Paper

Walsh-chirp sequences for wireless applications

Beata J. Wysocki, Tadeusz A. Wysocki, and Hans-Jürgen Zepernick

Abstract — This paper deals with a new method to design polyphase spreading sequences for DS CDMA wireless applications. The method is based on weighting symbols of the orthogonal Walsh sequences by the complex factors being symbols of baseband chirp sequences. The resulting sequences possess good aperiodic correlation properties, while maintaining the orthogonality. Because of the parametric design, the sequences can be optimized to achieve desired characteristics.

Keywords — spread spectrum communications, polyphase spreading sequences, multi-access interference, Walsh sequences, aperiodic correlation functions.

1. Introduction

Several families of complex spreading sequences have been proposed in literature, with some of them, e.g. [1] allowing for a good compromise between autocorrelation (AC) and cross correlation (CC) properties or even achieving orthogonality in the case of perfect synchronization [2]. Because for the downlink (base station to mobile transmission) the conditions for synchronous operation can be met, orthogonality would allow for cancellation of multi-access interference (MAI) for the down-link, and for simpler receivers in mobile terminals. This is not the case for an uplink transmission, where synchronization at the base station of arrival times of signals from different mobile terminals is very difficult, if possible to achieve.

An MAI impact on the system performance has been studied in [3], and theoretical formulae for an equivalent signal-to-noise ratio (SNR) approximation are given there. However, for short spreading sequences, these formulae can be regarded as very rough estimates, only. A reasonable approach to compare different sequence sets in such a case is to obtain error performance simulating the DS CDMA system under the same assumptions as those used in [3] for derivation of the SNR formulae.

In the paper, we propose a method to design sets of orthogonal polyphase sequences obtained by modification of orthogonal Walsh sequence. It is achieved by weighting symbols of the Walsh sequences by the complex factors obtained from the superposition of baseband chirp sequences [4]. Because of the parametric design, the Walsh-chirp sequences can be optimized to achieve the desired characteristics. In the numerical example, we show that the designed sequences can possess good aperiodic cross correlation and autocorrelation properties, enabling an increase in the number of simultaneous users compare to the number of users achievable, if Gold or Gold-like [5] sequences of the similar length are employed.

We also compare the performance of Walsh-chirp sequences with the sequences designed using the method described in [2]. For the same sequence length of 16, there are only 8 such sequences and up to 16 sequences using our method. In addition, the simulated bit-error-rate (BER) performance, once again indicate superiority of the Walsh-chirp sequences compared to the sequences designed using the method described in [2].

2. Poly-chirp sequences

For chirp modulation, an elementary phase pulse is given by [6]:

$$q_p(t) = \begin{cases} \frac{t^2}{2T^2} - \frac{t}{2T}, & 0 < t \leq T \\ 0, & \text{otherwise} \end{cases}, \tag{1}$$

where T is the duration of the pulse. Thus, a baseband chirp pulse $b(t)$ is of the form:

$$b(t) = \begin{cases} \exp\left[j2\pi h q_p(t)\right], & 0 < t \leq T \\ 0, & \text{otherwise} \end{cases}. \tag{2}$$

Discretizing the pulse $b(t)$ by substituting n for t, and N for T, we can write a formula defining a complex polyphase chirp sequence

$$\left\{\hat{b}_n(h)\right\} = \left(\hat{b}_n(h);\quad n = 1, 2, \ldots, N\right), \tag{3}$$

where:

$$\hat{b}_n(h) = \exp\left[j2\pi h b_n\right];\quad n = 1, 2, \ldots, N \tag{4}$$

$$b_n = \frac{n^2 - nN}{2N^2}, \tag{5}$$

with h being an arbitrary nonzero real constant.

The latest result can be generalized to obtain poly-chirp sequences. In order to do so, let us define a pulse referred to as a chirp pulse of the order s, if and only if the first time derivative of its instantaneous frequency (the angular acceleration) is a step function with the number of time intervals where it is constant being equal to s. Based on the above definition, we can derive the formulae for the elementary phase pulses for baseband chirps of any order. Again, substituting n for t, and N for T, we can get then formula defining a complex poly-chirp sequence of order s.

