Paper

Auction Models Supporting End-to-End Connection Trading

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Abstract—The paper concerns bandwidth allocation problem on the telecommunication market where there are many sellers and buyers. Sellers offer the bandwidth of telecommunication links. Buyers are interested in the purchase of the bandwidth of several links that makes up an end-to-end connection between two nodes of telecommunication network. We analyze three auction models supporting such a bandwidth exchange: NSP (network second price), BCBT (model for balancing communication bandwidth trading) and BCBT-CG which is a modification of BCBT that applies column generation technique. All of these models concern divisible network resources, treat bandwidth of telecommunication links as an elementary commodity offered for sale, and allow for purchasing bandwidth along multiple paths joining two telecommunication nodes. All of them also aim at maximizing the social welfare. Considered auction models have been compared in the respect of economic and computational efficiency. Experimental studies have been performed on several test instances based on the SNDlib library data sets.

Keywords—bandwidth auctions, divisible commodities, end-to-end connections, multi-commodity trade, multi-path routing.

1. Introduction

In this paper we consider a bandwidth market [1], [2] on which many sellers and many buyers are interested in the exchange of the links' bandwidth. The bandwidth of link is an elementary network resource that allows for transmitting some amount of data between two telecommunication nodes over given period of time. Telecommunication network consists of many nodes connected by numerous links. Therefore, on the bandwidth market there are typically many different network resources that may be offered for sale by different sellers, e.g., companies laying cables, network providers and other telecommunication link owners. Generally, the buyers, such as network providers, service providers and geographically spread organizations want to purchase the bandwidth of several links to realize specific network services.

Here, we focus on the case, in which buyers are interested in obtaining end-to-end connections. The end-to-end connection is a network service that allows for transmitting data between two arbitrary nodes in the telecommunication network. These nodes do not have to be directly connected by single link, but they may be joined by a path consisting of many links. Thus, in order to provide an end-to-end connection with predefined capacity, path(s) joining source and destination nodes of this end-to-end connection have to be determined and bandwidth of those path(s) has to be allocated to this end-to-end connection.

Each buyer or seller participates in the bandwidth exchange in order to achieve one's individual goals. The buyer derives utility from getting the end-to-end connection and he wants to purchase this end-to-end connection for the minimum price. The difference between buyer's utility and the end-to-end connection price defines one's net benefit from the trade. On the other hand the seller incurs the cost of network resource and he wants to sell the network resource for the maximum price. The difference between the network resource price and the seller's cost defines one's net benefit from the trade. Rational market participants aims at maximization of their net benefits.

The sum of all market participants' net benefits is called social welfare. From the global point of view it is desirable to ensure economic efficiency of bandwidth market in terms of social welfare. In other words, the problem is to determine the allocation of network resources offered for sale to end-to-end connections offered for purchase that will result in the maximum social welfare. This problem is complicated by the fact that the buyers' utilities and sellers' costs are their private information and market participants may not have interest in sincerely eliciting this information.

Currently, the bandwidth market is organized on the basis of bilateral agreements. This means that the trade is carried out by making deals between one seller and one buyer that have to negotiate with each other the contract terms. Because no one is encouraged to reveal one's private information, these negotiations are often very complex and time consuming. Also, the details about transactions are rather privately held than publicly announced information. This makes the whole trading process non-transparent and it limits access to important market related data (e.g., prices). If there are many paths realizing specific end-to-end connection, the buyer having limited information about links' prices may have difficulties in determining the cheapest one. Moreover, bilateral agreements do not support buyer in obtaining end-to-end connection when this end-to-end connection cannot be provided by a single seller. In such a case the buyer must independently negotiate with several owners of links to realize desired end-to-end connection. This leads to the risk of purchasing incomplete set of links if the trade negotiations fail with one of seller, whereas agreements with other sellers would be drawn up and signed.

Thus, there is a need for more sophisticated organization of bandwidth market. Some authors [3], [4] claim that the introduction of new forms of bandwidth trading is only a matter of time. One of the key aspects that may facilitate this process is a development and application of new market mechanisms, such as bandwidth auctions [4].

