Paper

Analytical properties of a stochastic teletraffic system with MMPP input and an access function

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Abstract — Stochastic modeling of teletraffic systems with restricted availability and correlated input arrival rates is of great interest in GoS (grade of service) analysis and design of certain telecommunication networks. This paper presents some analytical properties of a recursive nature, associated with the infinitesimal generator of the Markov process which describes the state of a teletraffic system with MMPP (Markov modulated Poisson process) input traffic, negative exponentially distributed service times, finite queue and restricted availability defined through a loss function. Also the possible application of the derived properties to a direct method of resolution of the linear system, which gives the stationary probability distribution of the system, will be discussed.

Keywords — stochastic analysis of telecommunication networks, teletraffic theory, GoS analysis of overflow teletraffic systems, queuing systems.

1. Introduction

Problems of performance analysis in telecommunication networks led in the past to the concept of restricted availability systems in which the connection paths may be such that an incoming call may be unsuccessful even when there are still idle circuits in the destination group. In classical studies [1] of teletraffic link systems "loss functions" were used to represent in simple mathematical terms the effects of the restricted availability with respect to the arriving calls for service. This function $(w(\upsilon))$ is defined as the conditional probability that a call arriving when there are υ occupied servers, is rejected. In particular this concept was used for calculating the blocking probability of restricted availability overflow systems arising in teletraffic networks with alternative routing. Although these systems typically did not have queuing facilities, modern technologies may provide systems with limited waiting room (say k queuing positions in a buffer). We also may consider teletraffic systems where decisions regarding the acceptance or rejection of a call are of a probabilistic nature and based on the number of calls already in progress (see example in [2]) or waiting for service, mechanism which could be also represented by some specific type of loss function. An example could be the case of "load sharing" [3] schemes of adaptive dynamic routing in multiexchange networks in which calls rejected by a given route are offered to alternative routes according to a set of probabilities which are a function of the states (number of occupations) of the individual groups of channels in the different links of the network.

On the other hand a number of studies [4–7] suggest that the MMPP could be used successfully for modeling certain types of superposition of complex teletraffic flows, including packetized voice and packet data traffic as well as video sources traffic in ATM networks. In particular the MMPP is the exact model for the superposition of independent IPP (interrupted Poisson processes), representing overflow traffics resulting from the overflow of Poisson inputs in loss systems with exponential distribution of the service times (model of great interest in circuit-switched networks with alternative routing).

The m-MMPP point process may be defined as a doubly stochastic Poisson process where the intensity process $\{\lambda(t),\, t \geq 0\}$ is governed by an ergodic Markov process, with m states, i.e.:

$$\lambda(t) := \lambda_{I(t)}\,,$$

where the R.V. (random variable) $I(t)$ indicates the state, at instant t, of an ergodic Markov process. When $I(t) = f$, $f = 1, \ldots, m$, the MMPP is said to be in phase f.

The MMPP is also a particular case of the "Versatile Markovian Point Process" model in [8] and may also be treated as a particular case of the Markovian arrival process model, see [9] and [10].

In a previous work [11], the exact analysis of a loss system with a m-MMPP input, a finite queue of capacity k, N servers with negative exponential service times and a loss function $\omega(\upsilon) := 1-\alpha_\upsilon$, was performed. The extension of this work by considering the exact analysis of a system with finite queuing capacity whose inputs are defined from a number of independent MMPPs each being subject to a particular "access function" is given in [12].

The analysis of such systems, including the characterization of the associated key processes (describing the system state, the overflow traffic and the carried traffic) is expressed in terms of the infinitesimal generator of the Markov process which describes the state of the system, Q. This paper presents some analytical properties of a recursive nature, associated with that infinitesimal generator. The considered loss function of the system may in general depend both on the number of occupations and the phase of the input MMPP. The paper begins by reviewing the basic features of the ergodic Markov process which represents the

$$Q = \begin{bmatrix} A=\underline{\alpha}(0)\Lambda & \alpha(0)\Lambda & & & & & \\ \mu I & A-\underline{\alpha}(1)\Lambda-\mu I & \underline{\alpha}(1)\Lambda & & & & \\ & & \dots & & & & \\ & & N\mu I & A-\underline{\alpha}(N)\Lambda-N\mu I & \underline{\alpha}(N)\Lambda & & \\ & & & N\mu I & A-\underline{\alpha}(N+1)\Lambda-N\mu I & \underline{\alpha}(N+1)\Lambda & \\ & & & & & \dots & \\ & & & & & N\mu I & A-N\mu I \end{bmatrix} \quad (2)$$

