Paper

Availability of WDM Multi Ring Networks

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Abstract—This paper presents a new approach to modeling the availability of the networks composed of multiple interconnected rings in two nodes. For availability modeling method algebraic formulation is used. Using this method, through the availability of multiple ring networks consisting of two and three-rings connected to two nodes, a general expression for the availability between two terminals of multiple ring networks is derived. To perform the expression some real assumptions were taken and the analytical calculation showed that the use of these expressions under these assumptions provides real values for availability between the two terminals of multiple ring networks. Information on the availability of links and nodes is taken from previously published works.

Keywords—algebraic, availability, multiple, WDM.

1. Introduction

Telecommunications services have an important role in the development of modern society. Rapid growth in demand for data transmission and Internet traffic require high-capacity transmission systems. The only transmission media that can meet the needs of such a large capacity is an optical fiber. Unlike other media only the optical fiber has a large unused capacity that can effectively utilize the technologies based on wavelength division multiplexing (WDM).

In such networks, disconnection for any reason, i.e. equipment failure or human error, can cause isolation in terms of telecommunications and profit losses for users and network operators. Therefore, the availability of telecommunication networks is becoming a very important factor for network operators. The basic problem is how to ensure the survival of links between two nodes within large networks in terms of failure and how to ensure high availability. The development of models of reliability and availability of single selfrenewable WDM rings have been dealt with by the authors in [1] using optical add/drop multiplexers in the nodes. There are a lot of papers dealing with the calculation of the availability using the shared-path protection in which it was concluded that the use of these methods guaranteed the achievement of availability and spare capacity utilization, which is especially important for operators as service providers [2]. M. Clouqueur and W. D. Grover [3] investigated how to design a network that is able to restore the connection of all cases of single link failure between any two nodes in the ring (span protection) with less investment. They also concluded that some linked networks are very robust to the simultaneous double link failures. Some authors investigated how the availability of services depends on the total capacity and on the protection resources allocated amount necessary for mastering the protection in the event of failure to the work path [4], [5].

In this paper, the authors will develop models and derive expression for availability between the two terminals for multiple ring networks, starting from a network consisting of two and three-rings connected to two nodes examples.

2. Mathematical Basis for Formulation of Network Models

2.1. General Remarks on Availability

Availability is defined as probability that a system is correct at some point in time T, provided that it properly worked at time $T = 0$ and that defective conditions appear (maintenance or failures), but are always repeared and the system is operative again. Also, the availability of a system at point in time is defined as the ratio of time for which the system is correct in relation to the total time. The Mean Time To Failure (MTTF) is defined as $1/\lambda$, where λ is intensity of the failure. It is usually expressed in Failure In Time (FIT), and 1 FIT means 1 failure in 10^9 hours. Using Mean Time To Repair (MTTR), the availability A can be calculated by [6]:

$$A = \frac{\text{MTTF}}{\text{MTTF}+\text{MTTR}} . \quad (1)$$

Similarly, the unavailability U is defined as probability complementary to the availability, i.e.

$$U = 1 - A , \quad (2)$$

and it is likely that the system is not working properly at some point in time. When reporting about system/network performances, the unavailability is often expressed as Mean Down Time (MDT) in minutes per year, i.e.

$$\text{MDT} = 365 \cdot 24 \cdot 60 \cdot U . \quad (3)$$

2.2. Methods of Algebraic Formulation

Although there are several methods suitable for modeling network availability, i.e. elementary paths enumeration, in this paper the algebraic formulation method is used [6], [7].

Instead of working with probabilities, in this paper the emphasis is on algebraic objects (polynomials) and their appropriate transformation. Let a network $G = (N, E)$ be a directed network with source node s and the destination node t (N – nodes, E – edges). Suppose that the nodes in the network G are ideal and that links $m = |E|$ are subject to failures that occur randomly and are repairable. Let each link $k = 1, 2, \ldots, m$ be corresponds with the variable x_k. Furthermore, let's x_i an event for $1 \leq i \leq m$, which indicates that the link e_i is in working order then its availability is $A_i = p[x_i]$. Events $x_1, x_2, \ldots, x_m$ are independent. If x_1 and x_2 are events then $x_1 + x_2$ are union of mutually exclusive events and $x_1 \cdot x_2$ the case of two independent events. The goal is to calculate the availability of the two terminals s and t as represented by a polynomial function of x elements:

$$A_{st}(x) = A_{st}(x_1, x_2, \ldots, x_m) . \tag{4}$$

which gives the exact availability between the two terminals when the appropriate variables x_k are replaced by the appropriate links availability A_k. To calculate the availability the two new operators is introduced:

- $\otimes$ is used to represent the series combination of two or more elements,
- $\oplus$ is used to represent the parallel combination of two or more elements.

