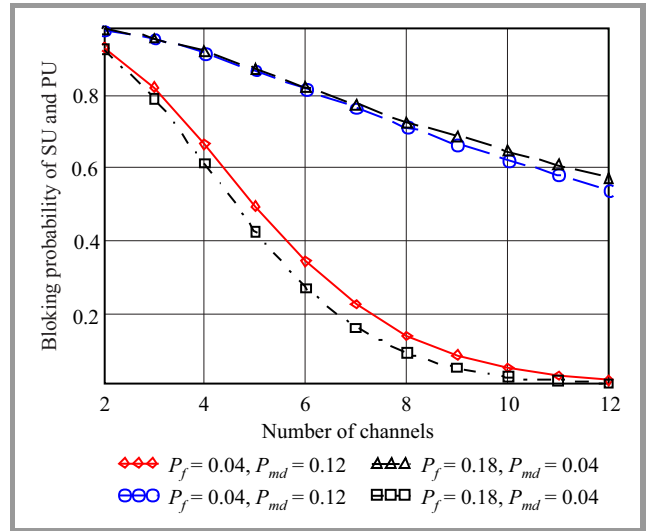


Fig. 8. Blocking probabilities of SU and PU with the variation of channel.

time the rate of decrement of blocking probability with increase in the channel number is more rapid for PU for the same reason. From the profile of blocking probability one shall notice that  $B_{PU} \approx 0$  for  $n > 12$  but  $B_{SU} > 0.35$ , for the same channel condition. The blocking probability is more sensitive to  $P_{md}$  than  $P_f$ . This is also visualized in Fig. 8.

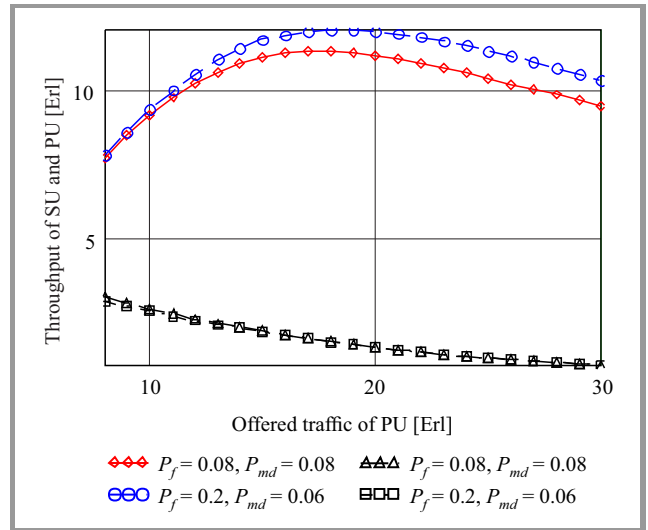


Fig. 9. Throughput of SU and PU with the variation of offered traffic.

Figure 9 shows that the throughput variation versus offered traffic of PU, under the conditions:  $n = 8$ , offered traffic of PU  $A_p = 8 \dots 30$  Erl, offered traffic of SU  $A_{s\_original} = 6$  Erl, probability of misdetection  $P_{md} = 0.08$  and probability of false alarm  $P_f = 0.08$ . The graph shows that the PU throughput increases within offered traffic of PU and attains at a maximum value then starts to decrease. At low offered traffic, most of the channels remains idle, again at higher offered traffic the blocking probability in-

creases, hence throughput lowers. Therefore, there must be an optimum offered traffic with maximum throughput. There is a tradeoff between idle channel and blocking probability visualized in Fig. 9. The impact of  $P_{md}$  is more prominent than  $P_f$  the throughput always decreases with increase in the offered traffic of PU because of its forced termination.

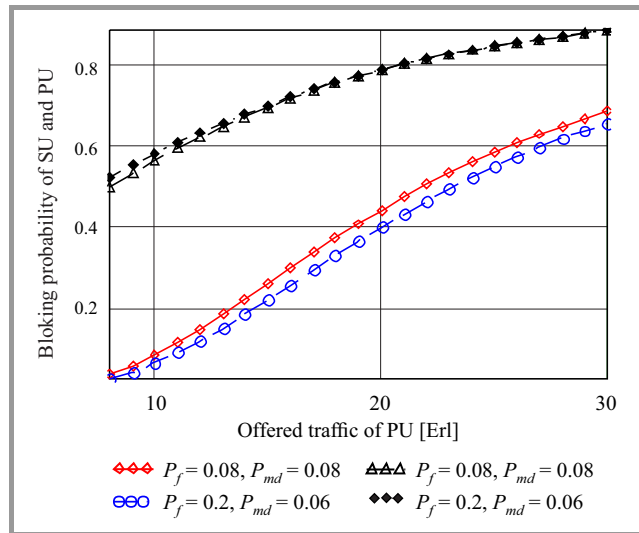


Fig. 10. Blocking probability of SU and PU with the variation of offered traffic.

Finally Fig. 10 shows that the variation of blocking probability of both PU and SU against the offered traffic of PU under following conditions: the number of channels  $n = 8$ , offered SU traffic  $A_{s\_original} = 6$  Erl, probability of misdetection  $P_{md} = 0.08$  and probability of false alarm  $P_f = 0.08$ . The graph shows that the blocking probability of both type of user increases with the PU offered traffic. The blocking probability is much higher for SU than PU like before, but the rate of  $B_{PU}$  increment is more prominent than  $B_{SU}$  since offered PU traffic affects the PU users directly and to SU indirectly.

## 4. Conclusion

In this paper the performance of a cognitive radio network using M/M/n/k traffic model based on 2-D Markov Chain is analyzed. The results shows the performance of the network varying different traffic parameters, provide expected results. Still the scope to use M/G/1/K traffic of packet switch network of finite queue has to be researched. In this case two dimensional traffic models using state transition chain will not be possible because of general PDF service time. The authors can apply tabular form of 2-D traffic model of M/G/1/K to pave the way for evaluating packet loss PU and SU probability. Finally the impact of fading channel on false alarm and misdetection can also be included on the traffic model to observe the performance under small scale fading environment. One of the major components of 5G mobile communications is the concept

of cognitive radio under each node-B to increase the carried such a network traffic. The proposed traffic model will be helpful for developing the next generation wireless networks.

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