system state by describing the structure of its infinitesimal generator Q obtained from previous work of the authors. Next, some recursive formulae for the matrix which contains the basis of the vector space of the solutions associated with a submatrix of a matrix of the type of Q, are derived by exploring the diagonal block structure of this type of matrices. These properties are then applied to Q having in mind its specific block structure. Also the possible application of the derived properties to a direct method of resolution of the linear system which gives the stationary probability distribution of the system, will be considered. Some numerical examples of application of such a direct method will be presented in order to illustrate its potential advantages and limitations.

2. Characterization of the system

Let us consider the stochastic service system represented in Fig. 1, with m-MMPP input, finite queue k, N servers, negative exponentially distributed service times (with mean μ^{-1}) and a loss function $\omega(\upsilon, f) = 1 - \alpha(\upsilon, f)$, $\upsilon = 0, \dots, N+k,\ f = 1, \dots, m$.

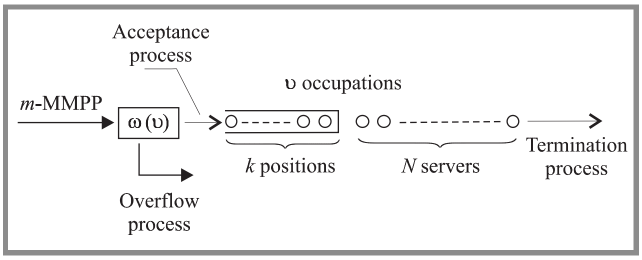

Fig. 1. Stochastic service system.

Note that the input m-MMPP may represent itself the superposition of a number of independent m_r-MMPPs, and the access function $\alpha(\upsilon, f)$ enables to represent the conditional probability of an arrival being accepted when the system is in state (υ, f), where υ is the number of occupations and f is the current phase of the input process (this general case was analysed in [12]), assuming that each m_r-MMPP has a particular access function $\alpha_r(\upsilon)$. The details of the analysis of the system, namely the characterization of the overflow process, the acceptance process and the termination process are given in [11] and [12].

The stochastic process $\{X_t, t \geq 0\}$ which describes the system state at instant t, has the state space:

$$I = \{i = (\upsilon, f),\ \upsilon = 0, \dots, N+k,\ f = 1, \dots, m\} \quad (1)$$

and is an ergodic Markov process. X_t is characterized by the infinitesimal generator [12] – see Eq. (2), shown at the top of this page, where:

$$\underline{\alpha}(\upsilon) = \mathrm{diag}\big(\alpha(\upsilon,1), \dots, \alpha(\upsilon,m)\big),$$
$$\Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_m),$$

λ_f is the intensity of the input MMPP in phase f and A is the infinitesimal generator of the ergodic Markov process governing the intensity process of the input MMPP.

An essential element of the system analysis or of any system with similar infinitesimal generator is the stationary measure π of Q (stationary probability distribution):

$$\pi = [\pi_i], \quad i \in I, \quad (3)$$

such that:

$$\begin{cases} \pi Q = 0 \\ \pi e = 1 \end{cases},$$

where e is the column matrix $e = [1, \dots, 1]^T$.

Note that the most relevant GoS parameters of this type of system, namely the call congestion and the waiting probability, may be expressed in terms of π (see [11] and [12]). In this paper a recursive formula for π will be derived which beyond its analytical value may also be used for a direct resolution of the linear system (3).