The union of arbitrary events $x_1 \oplus x_2$ and "product" of arbitrary events $x_1 \otimes x_2$ cannot be developed from numerical values, because it depends on the individual statistical correlation or the correlation between two events as opposed to union of two mutually exclusive events, or the product of independent events whose probability is easily calculated [8]. Most of the algebraic approaches for availability exact calculation between the two terminals using the fact that $\otimes$ is commutative and the fact that $x_1 \otimes x_2 = x_i$ and attenuates any repetition of events in $\otimes$ product and authors replace them by ordinary product.

Firstly, the case where links a and b are connected in series is considered. If the link failures are independent, the availability of such a structure is equal to the availability of products A_a and A_b. However, there is a possibility that links contain some common elements. Suppose, for example, that link a contains components $x_1x_3x_7$ and link b components $x_2x_3x_5$. Let A denote the event where components 1, 3 and 7 are correct and if failures of components are mutually independent, availability is $p(A) = A_1 \cdot A_3 \cdot A_7$ and let B denote the event that the components 2, 3 and 5 are correct and the availability of B is equal to $p(B) = A_2 \cdot A_3 \cdot A_5$. Now AB is an event where components 1, 2, 3, 5 and 7 are correct and one link that replaces a and b will be marked as $x_1x_2x_3x_5x_7$:

$$\begin{aligned} x_1x_3x_7 \otimes x_2x_3x_5 &= x_1x_2(x_3 \otimes x_3)x_5x_7 = \\ &= x_1x_2x_3x_5x_7 . \end{aligned}$$

Replacing the variable by adequate availabilities gives the availability of a replacement link

$$A_s = p(AB) = A_1 \cdot A_2 \cdot A_3 \cdot A_5 \cdot A_7 .$$

Another operator $\oplus$ is applied to links that are connected in parallel, in this case, links a and b. If elements are working independently with an availability A_a and A_b their replacement link will have availability

$$A_a + A_b - A_a \cdot A_b .$$

This formula can be expanded to include possible dependence. For example, let link a include components $x_1x_3x_7$ and link b components $x_2x_3x_5$. Parallel structure will function properly if either link is correct, and the

$$p(A \cup B) = p(A) + p(B) - p(AB) ,$$

so that one link that replaces a and b will have the variable

$$\begin{aligned} &x_1x_3x_7 \oplus x_2x_3x_5 = \\ &= x_1x_3x_7 + x_2x_3x_5 - x_1x_2(x_3 \otimes x_3)x_5x_7 . \end{aligned}$$

As $x_i \otimes x_i = x_i$ then $x_3 \otimes x_3 = x_3$ so it gets

$$\begin{aligned} &x_1x_3x_7 \oplus x_2x_3x_5 = \\ &= x_1x_3x_7 + x_2x_3x_5 - x_1x_2x_3x_5x_7 . \end{aligned}$$

Replacing the variables again by the adequate availability gives the availability of alternative link

$$p(A \cup B) = A_1 \cdot A_3 \cdot A_7 + A_2 \cdot A_3 \cdot A_5 - A_1 \cdot A_2 \cdot A_3 \cdot A_5 \cdot A_7 .$$

General algebraic "sum" of two polynomials f and g is defined as:

$$f \oplus g = f + g - (f \otimes g) . \tag{5}$$

Let S represent the set of all polynomials that can arise by a combination of monomer using these two operators, then let $(S, \oplus, \otimes)$ form a distribution lattice with the smallest element 0 (zero polynomial) and the greatest element 1 (unit polynomial). For $f, g, h \in S$ to apply the following axioms [9]:

$$\begin{aligned} &f \oplus f = f \\ &f \oplus 0 = f \\ &f \oplus g = g \oplus f \\ &f \otimes f = f \\ &f \otimes 1 = f \\ &f \otimes g = g \otimes f \\ &f \otimes (g \otimes h) = (f \otimes g) \otimes h \\ &f \oplus (g \oplus h) = (f \oplus g) \oplus h \\ &f \oplus (g \otimes h) = (f \oplus g) \otimes (f \oplus h) \\ &f \otimes (f \oplus g) = f \\ &f \oplus (f \otimes g) = f \end{aligned} \tag{6}$$

