Paper

Bandwidth Trading: A Comparison of the Combinatorial and Multicommodity Approach

Kamil Kołtyś, Piotr Pałka, Eugeniusz Toczyłowski, and Izabela Żółtowska

Abstract—Since the telecommunication market becomes more complex and dynamic, a strong need for a new, efficient and flexible bandwidth trading mechanisms appears. We believe that good mechanisms, that allow effective and fair allocation of bandwidth between market participants will help to develop the real competitive bandwidth market. In this paper we compare two different double-sided bandwidth auction mechanisms, that seem to be well suited approaches for trading indivisible units of bandwidth: combinatorial auction c-SeBiDA and multicommodity mechanism BACBR-I. The c-SeBiDA mechanism considers two types of commodities: inter-node links and paths consisting of particular links. Market participants may bid a single link, or a bundle of links, constituting a specific path. The BACBR-I mechanism is a multicommodity exchange model, that allows bidders to place buy offers not only for individual or bundled links, but rather for end-to-end connections. Therefore, it is the decision model that allocates the most efficient links to connections. We run a large set of experiments to test the allocation and computational efficiency obtained under both approaches.

Keywords—bandwidth allocation, combinatorial auction, computational efficiency, indivisible resources, multicommodity trade.

1. Introduction

We consider a multilateral network resources market. The market is supplied by many participants such as companies laying cables, network providers and other telecommunication link owners. The customers of the market are service providers (ISP, ASP, etc.), geographically spread organization and also network providers who want to expand their network coverage. We assume that sellers offer single telecommunication links and buyers want to purchase bandwidth between two nodes that may not necessarily be directly connected by single link. Requirement of trading end-to-end connections makes the allocation problem combinatorial, because bandwidth demand can be realized by several network links.

After the debacle of Enron Broadband Services in fall 2001 the development of organized market for bandwidth slowed down. Currently, the dominating form of bandwidth trading are bilateral agreements in which two participants negotiate the contract terms. The negotiations are complex, nontransparent and time consuming. This form of bandwidth trading requires a business relationships and often it is inefficient both globally and individually (especially for participants that have not relevant business relationships). The buyer that wants to purchase bandwidth between two nodes connected by a sequence of links owned by different providers must independently negotiate with all of them. If the negotiation fails with one of them (whereas agreements with other sellers would be drawn up and signed), the buyer will get useless bandwidth as it will not ensure the connection between selected nodes. Also even if the buyer manage to purchase bandwidth along some path connecting chosen nodes, there is a risk that this path would not be the cheapest one from all existing paths between this nodes. Thus there is a need of designing more sophisticated market mechanisms that will not have such severe drawbacks that are involved with bilateral agreements. Lately analysis of bandwidth market collapse in 2001 gives promise of emerging new forms of bandwidth trading in the future thanks to especially technologies like global managed private line (GMPLS) and automatic switched optical network/automatic switched transport network (ASON/ASTN) [1], [2].

In this paper we focus on auction based market mechanisms. We analyze two bandwidth auctions: combinatorial sellers' bid double auction (c-SeBiDA) [3] and model for balancing aggregated communication bandwidth resources with indivisible constraint (BACBR-I) that is an extension of balancing aggregated communication bandwidth resources (BACBR) model [4]. Our aim is to compare the allocation and computational efficiency of aforementioned mechanisms. In Section 2 we present considered mechanisms in terms of general properties of auction and the applied approach of supporting end-to-end connection trading. In Section 3 we formulate mathematical models of c-SeBiDA and BACBR-I. Section 4 contains experimental results. Section 5 summarizes our findings.

2. Bandwidth Auction Properties

2.1. General Properties

There are different types of auctions. The c-SeBiDA and BACBR-I mechanisms can be classified according to the auction taxonomy presented in [5] as single-round, socially efficient and double-sided auction of indivisible goods.

