Paper

Analysis of the System with Vacations under Poissonian Input Stream and Constant Service Times

Maciej Sosnowski[a,b] and Wojciech Burakowski[a,b]

[a] National Institute of Telecommunications, Warsaw, Poland

[b] Warsaw University of Technology, Warsaw, Poland

Abstract—In the paper approximate formulas for the mean waiting times and the buffer dimensioning in the system with vacations fed by the stream of Poissonian type with constant service times is shown. Furthermore, in the considered system the time intervals of the availability/not-availability of the service are constant and are run alternately according to the assumed cycle. More precisely, presented approach begin with derivation of the mean waiting times and, on the basis of this, the required buffer size for guaranteeing the losses less than predefined value is estimated. The accuracy of the presented analytical formulas is on a satisfactory level. The formulas were used for the System IIP dimensioning.

Keywords—approximation analysis, buffer dimensioning, mean waiting times, system with vacations.

1. Introduction

The paper studies the system with vacations fed by the stream of Poissonian type with constant service times. Furthermore, in the considered system the time intervals of the availability/not-availability of the service are constant and are run alternately according to the assumed cycle. The analysis of this system focuses on derivation of the analytical formulas to estimate the mean waiting times and next, on the basis thereof, to estimate required buffer size to satisfy assumed predefined level of losses.

The considered system well models a part of the IIP System [1] based on virtualized network infrastructure that corresponds to the organization of virtual links established for particular Parallel Internets (PIs). These Parallel Internets should work in isolation. For establishing separate virtual links delegated to particular PIs, access to a physical link by a cycle-based scheduler, as shown in Fig. 1 is managed.

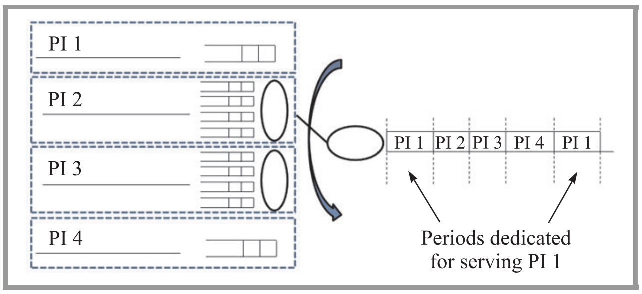

Fig. 1. Cycle-based scheduler for creating virtual links for 4 Parallel Internets working in isolation.

According to the best knowledge of the authors, such system was not analyzed in the literature. Most of papers, as i.e. [2], [3], [4], deal with TDMA systems, in which data are transmitted only in the chosen time-slots.

2. The System

2.1. The System with Vacations

The considered queuing system is depicted in Fig. 2. The system belongs to the family of systems with vacations. It means that periodically the system is in active and vacation periods. During the active periods (T_A) the packets are served while during the vacation periods (T_V) the service is not available. Moreover, we assume an infinite buffer size in the system. The queuing discipline is assumed to be FIFO.

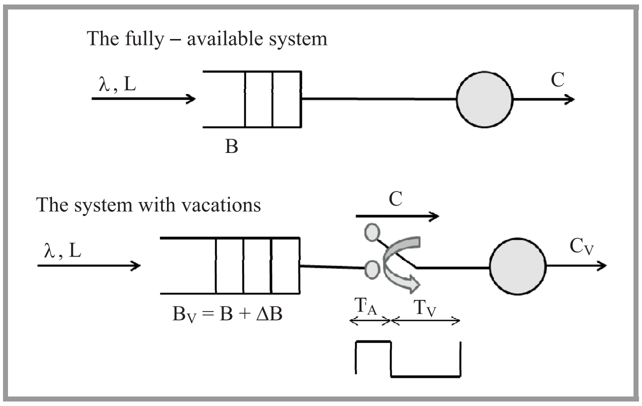

Fig. 2. Comparison of the systems: λ – arrival rate (Poissonian stream), L – packet size, B and B_V – buffers sizes, T_A – active period, T_V – vacation period, C and C_V – the output links rates.