In order to find a connection between the algebraic structures and network availability P_{st} is defined as a set of all

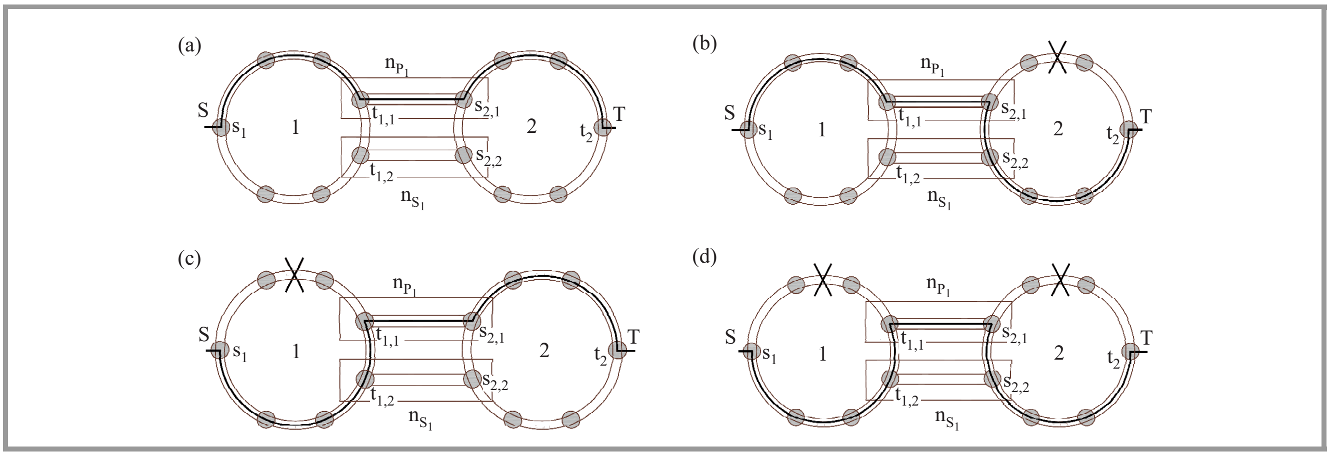

Fig. 1. Working paths and recovery: (a) without fault, (b) fault within the rings 2, (c) fault in side the ring 1, (d) fault with in two rings.

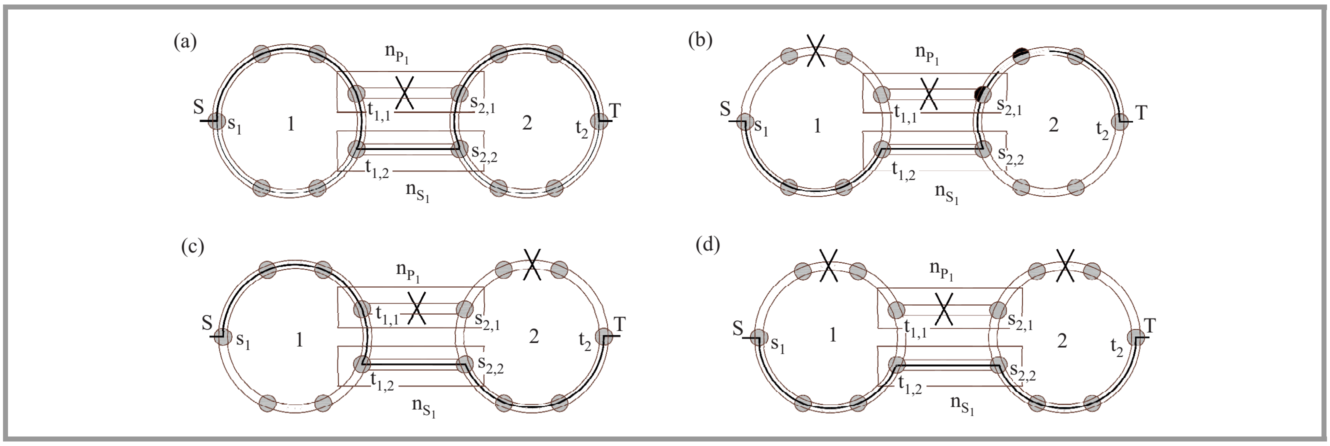

Fig. 2. Path of renewal: (a) without fault, (b) fault within the rings 1, (c) fault in side the ring 2, (d) fault with in two rings.

