

# Single Hysteresis Model for Limited-availability Group with BPP Traffic

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**Abstract**—This paper presents a single hysteresis model for limited-availability group that are offered Erlang, Engset and Pascal traffic streams. The occupancy distribution in the system is approximated by a weighted sum of occupancy distributions in multi-threshold systems. Distribution weights are obtained on the basis of a specially constructed Markovian switching process. The results of the calculations of radio interfaces in which the single hysteresis mechanism has been implemented are compared with the results of the simulation experiments. The study demonstrates high accuracy of the proposed model.

**Keywords**—*hysteresis mechanism, limited-availability group, multiservice BPP traffic, threshold models, WCDMA radio interface.*

## 1. Introduction

Many network systems make use of traffic management mechanisms that aim at an increase in the traffic capacity of the network. Such mechanisms are to be primarily found in access networks that are characterized by low capacity of resources. A good example of the above is provided by, for example, 2G, 3G, and 4G radio access networks in which radio interface capacities are very limited. Traffic management mechanisms in these systems usually employ such mechanisms as [1]: resource reservation, partial limitation of resources, priorities, traffic overflow, non-threshold compression and threshold compression. The operation of the reservation mechanism is based on making the capabilities of the reservation of resources for pre-selected call classes dependent on the load level of the system [2], [3]. Many operators take advantage of the mechanism of partial limitation of resources that limits the number of serviced calls of appropriate traffic classes to a predefined value [4]. Priorities are designated to particular classes of calls. Prioritized calls can – in the case of the lack of free resources – effect a termination of service for calls with lower priority [1]. The overflow mechanism is one of the oldest mechanisms used in telecommunications [5], [6]. When the mechanism applies, calls that cannot be serviced in a given system due to its current occupancy level are redirected to other systems that still have free resources. Non-threshold compression is, in turn, based on a possibility of making the throughput of serviced calls of selected classes decreased in order

to obtain free resources for servicing new calls [1]. This mechanism forms a basis of the High Speed Packet Access technology (HSPA) in Universal Mobile Telecommunications System (UMTS) networks [7].

In the threshold compression mechanism, the bit rate allocated to a new call depends on the load of the system. The mechanism is used to service elastic and adaptive traffic [8]. The first model of a threshold system, the so-called Single Threshold Model (STM), was devised in [9] and concerned a system that was called Single Threshold System (STS). Works [8], [10] considers systems with a number of independent thresholds, the so-called Multi Threshold Systems (MTS) and the corresponding analytical models, the so-called Multi Threshold Models (MTM). Paper [11], describe a variant of the single-threshold system – Single Hysteresis System (SHS) and the corresponding analytical model – Single Hysteresis Model (SHM). In SHS, two thresholds, in place of one, are introduced. The operation of each of the thresholds is dependent on the direction of changes in the load in the system. The introduction of hysteresis is followed by a more stable operation of the system, which can be proved by a decreased number of transitions between areas with high and low load.

The present paper for the first time proposes a SHM for limited-availability group [12] with traffic streams of BPP type. In the paper [13] a SHM was presented only for full-availability group. The very name – BPP [4], [10] stems from the names of the types of call streams, (Bernoulli, Poisson and Pascal) that comprise Engset, Erlang and Pascal traffic, respectively.

The paper is structured as follows. Section 2 presents analytical models of the BPP traffic. Section 3 discusses STM and structure of limited-availability group, whereas Section 4 describes SHS for limited-availability group with BPP traffic. In Section 5, the results of the analytical calculations are compared with the results of simulation experiments of two different structures of systems. Section 6 presents the conclusions resulting from the study.

## 2. Multi-Service BPP Traffic

Multi-service traffic is a mixture of different traffic streams that are differentiated from one another by the number of allocation units, the so-called Basic Bandwidth Units (BBU) [3] that are necessary to set up a connec-

tion in the system. Traffic streams can be generated by an infinite (Erlang) or finite (Engset and Pascal) number of traffic sources. The intensity of Erlang traffic of class  $i$ , generated in the occupancy state of the system  $n$  BBUs, can be expressed by the following formula:

$$A_i(n) = A_i = \lambda_i / \mu_i = \text{const}, \quad (1)$$

where:  $\lambda_i$  – the average call intensity of calls of class  $i$ ,  $\mu_i$  – the average intensity of service of calls of class  $i$ .

The intensity of Engset traffic of class  $j$  and that of Pascal traffic of class  $k$  depend on the state of the system and is defined in the following way [10]:

$$A_j(n) = [N_j - y_j(n)] \alpha_j = [N_j - y_j(n)] \frac{\gamma_j}{\mu_j}, \quad (2)$$

$$A_k(n) = [N_k + y_k(n)] \alpha_k = [N_k + y_k(n)] \frac{\gamma_k}{\mu_k}, \quad (3)$$

where:  $N_c$  – the number of traffic sources of class  $c$ <sup>1</sup>,  $y_c(n)$  – the number of calls of class  $c$ , serviced in state  $n$ ,  $\alpha_c$  – traffic intensity from one free source of class  $c$ :

$$\alpha_c = \gamma_c / \mu_c, \quad (4)$$

where  $\gamma_c$  is the call intensity of calls from one free source of class  $c$ .