Additional assumptions of the system are the following:

- the packets arrive to the system according to the Poisson process with the rate λ,
- the active (T_A) and vacation (T_V) periods are constant and they alternate,

- the link capacity is equal to C_V b/s,
- packet size (L) and, as a consequence, service times (h_v) of the packets are constant ($h_v = L/C_V$),
- the length of the active period T_A is multiples of h_v ($T_A = nh_v,\ n = 1, 2, \dots$).

In this system, the available capacity for the considered stream, denoted by C, is:

$$C = \frac{C_V T_A}{T_A + T_V}. \tag{1}$$

2.2. Fully Available System

This analysis refers to the fully available system with Poissonian input and deterministic service time, the system $M/D/1$. Especially, the formula for mean waiting times in such system will be exploited when the arrival rate λ, packets size L, and output link C (see Eq. (1), equivalent to the available link rate in system with vacations) are the same for both considered system. It should be noted that the average load (in the system with vacation, during the active period) ρ in both systems is also the same. The difference between the system with and without vacations is the service time. In the fully available system the service time is $h = L/C$.

For the M/D/1, the mean waiting time $E[W_F]$ for well-known Pollachek-Khinchin formula is:

$$E[W_F] = \frac{\rho h_{res}}{1-\rho}, \tag{2}$$

where $\rho = \lambda h$ and h_{res} is the residual service time (in the case of $h = constant$, $h_{res} = h/2$).

3. Analysis

3.1. Mean Waiting Time in the System with Vacations

A brief description of the approach to calculate mean waiting times for the considered system with vacations can be found in [5].

The analysis starts from the moment of the test packet arrives to the system. Thanks to the PASTA (Poisson Arrivals See Time Averages) principle, this test packet sees the system at a random moment. This packet can arrive when the system is on the active period or on the vacation period. When the packet arrives during the vacation period it cannot be served immediately even if there are no other packets in the system, but it should wait for a transmission at least, if no other packets are in the system, by the remaining time of the period T_V. On the other hand, when the packet arrives during the active period it can be served immediately (when there are no other tasks in the system) when the remaining part of this period is not less than h_v. This period is called a pure active period. Let's define:

$$P_V = \frac{T_V}{T_V + T_A}, \quad P_{A'} = \frac{T_A - h}{T_V + T_A}, \quad P_{h_v} = \frac{h_v}{T_V + T_A}, \tag{3}$$

where P_V, $P_{A'}$, P_{h_v} denotes the probability that a packet arrives during the vacation period, the active period (without the last part equal h_v), and the last part (equal h_v) of the active period, respectively.

The approximate formula for the mean waiting time has the following form:

$$\begin{aligned} E[W_V] \ = \ & P_{A'} E[W_F] + P_V (T_{V_{res}} + E[W_F]) + \\ & + P_{h_v} (h_{v_{res}} + T_V + E[W_F]), \end{aligned} \tag{4}$$

where $E[W_F]$ is calculated by Eq. (2), $T_{V res}$ is the residual time of the vacation time ($T_{V res} = \frac{T_V}{2}$) and $h_{v_{res}}$ is the residual time of the service packet time ($h_{v_{res}} = \frac{h_v}{2}$).

Equation (4) can be simplified to:

$$E[W_V] = E[W_F] + \frac{(T_V + h_v)^2}{2(T_A + T_V)}. \tag{5}$$

For the limit case, when $T_V = 0$, the mean waiting time is the same as in the fully available system. On the other hand, when T_V tends to infinity, the value of the mean waiting time also tends to infinity.

Unfortunately, the Eq. (5) is not proved in a clear mathematical way, but it is only deduced. It was assumed that if the task arrives during the pure active period, it expects a similar delay as in the fully available reference system. It happens with probability $P_{A'}$. Furthermore, when the task arrives at the periods when it cannot be transmitted immediately (even when no other tasks are in the system), it should wait for its transmission when the active period starts. So, in this case, we deduce that the packet will wait by the time to the moment when the active period starts plus the service times of the packets being in the system already.

