

Pricing Rules Comparison in the Context of Bandwidth Trade

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Abstract—In this paper we compare two pricing rules in the context of bandwidth trade. Allocation and pricing rules, together with a set of signals received from independent agents, constitute a market mechanism. In the paper we analyze two pricing rules: well known Vickrey-Clarke-Groves rule (VCG) and the parametric pricing rule (PPR). We apply these pricing rules to the allocation rule specified by the balancing communication bandwidth trade model (BCBT).

Keywords—communication bandwidth trade, mechanism design, pricing rule.

1. Introduction

Bandwidth market, in the context of independent traders (in this paper we will call them agents), determines a game among these agents. The finite group of agents interacts. The set of agents is denoted $\mathcal{S} = \{1, 2, \dots, n\}$ and generic agent is represented as i . Every agent tries to maximize her individual profits. Thus, the aims of particular agents are inconsistent. However, the market designer can influence the behavior of particular agents by applying specific rules to the game. Thus, the market designer tries to achieve the overriding goals. In the context of market games, such rules are allocation and pricing rules. Applying this specific rules to the game among independent agents results in the market mechanism.

Every agent is characterized by her preferences. We call this preferences the agent's type T_i . We usually assume that the agent knows only her type, but not those of other agents. The mechanism designer also does not know agents' types. We call such mechanism informationally decentralized [1].

Agent reports the signal θ_i to the mechanism. The signal, in the context of market mechanism, can be understood as a buying/selling offer. Signal is reported on the basis of the strategy function. The strategy function depends on the agent's type, behavior of the other agents and on the mechanism rules. Thus, the signal reported by the particular agent, can differs from this agent's type.

The allocation rule determines the allocation of the offers. It divides the offers for the accepted (also partially accepted) and rejected. Pricing rule sets a vector of prices for traded commodities. The mechanism receives the signals from particular agents and performs the allocation of commodities and determines their valuation – in accordance with the allocation and pricing rules.

2. Desired Mechanisms Properties

Mechanism is constructed in order to fulfill the desired results. Particular agents try to achieve their maximal individual profits, thus their goals are inconsistent. Moreover, goals of particular agents are also inconsistent with global goals, desired by the mechanism designer. Mechanism theory considers the set of the most desired properties. The most important mechanism properties are: incentive compatibility, individual rationality, Pareto-efficiency and budget balance [2].

Incentive compatibility property holds if no agent has incentives to report signal different from her type. In the other words, the incentive compatibility property holds, if no agent has incentives to report untruthful offer. Mechanism is individually rational, if no agent loses from participation in such mechanism. Such property is also called the voluntary participation property. The voluntary participation property means that if agent loses she can choose not to participate in given mechanism. Results of the mechanism are Pareto-efficient, if we can not improve results (i.e., payment) for one agent without making results for some other agents worse off. In the other words, mechanism results are Pareto-efficient if such a results are not Pareto-dominated by other results. Mechanism has balanced budget if sum of sellers expenses is equal to sum of buyers incomes. In the other words, mechanism is budget balanced, if there is no need to surcharge mechanism, and the mechanism does not give us additional profits.

The most desired mechanism should fulfill all listed properties. However, several theorems [3]–[5] (so called impossibility theorems) states, that it is impossible that single mechanism holds all properties. Therefore, we need to decide for the subset of desired mechanism properties, which should hold for considered mechanisms.

In the following parts of the paper, we introduce and compare two mechanisms for the bandwidth trade problems. These mechanisms are Pareto-efficient, individually rational and incentive compatible. However, we allow for the lack of budget balance.

3. Mechanisms Review

Many papers deal with the bandwidth trade problems. We will describe a number of them using the mechanism theory terminology.

In this section we review the variety of bandwidth trading mechanisms presented in the literature. There are three classes of such approaches:

- simultaneous, single link auctions [6], [7] – here we discuss the most recent mechanism MIDAS [6];
- combinatorial auctions [8], [9], with the c-SeBiDA combinatorial double auction as a good representative;
- the family of multicommodity market models (M^3), with basic BCBT market model [10].