Single-round auction are conducted in a single step. Bandwidth market participants submit their offers and the auc-

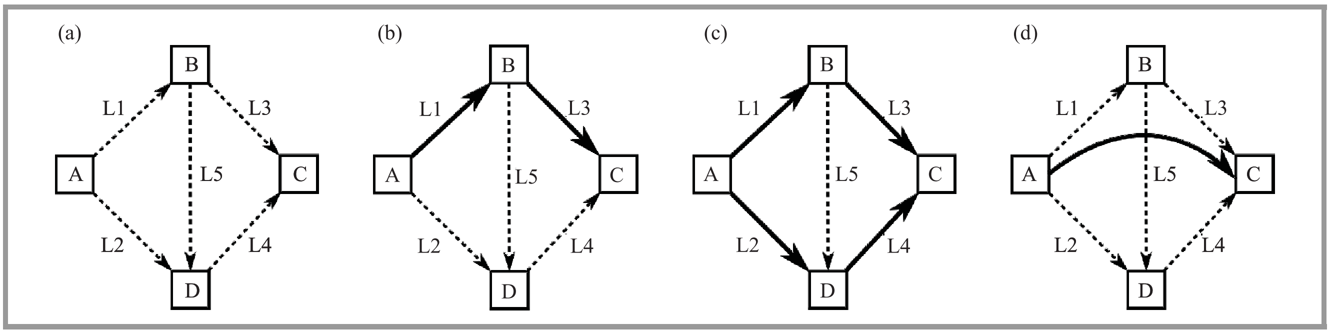

Fig. 1. Different types of supporting end-to-end connection trading: (a) network topology; (b) explicit single path specification; (c) explicit set of admissible paths specification; (d) implicit all possible path specification.

tion mechanism determine an allocation. Other type of auction is progressive one that is carried out in rounds. Progressive auction of bandwidth is proposed in [6].

Double-sided auction concerns multilateral exchange. It can be applied on the bandwidth market where there are many sellers and many buyers. Models for one-sided bandwidth auction (with one seller and many buyers) can be found in [5]–[7].

Auction is socially efficient when it aims at maximizing social welfare. Social efficiency is usually a goal of market mechanisms for bandwidth trading, especially for double-sided auctions. In case of one-sided bandwidth auction also other goals are taken into account, i.e. maximization of seller's revenue [5], [6].

Indivisible goods are integral and cannot be exchanged partially. Bandwidth may be treated as indivisible (modular) commodity. In real networks links often consist of modules that refer to specific transmission and encoding or framing schemes. This modules have determined capacity, i.e., T1 – 1.52 Mbit/s, E1 – 2.04 Mbit/s, OC-3 – 155.52 Mbit/s [8]. Above data transfer standards tend to form standardized contracts on bandwidth market that also use pre-specified amounts of bandwidth [9]. However there are market mechanisms that assume that bandwidth is available in any real fraction of Mbit/s [7], [10].

2.2. Supporting End-to-End Connection Trading

In case of bandwidth auction the essential property is how it supports end-to-end connections trading. We can distinguish three approaches: explicit single path specification, explicit set of admissible paths specification, implicit all possible path specification. All this approaches are illustrated in Fig. 1. Consider network presented in Fig. 1(a) and suppose that buyer wants to purchase bandwidth between nodes A and C. This end-to-end connection has three possible realizations by following sequences of links: L1–L3, L2–L4 and L1–L5–L4.

The first way of supporting end-to-end connection trading relies on explicitly specifying a single path that connects selected nodes. In considered example it can be a path L1–L3 (see Fig. 1(b)). The mechanism should guarantee that the same amount of bandwidth will be allocated at link L1 and L2. This approach is employed by the c-SeBiDA that enables the buyer to submit offer concerning bundle of links constituting particular path. Thus c-SeBiDA is a combinatorial auction. However, explicit single path specification can be also implemented in different manner, i.e., by simultaneous multi-unit dutch auctions [6].

The second approach is more flexible than the first one because it enables to specify a set of admissible paths that can be used to realize end-to-end connection. In considered example buyer may stipulate two paths L1–L3 and L2–L4 (see Fig. 1(c)). The mechanism may allocate different amount of bandwidth at each path, i.e., 90% of demanded bandwidth at path L1–L3 and the rest 10% at path L2–L4, but the summary bandwidth allocated at all paths must be not greater than the buyer's demand. This way of supporting end-to-end connection trading is proposed in [5], [7].