Equation (4) and (5) does not take into account the situation, e.g., when a task should wait a number of active periods until it starts transmission. Therefore, waiting times calculated from Eq. (5) will be lower than exact value.

Nevertheless, the Eq. (5) is relatively simple and it takes into account in a direct way the impact of the length of active and vacation periods on the packet delay.

In Tables 1 and 2, the values of the mean waiting times for the systems with vacations differing in lengths of active and vacation periods under different traffic load ρ and various T_A/T_V relations is presented.

It can be observed that for cases presented in Table 1, analytical results are very close to simulation results and the difference is only by few percent. For cases presented in Table 2, a bit less accuracy of the analytical results compared to the simulation can be observed, but the dif-

Table 1
Comparison of mean waiting times (short cycles)

	$T_A/T_V = 2h_v/4h_v$			$T_A/T_V = 10h_v/20h_v$		
ρ	Anal.	Sim.	Diff.	Anal.	Sim.	Diff.
0.2	2.5	2.4	3%	7.7	7.9	–2%
0.4	3.1	3.0	4%	8.4	8.6	–3%
0.6	4.3	4.2	3%	9.6	9.8	–2%
0.8	8.1	7.9	3%	13.4	13.3	0%
0.9	15.6	15.4	1%	20.9	20.6	1%
0.94	25.6	25.2	2%	30.9	30.6	1%
0.96	38.1	37.3	2%	43.4	43.5	0%

Table 2
Comparison of mean waiting times (long cycles)

	$T_A/T_V = 50h_v/100h_v$			$T_A/T_V = 100h_v/200h_v$		
ρ	Anal.	Sim.	Diff.	Anal.	Sim.	Diff.
0.2	34.4	36.5	–6%	67.7	72.2	–6%
0.4	35.0	39.4	–11%	68.3	77.8	–12%
0.6	36.3	42.7	–15%	69.6	84.4	–18%
0.8	40.0	47.5	–16%	73.3	92.3	–21%
0.9	47.5	54.6	–13%	80.8	100.1	–19%
0.94	57.5	63.9	–10%	90.8	110.0	–17%
0.96	70.0	76.1	–8%	103.3	122.2	–15%

ference is still on the satisfactory level. The difference is about 15% for most of the studied cases. The accuracy of Eq. (5) was also verified for other values of cycle durations and the results were similar to the ones presented above.

We can conclude that the Eq. (5) gives very accurate results for the system with vacations if at least one of these two conditions is met: cycle is short ($\sim$15 h max.) or T_A/T_V ratio is small ($\sim$1/4 max.). The results are also accurate if both conditions are close to these borders (i.e., $T_A/T_V = 10h_v/20h_v$).

3.2. *Buffer Dimensioning*

At present, an approximation for buffer dimensioning in the system with vacations is shown. The target is to dimension the buffer size as small as possible to assure that packet losses are less than a predefined value P_{loss}, e.g., $P_{loss} \leq 10^{-3}$. In order to do it in an exact way, the queue size distribution should be known. The presented approach assumes that only knowledge of the mean waiting times in the system, as calculated from Eq. (5) is available. Of course, the well-known Marcov's inequality can be used, see Eq. (6)), but it was checked that it gives an essential over-dimensioning of the buffer size and, as a consequence, the results are not reported in the paper.

Marcov's inequality:

$$P(X \geq n) \leq \frac{E[X]}{n}, \qquad (6)$$

where $E[X]$ is the mean value of the random variable X.

Therefore, the approach investigated in the paper assumes an approximation of the queue size distribution, which is described by only one parameter. In this case, queue size distribution in the system with vacation can be approximated by a M/M/1 queue size distribution.

3.2.1. Queue State Distribution for the System with Vacations – Simulation Results

In this subsection the queue state distribution for selected system with vacations differing in the lengths of active and vacations periods is shown.