### 3. Single Threshold System

#### 3.1. Single Threshold System – Working Idea

Assume that in the system for pre-defined call classes one threshold  $Q$ , has been introduced in [9]. Figure 1 shows the operation of the system with single threshold with the

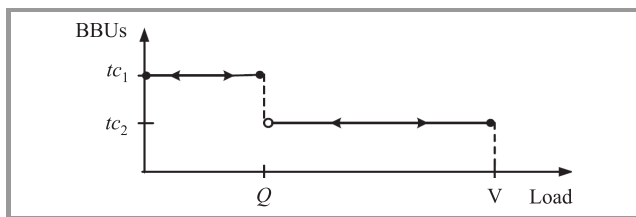


Fig. 1. Single Threshold System – working idea.

example of calls of one class  $c$ . If the load of the system is lower than the adopted value  $Q$  ( $0 \leq n \leq Q$ ), the Call Admission Control (CAC) function allows for a new call of class  $c$  with the maximum number of BBUs, equal to  $t_{c,1}$ , to be serviced. Following an increase in the load in the system and after exceeding the threshold  $Q$ , within the area of

<sup>1</sup> In the adopted notation, the indexes  $i$ ,  $j$ , and  $k$  are used to denote classes of Erlang, Engset and Pascal traffic, respectively. The index  $c$  is in turn used to consider traffic related to any traffic class.

maximum load ( $Q < n \leq V$ ), the CAC function admits for service a call of class  $c$  with the minimum number of BBUs, equal to  $t_{c,2}$ .

#### 3.2. Model of Limited-availability Group

The limited-availability group is the model of communication system that consists of  $k$  identical separated links [12]. Each link has the capacity equal to  $v$  BBUs. Thus, the total capacity of the system  $V$  is equal to  $V = kv$  BBUs. The system services a call – only when this call can be entirely carried by the resources of an arbitrary single link. Thus, limited-availability group is an example of the system with state-dependent service process. Figure 2 shows the model of the limited-availability group [10].

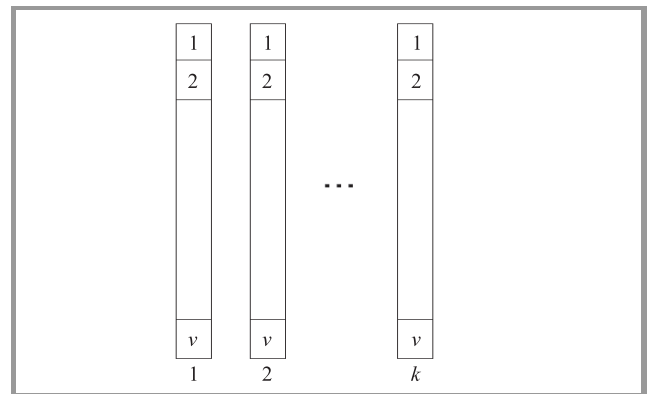


Fig. 2. Model of the limited-availability Group.

#### 3.3. Adaptive and Elastic Traffic in SHS

In systems servicing the so-called adaptive traffic [8] the change applies only to the number of BBUs necessary to set up a connection of a given class. The assumption is that traffic of this type requires sending of all data, while a decrease in the number of allocated BBUs will be followed by a deterioration of the Quality of Service (QoS) parameters. A good example of the above is the voice service “full-rate” and “half-rate” in the GSM network. Elastic traffic [8] requires all data to be transferred, thus a decrease in the allocated number of BBUs will be followed by an increase in the service time, i.e., the parameter  $1/\mu_c$ . HSDPA traffic in the UMTS network is an example of the above. Thus, the service of elastic traffic causes the value of offered traffic in particular load areas to be changed. Therefore, in the case of the service of elastic traffic, Formulas (1), (2) and (3) can be rewritten in the following way:

$$A_{i,s}(n) = A_i = \lambda_i / \mu_{i,s} = \text{const}, \quad (5)$$

$$A_{j,s}(n) = [N_j - y_{j,s}(n)] \alpha_{j,s} = [N_j - y_{j,s}(n)] \frac{\gamma_j}{\mu_{j,s}}, \quad (6)$$

$$A_{k,s}(n) = [N_k + y_{k,s}(n)] \alpha_{k,s} = [N_k + y_{k,s}(n)] \frac{\gamma_k}{\mu_{k,s}}, \quad (7)$$

where  $s$  indicates load area. We can distinguish two load areas in STM:  $s = 1$  for  $n \in \langle 0; Q \rangle$  and  $s = 2$  for  $n \in \langle Q; V \rangle$ .