For example, a double-chirp sequence $\{d_n\}$ is given by [4]:

$$d_n = \begin{cases} \frac{2n^2}{N^2} - \frac{n}{N}, & 0 < n \le N \\ -\frac{2n^2}{N^2} + \frac{3n}{N} - 1, & \frac{N}{2} < n \le N \\ 0, & \text{otherwise} \end{cases} \qquad (6)$$

and the complex double chirp sequence elements $\hat{d_n}$ are therefore given by:

$$\hat{d_n} = \exp[j2\pi h d_n]; \quad n = 1, 2, \ldots, N. \qquad (7)$$

Another class of sequences can be obtained if a superposition of chirp sequences of different orders is used to create complex polyphase sequences.

3. Walsh-chirp sequences

Let us consider here the sequences, $\{\hat{o}_n^{(i)}\}$, $i = 1, 2, \ldots, M$ having their elements $\hat{o}_n^{(i)}$ given by:

$$\hat{o}_n^{(i)} = w_n \hat{h}_n^{(i)}, \quad n = 1, 2, \ldots, N, \qquad (8)$$

where

$$w_n = \exp\left[j2\pi(c_1 b_n + c_2 d_n)\right], \qquad (9)$$

b_n and d_n are defined by Eqs. (5) and (6), respectively, c_1, c_2 are two real constants, and $\hat{h}_n^{(i)}$ are the elements of orthogonal sequences $\{\hat{h}_n^{(i)}\}$, $i = 1, \ldots, M$.
Because of Eq. (9), we have

$$w_n w_n^* = 1; \quad n = 1, \ldots, N, \qquad (10)$$

where w_n^* denotes the complex conjugate of w_n. Hence, it is easy to show that the sequences $\{\hat{o}_n^{(i)}\}$ are also orthogonal, as long as the factors w_n are kept constant for $i = 1, 2, \ldots, M$.
Therefore, choosing the sequences $\{\hat{h}_n^{(i)}\}$ as the sequences obtained from the orthogonal Walsh functions, we can produce a set of the orthogonal complex Walsh-chirp sequences. The cross- and autocorrelation performance of the set depends not only on the Walsh functions chosen in the first place, but also on the values of the parameters c_1 and c_2. In the next Section, we show that these parameters can be optimized to achieve the desired characteristics.

4. Numerical example

In order to design a set of Walsh-chirp sequences of length 16, let us consider a set of 13 Walsh sequences given in Table 1. Then, we find the values of the coefficients c_1 and c_2 which minimize the mean square value of the aperiodic CC (MSACC) [1] for the whole set.
For that purpose, we calculated the MSACC for $0 \le c_1 \le 30$ and $0 \le c_2 \le 30$, with the grid of 0.2. In the investigated region, it reaches the minimum of 0.8532 for $c_1 = 15.8$ and $c_2 = 24.4$. For those values of c_1 and c_2 the mean square value of the aperiodic AC (MSAAC) [1] is equal to 1.5962. The orthogonality of the designed sequence set is clearly visible in Fig. 1, where we plotted two example aperiodic CC functions (ACCFs).

Table 1
Set of 13 Walsh sequences

No.	Binary spreading sequence
1	- - - - + + + + - - - - + + + +
2	- - + + + + - - - - + + + + - -
3	+ + - - - - + + - - + + + + - -
4	+ + - - + + - - - - + + - - + +
5	- - + + - - + + - - + + - - + +
6	- + + - - + + - - + + - - + + -
7	+ - - + + - - + - + + - - + + -
8	+ - - + - + + - - + + - + - - +
9	- + + - + - - + - + + - + - - +
10	- + - + + - + - - + - + + - + -
11	+ - + - - + - + - + - + + - + -
12	+ - + - + - + - - + - + - + - +
13	- + - + - + - + - + - + - + - +

To assess the usefulness of the designed sequence set, we simulated operation of the DS CDMA system utilizing these sequences. We have assumed that data transmitted in any of the active channels is random, grouped into 1000 packets of 524 bits. The system has been considered as an asynchronous one, with only the examined channel kept synchronized to the corresponding reference sequence generated in the receiver, while the interfering m channels have been randomly delayed with respect to the examined channel.