elementary *s*-*t* paths P in the network G. It also defines the value of path $v(P)$ as a variables product representing the links along the path P

$$v(P) = \otimes \prod \{x_k : k \in P\} . \tag{7}$$

Availability in this case is the sum of values $v(P)$ over all elementary paths from s to t

$$A_{st}(\mathrm{x}) = \oplus \sum_{P \in P_{st}} v(P) . \tag{8}$$

In the $\oplus$ operation each link was included as many times as the number of occurrences in the set P_{st}, of course, since this operator is $\otimes$, before applying the simple product, repetitions are eliminated.

3. Availability of Multi Ring Networks

Generally, in the multiple ring networks the transmitted signal between two nodes S and T passes N_R rings and interconnecting nodes. The path through, which the signal passes during normal operation is called a work path. When analyzing renewal paths, only those paths that include rings through, which the working path is passing are taken. Although there are more interconnection rings that could provide connectivity between nodes S and T in the case of failure on the working path, those are not analysed because path recovery requires considerable amount of time. In this way only subnets, which are comprised of interconnected rings over the working path are examined, while all other combinations are not analyzed.

In this paper, the models of availability were developed and expressions derived for the availability between two terminals of network consisting of two and three rings that are connected to two nodes. The expression valid for the network of two and three rings is also valid for multiple rings network comprised of N_R as demonstrated by mathematical induction.

3.1. Availability of Network of Two Rings

Another focus is the network comprised of two rings. As possible failure sites ring 1, ring 2 and the interconnecting