#### 3.4. Occupancy Distribution in Limited-availability Group with STM and BPP Traffic

The occupancy distribution in the limited-availability group with single threshold mechanism and BPP traffic can be determined on the basis of the model worked out in [9] for STM with Erlang traffic and in [10] for MTM with BPP traffic. According to this model, the occupancy distribution in considered model can be rewritten as follows:

$$n[P_n]_Q^{(V)} = \sum_{s=1}^2 \left\{ \sum_{i \in M_1} A_{i,s}(n-t_{i,s}) t_{i,s} \sigma_{i,s,Total}(n-t_{i,s}) [P_{n-t_{i,s}}]_Q^{(V)} + \sum_{j \in M_2} A_{j,s}(n-t_{j,s}) t_{j,s} \sigma_{j,s,Total}(n-t_{j,s}) [P_{n-t_{j,s}}]_Q^{(V)} + \sum_{k \in M_3} A_{k,s}(n-t_{k,s}) t_{k,s} \sigma_{k,s,Total}(n-t_{k,s}) [P_{n-t_{k,s}}]_Q^{(V)} \right\}, \quad (8)$$

where:  $n[P_n]_Q^{(V)}$  – probability of  $n$  BBUs being busy in STS with capacity  $V$  BBUs,  $M_x$  – a set of call classes of Erlang ( $x = 1$ ), Engset ( $x = 2$ ) and Pascal calls ( $x = 3$ ), respectively,  $t_{c,s}$  – the number of BBUs required to set up a connection of class  $c$  in load area  $s$ ,  $A_{c,s}(n)$  – the average traffic intensity of class  $c$  offered to the system in the occupancy state  $n$  that belongs to the load area  $s$ . For adaptive traffic, this parameter is determined by Eqs. (1)–(3); for elastic traffic, by Eqs. (5)–(7).  $\sigma_{c,s,Total}(n)$  – conditional transition coefficient that determines which part of the input call stream in the threshold area  $s$  will be transferred between the states  $n$  and  $n + t_{c,s}$ :

$$\sigma_{c,s,Total}(n) = \sigma_{c,s,LAG}(n) \cdot \sigma_{c,s}(n), \quad (9)$$

where  $\sigma_{c,s,LAG}(n)$  is a conditional transition probability which determines the part of class  $c$  arrival stream which is transferred between states  $n$  and  $n + t_{c,s}$ ,  $\sigma_{c,s}(n)$  – conditional transition probability that in Eq. (8) is a switching coefficient between appropriate load areas.

The conditional transition probability can be determined with the help of following equation [15]:

$$\sigma_{c,s,LAG}(n) = 1 - \frac{F(V-n, k, t_{c,s} - 1, 0)}{F(V-n, k, v, 0)}, \quad (10)$$

where  $F(x, k, v, t)$  is the number of possible allocations of  $x$  free BBUs in  $k$  links, calculated with the assumption that the capacity of each link is equal to  $v$  BBUs and each link has at least  $t$  free BBUs:

$$F(x, k, v, t) = \sum_{i=0}^{\lfloor \frac{x-kt}{v-t+1} \rfloor} (-1)^i \binom{k}{i} \binom{x-k(t-1)-1-i(v-t+1)}{k-1}. \quad (11)$$

The threshold mechanism introduces dependence between the traffic stream and the current state of the system. This dependence can be determined as follows. For traffic classes that do not undergo the threshold mechanism, this parameter always takes on the value equal to one:

$$\sigma_{c,s}(n) = 1. \quad (12)$$

For all traffic classes that undergo the threshold mechanism, the value of the parameter  $\sigma_{c,s}(n)$  is defined in the following way:

$$\sigma_{c,1}(n) = \begin{cases} 1 & \text{for } n \leq Q, \\ 0 & \text{for } n > Q, \end{cases} \quad \sigma_{c,2}(n) = \begin{cases} 0 & \text{for } n \leq Q, \\ 1 & \text{for } n > Q. \end{cases} \quad (13)$$

To determine the occupancy distribution in STM according to Eq. (8) it is necessary to determine the values of intensities of offered Engset  $A_{j,s}(n)$  and Pascal  $A_{k,s}(n)$  traffic streams in individual states of the service process. These values can be determined on the basis of the parameter  $y_{c,s}(n)$ , i.e., the number of calls of a given class serviced in state  $n$  that belong to the load area  $s$ . This parameter can be approximated by the average number of calls of a given class that are serviced in the occupancy state  $n$  [1], [10]:

$$y_{c,1}(n) = \frac{A_{c,1}(n-t_{c,1}) \sigma_{c,1,Total}(n-t_{c,1}) [P_{n-t_{c,1}}]_Q^{(V)}}{[P_n]_Q^{(V)}}, \quad \text{for } n \leq Q + t_{c,1}, \quad (14)$$

$$y_{c,2}(n) = \frac{A_{c,2}(n-t_{c,2}) \sigma_{c,2,Total}(n-t_{c,2}) [P_{n-t_{c,2}}]_Q^{(V)}}{[P_n]_Q^{(V)}}, \quad \text{for } n > Q + t_{c,2}. \quad (15)$$

Notice that in order to determine the occupancy distribution by Eq. (8) in STM it is necessary to determine the values  $y_{c,s}(n)$ . These parameters can be determined on the basis of Formulas (14) and (15) which in turn require the knowledge of the distribution (8). Therefore, the determination of the occupancy distribution in STM requires a construction of a special iterative program which is discussed in detail in [10].

After the determination of the occupancy distribution in STM it is possible to determine blocking probabilities for individual call classes:

$$E_c = \sum_{n=V-t_{c,2}+1}^V (1 - \sigma_{c,2,Total}(n)) [P_n]_Q^{(V)}. \quad (16)$$

Formula (16) expresses the sum of blocking states for calls of class  $c$  in the highest ( $s = 2$ ) load area.