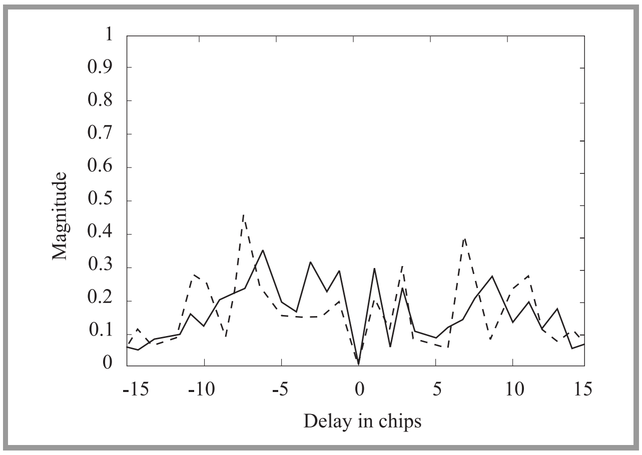

Fig. 1. Magnitudes of the ACCFs between the sequences: (1,10) – solid line, and (4,11) – dashed line.

Those random delays, τ_i, $i = 1, \ldots, m$, have been chosen as integer multiples of 0.5, and satisfying the condition:

$$0 \le \tau_i < N. \qquad (11)$$

Since in a real system phases of the generators used in all of the transmitting terminals can be different, we multiplied each of the interferers' signals by a coefficient:

$$\rho_i = \exp(j\phi_i), \tag{12}$$

where ϕ_i is a constant chosen randomly from the interval $[0, 2\pi]$.

In order to simplify the simulations, we have kept those randomly chosen coefficients, τ_i, ϕ_i, $i = 1, \ldots, m$, constant throughout the transmission of a single packet in the examined channel, with drawing of them repeated before simulation of every new transmission of a single packet.

For each of the simulated packet transmissions in the examined channel, the sequences used by the interferers has been chosen randomly from the set of all possible spreading sequences utilized in the system, disregarding the one used by the channel under examination.

Apart from the presence of MAI, we have assumed the presence of white Gaussian noise in the channel. We performed the simulations for $E_b/N_0 = 20$ dB, and $E_b/N_0 = 8$ dB, where N_0 is single sided power spectral density of white nose, and E_b is energy per information bit. To avoid being drawn into considering the problems associated with the "near-far-effect", we have assumed powers of all signals arriving at the receiver kept at the same level.

The achieved BER is plotted in Fig. 2 versus the number of interfering channels. For the comparison, we present there also BER characteristic obtained in the case when the designed sequence set is replaced by the set of 15-chip Gold-like sequences [5].

From the results presented in Fig. 2, it is clearly visible that the system utilizing Walsh-chirp sequences significantly outperforms the one utilizing Gold-like sequences, allowing for considerably more simultaneous users in the system at the same level of BER.

Then, we have compared performance of the Walsh-chirp sequence with the orthogonal polyphase spreading sequences proposed in [2] for the length $N = 16$. The set of sequences in [2] is defined by $U_{m,n}(N) = \{u_m : 1 \le M < N\}$, while the ith element of a given sequence u_M is defined by

$$u_M(i) = (-1)^{M_i} \exp\left[\frac{j\pi(iM^m + i^n)}{N}\right]; \quad 1 \le i < N, \tag{13}$$

where m is any positive nonzero integer, n is a real number, while M and N are relatively prime numbers.

In our case, $N = 16$, the parameter M can take only the values $M \in \{1, 3, 5, 7, 9, 11, 13, 15\}$, so the maximum number of sequences is equal to 8. There is no general method given in [2] for finding the values of the parameters m, and n. Following [2], we chose the value of $m = 1.0$, and calculated MSACC and the MSAAC as functions of the parameter n. The obtained plots are given in Fig. 3.

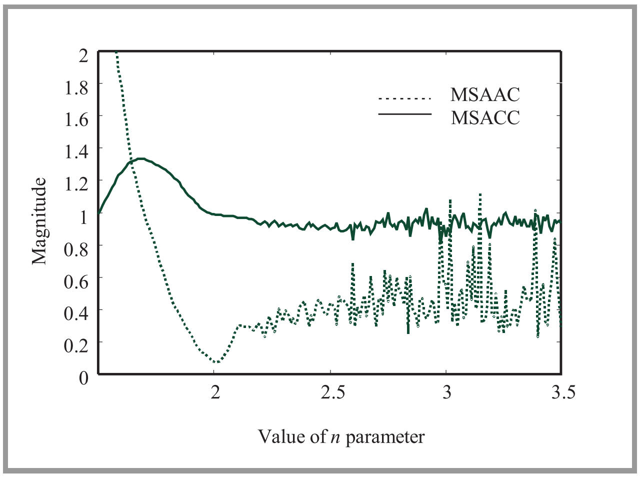

Fig. 3. Plots of MSAAC and MSACC as a function of the parameter n for the sequence set defined using formula (13).