2. Bandwidth Auction Models

Auction is one of the oldest way of performing the trade on the market. It is characterized by the fact that it defines the kind of offers the auction participants may submit to report their willingness of selling or purchasing commodities being traded and that it defines the formal rules that allow on the basis of submitted offers to determine what commodities are sold or bought by each auction participant. The formal rules of the auction are typically divided into the allocation rule that determines the amount of commodities being exchanged between each buyer and seller, and the pricing rule that sets revenues of sellers and payments of buyers.

2.1. Overview of Bandwidth Auctions

In literature, there are many auction models that support end-to-end connection trading on the bandwidth market. These models make different assumptions about the bandwidth auction.

One of the most important assumptions relates to the number of network providers that may participate in the bandwidth auction as a seller. One group of the models [5]–[10] concerns one-sided auction, in which the bandwidth of all telecommunication links is offered for sale by one auctioneer to many buyers. Mostly, the auctioner is, or acts on behalf of, a provider that owns or manages the telecommunication network. Although some of these models [8]–[10] can be used to trade network resources owned by different providers, they require that all of them pass their true private information to the auctioneer. Therefore, these auction models do not take into account that the network resource providers may act strategically competiting with each other.

However, proceeding bandwidth market liberalization favors competition between providers and it seems that above assumption may be too restrictive. So, there is a need of market mechanisms that would allow to perform the bandwidth exchange between many sellers and many buyers. It is even believed that the development of double auctions supporting bandwidth trading is one of the most promising new research directions [11]. So far, there are a few models for double auction that supports end-to-end connection trading [12]–[17].

The another important assumption concerns the divisibility of network resources. Some auction models allow for trading bandwidth in modules of predefined capacity [10], [14]–[17] while others treat network resources as fully divisible commodities [5]–[9], [12], [13]. Both these assumptions may be resonable depending on the telecommunication technology. In the lower layers of telecommunication network, the links' capacities have often modular character, e.g., optic fibres, SDH modules. In the higher layers, the bandwidth of links may be divided in almost every real fraction of Mbit/s, e.g., ATM virtual paths, IP flows.

The last assumption, mentioned here, relates to the way of supporting end-to-end connection trading. Most of the proposed auction models require that the buyer specifies the single path to be used to realize desired end-to-end connection. These auction models ensure that the same amount of bandwidth will be allocated to the buyer at each link constituing specified path. A more flexible approach from the buyer point of view is applied in the Network Second Price model (NSP) [12]. The NSP model allows the buyer to specify many paths that can be used to realize desired end-to-end connection. The NSP model considers all this paths when allocating bandwidth to particular end-to-end connection, so the buyer can increase the chance of purchasing end-to-end connection with predefined capacity by specifying many paths. The Kelly's model [5] and the model for balancing communication bandwidth trading (BCBT) [13] are even more flexible than NSP, because they allow the buyer to submit an offer for commodity representing a demand for end-to-end connection. In these models buyer does not have to specify any paths, but only source and destination nodes of the end-to-end connection. Therefore, these models seem to be more convenient to the buyer as one does not have to know the network topology.

In paper [18] double auction models concerning bandwidth as a modular commodity have been analyzed. Here we compare double auction models supporting end-to-end connection trading that treat bandwidth as fully divisible commodity. To this group of auction models belong the BCBT and NSP models that are known from the literature and also the BCBT-CG model that is here proposed.

2.2. Comparision of the BCBT and NSP Models

The BCBT and NSP models relate to the auctions that can be classified as sealed-bid single-round double auctions. This means that in both auction models sellers and buyers submit their offers knowing nothing about the offers of other auction participants and the auction mechanism determines the allocation, and pricing only on the basis of submitted offers. In both models the bandwidth of telecommunication links is an elementary, fully divisible commodity. Let E be a set of all telecommunication links. In NSP model there is no information about which network nodes are connected by particular link $e \in E$. In BCBT model there is a set V denoting all telecommunication nodes. For each link $e \in E$ and each node $v \in V$ the parameter a_{ve} defines if node v is a source ($a_{ve} = 1$) or destination ($a_{ve} = -1$) node of link e or that the node v is not incident to link e ($a_{ve} = 0$). Thus BCBT model has a full information about the network topology.