The last approach is the most flexible from the buyer point of view as it enables implicitly specification of all possible paths by submitting offer directly at pair of nodes posing the source and target of end-to-end connection. In considered example, buyer specifies in the offer only source and target of end-to-end connection – appropriately nodes A and C (see Fig. 1(d)). The mechanism itself decides which links are used for realizing this demand. The end-to-end connection between nodes A and C may be realized by several sequence of links, i.e., 40% of demanded bandwidth is served by L1–L3, 50% by L2–L4 and 10% by L1–L5–L4. The BACBR-I applies this approach. It defines two types of commodities: links that are offered for sale and end-to-end connections which are the subject of demand. The buyer that submits an offer for end-to-end connection gets bandwidth at links that generally pose a flow between two selected nodes.

2.3. Auction Rules

Auction mechanism specifies information that market participants must include in their offers. This information is leveraged by two rules of mechanism: allocation rule and pricing rule. Allocation rule decides which offers are accepted. Pricing rule defines the buyers' charges and sellers' incomes. Both this rules are necessary for clearing the market.

The c-SeBiDA and BACBR-I are single-round, socially efficient and double-sided auction of bandwidth considered as indivisible good. The essential difference between this two mechanisms is in the way of supporting trading end-to-end connections. Our goal is to study how this different approaches affects allocation and computational efficiency of this mechanism. Thus further we analyze only allocation rule of considered mechanisms because it is the one that implements the method of end-to-end connection trading and responds for determining optimal value of the social welfare. Nonetheless, it is worth to mention that pricing rule also affects mechanism efficiency as it decides if mechanism gives incentives for truthful bidding. Here we assume that market participants are truthful, so the allocation rule is maximizing substantial social welfare.

3. Mathematical Models

3.1. Notation

The c-SeBiDA and BACBR-I allocation rules can be formulated as mixed integer linear problems. Below we present notation used in both mathematical programming models:

sets:

B	buy offers,
S	sell offers,
E	network links,
$S(e)$	sell offers concerning link $e \in E$, $S(e) \subseteq S$,
D	end-to-end connections[1],
$B(d)$	buy offers concerning end-to-end connection $d \in D$, $B(d) \subseteq B$[1],
V	network nodes[1]

parameters:

$z_m^{\max}$	maximum units of bandwidth offered for purchase according to buy offer $m \in B$,
E_m	unit price of buy offer $m \in B$,
$z_l^{\max}$	maximum units of bandwidth offered for sale according to sell offer $l \in S$,
S_l	unit price of sell offer $l \in S$,
b_{em}	$= 1$ if link $e \in E$ belongs to bundle for which buy offer $m \in B$ is submitted, $= 0$ otherwise[2],
M_d	indivisible unit size of end-to-end connection $d \in D$[1],
M_e	indivisible unit size of link $e \in E$[1],
a_{ve}	$= 1$ if node $v \in V$ is source of link $e \in E$, $= -1$ if node $v \in V$ is target of $e \in E$, $= 0$ otherwise[1],
s_d	source of end-to-end connection $d \in D$[1],
t_d	target of end-to-end connection $d \in D$[1];

variables:

z_m	realization of buy offer $m \in B$,
z_l	realization of sell offer $l \in S$,
x_{ed}	bandwidth flow serving end-to-end connection $d \in D$ allocated to network link $e \in E$[1].

Both c-SeBiDA and BACBR-I collect all buy (B) and sell (S) offers. They require that each buy offer $m \in B$ contains the maximum buy unit price E_m and the maximum units of bandwidth offered for purchase $z_m^{\max}$. Similarly each sell offer $l \in S$ has to include the minimum sell unit price S_l and the maximum units of bandwidth offered for sale $z_l^{\max}$. According to this two parameters of submitted offers considered mechanisms determine the optimal allocation specified by variables z_m and z_l.

3.2. The c-SeBiDA Model

The c-SeBiDA model assumes that sell and buy offers concern network links (E). Sell offer regards single link. For each link many sell offers can be submitted ($S(e)$). Buy offer is combinatorial and it regards bundle of links (defined by parameters b_{em}).