For simulating BER performance we took $n = 2.5$ where there is a reasonable compromise between the values of MSACC and MSACC equal to 0.8968 and 0.4394, respectively. The achieved BER is plotted in Fig. 4 versus the number of interfering channels. Once again, it is visible

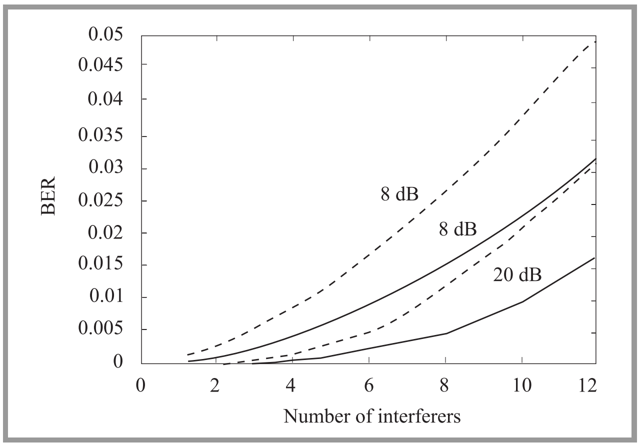

Fig. 2. BER as a function of the number of interfering channels; 16-chip Walsh-chirp sequences – solid lines, 15-chip Gold-like sequences – dashed lines.

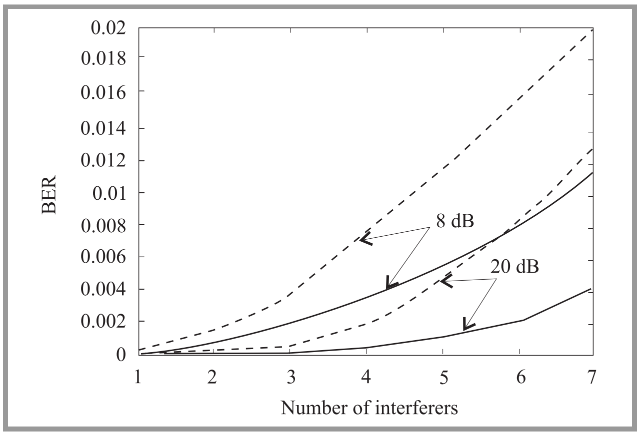

Fig. 4. BER as a function of the number of interfering channels; 16-chip Walsh-chirp sequences – solid lines, 16-chip sequences designed according to [2] – dashed lines.

that the Walsh-chirp sequences outperform sequences proposed in [2] for $N = 16$.

5. Conclusions

In the paper, we introduced a new method to design sets of orthogonal polyphase sequences. The method is based on utilizing a binary orthogonal sequence set and weighting the symbols of binary sequences with the complex coefficients obtained from the superposition of baseband chirp sequences. The resultant polyphase sequences can be optimized to achieve desired correlation properties of the set. As indicated by the simulations, error performance of a DS CDMA system utilizing the sequence set developed in the numerical example is significantly better than in the case of a system employing Gold-like sequences of a similar length, or orthogonal polyphase sequences of the same length, $N = 16$. Further on, one can apply the method proposed in this letter together with the sequences proposed in [2], i.e. instead of Walsh sequences, the sequences proposed in [2] can be modified using chirp pulses.

References

[1] I. Oppermann and B. S. Vucetic, "Complex spreading sequences with a wide range of correlation properties", *IEEE Trans. Commun.*, vol. COM-45, pp. 365–375, 1997.

[2] I. Oppermann, "Orthogonal complex-valued spreading sequences with a wide range of correlation properties", *IEEE Trans. Commun.*, vol. COM-45, pp. 1379–1380, 1997.

[3] M. N. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication". Part I: "System analysis", *IEEE Trans. Commun.*, vol. COM-25, pp. 795–799, 1977.

[4] B. J. Wysocki and T. A. Wysocki, "Chirp sequences for wireless data networks", in *Proc. 5th Int. Symp. Commun. Theory Appl.* (ISCTA'99), Ambleside, U.K., 11–16 July 1999, pp. 202–204.