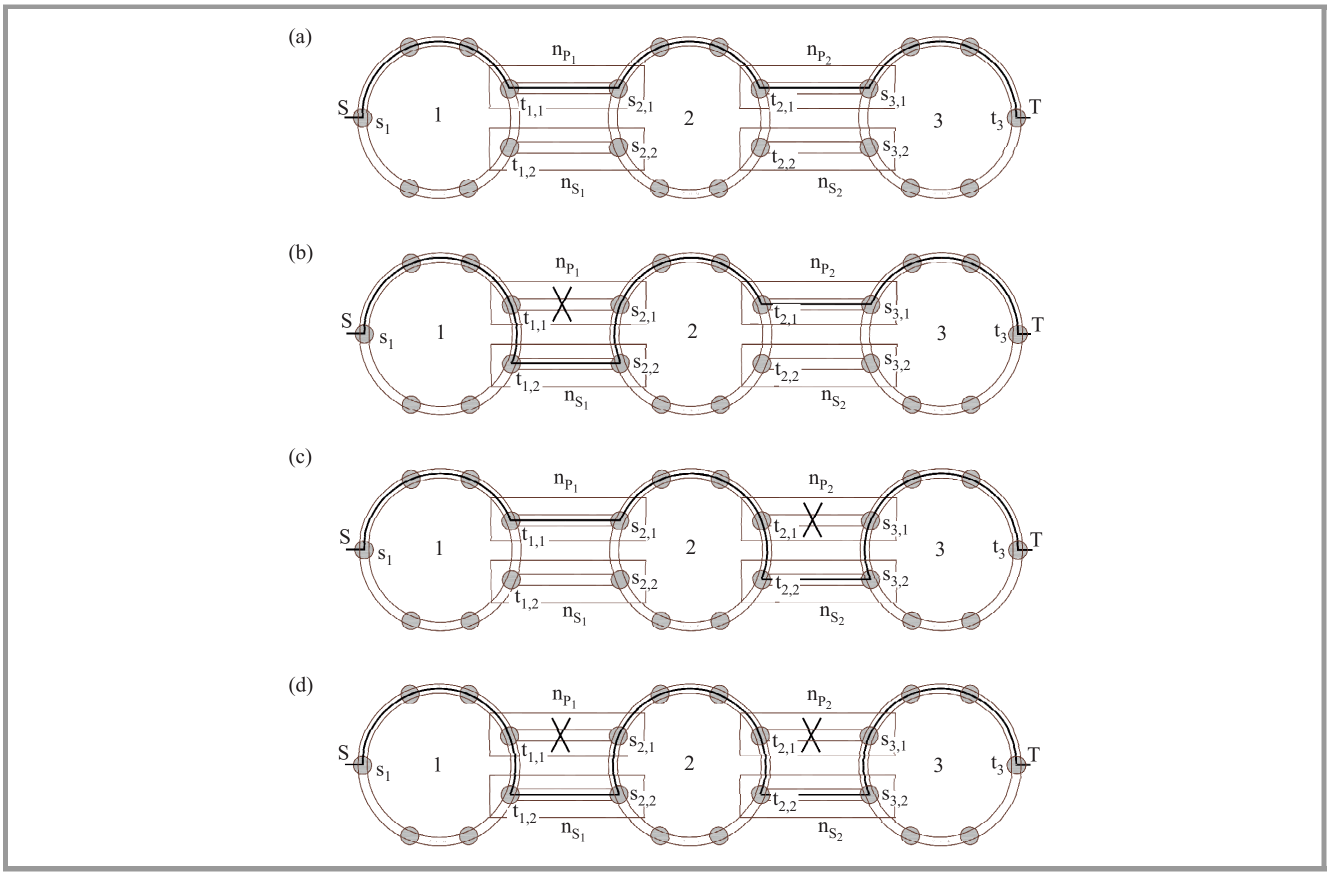

Fig. 3. Three ring network with: (a) no failures, (b) failure of the first interconnect node, (c) failure of another interconnect node, (d) failure of both interconnect nodes.

node are taken so that there is a total of 7 combinations or seven renewal paths:

$$\binom{3}{1}+\binom{2}{2}+\binom{3}{3}=3+3+1=7\,.$$

Renewal paths between nodes S and T need not be mutually exclusive because a parallel sum eliminates identical (duplicate) paths. For a network comprised of two rings there is a work path with no failures and 7 renewal paths which are the result of failures at work paths of rings 1 and 2 and primary interconnecting node. Suppose that each ring is comprised of N links where there are m working and that the connection between $t_{1,1}-t_{1,2}$, and $s_{2,1}-s_{2,2}$ is achieved with one link.

Figure 1 shows working path and recovery paths, but with always correct primary interconnection node.

Figure 2 shows the paths of renewal, but always with a faulty primary interconnect node.

The value of path in failure-free (P_0) case is

$$v_{st}(P_0)=(x_{l_{r1}})^m\otimes x_{P1}\otimes(x_{l_{r2}})^m\,,$$

where $x_{l_{r1}}$ and $x_{l_{r2}}$ denote links that belong to the first (index r_1) and second ring (index r_2).

For each of these 7 paths the renewals are also defined as the values of paths that are marked with $v_{st}(P_1)$ (Fig. 1b) to $v_{st}(P_7)$ (Fig. 2d).

Availability is a "parallel sum" of path values when there are no failures and values of all renewal paths, which are the consequence of failures, and therefore

$$A_{s_1t_2}(\mathrm{x})=\oplus\sum_{P_i\in P_{st}}v_{st}(P_i)\,.$$

Since the true availability between two nodes of an individual ring for the standard size of rings is approximately equal to the minimum availability [10], so in this case availability between the two terminals for the network composed of two rings is

$$A_{ST}(\mathrm{x})=(X_{\min_1}\cdot X_{\min_2})\otimes\left[x_{P1}+x_{S1}-(x_{P1}\otimes x_{S1})\right]\,.$$

Assuming that the rings and interconnecting nodes also have no common elements, sign $\otimes$ could be elided and the common sign of multiplication might be used

$$A_{ST}(\mathrm{x})=\left(\prod_{i=1}^{2}X_{\min_i}\right)\cdot\left[x_{P1}+x_{S1}-x_{P1}\cdot x_{S1}\right]\,.$$