### 3.5. Modified Threshold Values

Consider a STS in which the  $Q$  threshold for  $c$  class calls has been introduced (Fig. 1). In the model, the occupancy states of the system are divided into two load areas. Assume that the mode of operation of STS depends on the direction of the load change. To analyze the system two scenarios for its operation can be considered [11].

The first scenario assumes that the load of the system increase. This situation for class  $c$  corresponds to Fig. 3a. It is assumed that the number of demanded BBUs changes after exceeding the threshold  $Q$  from the value  $t_{c,1}$  BBUs in the area ( $n \leq Q$ ) to the value  $t_{c,2}$  BBUs in the area ( $Q < n \leq V$ ). In such system, the occupancy distribution can be approximated by Eq. (8) determined for the STM described in Section 3.4.

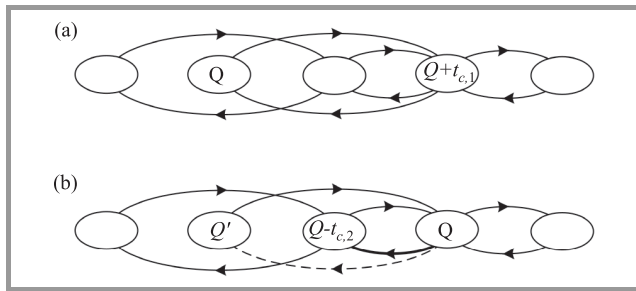


Fig. 3. Threshold for the scenarios: (a) – first, (b) – second.

The second scenario assumes that the loads of the system decrease. According to the definition of the threshold, it is the last state in which a call that demands a reduced number of BBUs ( $t_{c,2}$ ) can appear. This transition is marked in Fig. 3b with bold line. In order to determine the occupancy distribution, the so-called residual traffic, marked with dotted line, has to be additionally considered. Residual traffic is traffic that results from calls admitted in the lower load area ( $n \leq Q$ ) that have not yet been terminated before the system has been transferred from the lower load area to the higher load area ( $Q < n \leq V$ ). The relation between threshold values for the first and the second scenarios can be determined on the basis of relation [11]:

$$Q' = Q - t_{c,2} - 1, \quad (17)$$

where  $Q'$  defines the threshold for the second scenario that corresponds to the threshold  $Q$  for the first scenario.

## 4. Single Hysteresis System

### 4.1. Single Hysteresis System – Working Idea

The operation of the single hysteresis system for calls of one class  $c$  is shown in Fig. 4a. In STS, one threshold  $Q$  (Fig. 1) has been introduced, while in SHS a pair of thresholds  $Q_1, Q_2$  is introduced. With a change in the load from low to high, threshold  $Q_1$  operates. With a change from

high to low load, thresholds  $Q_2$  is used. Between  $Q_1$  and  $Q_2$  transition areas appear that form hysteresis. In SHS, two, partly overlapping, load areas can be distinguished. In the low load area ( $s = 1, 0 \leq n \leq Q_1$ ), the Call Admission Control function (CAC) admits for service a new call of class  $c$  with the maximum number of BBUs, equal to  $t_{c,1}$ . In the high-load area ( $s = 2, Q_2 < n \leq V$ ) the CAC function admits a new call of class  $c$  with a lowest number of BBUs, equal to  $t_{c,2}$ .

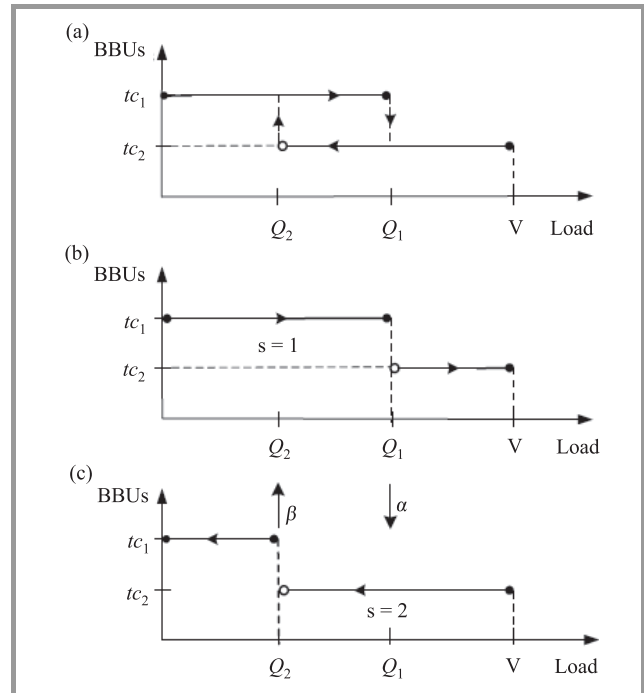


Fig. 4. System: (a) with single hysteresis mechanism and (b)–(c) its decomposition into STM components.