3.1. Simultaneous, Single Link Auctions

In the simultaneous, separate auctions for individual links an agent that wants to buy a certain path must put simultaneous bids at all relevant auctions. Then special, iterative mechanisms are required to coordinate individual links. This aspect, as well as possible suboptimality are the main roots of our criticisms for these methods. The review of the auction mechanisms dealing with the problem of coordination of simultaneous, single link auctions is presented in [6]. Authors point out several drawbacks of already proposed mechanism, such as the convergence problem and lack of incentive for submitting truthful bids. In the [6] the simultaneous multi-unit Dutch auction one for each link mechanism (MIDAS) is proposed. The allocation rule of the MIDAS derives from the generalized Vickrey auction [11], but is carried out by the simultaneous Dutch auctions. The payment rule of the mechanism is equivalent to Vickrey-Clarke-Groves payment rule. However, we have to remember that incentive compatibility property is satisfied only when allocation rule is efficient, which is not always true in the case of MIDAS mechanism. Thus, even though the mechanism may seem to be simple and scalable, the complicated synchronization, that requires full information makes it impractical in our opinion.

3.2. Combinatorial Auctions

Combinatorial auctions are designed for trading on dependent commodities. One particular auction model that appears in the context of bandwidth market is the combinatorial seller's bid double auctions (c-SeBiDA) [8]. The c-SeBiDA considers two types of commodities: inter-node links and paths consisting of particular links. Agents may bid a single link or a bundle of links constituting specific path. Allocation rule ensures that the same indivisible amount of bandwidth is assigned to all links constituting buyers path, thus a buyer has no risk of buying different amount of bandwidth on some required links. The c-SeBiDA auction has several valuable properties, such as the maximization of the global economic wealth. However, similarly to the approaches concerning on simultaneous auctions, buyers bids must specify the particular links that constitute a desired path. This may lead to welfare inefficiency. Welfare inefficiency corresponds to a situation

where the social welfare obtained is not maximum possible to reach – we can imagine the allocation rule, which allocates resources in a better way.

The c-SeBiDA mechanism is individually rational. Its results are Pareto-efficient and it has balanced budget. However, such a mechanism does not hold incentive compatibility property – particular agents can derive unreasonable profits from this mechanism.

3.3. Multicommodity Market Models

In the multicommodity auction models, the efficient market balance is obtained in the effect of joint optimization of many elementary buy and sell offers. Multicommodity means that market entities can trade with bundles (packages) of different commodities. The balancing communication bandwidth trade (BCBT) model proposed in [10] allows bidders to place buy offers not only for bundled links, but rather for end-to-end connections. Therefore, no buyer does not have to know which links to choose to best allocate the demanded capacity. It is the decision model that allocates the most efficient links to paths.

We assume that the communication network consists of nodes connected by links. The inter-node link represents a network resource (bandwidth), that can be an elementary commodity offered for sale on the bandwidth market. However, network resources being traded can be more complex and can be composed of many parallel links, or end-to-end node connections represented by paths or subnetworks.

Each buy offer concerns a point-to-point bandwidth connection between a pair of specified locations in a communication network. The locations form the set of network nodes \mathcal{V} . The connections (and links) are unidirectional, i.e., they have source and sink nodes.

The objective of BCBT model is the maximization of total economic welfare Eq. (1), which is the total surplus of all buyers and sellers. Constraints (2) and (3) set upper and lower bounds on particular network links (x_e) and particular end-to-end network demands (x_d). The non-negative variable x_{ed} constraint (4) is interpreted as a bandwidth capacity allocated to network link e to serve end-to-end demand d . Also, the sum of capacities allocated to all network demands $\sum_{d \in \mathcal{D}} x_{ed}$ served by particular network link e , should not exceed the realization x_e of the link constraint (5). Finally, the sum of all capacities, provided with incidence matrix a_{ve} , allocated to all network links, serving particular network demand, should not exceed the realization of the end-to-end demand x_d Eq. (6):

$$\hat{Q} = \max \left(\sum_{d \in \mathcal{D}} E_d x_d - \sum_{e \in \mathcal{E}} S_e x_e \right), \quad (1)$$