3. Analytical properties

3.1. Preliminary analysis and results

Let us consider the linear system (3), where Q is the square matrix composed of $S+1$ rows (and columns) of square blocks of order m:

$$Q := \begin{bmatrix} A_0 & C_0 & & & & \\ M_1 & A_1 & C_1 & & & \\ & & \dots & & & \\ & & M_i & A_i & C_i & \\ & & & & \dots & \\ & & & & M_S & A_S \end{bmatrix}. \quad (4)$$

It is assumed that Q is the infinitesimal generator of an ergodic jump Markov process (of stationary measure π), and the matrices $M_1, \ldots, M_S$ are regular.
Let us now consider the submatrix Q' with $(S+1)$ block rows and S block columns, obtained from Q by eliminating the block column S:

$$Q' := \begin{bmatrix} A_0 & C_0 & & & & & \\ M_1 & A_1 & C_1 & & & & \\ & & \cdots & & & & \\ & & M_i & A_i & C_i & & \\ & & & & \cdots & & \\ & & & & M_{S-1} & A_{S-1} \\ & & & & & M_S \end{bmatrix} . \quad (5)$$

Since the square submatrix Q'':

$$Q'' := \begin{bmatrix} M_1 & A_1 & C_1 & & \\ & & \cdots & & \\ & & M_{S-2} & A_{S-1} & C_{S-2} \\ & & & M_{S-1} & A_{S-1} \\ & & & & M_S \end{bmatrix} \quad (6)$$

of order $S.m$ is regular (because all its diagonal blocks $M_1, \ldots, M_S$, are regular), we conclude that Q' has also rank $S.m$ (Q' has $S.m$ columns). Therefore the set s of solutions of the homogeneous system:

$$uQ' = 0 \quad (7)$$

constitutes a vector space of dimension $(S+1)m - Sm = m$. Let $T := \{x_0, \ldots, x_{m-1}\}$ be a basis of s, and consider the rectangular matrix:

$$X := \begin{bmatrix} x_0 \\ \cdots \\ x_{m-1} \end{bmatrix} = \left[X_0 \,|\, X_1 \,|\, \ldots \,|\, X_S\right], \quad (8)$$

where every square block X_i has order m.
From Eqs. (7) and (8):

$$XQ' = 0 . \quad (9)$$

Obviously $\pi \in s(\pi Q = 0 \Rightarrow \pi Q' = 0)$. Then we have:

$$\pi = \gamma X , \quad (10)$$

where $\gamma := [\gamma_0, \ldots, \gamma_{m-1}]$ is a row matrix of order m.
Representing by I_k the identity matrix of order k, putting:

$$X_0 = I_m \quad (11a)$$

(which guaranties T as a basis of s) and multiplying the first block column of Q' by X we obtain:

$$X_0A_0 + X_1M_1 = 0 \Leftrightarrow X_1 = -X_0A_0M_1^{-1} = -A_0M_1^{-1}. \quad (11b)$$

Using now the second block column:

$$X_0C_0 + X_1A_1 + X_2M_2 = 0 \Leftrightarrow X_2 = -(X_0C_0 + X_1A_1)M_2^{-1}. \quad (11c)$$

For the $(i-1)$th block column:

$$X_i = -(X_{i-2}C_{i-2} + X_{i-1}A_{i-1})M_i^{-1}, \quad i = 2, \ldots, S. \quad (11d)$$

Therefore Eqs. (11) allow us to obtain X recursively.
Let us now designate by Q_i, $i = 0, \ldots, S$, the ith block column of matrix Q. From (9) we have $XQ_i = 0$, $i \in \{0, \ldots, S-1\}$. However the ergodicity of the Markov jump process referred to above implies, as is well known, that the set of solutions of Eq. (3) constitutes a vector space of dimension 1. Then we have, for $m > 1$ (the case $m = 1$ is trivial because π becomes directly determined from (11)):

$$XQ_S \neq 0 . \quad (12)$$

Therefore (12) in conjunction with Eqs. (10) and (3) allows us to say that γ can be obtained by resolving the m-order system:

$$\gamma(XQ_S) = 0 . \quad (13)$$

Explicitly:

$$\gamma(X_{S-1}C_{S-1} + X_SA_S) = 0 . \quad (14)$$

This system is obviously singular but its vector space of solutions has dimension 1. By introducing γ in (10), π becomes known in terms of X after the normalization $\pi e = 1$.