### 4.2. Occupancy Distribution in SHS

Figures 4b–c show a decomposition of SHM (Fig. 4a) into two STMs, where  $s$  indicates the area of the considered load. STMs, models are selected in such a way as to have the corresponding load area as high as possible. The arrows between Figs. 4b,c indicate possible transitions between neighboring STMs. The arrows between STM<sub>1</sub> (Fig. 4b) and STM<sub>2</sub> (Fig. 4c) indicate that the instance of exceeding of threshold  $Q_1$  triggers a change from the STM<sub>1</sub> model to STM<sub>2</sub>, whereas the instance of exceeding of threshold  $Q_2$  is followed by a change from the STM<sub>2</sub> model to STM<sub>1</sub>. The parameters  $\alpha$  and  $\beta$  that correspond to the arrows define intensities of the transitions between appropriate STMs. They are determined by values of streams that exceed indicated thresholds. How these parameters are determined will be presented in the Section 5.

The transition STM<sub>2</sub> → STM<sub>1</sub> is aligned with the direction of the change in the load from high to low, therefore this transition will be described by STM<sub>2</sub> with the threshold that corresponds to the second scenario (Sec-



tion 3.4), i.e., threshold  $Q'_2$  (Eq. (17)). Hence, the thresholds in the considered system can be written in the following way:

$$Q_x = \begin{cases} Q_x & \text{for odd } x, \\ Q'_x = Q_x - t_{c,s} - 1 & \text{for even } x, \end{cases} \quad (18)$$

where  $t_{c,s}$  is the number of BBUs that is necessary to set up a connection of class  $c$  in the load area  $s$ , if threshold  $Q_x$  defines the transition  $STM_s \rightarrow STM_{s-1}$ .

Let's denote the occupancy distributions in  $STM_s$  presented in Fig. 4b–c with the symbols  $[P_n]_{Q_1}^{(V)}$  and  $[P_n]_{Q'_2}^{(V)}$ . These distributions can be determined on the basis of Eq. (8) for appropriate pairs of thresholds adopted for a given STMs (Eq. (18)). The occupancy distribution in SHM  $[P_n]_{H_1, H_2}^{(V)}$  can be modeled on the basis of the weighted sum of the occupancy distributions in  $STM_s$  into which the SHM under consideration is decomposed [12]:

$$[P_n]_{H_1, H_2}^{(V)} = P(1)[P_n]_{Q_1}^{(V)} + P(2)[P_n]_{Q'_2}^{(V)}, \quad (19)$$

where  $P(s)$  is the probability that SHS stays in the load area  $s$ , that corresponds to the average time the system spends in this particular load area.

### 4.3. Switched Process in SHM

Probabilities  $P(s)$  can be determined on the basis of the two-state Markov process [11], whose diagram is presented in Fig. 5. This process is an analytical model for switches between appropriate load areas. The states in the diagram correspond to the execution of the service process in a given load area (described by a corresponding  $STM_s$ ), whereas the parameters  $\alpha$  and  $\beta$  denote the intensities of transitions between the appropriate load areas.

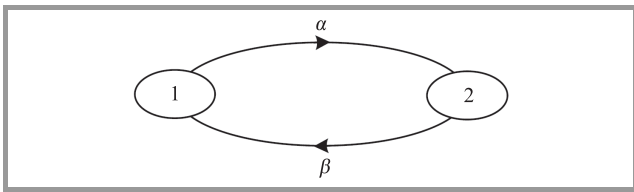


Fig. 5. Markovian switching process in SHM.

On the basis of the process presented in Fig. 5, it is possible to add and solve in a convenient way the state equations. The solution is expressed with the following formulas:

$$P(1) = \frac{\beta}{\alpha + \beta}, \quad P(2) = \frac{\alpha}{\alpha + \beta}. \quad (20)$$

The intensities of transition  $\alpha$  determine the transitions in the direction lower load  $\rightarrow$  higher load and are the sum of

all traffic streams that exceed the appropriate thresholds. Thus:

$$\alpha = \sum_{n=Q_1-t_{\max}+1}^{Q_1} \sum_{c \in M_1 \cup M_2 \cup M_3} A_{c,s}(n) t_{c,s} \varphi_{c,s}(n). \quad (21)$$

The parameter  $\varphi_{c,s}(n)$  is calculated in the following way:

$$\varphi_{c,s}(n) = \begin{cases} 1 & \text{for } n > Q_x - t_{c,s}, \\ 0 & \text{for } n \leq Q_x - t_{c,s}. \end{cases} \quad (22)$$

The intensities of transition  $\beta$  determine transitions in the direction higher load  $\rightarrow$  lower load and are the sum of all service streams that exceed appropriate thresholds. Therefore:

$$\beta = \sum_{n=Q_2}^{Q_2+t_{\max}-1} \left\{ \sum_{c \in M_1 \cup M_2 \cup M_3} y_{c,s}(n) t_{c,s} \varphi_{c,s}(n) + \sum_{c \in M_1 \cup M_2 \cup M_3} y_{c,s-1}(n) t_{c,s-1} \varphi'_{c,s-1}(n) \right\}. \quad (23)$$

The parameters  $\varphi_{c,s}(n)$  and  $\varphi'_{c,s}(n)$  are calculated as follows:

$$\varphi_{c,s}(n) = \begin{cases} 1 & \text{for } n < Q_x + t_{c,s}, \\ 0 & \text{for } n \geq Q_x + t_{c,s}, \end{cases} \quad (24)$$

$$\varphi'_{c,s}(n) = \begin{cases} 1 & \text{for } n < Q_x + t_{c,s-1} - t_{c,s}, \\ 0 & \text{for } n \geq Q_x + t_{c,s-1} - t_{c,s}. \end{cases} \quad (25)$$

The second sum within the brace bracket in Formula (23) includes residual traffic of class  $c$  that is serviced in area  $s$  (Section 3.5).