subject to:

$$0 \leq x_d \leq h_d, \quad \forall d \in \mathcal{D}, \quad (2)$$

$$0 \leq x_e \leq y_e, \quad \forall e \in \mathcal{E}, \quad (3)$$

$$\sum_{d \in \mathcal{D}} x_{ed} \leq x_e, \quad \forall e \in \mathcal{E}, \quad (4)$$

$$0 \leq x_{ed}, \quad \forall e \in \mathcal{E}, \quad \forall d \in \mathcal{D}, \quad (5)$$

$$\sum_{e \in \mathcal{E}} a_{ve} x_{ed} = \begin{cases} x_d & v = s_d \\ 0 & v \neq s_d, t_d \\ -x_d & v = t_d \end{cases}, \forall v \in \mathcal{V}, \forall d \in \mathcal{D}, \quad (6)$$

where:

indices:

$d \in \mathcal{D}$ buy offers – demands for bandwidth,
 $v \in \mathcal{V}$ network nodes,
 $e \in \mathcal{E}$ sell offers – network resources;

parameters:

a_{ve} = 1 if link e originates in node v ,
 = -1 if e terminates in node v ,
 = 0 otherwise,
 s_d source node for demand d ,
 t_d sink node for demand d ,
 h_d required capacity of demand d ,
 E_d offered unit price for demand d ,
 y_e offered capacity of network link e ,
 S_e offered unit price for network link e ;

variables:

x_{ed} bandwidth flow serving demand d allocated to network link e ,
 x_d contracted bandwidth capacity for demand d ,
 x_e contracted bandwidth capacity for network link e .

The x_e and x_d are, respectively, values of realized bandwidth on the link e and on the demand d . They are also the accepted offers for link e and demand d – in the BCBT model sell offers correspond network links and buy offers correspond demand paths resulting in a multigraph.

As stated before, we will identify offerers with agents, and the submitted offers with signals send to mechanism. Because single offer relates to single link or single path, we can identify the offers with the network resources. Thus, let us define the set of agents as the sum of the sets $\mathcal{S} = \mathcal{E} \cup \mathcal{D}$. Also let us define the signal θ_i sent by the i th agent as the tuple: offered price and offered capacity. When i th agent represents bandwidth seller, such tuple is equal to $\theta_i = \langle S_i, y_i \rangle$, otherwise such tuple is equal to $\theta_i = \langle E_i, h_i \rangle$. Also the allocation results we will denote as $\mathbf{x} = (x_1, \dots, x_i, \dots, x_n)$.

Let us note that the BCBT model does not define any pricing rule, it defines only allocation rule. So we will treat the BCBT model as the allocation rule. This also applies to other bandwidth trade models from the BCBT family. Therefore, we need to propose the pricing rules to the base BCBT allocation rule. As the result we obtain two market mechanisms presented in the next sections.

4. Analyzed Pricing Rules

As we stated before, we analyze and compare two pricing rules in the context of bandwidth trade. These rules are the Vickrey-Clarke-Groves (VCG) rule (mechanism) and the parametric pricing rule (PPR).

4.1. Vickrey-Clarke-Groves Pricing Rule

VCG pricing rule – or rather VCG mechanism was introduced in the papers [12]–[14]. VCG mechanism does not define allocation rule, it only states that applying VCG pricing rule to efficient allocation rule creates VCG mechanism. An allocation rule is said to be efficient if it maximizes “social welfare”, treated as the aggregation of particular agents’ utility functions. Thus, the BCBT is an efficient allocation rule.

For the sake of simplicity, let us assume, that every agent submits one offer. VCG pricing rule sets payoff for every agent. Payoff for i th agent is defined as the opportunity cost that the presence of i th agent introduces to all other agents.