Remark. The present analysis is also applicable to systems with Q having the same general properties, but with the form:

$$Q = \begin{bmatrix} A_{00} & A_{01} & A_{02} & \ldots & A_{0S} \\ M_1 & A_{11} & A_{12} & \ldots & A_{1S} \\ & M_2 & A_{22} & \ldots & A_{2S} \\ & & & \cdots & \\ & & & M_S & A_{SS} \end{bmatrix} . \quad (15)$$

In this case, (11) becomes:

$$X_i = -\left(\sum_{j=0}^{i-1} X_jA_{j,i-1}\right) M_i^{-1}, \; i = 1, 2, \ldots, S . \quad (16)$$

3.2. Application to the system

In the case of the matrix Q of the system shown in Fig. 1 we have $S = N + k$ and:

$$\begin{aligned} M_i &= f(i)\mu I_m, & i &= 0, \ldots, S \\ C_i &= \underline{\alpha}(i)\Lambda, & i &= 0, \ldots, S-1 \\ C_S &= 0 & & \\ A_i &= A - M_i - C_i, & i &= 0, \ldots, S , \end{aligned} \quad (17)$$

where $\alpha(i)$ and Λ are diagonal of order m, I_m is the identity matrix of order m, A is singular of order m and:

$$f(i) := \begin{cases} i & \text{if } i < N \\ N & \text{if } i \geq N \end{cases} .$$

Proposition. For the case of matrix Q (2) and making $X_0 = I_m$, X is given recursively by:

$$X_i = \left\lfloor X_{i-1}C_{i-1} - \left(\sum_{j=0}^{i-1} X_j\right) A \right\rfloor M_i^{-1}, \quad i = 1,\dots,N+k. \tag{18}$$

Proof (by induction):

1. From Eqs. (11b) and (16) (for $i = 1$):
$$X_1 = -A_0 M_1^{-1} = (C_0 - A) M_1^{-1}$$
which satisfies Eq. (18), taking into consideration that $X_0 = I_m$.
2. From Eq. (11d):
$$X_{i+1} = -\left(X_{i-1}C_{i-1} + X_i A_i\right) M_{i+1}^{-1}. \tag{19}$$
Substituting (17) in (19):
$$X_{i+1} = \left(-X_{i-1}C_{i-1} - X_i A + X_i C_i + X_i M_i\right) M_{i+1}^{-1}.$$
Introducing (18):
$$X_{i+1} = \left\lfloor -X_{i-1}C_{i-1} - X_i A + X_i C_i + X_{i-1}C_{i-1} + \right.$$
$$\left. -\left(\sum_{j=0}^{i-1} X_j\right) A \right\rfloor M_{i+1}^{-1} = \left[X_i C_i - \left(\sum_{0}^{i} X_j\right) A\right] M_{i+1}^{-1}.$$
□

From Eq. (18), with $i = S$ we have:

$$X_S M_S = -\left(\sum_{j=0}^{S-1} X_j\right) A + X_{S-1} C_{S-1}$$

or:

$$\left(\sum_{j=0}^{S-1} X_j\right) A = X_{S-1} C_{S-1} - X_S M_S\,.$$

By adding $X_S A$ to both sides we obtain:

$$\left(\sum_{j=0}^{S} X_j\right) A = X_{S-1} C_{S-1} - X_S M_S + X_S A$$

or, taking into consideration that $A_S = A - M_S$:

$$\left(\sum_{j=0}^{S} X_j\right) A = X_{S-1} C_{S-1} + X_S A_S$$

this implies, taking (15) into consideration, that the m-order system:

$$\gamma\left[\left(\sum_{j=0}^{S} X_j\right) A\right] = 0 \tag{20}$$

may be used for obtaining γ.
The result (20) can also be derived from stochastic considerations, noting that $u := \sum_{0}^{N+k} \pi_\upsilon$ is the stationary probability measure of the underlying Markov jump process of the input MMPP.

3.3. The case $m = 2$

In this case, formulae (18) and (20) can be simplified. In fact since A is the infinitesimal generator of a Markov jump process, the sum of the elements of each row is zero. In other words, the first and the second columns of A have symmetrical elements. Let

$$A_0 := \begin{bmatrix} a_{0,0} \\ a_{0,1} \end{bmatrix}, A_1 := \begin{bmatrix} a_{0,1} \\ a_{1,1} \end{bmatrix},$$
$${}_iR := \sum_{j=0}^{i} X_j,\ i = 0,\dots,N+k \tag{21}$$