The NSP and BCBT models allow the seller to submit the sell offer for particular link. With each link $e \in E$ there can be many sell offers involved that form the set $S(e)$. The set of all sell offers is denoted by $S = \cup_{e \in E} S(e)$. In both models each sell offer $l \in S$ is characterized by two parameters: S_l – the minimum unit price at which seller is willing to sell the bandwidth of link, and $x_l^{\max}$ – the maximum amount of bandwidth offered for sale by the seller.

Considered auction models differ substantially in the commodity type for which the buy offer can be submitted. Let D denote the set of all buy offers. In the NSP model, the buy offer is related with a set of possible paths. The buyer that submits the buy offer $d \in D$ specifies the set of possible paths P_d. For each path $p \in P_d$ and each link e he has to define a binary parameter b_{edp} that equals 1 if link e belongs to path p and equals 0 otherwise. This means that the buyer must know the network topology in order to correctly specify the paths. In the case of BCBT the buy offer is involved with a commodity that represents a demand for end-to-end connection. In the buy offer d the buyer specifies only a source node s_d and a destination node t_d. In both NSP and BCBT models the buyer also has to define in the buy offer d two parameters: E_d – the maximum unit price at which the buyer is willing to buy the bandwidth of end-to-end connection, and $x_d^{\max}$ – the maximum amount of bandwidth offered for purchase by the buyer.

The allocation rules of NSP and BCBT models decide, which offers are accepted aiming at social welfare maximization. In other words, both allocation rules match sell and buy offers allocating bandwidth of links offered for sale to end-to-end connections offered for purchase in order to achieve the maximum sum of all market participants' net benefits. Both allocation rules allow for multipath routing, i.e., each end-to-end connection may be realized by several paths. The essential difference between NSP and BCBT allocation rules is that the allocation rule of NSP has given the predefined paths for each end-to-end connection while the allocation rule of BCBT itself has to determine the paths for each end-to-end connection. Note that the BCBT model that has full information about network topology, considers all paths that can be used to realize particular end-to-end connection. In the case of NSP model the allocation rule is restricted to paths specified by the buyers. Therefore, for given submitted offers, the NSP model takes into account only some subset of all allowable paths that can be generated in the case of BCBT model. The allocation rules of NSP and BCBT models can be formulated as linear programming problems with the same objective functions. It is worth to mention that the LP problem defining NSP allocation rule is a restriction of LP problem defining BCBT allocation rule. Thus, assuming that for the buyers it is indifferent what paths are used to satisfy their demands for end-to-end connections, the allocation obtained by the NSP model cannot be better in terms of the social welfare than the one given by the BCBT model.

The NSP and BCBT auction models define also different pricing rules. NSP adapts VCG-style pricing [12] while BCBT determines the clearing prices on the basis of dual prices of LP formulation of its allocation rule. Further we will focus on the comparison of NSP and BCBT allocation rules, so we do not discuss here the details of both pricing rules.

2.3. The BCBT-CG Auction Model

In this paper we propose the BCBT-CG model. The BCBT-CG model is a modification of BCBT model that differs from BCBT only in the way of determining the optimal allocation. The allocation rules of BCBT-CG and BCBT models are equivalent in the respect of social welfare.

In BCBT model the allocation is determined by solving a LP problem, in which all possible paths for each end-to-end connection are considered at once. As opposed to this approach, in BCBT-CG the allocation problem is decomposed into the master problem and the subproblem using column generation technique.

The master problem is a restriction of the allocation problem defined by BCBT model in which for each buy offer a set of allowable paths is specified. Thus, in the master problem of BCBT-CG model for each buy offer d there is defined a set of paths P_d with each path described by binary parameters b_{edp} like in the NSP model. The aim of the master problem is to determine the optimal (i.e., providing maximum social welfare) bandwidth allocation assuming that end-to-end connection can be realized only by the predefined paths.