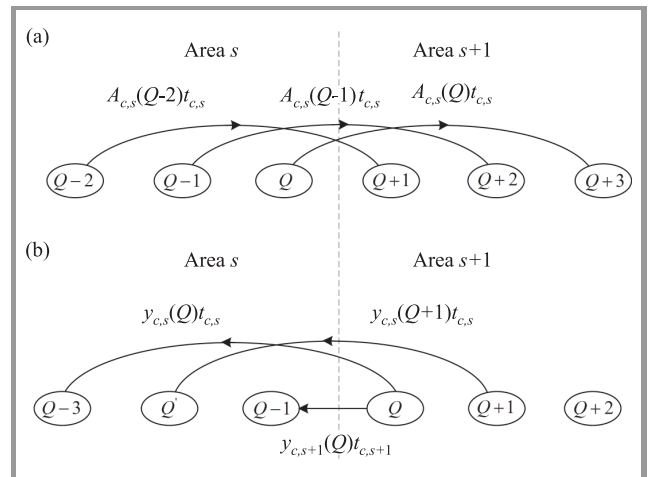


Fig. 6. Interpretation of passages: (a)  $STM_s \rightarrow STM_{s+1}$  and (b)  $STM_{s+1} \rightarrow STM_s$ .

Figure 6a shows traffic streams of class  $c$  for the transition  $STM_s \rightarrow STM_{s+1}$ , whereas Fig. 6b presents service streams for the transition  $STM_{s+1} \rightarrow STM_s$ . The accompanying assumption is that  $t_{c,s} = 3$  and  $t_{c,s+1} = 1$ .

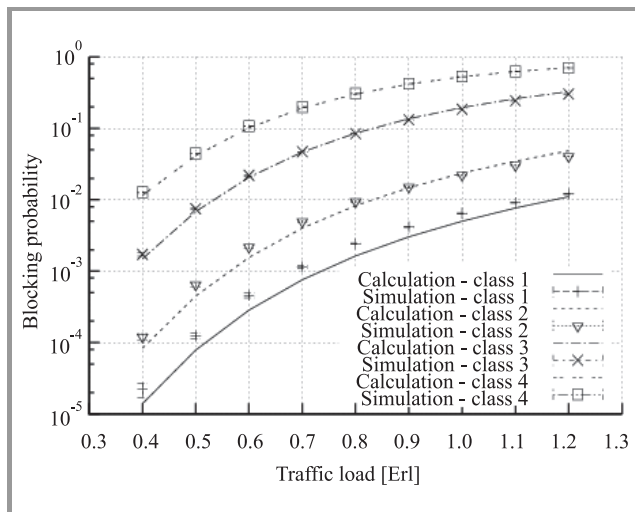
## 5. Numerical Study

The presented method for a determination of the blocking probability in limited-availability systems with hysteresis mechanisms is an approximate method. In order to confirm the adopted assumptions, the results of the analytical calculations were compared with the simulation data. The research was carried for two systems.

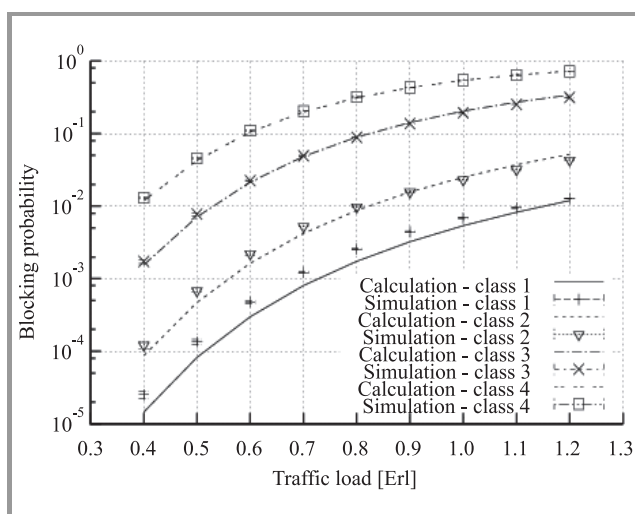
The study was carried out for users demanding a set of four traffic classes. In the examined WCDMA interface with virtual links it was assumed that the SHS was applied to the second traffic class. The structure of traffic offered to considered systems can be described in the following way:

- the number of BBUs required by calls of particular classes:

$$t_{1,1} = 53 \text{ BBUs}, t_{2,1} = 257 \text{ BBUs}, t_{2,2} = 129 \text{ BBUs}, \\ t_{3,1} = 503 \text{ BBUs}, t_{4,1} = 1118 \text{ BBUs}.$$



**Fig. 7.** Blocking probability in SHS with Engset traffic streams (System 1,  $N_1 = 1000$ ,  $N_2 = 1000$ ,  $N_3 = 1000$  and  $N_4 = 1000$ ).



**Fig. 8.** Blocking probability in SHS with Erlang (class 1), Engset (class 2,  $N_2 = 1000$ ) and Pascal (classes 3 and 4,  $N_3 = N_4 = 1000$ ) traffic streams (System 1).