since $A_0 = -A_1$, this implies ${}_iRA_0 = -{}_iRA_1$. So, this kind of symmetry of matrix A is transmitted to the matrices ${}_iRA$ and these matrix products are simplified.
The space of solutions of the singular system (20) has dimension 1. So we may arbitrate $\gamma_0 = 1$ and put:

$$R :=_{N+k} R := \begin{bmatrix} r_{0,0} & r_{0,1} \\ r_{1,0} & r_{1,1} \end{bmatrix},$$
$$RA = \begin{bmatrix} r_{0,0}a_{0,0} + r_{0,1}a_{1,0} & -\left(r_{0,0}a_{0,0} + r_{0,1}a_{1,0}\right) \\ r_{1,0}a_{0,0} + r_{1,1}a_{1,0} & -\left(r_{1,0}a_{0,0} + r_{1,1}a_{1,0}\right) \end{bmatrix} \tag{22}$$

then:

$$\gamma(RA) = [1, \gamma_1] \times$$
$$\times \begin{bmatrix} r_{0,0}a_{0,0} + r_{0,1}a_{1,0} & -\left(r_{0,0}a_{0,0} + r_{0,1}a_{1,0}\right) \\ r_{1,0}a_{0,0} + r_{1,1}a_{1,0} & -\left(r_{1,0}a_{0,0} + r_{1,1}a_{1,0}\right) \end{bmatrix} = 0$$

and

$$\gamma_1 = -\frac{r_{0,0}a_{0,0} + r_{0,1}a_{1,0}}{r_{1,0}a_{0,0} + r_{1,1}a_{1,0}} = -\frac{r_{0,0} + \alpha r_{0,1}}{r_{1,0} + \alpha r_{1,1}},$$
$$\alpha := \frac{a_{1,0}}{a_{0,0}}. \tag{23}$$

4. Calculation of the probability distribution

An obvious application of the recursive formula (18) is the resolution of the linear system (3).
In [13], an iterative method for solving a system which is a particular case of the one under consideration (with full availability which corresponds to $\alpha(\upsilon, f) = 1$, for all (υ, f), was presented. This method results from the application of the general procedure for constructing iterative methods (see [14], p. 532):

$$\pi' Q = 0 \Leftrightarrow \pi' R = -\pi'(Q - R) \Leftrightarrow$$
$$\Leftrightarrow \pi' = \pi'(Q - R)(-R)^{-1} \tag{24}$$

and

$$\pi'^{(n)} = \pi'^{(n-1)}\,(Q - R)\,(-R)^{-1}\,, \tag{25}$$

where $\pi'^{(n)}$ is the value of π' after the nth iteration. This scheme converges if the spectral radius of $(I-QR)^{-1}$ is less than 1 ([14], theor. 8.2.1).
In [13] it is considered:

$$R := I_{N+k+1} \otimes (A - \Lambda - N\mu I_m), \tag{26}$$

where $\otimes$ represents Kronecker product.
Putting

$$M = -(A - \Lambda - N\mu I_m) \tag{27}$$

then

$$(-R)^{-1} = I_{N+k+1} \otimes M^{-1}. \tag{28}$$

Introducing (27) and (28) in (25) the following iterative method is now obtained for the system (3), with Q given by (2):

$$\begin{cases} \underline{\pi}_0'^{(n)} = \Big\{ \underline{\pi}_0'^{(n-1)} \Big[\big(I - \underline{\alpha}(0)\big)\Lambda + N\mu I \Big] + \\ \quad + \underline{\pi}_1'^{(n-1)} \mu I \Big\} M^{-1} \\ \dots \\ \underline{\pi}_i'^{(n)} = \Big\{ \underline{\pi}_{i-1}'^{(n-1)} \underline{\alpha}(i-1)\Lambda + \underline{\pi}_i'^{(n-1)} \Big[\big(I - \underline{\alpha}(i)\big)\Lambda + \\ \quad +(N-i)\mu I \Big] + \underline{\pi}_{i+1}'^{(n-1)} (i+1)\mu I \Big\} M^{-1} \\ \dots \\ \underline{\pi}_N'^{(n)} = \Big\{ \underline{\pi}_{N-1}'^{(n-1)} \underline{\alpha}(N-1)\Lambda + \underline{\pi}_N'^{(n-1)} \Big[\big(I - \underline{\alpha}(N)\big)\Lambda \Big] + \\ \quad + \underline{\pi}_{N+1}'^{(n-1)} N\mu I \Big\} M^{-1} \\ \dots \\ \underline{\pi}_{N+j}'^{(n)} = \Big\{ \underline{\pi}_{N+j-1}'^{(n-1)} \underline{\alpha}(N+j-1)\Lambda + \underline{\pi}_{N+j}'^{(n-1)} \times \\ \quad \times \Big[\big(I - \underline{\alpha}(N+j)\big)\Lambda \Big] + \underline{\pi}_{N+j+1}'^{(n-1)} N\mu I \Big\} M^{-1} \\ \dots \\ \underline{\pi}_{N+k}'^{(n)} = \Big\{ \underline{\pi}_{N+k-1}'^{(n-1)} \underline{\alpha}(N+k-1)\Lambda + \underline{\pi}_{N+k}'^{(n-1)} \Lambda \Big\} M^{-1}. \end{cases} \tag{29}$$