Set of agents sends a vector of signals $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$ to the mechanism. Let us define vector $\boldsymbol{\theta}_{-i} = (\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_n)$, which contains the set of all signals but signal θ_i . The $Q(\boldsymbol{\theta})$ is the economic welfare obtained by the allocation rule with all the signals $\boldsymbol{\theta}$. The $Q(\boldsymbol{\theta}_{-i})$ is the economic welfare, obtained by the allocation rule without i th signal. So, the payment for i th agent is equal to:

$$I_i = Q(\boldsymbol{\theta}) - Q(\boldsymbol{\theta}_{-i}) \quad (7)$$

We can determine prices for each agent (if her offer is accepted). If the agent i submitted the selling offer, the price for her is equal to Eq. (8), otherwise, the buying price for i th agent is equal to Eq. (9). Prices for agents which offers were rejected are negligible.

$$\pi_i^S = \frac{S_i x_i + I_i}{x_i}, \quad (8)$$

$$\pi_i^K = \frac{E_i x_i - I_i}{x_i}. \quad (9)$$

Applying the VCG pricing rule to the BCBT allocation rule results in the VCG mechanism. Properties of such mechanism are as follows: it fulfills incentive compatibility property, results of the mechanism are Pareto-efficient, individual rationality is also fulfilled. Unfortunately, there is a lack of budget balance.

4.2. Parametric Pricing Rule

Second considered pricing rule is the parametric pricing rule [15]. This rule sets prices accordingly to modified Vickrey double auction (MVDA) [16]. The MVDA mechanism was designed for double auction with indivisible commodities. It sets the differentiated buying and selling prices in such a way, that there is no agent, which has incentives to deviate from her type.

Parametric pricing rule uses parametric analysis performed in the respect to the mathematical model of the allocation rule. Parametric analysis is based on repeatedly performed sensitivity analysis. Sensitivity analysis also is performed in the respect to the mathematical model of the allocation rule. Sensitivity analysis provides us with results, which

tell us how much the i th offer price can be changed without changing the commodities allocation. Let us denote results performed in step κ of parametric analysis as $s_i^{(\kappa),+}$ for the selling prices, and as $e_i^{(\kappa),-}$ for the buying prices.

Given i th price (for the sake of simplicity let us assume that it is selling price) is increased of the value $s_i^{(\kappa),+} + \varepsilon$ (where $0 < \varepsilon \ll 1$). Afterwards, the allocation model is solved again and the allocation of the commodities changes. Particular steps of parametric analysis set more beneficial price for i th offer, nevertheless accepted volume of i th offer decreases. The analysis is performed until given i th offer is rejected. On the basis of i th offer price in the last step (let us denote the number of the last step by κ^*) the individual price (π_i^S or π_i^K) for such offer is set.

Parametric pricing rule sets individual prices for each offer. Combined with the BCBT allocation rule constitutes a mechanism. This mechanism fulfills incentive compatibility property, its results are Pareto-efficient, individual rationality is also fulfilled.

Selling prices set by the parametric pricing rule are not lesser than buying prices. Thus the budget balance property does not hold for mechanisms with the parametric pricing rule.

4.3. Imbalance Reduction

We propose the algorithm to reduce the budget imbalance. The main idea of this algorithm is to change the stop criterion of the parametric pricing rule. Stop criterion of the parametric rule generally implies that the parametric analysis will be carried out until the rejection of given offer. We propose change of the stop criterion – the analysis will be carried out until given offer is profitable:

$$I_i = x_i(\pi_i^S - S_i^0) \quad \forall i \in E, \quad (10)$$

$$I_i = x_i(E_i^0 - \pi_i^K) \quad \forall i \in D. \quad (11)$$

However, to perform such algorithm modification, we have to calculate profit for every agent. Equations (10) and (11) represent the profit of agents. Let us notice, that the rules to calculate the profits, need to know agents' types. However, in the previous sections we have assumed that types are private knowledge of particular agents. Nevertheless, we can assume that required value of S_i^0 or E_i^0 (prices that correspond to the agents' types) belongs to certain intervals. Such intervals can be determined on the basis of expert and common knowledge, historical data, etc. Therefore, let us assume that these values belong to the following intervals $S_i^0 \in \ll S_i^0, \bar{S}_i^0 \gg$ and $E_i^0 \in \ll \underline{E}_i^0, \bar{E}_i^0 \gg$.