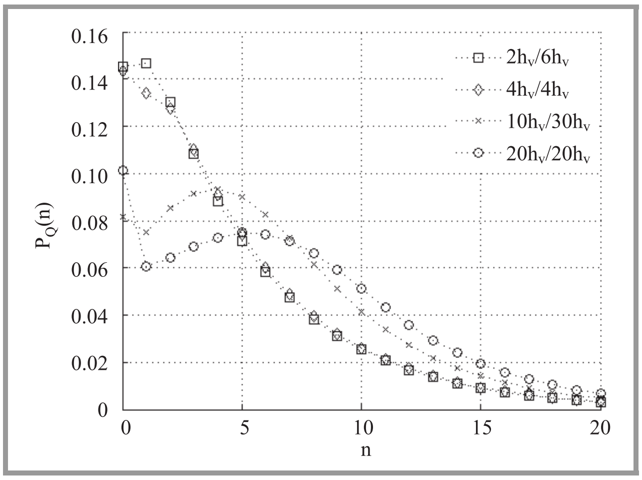

Fig. 3. Queue state distribution obtained from simulation ($n = 0 \ldots 20$), $\rho = 0.9$.

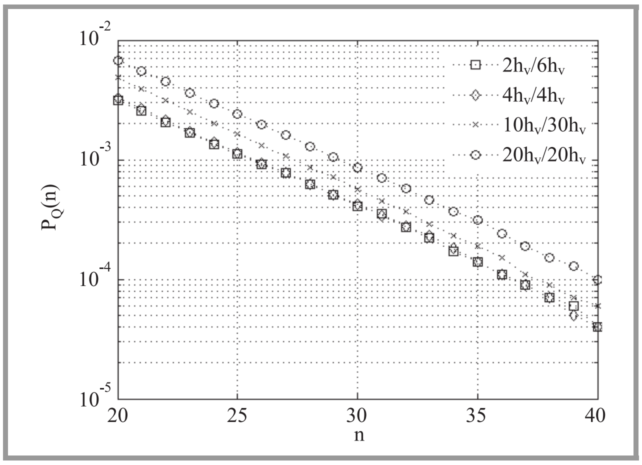

Fig. 4. Queue state distribution obtained from simulation ($n = 20 \ldots 40$), $\rho = 0.9$.

In Figs. 3 and 4 one can observe queue state distribution in the system with vacations for different values of T_A/T_V.

In the presented curves the confidence intervals at the 95% level are negligibly small and they are not depicted. The simulation results show that the approximation of the presented characteristics by the geometric distributions is justified although one can observe the differences in the first phase of the curves.

3.2.2. Approximation by the M/M/1 Queue Size Distribution

As it is known, the system state distribution follows the geometric distribution in the $M/M/1$ system. Queue is empty if 0 or 1 task in system, therefore the queue state distribution has the following form:

$$\begin{cases} P_Q(0) = P(0) + P(1) \\ P_Q(n) = P(n+1),\ n > 0 \end{cases} \tag{7}$$

where: $P(n) = \rho_g^n(1-\rho_g)$ – probability, that system is in the n state, ρ_g – server load.
Therefore,

$$\begin{cases} P_Q(0) = 1 - \rho_g^2 \\ P_Q(n) = \rho_g^{n+1}(1-\rho_g),\ n > 0 \end{cases} \tag{8}$$

So, the mean queue state in the $M/M/1$ system is:

$$E[n] = \sum_{n=0}^{\infty} n P_Q(n) = \frac{\rho_g^2}{1-\rho_g}. \tag{9}$$

Then ρ_g (the parameter of the M/M/1 queue state distribution) can be calculated from:

$$\rho_g = \frac{\sqrt{(E[n])^2 + 4E[n]} - E[n]}{2}, \tag{10}$$

where $E[n] = E[n_V] = \lambda E[W_V]$ and $E[W_V]$ is done by Eq. (5).

In Figs. 5–8 the comparisons between queue state characteristics obtained by the simulation and by approximation of the M/M/1 queue state distribution is shown. These results correspond with the exemplary system with vacations when $T_A/T_V = 10h_v/30h_v$ with the load of $\rho = 0.6$ and $\rho = 0.9$.