For given buy offer d let us denote by the variable x_{dp} the amount of bandwidth allocated at path $p \in P_d$. Then the variable $x_d = \sum_{p \in P_d} x_{dp}$ is the total amount of bandwidth allocated to end-to-end connection involved with buy offer d. Moreover, let us define the variable x_l that indicates the amount of bandwidth sold at link involved with sell offer l. Note that the variables x_d and x_l also denote the realization volume of buy offer d and sell offer l, respectively. Then, the master problem of BCBT-CG can be formulated as following LP problem:

$$\hat{Q} = \max \left(\sum_{d \in D} E_d x_d - \sum_{l \in S} S_l x_l \right), \quad (1)$$

$$\sum_{d \in D} \sum_{p \in P_d} b_{edp} x_{dp} \le \sum_{l \in S(e)} x_l, \quad \forall_{e \in E}, \quad (2)$$

$$x_d = \sum_{p \in P_d} x_{dp} \quad \forall_{d \in D}, \quad (3)$$

$$0 \le x_d \le x_d^{\max}, \quad \forall_{d \in D}, \quad (4)$$

$$0 \le x_l \le x_l^{\max}, \quad \forall_{l \in S}, \quad (5)$$

$$0 \le x_{dp}, \quad \forall_{d \in D}, \forall_{p \in P_d}. \quad (6)$$

The objective function (1) aims at the maximization of the difference between buyers' payments and sellers' revenues according to the buy and sell prices specified in the offers. So, assuming that the offers are sincere, i.e., they represent real private information of the auction participants, the objective function ensures that for given predefined set of paths P_d the optimal solution of the master problem gives the allocation with the maximum social welfare. Constraints (2) guarantee that for each link the total amount of bandwidth of this link allocated to all paths predefined for buy offers cannot be greater than the sum of realization volumes of all sell offers related to this link. Next group of constraints (3) state that each buy offer realization volume equals to the sum of bandwidth allocated at all paths specified for this offer. Constraints (4) and (5) define allowable realization volumes of buy and sell offers, respectively.

The column generation subproblem assumes that for each link there is defined a unit price, i.e., the price, at which one unit of link's bandwidth is sold/purchased. The subproblem relies on calculating for each end-to-end connection the cheapest path according to given links' prices. This can be done by an arbitrary shortest path algorithm.

Here we formulate the subproblem as a LP problem. Let the variable x_{ed} denote the amount of bandwidth of link e allocated to end-to-end connection involved with buy offer d. Let the parameter $\hat{\lambda}_e$ indicate the unit price of link e. Then the solution of subproblem can be obtained by solving following LP problem:

$$\min \sum_{d \in D} \sum_{e \in E} \hat{\lambda}_e x_{ed}\,, \tag{7}$$

$$\sum_{e \in E} a_{ve} x_{ed} \;=\; \begin{cases} 1 & v = s_d \\ 0 & v \neq s_d, t_d \\ -1 & v = t_d \end{cases} \,, \quad \forall_{v \in V, d \in D}\,, \tag{8}$$

$$0 \;\leq\; x_{ed}, \quad \forall_{d \in D}, \forall_{e \in E}\,. \tag{9}$$

The objective function (7) minimizes the total cost of the realization of all end-to-end connections according to link's prices $\hat{\lambda}_e$. Equations (8) define flow conservation constraints that must be met for each end-to-end connection. Simplex algorithm determines vertex solution $\hat{x}_{ed}$ of above LP problem giving for each buy offer d the cheapest path p defined by $b_{edp} = \hat{x}_{ed}$ with unit buy price equal to $\sum_{e \in E} \hat{\lambda}_e \hat{x}_{ed}$.

The complete allocation rule of the BCBT-CG is defined by following iterative algorithm based on the column generation technique that exploits above definitions of the master problem Eqs. (1)–(6) and the subproblem Eqs. (7)–(9):

1. For each buy offer d initialize a set of predefined paths P_d, e.g., set P_d may include one path p being a solution of the cheapest path subproblem with links' prices $\hat{\lambda}_e = \min_{l \in S(e)} S_l$.

2. Solve master problem for given P_d: determine optimal allocation $\hat{x}_l$, $\hat{x}_d$, $\hat{x}_{dp}$ and optimal values of dual prices $\hat{\lambda}_e$ i $\hat{\omega}_d$ corresponding to constraints (2) and (3), respectively.