The main idea of the algorithm for improve the budget balance is to limit price for particular participants. Limitation of price shall be made by reducing the number of steps of the parametric analysis. The algorithm retains good mechanism properties: incentive compatibility, individual rationality and Pareto-efficiency, and reduces the budget imbalance.

5. Experimental Studies

First, on the simple case study, we show differences and similarities between the VCG and the PPR pricing rules. Next, we present a series of experiments. Such experiments focus on three networks from the SNDlib [17] repository. On such experiments we compare the imbalance measure.

5.1. Simple Case Study

The simple case study was performed on the exemplary four-node network (see Fig. 1).

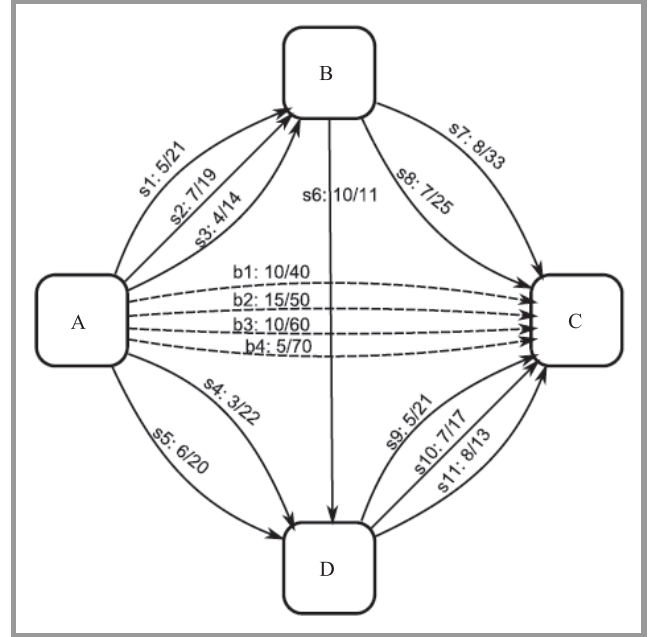


Fig. 1. Four-node network. Solid lines represent links, dotted lines represent demands. The notation is following: $o:v/p$, where o means offer ID, v means offered bandwidth volume and p means offer price for unit of the bandwidth.

In Table 1 we can see prices, set by the mechanisms applying parametric rule and the VCG pricing rule, to particular agents. We can also see results of allocation rule. Some of prices are the same for VCG and PPR, some prices are greater for VCG, some are greater for PPR, nevertheless all the prices are very similar.

Both mechanisms (applying the parametric pricing rule and applying the VCG pricing rule) do not meet the budget balance property. To compare such mechanisms, we propose the measure – relative budget imbalance ratio Eq. (12). Proposed measure reflects the degree of non-compliance with budget balance property. The SI is the total seller's incomes $SI = \sum_{i \in E} S_i x_i$, the BE is the total buyer's expenses $BE = \sum_{i \in D} E_i x_i$:

$$RBUT = \frac{SI - BE}{SI + BE}. \quad (12)$$

Relative budget imbalance ratio for the mechanism applying the parametric pricing rule is equal $RBUT = 3.79\%$, and

Table 1
Prices comparison for mechanisms applying the parametric pricing rule and VCG pricing rule

Nodes	Offer e	Offered vol. S_e	Acc. vol. y_e	Offer price x_e	VCG π_e^S	Comp.	PPR π_e^S
	–	[Mbit/s]		[Euro/Mbit/s]			
A-B	s1	5	2	21	22	=	22
	s2	7	7	19	21.57	>	21
	s3	4	4	14	21.25	>	21
A-D	s4	6	6	20	32.50	>	32
	s5	3	3	22	32	=	32
B-D	s6	10	6	11	13.33	<	14
B-C	s7	7	7	29	30.54	<	31
	s8	8	0	32	–		–
D-C	s9	7	7	17	19.57	<	20
	s10	8	8	13	19.75	<	20
	s11	5	0	21	–		–
	d	E_d	h_d	x_d	π_d^K		π_d^K
	–	[Mbit/s]		[Euro/Mbit/s]			
A-C	b1	10	10	60	49.80	<	50
	b2	15	7	50	47.17	>	47
	b3	5	5	70	50	=	50
	b4	10	0	40	–		–