One can observe that the approximation by M/M/1 queue state distribution gives larger values for the tail of the distribution. It is important since we want to dimension buffer size for rather low values of losses, e.g., 10^{-3} or less.

After some algebra, the final formula for buffer size dimensioning, is

$$B = \left\lceil \frac{\ln(P_{loss})}{\ln(\rho_g)} - 1 \right\rceil, \tag{11}$$

where $\lceil x \rceil$ denotes the minimum integral value greater or equal x.

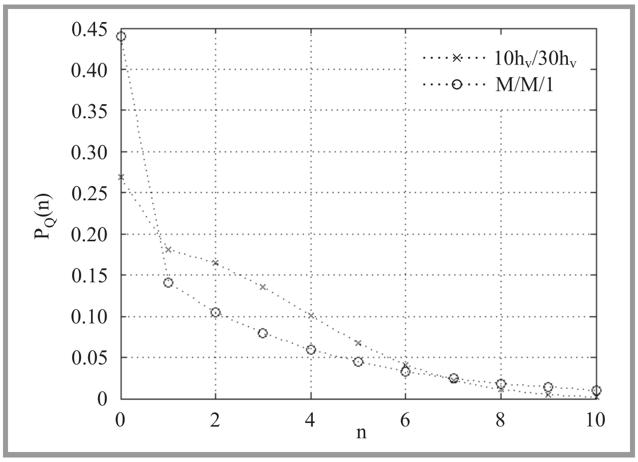

Fig. 5. Queue state distribution obtained from simulation compared with M/M/1 queue state distribution ($n = 0\ldots10$), $\rho = 0.6$.

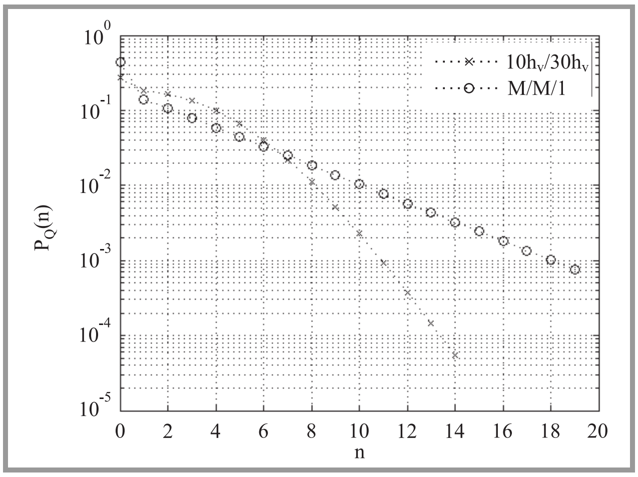

Fig. 6. Queue state distribution obtained from simulation compared with M/M/1 queue state distribution ($n = 0\ldots20$), $\rho = 0.6$.

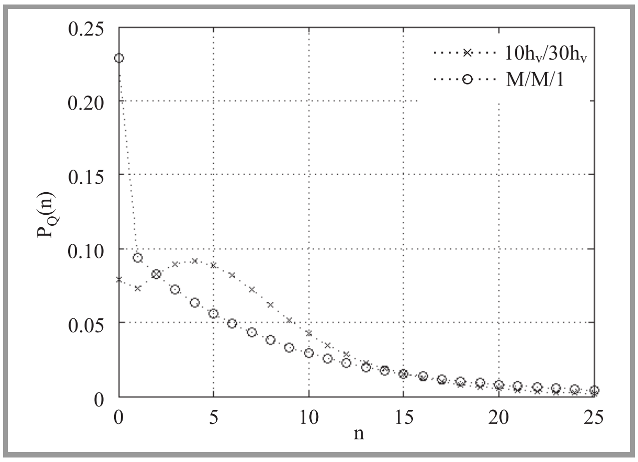

Fig. 7. Queue state distribution obtained from simulation compared with M/M/1 queue state distribution ($n = 0\ldots25$), $\rho = 0.9$.