As initial value, analogously to Meier [13], we may put:

$$\pi'^{(0)} = \big[(N+k+1).m\big]^{-1} e^T. \tag{30}$$

As an alternative one might apply the recursive scheme (18) for constructing a direct method of resolution of the system:

1. $X_0 = I_m$.
2. For $i = 1, \dots, S$, apply the recursion (18) in X_i.
3. Solve:

   $$\gamma\big(X_{S-1}\underline{\alpha}(S-1)\Lambda + X_{S-1}(A - M_S)\big) = 0$$

   with respect to γ, by any suitable method.
4. Compute $\pi' = \gamma X$ and finally $\pi = \dfrac{\pi'}{\pi' e}$.

This method has the disadvantage of any direct method: error propagation. However it has the advantage of its simplicity and efficiency in terms of implementation, which makes it attractive for systems with small dimension. This method may also be used to obtain a first approximate solution, which may then be improved through an iterative scheme such as (29). Note, on the other hand, that the method takes advantage of the particular block structure of Q.
For an interesting overview of numerical techniques for the resolution of sparse linear systems namely related to Markov processes analysis, see [15].

5. Computational experiments

In Table 1 some computational results are presented, obtained under the following conditions:

$$\mu = 1,\ k = 0,\ N = 160,\ m = 2,$$
$$\alpha(\upsilon) = \mathrm{diag}(1 - c^{N-\upsilon}, \dots, 1 - c^{N-\upsilon}),$$
$$\upsilon = 0, \dots, N-1,\ c = \frac{\lambda}{\mu N}$$

(where λ is the mean intensity of the input m-MMPP, and the choice of $\underline{\alpha}(\upsilon)$ corresponds to the classical "geometric group" approximation by Smith [17]),

$$A = \begin{bmatrix} -a_0 & a_0 \\ a_1 & -a_1 \end{bmatrix},\ \Lambda = \begin{bmatrix} l_0 & \\ & l_1 \end{bmatrix}.$$

Each row corresponds to a calculation of π by three different methods: using recursive formula only (column "recurs"), recursive formula refined by the iterative method (columns "refined" and "nitd") and iterative method only (columns "iterat" and "nitm"). Iterative schemes are stopped when $\max\Big\{\big|\pi_i^{(n)} - \pi_i^{(n-1)}\big|,\ i \in I\Big\} \le 10^{-6}$ (columns "nitd" and "nitm" present the number of iterations in the respective case). After calculation of π, the vector err = πQ is evaluated; columns "recurs", "refined", "iterat" present the maximum absolute values of this vector in the three cases:

$$\left.\begin{array}{l}\text{recurs}\\ \text{refined}\\ \text{iterat}\end{array}\right\} = \max\Big\{\big|\mathrm{err}_i\big|,\ i \in I\Big\} = \varepsilon_{\max}.$$

It can be seen that the recursion is sensitive to the "jitter" [16] of the input MMPP. In fact greater values of a_0 and a_1 (which imply increased "jitter") increases the recursion fragility, leading to unacceptable $\varepsilon_{\max}$ unless the refinement through the iterative procedure is applied. Another point to take into consideration concerns the relative values of l_0 and l_1; when l_0 approximates l_1, the input MMPP approximates the Poisson process and recursion efficiency increases. To illustrate this behavior some examples are shown where the input MMPP degenerates into a Poisson process ($l_0 = l_1$); in such examples $\varepsilon_{\max} = 0$. In the great majority of cases the recursion followed by