3. Solve the cheapest path subproblem for given links' prices $\hat{\lambda}_e$: for each buy offer d determine the cheapest path realizing relevant end-to-end connection as a path σ such that $b_{ed\sigma} = \hat{x}_{ed}$ for each link e.

4. If for each buy offer d the cheapest path σ fulfills following condition $\sum_{e \in E} \hat{\lambda}_e b_{ed\sigma} \geq \hat{\omega}_d$, then the allocation $\hat{x}_l$, $\hat{x}_d$, $\hat{x}_{dp}$ detemined in step 2 is optimal. Otherwise, for each buy offer d, for which the cheapest path σ fulfills condition $\sum_{e \in E} \hat{\lambda}_e b_{ed\sigma} < \hat{\omega}_d$, add path σ to the set P_d and go to the step 2.

At first, the set of paths realizing relevant end-to-end connection is initialized for each buy offer. In the second step the master problem is solved. In this way the optimal allocation is determined and the unit prices of links and end-to-end connections are set according to the optimal values of dual prices. In the third step the subproblem is solved and the cheapest path is found for each buy offer. In the fourth step for each buy offer it is checked if the unit price of the cheapest path is greater or equal to unit price of end-to-end connections. If all paths fulfil this condition, the allocation determined in second step is optimal and the algorithm stops. Otherwise, for each buy offer, for which the condition is not met, the cheapest path is added to the set of predefined paths and the algorithm goes to the second step where the next iteration begins.

3. Experimental Studies

The allocation rules of NSP, BCBT and BCBT-CG models have been compared in the respect of economic and computational efficiency. The experimental studies have been performed on several test instances concerning allocation problems on the bandwidth market.

3.1. Test Instances

Test instances for the allocation problems on bandwidth market have been based on the SNDlib library [19]. Although this library contains data sets for survivable fixed telecommunication network design problems, the information derived from it was very usefull in the preparation of test instances for allocation problems on the bandwidth market. Some data such as network topology (nodes and links) and demands for end-to-end connection have been directly applied in prepared test instances. Other data such as links' capacities, demands' volumes and distances between nodes have been used for the generation of the offers' parameters. Three data sets from the SNDlib library have been used: *sun*, *janos-us* and *giul39*. Figure 1 shows the network topologies given by considered data sets. Table 1 contains information about the number of nodes, links and end-to-end connections for these data sets.

Table 1
Number of nodes, links and end-to-end connections for each data set

Data set	Nodes	Links	End-to-end connections
sun	27	102	67
janos-us	26	84	650
giul39	39	172	1471