[5] P. Fan and M. Darnell, *Sequence Design for Communications Applications*. New York: Wiley, 1996.

[6] T. Wysocki, "Chirp modulation", in *Encyclopedia of Electrical and Electronics Engineering*, J. G. Webster, Ed. New York: Wiley, 1999, vol. 13.

Beata Joanna Wysocki graduated from Warsaw University of Technology receiving her M.Eng. degree in electrical engineering in 1991. In 1994, she started the Ph.D. study in the Australian Telecommunications Research Institute at Curtin University of Technology. In March 2000, she was awarded her Ph.D. for the thesis: "Signal Formats for Code Division Multiple Access Wireless Networks". During the Ph.D. studies, she was involved in a research project "Wireless ATM Hub" at the Cooperative Research Centre for Broadband Telecommunications and Networking, and worked as a research assistant at Edith Cowan University, within the ARC funded "CDMA with enhanced protection against frequency selective fading", and "Reliable high rate data transmission over microwave local area networks". Since October 1999 she has been with the Telecommunications & Information Technology Research Institute at the University Wollongong as a research fellow. Her research interests include sequence design for direct sequence (DS) code division multiple access (CDMA) data networks and error control strategies for broadband wireless access (BWA) systems.

e-mail: beata@snrc.uow.edu.au
University of Wollongong, T.I.T.R.
Northfields Av.
Wollongong NSW 2522, Australia

Tadeusz Antoni Wysocki received the M.Sc.Eng. degree with the highest distinction in telecommunications from the Academy of Technology and Agriculture, Bydgoszcz, Poland, in 1981. In 1984, he received his Ph.D. degree, and in 1990, was awarded a D.Sc. degree (habilitation) in telecommunications from the Warsaw University of Technology. In 1992, Dr. Wysocki moved to Perth, Western Australia to work at Edith Cowan University. He spent the whole 1993 at the University of Hagen, Germany, within the framework of Alexander von Humboldt Research Fellowship. After returning to Australia, he was appointed a Program Leader, Wireless Systems, within Cooperative Research Centre for Broadband Telecommunications and Networking. Since December 1998 he has been working as an Associate Professor at the University of Wollongong, NSW, within the School of Electrical, Computer and Telecommunications Engineering being a Program Leader of the Smart Networks Program within the Cooperative Research Centre for Smart Internet Technology. The main areas of Dr. Wysocki's research interests include: indoor propagation of microwaves, code division multiple access (CDMA), digital modulation and coding schemes, as well as mobile data protocols including those for ad hoc networks. He is the author or co-author of three books, over 100 research publications and nine patents. He is a Senior Member of IEEE.

e-mail: tad_wysocki@uow.edu.au
University of Wollongong
School of Electrical, Computer
and Telecommunications Engineering
Northfields Av.
Wollongong NSW 2522, Australia

Hans-Jürgen Zepernick received the Dipl.-Ing. degree form the University of Siegen, Germany, in 1987, and the Dr.-Ing. degree from the University of Hagen, Germany, in 1994, both in electrical engineering. From 1988 to 1989, he was a Research Engineer with the Radio and Radar Department at Siemens AG, Munich, Germany. Here he developed signal processing algorithms that are suitable for miniaturization of Bragg cell receivers. From 1989 to 1995, he was a Research Associate with the Institute of Telecommunications at the University of Hagen, where his work was focused on methods for performance analysis of error control coding schemes with application to mobile communication systems. In May 1995, Dr. Zepernick joined the Cooperative Research Centre for Broadband Telecommunications and Networking (CRC-BTN) in Perth, Australia, as a Research Fellow and conducted research on physical layer and data link layer topics for wireless ATM. Since 1997, he has been a Project Leader for the Coding and Modulation project at the CRC-BTN. Since 1999, Dr. Zepernick has been employed as a Program Leader for the Wireless Program at the Australian Telecommunications Cooperative Research Centre (ATCRC), Perth, Australia. In August 2000, he has been appointed the Deputy Director of the Australian Telecommunications Research Institute (ATRI), Perth, Australia. Dr. Zepernick is an author or co-author of some 50 technical papers. His research interests include radio channel characterization and modeling, coding and modulation, equalization, spread-spectrum systems, wireless networks and future generation wireless systems.

e-mail: hans@atri.curtin.edu.au
Curtin University of Technology, ATRI
GPO Box U1987, Perth WA 6845, Australia