Table 1
Computational results

N	l_0	l_1	a_0	a_1	λ	recurs	refined	nitd	iterat	nitm
160	80	0	0.1	0.1	40.0	4.8×10^{-2}	2.4×10^{-4}	62	1.6×10^{-4}	1864
160	80	0	0.1	1	72.7	1.1×10^{-3}	1.6×10^{-4}	3	2.8×10^{-4}	1380
160	80	0	0.1	10	79.2	0	2.4×10^{-12}	1	2.8×10^{-4}	997
160	80	0	1	0.01	0.8	3.4×10^{-1}	1.6×10^{-4}	2069	1.3×10^{-4}	2832
160	80	0	1	0.1	7.3	1.0×10^{0}	1.6×10^{-4}	1743	1.3×10^{-4}	2557
160	80	0	1	1	40.0	5.0×10^{-1}	1.6×10^{-4}	997	1.6×10^{-4}	1367
160	80	0	1	10	72.7	0	3.9×10^{-8}	1	2.8×10^{-4}	929
160	80	0	10	0.1	0.8	3.1×10^{1}	1.6×10^{-4}	2166	1.3×10^{-4}	2116
160	80	0	10	1	7.3	2.4×10^{1}	1.3×10^{-4}	1931	1.3×10^{-4}	1823
160	80	0	10	10	40.0	3.0×10^{1}	2.2×10^{-4}	1304	2.3×10^{-4}	1082
160	80	48	0.1	0.1	64.0	0	8.9×10^{-16}	1	2.1×10^{-4}	1148
160	80	48	0.1	1	77.1	0	2.2×10^{-15}	1	2.5×10^{-4}	1352
160	80	48	0.1	10	79.7	0	1.8×10^{-15}	1	2.5×10^{-4}	1006
160	80	48	1	0.1	50.9	0	8.9×10^{-16}	1	1.9×10^{-4}	1629
160	80	48	1	1	64.0	0	6.7×10^{-16}	1	2.1×10^{-4}	992
160	80	48	1	10	77.1	0	1.8×10^{-15}	1	2.5×10^{-4}	980
160	80	48	10	0.1	48.3	0	3.1×10^{-9}	1	1.9×10^{-4}	1415
160	80	48	10	1	50.9	0	3.8×10^{-8}	1	1.9×10^{-4}	1364
160	80	48	10	10	64.0	0	1.9×10^{-7}	1	2.4×10^{-4}	1071
160	80	80	0.1	0.1	80.0	0	8.9×10^{-16}	1	2.4×10^{-4}	931
160	80	80	0.1	1	80.0	0	1.3×10^{-15}	1	2.4×10^{-4}	1098
160	80	80	0.1	10	80.0	0	8.9×10^{-16}	1	2.4×10^{-4}	1013
160	80	80	1	0.1	80.0	0	2.2×10^{-15}	1	2.4×10^{-4}	1098
160	80	80	1	1	80.0	0	6.7×10^{-16}	1	2.4×10^{-4}	931
160	80	80	1	10	80.0	0	8.9×10^{-16}	1	2.4×10^{-4}	1003
160	80	80	10	0.1	80.0	0	8.9×10^{-16}	1	2.4×10^{-4}	1013
160	80	80	10	1	80.0	0	1.3×10^{-15}	1	2.4×10^{-4}	1003
160	80	80	10	10	80.0	0	4.8×10^{-13}	1	2.4×10^{-4}	931
160	160	0	0.1	0.1	80.0	1.1×10^{2}	1.2×10^{-4}	7175	1.6×10^{-4}	1680
160	160	0	0.1	1	145.5	3.0×10^{0}	3.0×10^{-4}	1091	4.0×10^{-4}	1382
160	160	0	0.1	10	158.4	1.4×10^{-2}	2.9×10^{-4}	44	4.2×10^{-4}	996
160	160	0	1	0.1	14.5	3.0×10^{1}	1.7×10^{-4}	2583	1.2×10^{-4}	3018
160	160	0	1	1	80.0	1.2×10^{2}	1.2×10^{-4}	1542	1.7×10^{-4}	963
160	160	0	1	10	145.5	2.0×10^{1}	5.5×10^{-4}	490	4.0×10^{-4}	1176
160	160	0	10	0.1	1.6	1.4×10^{2}	8.9×10^{-5}	2294	1.1×10^{-4}	2160
160	160	0	10	1	14.5	7.4×10^{1}	1.7×10^{-4}	1809	1.2×10^{-4}	1779
160	160	0	10	10	80.0	2.3×10^{1}	2.4×10^{-4}	1180	3.2×10^{-4}	631
160	160	96	0.1	0.1	128.0	0	1.9×10^{-9}	1	2.5×10^{-4}	1220
160	160	96	0.1	1	154.2	0	1.2×10^{-14}	1	3.5×10^{-4}	1383
160	160	96	0.1	10	159.4	0	1.3×10^{-15}	1	3.5×10^{-4}	1429
160	160	96	1	0.1	101.8	0	5.4×10^{-8}	1	2.3×10^{-4}	1771
160	160	96	1	1	128.0	0	1.0×10^{-8}	1	3.2×10^{-4}	1266
160	160	96	1	10	154.2	0	3.6×10^{-15}	1	3.4×10^{-4}	1082
160	160	96	10	0.1	96.6	8.3×10^{-1}	2.5×10^{-4}	692	2.3×10^{-4}	1277
160	160	96	10	1	101.8	1.5×10^{0}	2.5×10^{-4}	826	2.3×10^{-4}	1328
160	160	96	10	10	128.0	1.1×10^{-4}	5.6×10^{-5}	1	3.2×10^{-4}	1346
160	160	128	0.1	0.01	130.9	0	2.2×10^{-15}	1	3.1×10^{-4}	6168
160	160	128	0.1	0.1	144.0	0	1.3×10^{-15}	1	2.9×10^{-4}	1277
160	160	128	0.1	1	157.1	0	2.7×10^{-15}	1	3.3×10^{-4}	1380
160	160	128	0.1	10	159.7	0	1.3×10^{-15}	1	3.3×10^{-4}	1860
160	160	128	1	0.1	130.9	0	1.8×10^{-15}	1	2.7×10^{-4}	1449
160	160	128	1	1	144.0	0	1.8×10^{-15}	1	3.2×10^{-4}	1180
160	160	128	1	10	157.1	0	2.7×10^{-15}	1	3.3×10^{-4}	1083
160	160	128	10	0.1	128.3	0	3.6×10^{-15}	1	2.7×10^{-4}	1615
160	160	128	10	1	130.9	0	9.2×10^{-15}	1	2.7×10^{-4}	1568
160	160	128	10	10	144.0	0	3.1×10^{-15}	1	3.2×10^{-4}	1195