- traffic of particular classes was offered to the system in the following exemplary proportions:  $A_{1,1}(0)t_{1,1} : A_{2,1}(0)t_{2,1} : A_{3,1}(0)t_{3,1} : A_{4,1}(0)t_{4,1} = 1 : 1 : 1 : 1$ .
- the hysteresis thresholds are assumed to be equal to, respectively:  $Q_2 = 4000$  BBUs,  $Q_1 = 6500$  BBUs.

The research was carried for two systems described below:

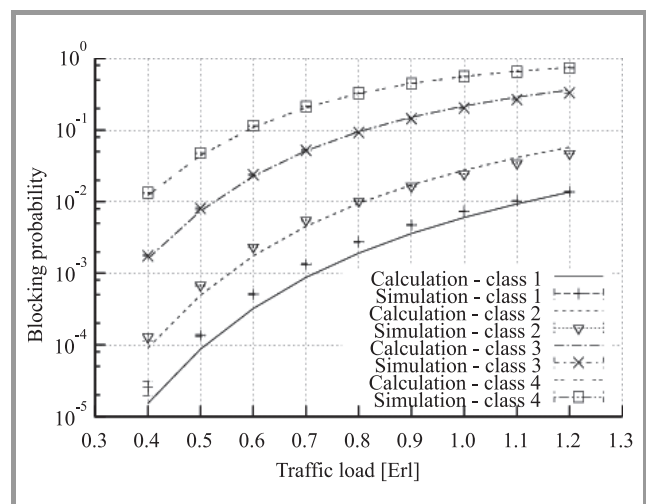
### System 1

- number of virtual links:  $k = 2$ .
- capacity of single virtual link:  $v = 4000$  BBUs.
- total capacity of system:  $V = 8000$  BBUs.

### System 2

- number of virtual links:  $k = 4$ .
- capacity of single virtual link:  $v = 2000$  BBUs.
- total capacity of system:  $V = 8000$  BBUs.

The results of the research study are presented in Figs. 7–12, depending on the value of traffic  $a$  offered to a single BBU. The results of the simulation are shown in the charts in the form of marks with 95% confidence intervals that have been calculated according to the t-Student distribution for the five series with 1,000,000 calls of each class. For each of the points of the simulation, the value of the confidence interval is at least one order lower than the mean value of the results of the simulation. In many a case, the value of the simulation interval is lower than the height of the sign used to indicate the value of the simulation experiment.



**Fig. 9.** Blocking probability in SHS with Pascal traffic streams (System 1,  $N_1 = 1000$ ,  $N_2 = 1000$ ,  $N_3 = 1000$  and  $N_4 = 1000$ ).

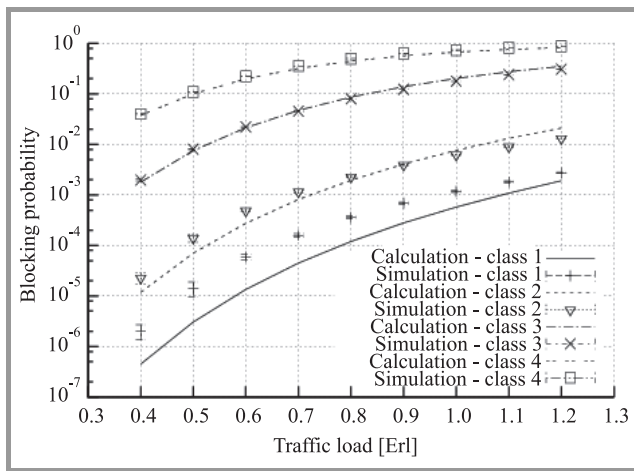
The results of the research study confirm high accuracy of the proposed SHM model for limited-availability group

## 6. Conclusions

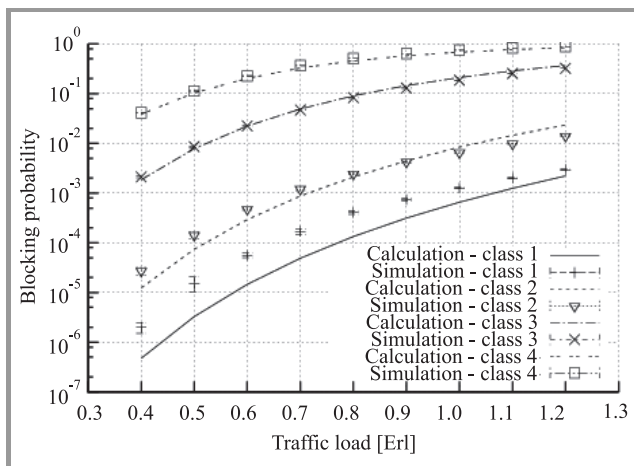
This paper proposes a new analytical model of SHS for limited-availability group to which a mixture of different BPP traffic streams is offered. The SHS, introduced into a given system, allows the blocking probability to be decreased for particular traffic classes and leads to a reduction in fluctuations in the load. The paper also presents a possibility of the application of SHS for traffic control in the UMTS network. All the presented simulation experiments for the considered systems confirm good accuracy of the proposed analytical SHM model for traffic streams of BPP type. Summing up, the single hysteresis mechanism can be successfully used in the call admission control function of communications and cellular networks.