3.2. Availability of Network of Three Rings

A similar analysis can be implemented for a network consisting of three rings as shown in Fig. 3, taking into account all the renewal paths that can occur as a result of failures of each ring parts, the interconnections of nodes and all their possible combinations. If the number of possible failures is 5, the number possible combinations total is

$$\binom{5}{1}+\binom{5}{2}+\binom{5}{3}+\binom{5}{4}+\binom{5}{5}=5+10+10+5+1=31\,.$$

To avoid examining all possible renewal paths, the authors take into consideration confirmed fact in Subsection 3.1 that all failures of individual rings work path renew within themselves, so that the analysis is brought down to the failures of interconnection nodes, which are only two and therefore the combinations number decreases significantly:

$$\binom{2}{1}+\binom{2}{2}=2+1=3\,.$$

If it is assumed that all the interconnecting nodes of the same type are equal (identical equipment) and assuming that the rings and interconnecting nodes have no common elements, simple multiplication sign can be written instead of sign $\otimes$

$$A_{ST}(\mathrm{x})=\left[\prod_{i=1}^{3}X_{\min_i}\right]\cdot\left[x_P+x_S-x_P\cdot x_S\right]^2.$$

If the analysis is conducted for a network consisting of $4,5,\ldots,N_R$ rings, there would be a general expression for the availability of two terminals of multiple ring network in which the rings are interconnected in two nodes:

$$A_{ST}(\mathrm{x})=\left[\prod_{i=1}^{N_R}X_{\min_i}(\mathrm{x})\right]\cdot\left[x_P+x_S-x_P\cdot x_S\right]^{N_R-1},\qquad(9)$$

which means that the availability between two nodes of multiple ring network is equal to the product of minimum availability of individual rings and the interconnections of nodes availability.

4. Numerical Results

Analysis results are presented on a network consisted of two rings to show that the difference is negligible when the actual and minimum availability for individual ring is calculated. Availability data for optical components shown in Table 1 are taken from [10].

Table 2 shows that for the typical size of rings (6–10 nodes), difference between the actual availability and the one obtained by taking a minimum availability for each individual ring are in the worst case of 0.0245 min/year, which is negligible so that the Eqs. (8) and (9) can be used to calculate availability. Note: W is the number of wavelength channel.

Even for larger rings with 14 and 16 nodes, those differences were of 0.1 min/year, which is also negligible. This is especially important for SLA contracts which set a minimum availability threshold and charge penalties for exceeding them.

Table 1
Availability data for optical components

Component	Symbol	Failure rate [FIT]
Booster amplifier	BOA	3200
Line amplifier	LOA	3200
Preamplifier	POA	3200
Multiplexer	MUX	25×W
Demultiplexer	DEMUX	25×W
Optical switch	OSW	1000
Fix transmitter	TRX	186
Tunable transmitter	TX	745
Fix receiver	RX	70
Tunable receiver	RCX	470
Switch	SW	50
Splitter	SPL	50
Cable (per km)	OC	100

5. Conclusion

This paper presents a new approach to modeling the availability of multiple ring network using an algebraic formulation of working with numeric values. It is shown that the paths need not be mutually exclusive because the use of operator $\otimes$ eliminates duplicate routes. The analysis showed that the availability calculation of multiple ring network, in which the rings are connected to two nodes, is actually reduced to a product of minimum availability of individual rings and the availability of the nodes interconnections. This is done with some realistic assumptions and these are: the rings do not have common elements, the interconnection nodes also have no elements in common and neither rings nor interconnecting nodes have also any common elements which are the most common cases in practice, otherwise the protection would not make sense. The analytical results have confirmed the theoretical analysis so that the derived general expression for the availability of the two terminals can be used to analyze and evaluate the multiple ring networks availability.

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Table 2
Comparison of availability between S and T when using the actual availability of the ring and the minimum availability of the ring

Number of ring nodes			The actual availability	Minimum availability	Difference MDT [min/year]
	$N=6$	$N=10$			
No. links of working path	$m=1$	$m=7$	0.9999752896	0.9999752482	0.0217901963
No. links of working path	$m=3$	$m=8$	0.9999752948	0.9999752482	0.0245122162
No. links of working path	$m=5$	$m=3$	0.9999752741	0.9999752482	0.0136182825
	$N=14$	$N=116$			
No. links of working path	$m=2$	$m=13$	0.9999750980	0.9999748389	0.1361543751

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