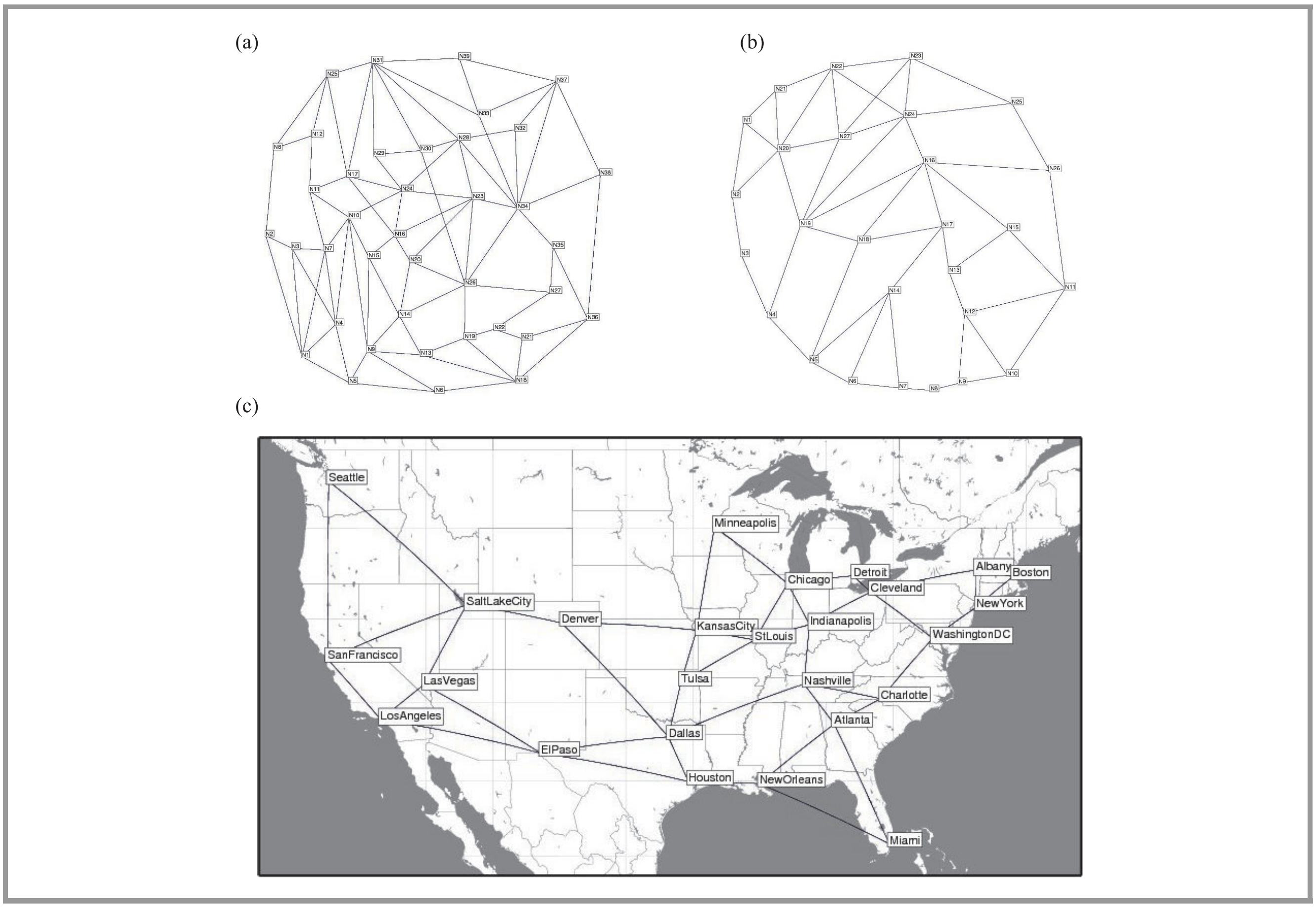

Fig. 1. Topologies for considered data sets: (a) sun; (b) giul39; (c) janos-us.

Data sets derived from the SNDlib do not include all data required to form full test instance of the allocation problem on the bandwidth market. Some data involved with offers has had to be generated. It has been assumed that:

- the unit price specified in offer concerning link/end-to-end connection is proportional to the distance between nodes connected by this link/demand,
- the total amount of bandwidth offered for sale (purchase) at particular link (end-to-end connection) is proportional to the link capacity (demand volume).

Table 2
Number of sell and buy offers for each test instance

Test instance	Sell offers	Buy offers
sun-2	206	129
sun-4	407	274
sun-6	619	411
janos-us-2	165	1287
janos-us-4	340	2666
janos-us-6	500	3916
giul39-2	330	2938
giul39-4	708	5869
giul39-6	1043	8803

Moreover, in the NSP model it is required that for each buy offer a set of paths is specified. It should be noted that the path specification has significant influence on the economic and computational efficiency of NSP model. Thus, in the case of NSP model we have decided to consider 100 variants of each test instance. In *k*-th variant for each buy offer the *k* cheapest paths (calculated according to the minimum unit sell prices specified in sell offers) is specified as a set of possible paths.

On the basis of each considered data set, a three test instances have been prepared with respectively 2, 4 and 6 offers on average submitted for each link/end-to-end connection. Table 2 presents the number of sell and buy offers generated for each test instance. As mentioned above, in the case of NSP model, for each test instance 100 variants of path specification for buy offers have been considered.

3.2. Computational Efficiency Analysis

For all test instances the allocation have been determined using NSP, BCBT and BCBT-CG auction models. All LP problems have been solved by means of CPLEX 12.1 on the computer with processor Intel Core2 Duo T8100 2.1 GHz, main memory 3 GB and 32-bit operating system MS Vista.