For comparison, the formula to dimension buffer size in the case of REM (Rate Envelope Multiplexing) multiplexing [6] is

$$\rho = \frac{2B}{2B - ln(P_{loss})}, \tag{12}$$

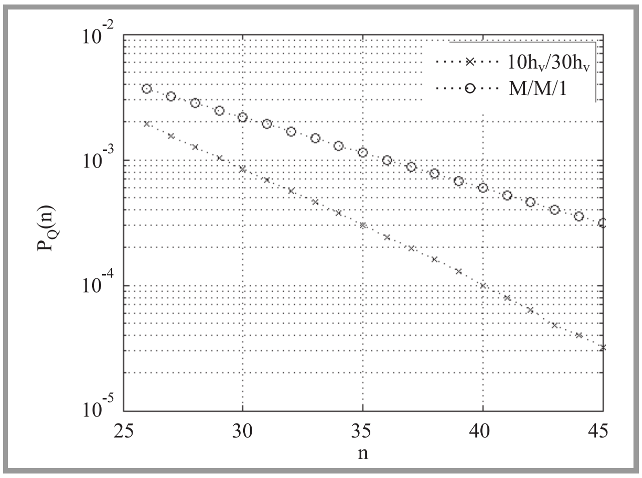

Fig. 8. Queue state distribution obtained from simulation compared with M/M/1 queue state distribution $(n = 26 \dots 45)$, $\rho = 0.9$.

and it can be transformed to

$$B = \left\lceil \frac{ln(P_{loss})}{2 - \frac{2}{\rho}} \right\rceil . \qquad (13)$$

3.3. Results

In this section results for buffer dimensioning in the system with vacations REM multiplexing are presented. In Table 3 we show the results (B) of required buffer size for the system without vacations and REM multiplexing. The buffer size $B_{opt.}$ is obtained by simulation and $B_{over.}$ – indicates the relative error.

Table 3
Comparison of buffer size for the M/D/1 system and $P_{loss} = 10^{-3}$

ρ	$B_{opt.}$	B	$B_{over.}$ [%]
0.6	6	6	0
0.8	12	14	17
0.9	22	32	45
0.95	39	65	67

$B_{opt.}$ – the buffer size that provides the loss probability on the 10^{-3} level, result from the simulation
B – the buffer size calculated from Eq. (11)
$B_{over.}$ – percentage oversize of the buffer B

Table 4 presents the results of the required buffer size for the selected systems with vacations assuming $P_{loss} = 10^{-3}$. The reported results say that presented approach gives always the over estimation of the required buffer size. This overestimation is about 100%. Thus, the results are rather positive taking into account that the method is based on the approximation of the mean waiting time value only.

Table 4
Measured loss probability in the system with vacations

ρ	$E[n]$ sim.	$E[n]$ anal.	ρ_g	$B_{opt.}$	B	$B_{over.}$ [%]	P_{loss}
$T_A/T_V = 4h_v/4h_v$							
0.6	0.92	0.92	0.6	8	13	63	4.30E-06
0.8	2.24	2.22	0.75	14	24	71	7.90E-06
0.9	4.76	4.75	0.85	24	42	75	1.87E-05
0.95	9.78	9.78	0.91	40	73	83	3.03E-05
$T_A/T_V = 2h_v/6h_v$							
0.6	0.86	0.91	0.59	7	13	86	3.20E-06
0.8	2.12	2.21	0.74	13	22	69	1.63E-05
0.9	4.64	4.74	0.85	23	42	83	1.40E-05
0.95	9.27	9.75	0.91	39	73	87	2.24E-05
$T_A/T_V = 20h_v/20h_v$							
0.6	2.49	2.1	0.74	13	22	69	0.00E+00
0.8	4.38	3.8	0.82	19	34	79	6.00E-07
0.9	7.18	6.53	0.88	29	54	86	5.10E-06
0.95	12.35	11.64	0.93	45	95	111	5.20E-06
$T_A/T_V = 10h_v/30h_v$							
0.6	2.22	2.25	0.75	12	24	100	0.00E+00
0.8	3.87	4	0.83	17	37	118	0.00E+00
0.9	6.34	6.75	0.88	27	54	100	4.00E-06
0.95	12.06	11.88	0.93	43	95	121	3.40E-06