the iterative procedure performs more efficiently then the "pure" iterative procedure. The "refined" recursion tends to be less efficient then the "pure" iterative method when intensity l_1 is close to 0, corresponding to the MMPP "degenerating" into a IPP and when the "jitter" has a significant increase, leading to a direct solution with great error. Many other computational experiments have confirmed these general trends.

6. Conclusions

Analytical properties of a recursive nature, associated with the infinitesimal generator of jump Markov processes describing certain teletraffic systems having a peculiar diagonal block structure, have been derived. These properties were applied to the infinitesimal generator of a system with MMPP input, negative exponentially distributed service times, finite queue and restricted availability defined through a loss function. The resulting recursive formulae may be applied as a direct scheme for the resolution of the linear system, which gives the stationary probability distribution of the system, in terms of which the main GoS parameters may be expressed. Numerical examples with systems of small dimension, suggest that the method error depends critically on the "jitteriness" of the input MMPP and the arrival intensities. Therefore it is recommended that the derived recursion be used to obtain a first approximate solution to be improved through an iterative scheme. Comparison of this "refined" recursive scheme with the "pure" iterative model in [13] indicate that the former performs more efficiently in most cases when the arrival intensities are all relatively far from zero and when the "jitterness" factor of the input MMPP is limited.

Finally note that the obtained recursive formulae are valid for any infinitesimal generator with the considered block structure. Possible application of the recursion to other Markovian stochastic systems with the same type of block structure might be envisaged as future work.

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