Table 3
The time of determining the optimal allocation

Test instance	NSP		BCBT	BCBT-CG
	$k = 5$	$k = \min\{k^*, 100\}$		
sun-2	0.11	0.02 (k^*=9)	0.23	3.36
sun-4	0.05	0.03 (k^*=4)	0.73	2.96
sun-6	0.09	0.41 (k^*=32)	2.42	3.31
janos-us-2	0.44	2.68 (k^*=38)	8.36	5.01
janos-us-4	0.64	8.71 (k^*=33)	38.92	11.82
janos-us-6	1.19	46.18 (k^*=84)	88.92	20.56
giul39-2	1.26	7.79 (k^*=8)	74.79	32.54
giul39-4	4.26	172.71 (k^* >100)	34.51	18.25
giul39-6	10.87	402.83 (k^* >100)	1162.69	176.33

Iterative algorithm constituing allocation rule of BCBT-CG model has been implemented in AIMMS 3.10.

In the case of NSP model, 100 variants of buy offers have been considered for all test instances. For each test instance there has been determined the smallest variant k^*, for which NSP gives the same value of social welfare as BCBT and BCBT-CG models. For each variant $k < k^*$ the allocation obtained by NSP is worse in terms of social welfare than the allocation given by BCBT (BCBT-CG) model. On the other hand, for variants $k \geq k^*$ the allocation obtained by NSP is equivalent in the respect of economic efficiency to allocation given by the BCBT (BCBT-CG) model.

Table 3 presents the time of determining the optimal allocation by each auction model. In the case of NSP model the results for two variants are shown: $k = 5$ and $k = k^*$. The values of k^* vary for different test instances and are given in the Table 3. As it can be seen, if the NSP model is applied then obtaining as efficient allocation as in the case of BCBT (BCBT-CG) model, it requires that the buyers specify quite many paths in their buy offers. The minimum is four paths, but there are test instances that requires 30 or more paths to be specified for each buy offer. For last two test instances, namely *giul39-4* and *giul39-6*, even 100 paths specified for each buy offer have not been enough to ensure that the NSP will result in the same social welfare as BCBT or BCBT-CG model. For these test instances the time for the NSP model in Table 3 is given for $k = 100 < k^*$.

From the obtained experimental results it follows that in the most of test instances the NSP model is faster than BCBT and BCBT-CG. If we consider only the variants of test instances with $k = 5$ then it turns out that NSP model is undoubtedly the fastest one. However, it should be noted that for variants with $k = 5$ only for the test instance *sun-4* the NSP model provide as efficient allocation as the BCBT or BCBT-CG model. Also, for the NSP model, the time of determining the k-th cheapest paths by the buyers is here not taken into account.

Considering variants of test instances with $k = k^*$, we can see that there are test instances for which BCBT or BCBT-CG requires less time than NSP in order to compute the optimal allocation. The allocation time for BCBT is shorter than in the case of NSP model for the test instance *giul39-4*. In turn, the BCBT-CG model is faster than NSP for three test instances: *janos-us-6*, *giul39-4* and *giul39-6*.

Comparing the allocation times for the BCBT and BCBT-CG models, we can see that for the larger test instances based on data sets *janos-us* and *giul39*, the BCBT-CG model requires less time to determine the optimal allocation than the BCBT model, and the difference is significant especially for the last test instance *giul39-6*. Thus, the computational efficiency of determining the optimal allocation can be improved by applying the BCBT-CG model instead of BCBT.

3.3. Economic Efficiency Analysis

The BCBT and BCBT-CG models are equivalent in the respect of economic efficiency as they give the allocations providing the same social welfare. In the case of NSP model, the value of resulting social welfare depends on the path specifications made by the buyers in their buy offers. The allocation obtained by NSP for variant k of given test instance will be at least as efficient as the one obtained for variant $k - 1$. Moreover, the social welfare provided by the NSP model cannot be higher than the one given by the BCBT or BCBT-CG model. Here, we analyze how the social welfare obtained by the NSP model changes in the relation to the opitmal allocation given by BCBT (BCBT-CG) considering the first 10 variants of each test instance.