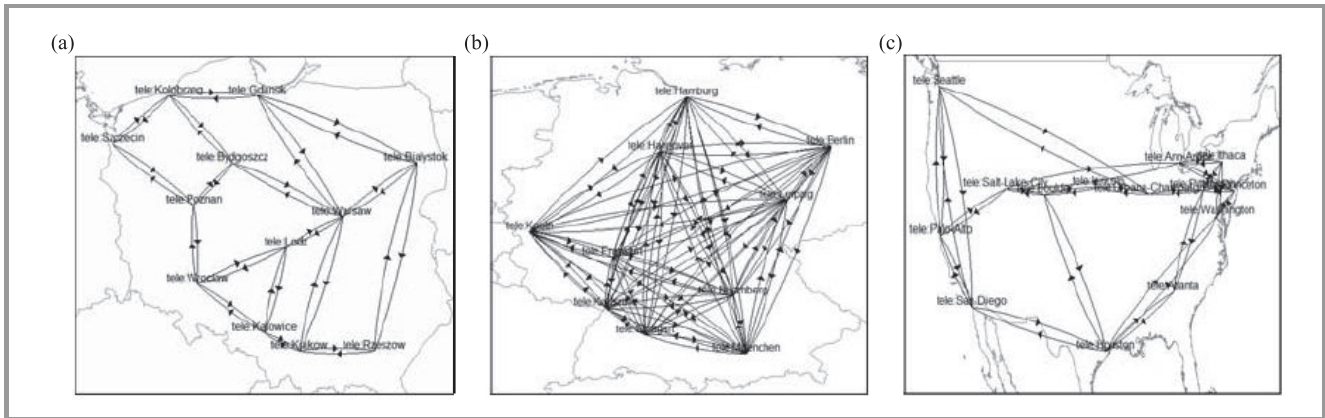


Fig. 2. Network instances [17]: (a) Polska; (b) dfn-bwin; (c) nobel-us.

for the mechanism applying the VCG pricing rule is equal to $RBUT = 3.64\%$ (see Table 1). We can see, that the measure is slightly better for mechanism applying the VCG pricing rule.

5.2. Series of Experiments

Three series of experiments were performed, each contained ten experiments. The experiments concern bandwidth trading performed on the networks taken from the SNDlib library [17]. We analyzed following network instances: Polska, dfn-bwin and nobel-us (Fig. 2). We generated offers for particular network resources (links and demands). Offer prices for particular links and demands was generated on the basis of absolute value of normal distribution, where mean of the distribution was equal to distance between end-to-end nodes. The maximal accepted

Table 2
Aggregated relative budget imbalance for the series of experiments performed on the three backbone networks

Agg. measure		VCG	PPR
Polska			
RBUT	min	0.28	0.37
	mean	0.33	0.45
	max	0.44	0.53
dfn-bwin			
RBUT	min	0.35	0.59
	mean	0.43	0.65
	max	0.47	0.73
nobel-us			
RBUT	min	0.34	0.46
	mean	0.38	0.52
	max	0.40	0.61

trade volumes were generated from absolute value of normal distribution.

In Table 2 we can see aggregated results of the experiments. As we can see, for analyzed cases, the mean value of the relative budget imbalance is lesser for the VCG pricing rule. Thus, we can state, that the VCG pricing rule is better than the parametric pricing rule in terms of the imbalance measure.

6. Summary

In the paper we have analyzed two pricing rules in the context of the balancing communication bandwidth trade allocation rule. These pricing rules are the VCG pricing rule and the parametric pricing rule. Both rules have good properties for multicommodity exchange with infrastructure constraints, specifically for the bandwidth trade. Both pricing rules have cost for adopting, which results from budget imbalance. The experiments show, that for the given data, budget imbalance measure is better for the VCG pricing rule.

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