The mathematical model of the c-SeBiDA is following:

$$\hat{Q} = \max\left(\sum_{m\in B} E_m z_m - \sum_{l\in S} S_l z_l\right), \tag{1}$$

$$0 \le z_m \le z_m^{\max}, \quad \forall_{m\in B}\,, \tag{2}$$

$$0 \le z_l \le z_l^{\max}, \quad \forall_{l\in S}\,, \tag{3}$$

$$z_m \in \mathbb{Z}, \quad \forall_{m\in B}\,, \tag{4}$$

$$\sum_{m\in B} b_{em} z_m \le \sum_{l\in S(e)} z_l, \quad \forall_{e\in E}\,. \tag{5}$$

The aim of c-SeBiDA is maximizing social welfare. Thus objective function is defined by Eq. (1), where $\hat{Q}$ denotes optimal value of social welfare. First two constraints (2) and (3) set lower and upper bounds of buy and sell offers realizations, respectively. Next constraint (4) ensures that buy offer realization is integral. Sell offers realization will be integral due to constraint (5) which assure that aggregated demand for link is not greater than aggregated supply of bandwidth for that link. More details about c-SeBiDA can be found in [3].

3.3. The BACBR-I Model

The BACBR-I model considers two types of commodities: network links (E) and end-to-end connections (D). Each link and end-to-end connection has predefined module size in which its bandwidth can be traded (respectively, M_e and M_d parameters). Sell offer regards single link and for each link many sell offers can be submitted ($S(e)$). Buy offer concerns single end-to-end connection and for each connection many buy offers can be submitted ($B(d)$). Because

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[1] Only relevant to BACBR-I model.

[2] Only relevant to c-SeBiDA model.

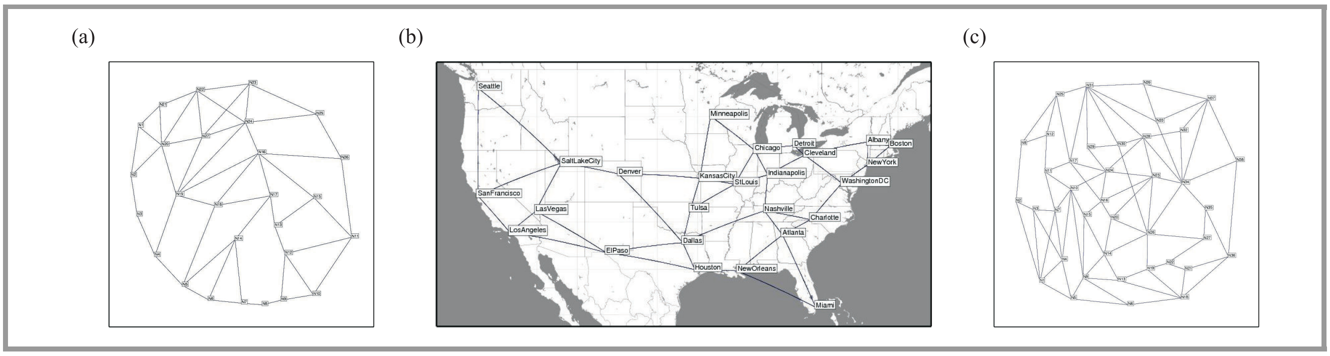

Fig. 2. Network topologies [11]: (a) network sun; (b) network janos-us; (c) network giul39.

BACBR-I itself chooses the links that realize particular end-to-end connection (x_{ed}), it must have an information about network topology. Thus, a set of network nodes (V) and incidence matrix elements (a_{ve}) are given. Also for end-to-end connections source (s_d) and target (t_d) nodes are specified.