Table 4 presents the ratio between the value of social welfare provided by NSP and the one given by the BCBT (BCBT-CG). Experimental results show that this ratio for all considered test instances is at least 95% just for k greater than 3. If $k = 5$, then the ratio is 99% or higher for almost all test instances except for two (*giul39-4* and *giul39-6*). So, if all buyers can anticipate and specify in the buy offers the five cheapest path realizing demanded end-to-end

Table 4
The ratio between the value of social welfare provided by NSP and BCBT (BCBT-CG)

Test instance	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$
sun-2	0.93	0.97	0.99	0.99	0.99	0.99	0.99	0.99	1	1
sun-4	0.93	0.98	0.99	1	1	1	1	1	1	1
sun-6	0.93	0.95	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
janos-us-2	0.89	0.95	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99
janos-us-4	0.95	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
janos-us-6	0.82	0.97	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99
giul39-2	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1	1	1
giul39-4	0.8	0.9	0.93	0.95	0.96	0.97	0.97	0.98	0.98	0.98
giul39-6	0.83	0.91	0.94	0.96	0.97	0.97	0.98	0.98	0.98	0.99

connections then it is highly possible that the NSP would determine the allocation, which is almost as economically efficient as the allocation given by BCBT.

However, it should be noted that the NSP model does not guarantee as efficient allocation as BCBT or BCBT-CG model. For some test instances, e.g., *giul39-4* and *giul39-6*, even if the byuers would know the sell prices of each link and specify the 100 least expensive paths in their offers the NSP model leads to the social welfare that is lower than the one that can be provided by the application of the BCBT (BCBT-CG) model (see Table 3).

4. Conclusions

In this paper we compare three auction models that treat bandwidth as divisible commodity and support end-to-end connection trading. One of these models, called BCBT-CG, is proposed here as a modification of BCBT model. The BCBT-CG allows for determining the allocation that is equivalent to the one obtained by the BCBT model. However, it applies as an allocation rule the iterative algorithm based on the column generation technique that has better computational efficiency than the BCBT allocation rule. Experimental studies verify that the BCBT-CG model can be used instead of BCBT to reduce the time of determining the optimal allocation.

Compared to BCBT and BCBT-CG models, the NSP model is less convenient from the buyer point of view as it requires from him to specify the paths realizing demanded end-to-end connection. In the case of NSP model, the buyer has to know the network topology and he bears the responsibility for choosing the appropriate set of paths, which would give him the best payoff. On the other hand, in the BCBT and BCBT-CG models the buyer must only specify the source and destination nodes of the desired end-to-end connection. The allocation rules of these auction models are responsible for determining the optimal paths.

From the experimental results it follows that the NSP model does not guarantee as efficient allocation as the BCBT and BCBT-CG models, even if all buyers specify the 100 least expensive paths in their buy offers. The merit of NSP is that it allows for the fast determination of almost optimal allocation requiring just a five cheapest paths to be specified by each buyer. However, the time of determining the allocation by the NSP increases with the number of paths specified in the buy offers and if there is many such a paths, the NSP may be slower than the BCBT and BCBT-CG models.

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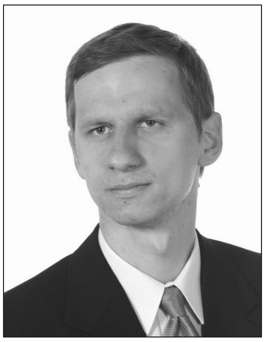

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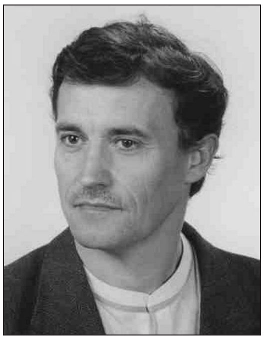

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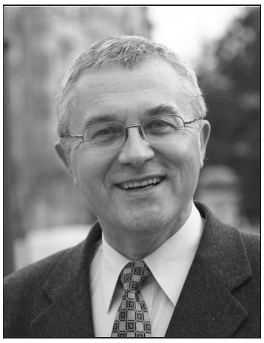

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