ρ_g – parameter of M/M/1 queue state distribution calculated from Eq. (10)
P_{loss} – measured loss probability for the buffer B (95% confidence intervals are on 10^{-7} level)

4. Summary

In the paper the analysis of the system with vacations fed by Poissonian stream was presented, constant service times and constant length of active and vacation periods. For this system the analytical approximate formulas for the mean waiting times and the buffer dimensioning was shown. The analytical results were compared with the simulation. The accuracy of the approximation is satisfactory.

The described methods were used to dimension virtual links in the IIP System build by the virtualization of the network infrastructure.

References

[1] W. Burakowski *et al.*, "Virtualized network infrastructure supporting co-existence of Parallel Internets", in *Proc. 13th ACIS Int. Conf. Softw. Engin., Artif. Intell., Netw. Parallel/Distrib. Comput. SNPD 2012*, Kyoto, Japan, 2012.

[2] S. S. Lam, "Delay analysis of a Time Division Multiple Access (TDMA) channel", *IEEE Trans. Commun.*, vol. 25, pp. 1489–1494, 1977.

[3] K. T. Ko and B. Davis, "Delay analysis for a TDMA channel with contiguous output and Poisson message arrival", *IEEE Trans. Commun.*, vol. COM-32, no. 6, pp. 707–709, 1984.

[4] I. Rubin, "Message delays in FDMA and TDMA communication channels", *IEEE Transactions Commun.*, vol. COM-27, pp. 769–777, 1979.

[5] M. Sosnowski and W. Burakowski, "Evaluation of mean waiting time in the system with vacations", Student Poster Session, *24th International Teletraffic Congress ITC 24*, Kraków, Poland, 2012.

[6] J. Roberts (Ed.), Information technologies and science: COST 224: *Performance evaluation and design of multiservice networks*, EUR 14152 en, 1992.

Maciej Sosnowski received his M.Sc. degree in Telecommunications from Warsaw University of Technology, Poland, in 2012. Since 2012 he has been Ph.D. student. He is a member of the TNT research group.

E-mail: M.Sosnowski3@itl.waw.pl
National Institute of Telecommunications
Szachowa st 1
04-894 Warsaw, Poland

E-mail: M.Sosnowski@tele.pw.edu.pl
Faculty of Electronics and Information Technology
Warsaw University of Technology
Nowowiejska st 15/19
00-665 Warsaw, Poland

Wojciech Burakowski received his M.SC. Ph.D. and D.Sc. degrees in Telecommunications from Warsaw University of Technology in 1975, 1982 and 1992, respectively. Now he works as Full Professor at the Institute of Telecommunications, Warsaw University of Technology and at the National Institute of Telecommunications, Warsaw, as an R&D Director. He also leads the TNT research group (tnt.tele.pw.edu.pl). Since 1990 he has been involved in several COST and EU Framework Projects. He is a member of Telecommunication Section of the Polish Academy of Sciences and an expert in 7 FR Programme. He was a chairman and a member of many technical programme committees of national and international conferences. He is the author or co-author of about 200 papers published in books, international and national journals and conference proceedings and about 80 technical reports. He supervised 15 Ph.D. dissertations. His research areas include new networks techniques, ATM, IP, heterogeneous networks (fixed and wireless), network architecture, traffic engineering, simulation techniques, network mechanisms and algorithms, and recently Future Internet. Currently, he leads the strategic national project "Future Internet Engineering" (www.iip.net.pl).

E-mail: W.Burakowski@itl.waw.pl
National Institute of Telecommunications
Szachowa st 1
04-894 Warsaw, Poland

E-mail: wojtek@tele.pw.edu.pl
Faculty of Electronics and Information Technology
Warsaw University of Technology
Nowowiejska st 15/19
00-665 Warsaw, Poland