The mathematical model of the BACBR-I is following:

$$\hat{Q} = \max\left(\sum_{m\in B} E_m z_m - \sum_{l\in S} S_l z_l\right), \qquad (6)$$

$$0 \le z_m \le z_m^{\max}, \quad \forall_{m\in B}\,, \qquad (7)$$

$$0 \le z_l \le z_l^{\max}, \quad \forall_{l\in S}\,, \qquad (8)$$

$$z_m \in \mathbb{Z}, \quad \forall_{m\in B}\,, \qquad (9)$$

$$z_l \in \mathbb{Z}, \quad \forall_{l\in S}\,, \qquad (10)$$

$$\sum_{d\in D} x_{ed} \le \sum_{l\in S(e)} M_e z_l, \quad \forall_{e\in E} \qquad (11)$$

$$0 \le x_{ed}, \quad \forall_{e\in E, d\in D} \qquad (12)$$

$$\sum_{e\in E} a_{ve} x_{ed} = \begin{cases} \sum\limits_{m\in B(d)} M_d z_m & v = s_d \\ 0 & v \neq s_d, t_d \\ -\sum\limits_{m\in B(d)} M_d z_m & v = t_d \end{cases}, \ \forall_{v\in V, d\in D} \qquad (13)$$

First four equations in BACBR-I model Eqs. (6)–(9) are the same as in the c-SeBiDA. It stems from the fact that BACBR-I also maximizes social welfare and restricts offers realizations according to their maximum volumes. Constraint (10) imposes that sell offer realization is integral. Next two constraints ensure that total bandwidth flow at particular link will not be greater than aggregated realizations of sell offers concerning this link, constraint (11) and that bandwidth flow at all links will be non-negative, constraint (12). Equation (12) is a flow conservation constraint that must be met for each end-to-end connection.

Comparing above models, both of them maximize social welfare on the basis of submitted offers that contain unit price and maximum volume. Both models treat bandwidth as indivisible good and support trading end-to-end connections. The BACBR-I and c-SeBiDA differs in the way of supporting end-to-end connection trading. Moreover, the BACBR-I is more comprehensive than c-SeBiDA, as it enables trading network resources consisting of modules with different size.

4. Experimental Studies

4.1. Test Instances

Experimental studies have been conducted on several test instances which are based on data from survivable network design library (SNDlib) available on the web site [11]. Although the SNDlib is a set of survivable fixed telecommunication network design problems, it provides information that is very important in bandwidth trading problems: network topology and a set of end-to-end connections. We consider three networks from SNDlib: sun, janos-us and giul39. Topologies of this networks are presented in Fig. 2. Table 1 contains the number of nodes, links and end-to-end connections for each considered network.

Table 1
Information about size of considered networks

Network	Nodes	Links	End-to-end connections
sun	27	102	67
janos-us	26	84	650
giul39	39	172	1471

Bandwidth allocation test instance besides aforementioned data requires specification of offers that are submitted for network resources. Offers have been generated according to the following rules:

- summarized bandwidth offered for sale (purchase) at link (end-to-end connection) equals the link capacity (end-to-end connection demand value) given by SNDlib;
- unit price of offer concerning link (end-to-end connection) is determined on the basis of the distance between nodes connected by this link (end-to-end connection) and some random factor that is used to differentiate prices of offers regarding the same link (end-to-end connection).

So also some other data from SNDlib such as nodes coordinates (used to calculate distance between nodes), links capacities, end-to-end connections demands come in useful for preparing test instances of bandwidth allocation problem.

For each network three offer variants have been generated with different average number of offers submitted for single link or end-to-end connection, respectively, 2, 4 and 6 offers per link or end-to-end connection. In all, nine test instances of bandwidth allocation problem have been prepared. All of them have been adjusted to the c-SeBiDA mechanism in which buy offers are submitted not for end-to-end connections, but for bundles of links. For each buy offer a sequence of links has to be specified that realizes suitable end-to-end connection. As a realization of end-to-end connection we choose randomly one of the three least expensive path realizations. Because end-to-end connection realization affects the social welfare obtained by the c-SeBiDA, we consider five variations of each test instance in which buy offers are submitted for different sequence of links realizing particular end-to-end connection. All test instances have been implemented in multicommodity market data model (M^3) [12].

4.2. Allocation Efficiency

The comparison of allocation efficiency obtained by both c-SeBiDA and BACBR-I mechanisms in all test instances is given in Table 2. Test instance is identified by network name and average number of offers submitted for single link or end-to-end connection. Table 2 does not contain the numerical values of social welfare achieved by both mechanisms, but only the relation between them assuming that social welfare determined by BACBR-I equals 1. Because in case of the c-SeBiDA mechanism we analyze five different variations of bundles of links generated for buy offers the maximium, average and minimum social welfare obtained by this mechanism in proportion to BACBR-I optimal allocation is presented.