## References

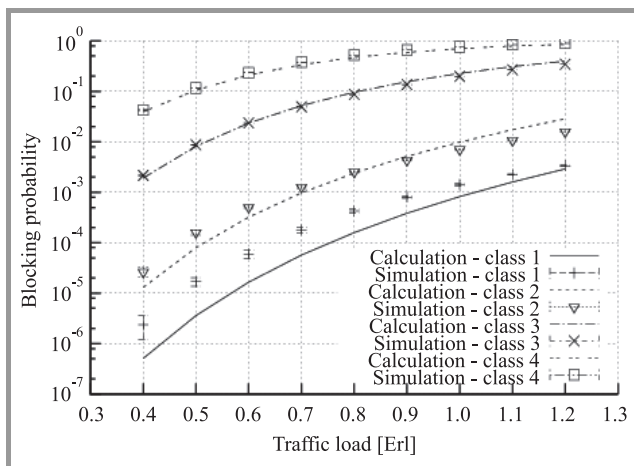
- [1] M. Stasiak, M. Głabowski, A. Wiśniewski, and P. Zwierzykowski, *Modeling and Dimensioning of Mobile Networks: from GSM to LTE*. Chichester: Wiley, 2011.
- [2] J. Roberts, "Teletraffic models for the Telecom 1 integrated services network", in *Proc. 10th Int. Telegraf. Congr. ITC 83*, Montreal, Canada, 1983, p. 1.1.2.
- [3] M. Pióro, J. Lubacz, and U. Körner, "Traffic engineering problems in multiservice circuit switched networks", *Comp. Netw. ISDN Sys.*, vol. 20, pp. 127–136, 1990.
- [4] V. Iversen, "Teletraffic Engineering and Network Planning", Lyngby, Technical University of Denmark, 2009.
- [5] Q. Huang and V. Iversen, "Approximation of loss calculation for hierarchical networks with multiservice overflows", *IEEE Trans. Commun.*, vol. 56, no. 3, pp. 466–473, 2008.
- [6] M. Głabowski, K. Kubasik, and M. Stasiak, "Modeling of systems with overflow multi-rate traffic", *Telecommun. Systems*, vol. 37, no. 1–3, pp. 85–96, 2008.
- [7] H. Holma and A. Toskala, *WCDMA for UMTS: HSPA Evolution and LTE*, 5th ed. New York, London: Wiley, 2010.
- [8] V. Vassilakis, I. Moscholios, and M. D. Logothetis, "Call-level performance modelling of elastic and adaptive service-classes with finite population", *IEICE Trans. Commun.*, vol. E91-B, no. 1, pp. 151–163, 2008.
- [9] J. Kaufman, "Blocking with retrials in a completely shared resource environment", *Perform. Evaluation*, vol. 15, no. 2, pp. 99–113, 1992.
- [10] M. Głabowski, A. Kaliszczan, and M. Stasiak, "Modeling product form state-dependent systems with BPP traffic", *Perform. Evaluation*, vol. 67, no. 2, pp. 174–197, 2010.
- [11] M. Sobieraj, M. Stasiak, J. Weissenberg, and P. Zwierzykowski, "Analytical model of the single threshold mechanism with hysteresis for multi-service networks", *IEICE Trans. Commun.*, vol. 95, no. 1, pp. 120–132, 2012.
- [12] M. Sobieraj, M. Stasiak, and P. Zwierzykowski, "Model of the threshold mechanism with double hysteresis for multi-service networks", in *Communications in Computer and Information Science*, vol. 291, A. Kwiecien, P. Gaj, and P. Stera, Eds. Springer, 2012, pp. 299–313.
- [13] M. Stasiak, M. Sobieraj, J. Weissenberg, and P. Zwierzykowski, "Single hysteresis model for multi-service networks with BPP traffic", in *Proc. 17th Polish Telegraf. Symp.*, Zakopane, Poland, 2012, pp. 53–58.
- [14] J. Roberts, U. Mocci, and J. Virtamo, Eds., "Broadband Network Teletraffic", Final Report of Action COST 242, Springer, 1996.
- [15] M. Stasiak, "Blocking probability in a limited-availability group carrying mixture of different multichannel traffic streams", *Annales des Télécommunications*, vol. 51, no. 11–12, pp. 611–625, 1996.



**Fig. 10.** Blocking probability in SHS with Engset traffic streams (System 2,  $N_1 = 1000$ ,  $N_2 = 1000$ ,  $N_3 = 1000$  and  $N_4 = 1000$ ).



**Fig. 11.** Blocking probability in SHS with Erlang (class 2), Engset (class 2,  $N_2 = 1000$ ) and Pascal (classes 3 and 4,  $N_3 = 1000$  and  $N_4 = 1000$ ) traffic streams (System 1).



**Fig. 12.** Blocking probability in SHS with Pascal traffic streams (System 2,  $N_1 = 1000$ ,  $N_2 = 1000$ ,  $N_3 = 1000$  and  $N_4 = 1000$ ).

with BPP traffic. Greater accuracy we can obtain for traffic classes which require the largest number of BBUs.



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