Table 2
Comparison of c-SeBiDA and BACBR-I allocation efficiency

Network	Offers	BACBR-I	c-SeBiDA		
			max	avg.	min
sun	2	1	0.77	0.75	0.73
	4	1	0.77	0.72	0.65
	6	1	0.83	0.81	0.81
janos-us	2	1	0.84	0.8	0.77
	4	1	0.85	0.82	0.79
	6	1	0.86	0.84	0.81
giul39	2	1	0.8	0.79	0.79
	4	1	0.78	0.78	0.77
	6	1	0.82	0.81	0.81

Allocation efficiency of the c-SeBiDA is on average about 80% of BACBR-I allocation efficiency. In the best case social welfare achieved by c-SeBiDA accounts for 86% of social welfare determined BACBR-I. In case of network sun with 4 offers per single link or end-to-end connection on average the c-SeBiDA obtains only 65% of social welfare provided by BACBR-I.

4.3. Computational Efficiency

Table 3 presents information about number of variables and constraints of mathematical models related to particular mechanism and test instance identified by network name and average number of offers submitted for single link or end-to-end connection. The BACBR-I mathematical model has more variables and constraints than the c-SeBiDA model. The difference is substantial for the largest network giul39 with average 2 offers per link or end-to-end connection. In this test instance the BACBR-I model has about 80 and 10 times more variables and constraints, respectively, than the c-SeBiDA model.

Table 3
Number of variables (var.) and constraints (con.) in c-SeBiDA and BACBR-I mathematical models

Network	Offers	BACBR-I		c-SeBiDA	
		var.	con.	var.	con.
sun	2	7169	2581	335	772
	4	7515	3273	681	1464
	6	7864	3971	1030	2162
janos-us	2	56052	19888	1452	2988
	4	57606	22996	3006	6096
	6	59016	25816	4416	8916
giul39	2	256280	64077	3268	6708
	4	259589	70695	6577	13326
	6	262858	77233	9846	19864

The comparison of computational efficiency of c-SeBiDA and BACBR-I is given in Table 4. The table presents the time of solving mixed-integer linear programming prob-

Table 4
Comparison of c-SeBiDA and BACBR-I allocation time [s]

Network	Offers	BACBR-I	c-SeBiDA
sun	2	0.7	0.02
	4	0.89	0.02
	6	0.92	0.02
janos-us	2	15.8	0.05
	4	15.93	0.03
	6	14.19	0.07
giul39	2	499.01	0.05
	4	526.39	0.09
	6	512.71	0.17

lems related to considered test instances. Optimization has been performed by CPLEX 9.1 on computer with processor Intel Core2 Duo T8100 2.1 GHz, main memory 3 GB and 32-bit operating system MS Vista.

The BACBR-I mechanism requires more time than c-SeBiDA to determine optimal allocation. It is meaningful in case of the largest network giul39, for which BACBR-I model must find optimal allocation of links bandwidth for great number of end-to-end connections. Complexity of this task is reflected by the large number of variables and constraints of the BACBR-I model. It is worth noting that the allocation time of both mechanism is not rising a lot with increase of average number of offers submitted for single link or end-to-end connection.

5. Summary

This paper compares two single-round, socially efficient and double-sided auctions of indivisible network resources that represents different approaches of supporting end-to-end connection trading. The c-SeBiDA is a combinatorial auction that requires explicit single path specification posing realization of particular end-to-end connection. The BACBR-I enables submitting buy offers for pair of nodes that are the source and target of end-to-end connection. The former mechanism requires that buyer knows a network topology and chooses appropriate links. Explicit bundle of links specification in buy offers affects allocation efficiency of the c-SeBiDA which provides on average 20% less social welfare than the BACBR-I mechanism. The BACBR-I itself allocates the bandwidth links to the buyer assuring connections between selected nodes. It provides highest allocation efficiency, however, it requires more time to determine optimal allocation than the c-SeBiDA.

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