Paper

Performance of CDMA systems with Walsh and PN coding

Piotr Gajewski and Jerzy Dołowski

Abstract — Wideband CDMA method has been recently investigated for its using in the future communication systems. Such systems capacity depends on many factors, the main is multi-access interference (MAI). MAI is an inherent CDMA property connected to correlation characteristics of codes used to access and to spreading. The influence on CDMA performance of these codes families and parameters is presented in this paper. The standard Gaussian approximation has been used for interference analysis. Bit error rate (BER) on detector output is a measure of interference influence on transmission quality. It can be used for $\mathbf{SNR_i}$ evaluation on receiver input for channel assignment and handover procedures. The results of interference analysis for the pseudo-noise sequences (M-sequences, Gold, de'Bruin and Jennings) as well as for the Walsh orthogonal mapping are presented in this paper.

Keywords — PN spreading, CDMA, multi-access interference.

1. Introduction

Recently, wideband code division multiple access (WCDMA) has become the most promising technology for the future personal communication systems (PCS) [1, 2]. This access scheme has a very good behaviour in the urban areas also in case of coexistence with the other cellular systems.

In DS-CDMA systems, channels are created using the specific code sequences. These sequences are used for the spectrum spreading, synchronisation as well as for information transmission. The number of logical channels used is limited by a number of accessible codes in suitable family and by some level of interference. In known standards (IS-95, IS-665), each base station uses the same sets of the code sequences but with different phase shifts [3]. In CDMA standard, the same channel can be used at neighbouring cells; it gives the channel reuse factor approximately equal to one. So, the system capacity is limited by required level of interference from other users from the current and neighbouring cells [4].

Additionally in CDMA, interference results from the nonzero correlation value between two various sequences of used code. The analysis of parameters of codes and their influence on the CDMA performance make possible a choice of codes for the CDMA system. The channel assignment to the particular call could be realised by calculation of the minimal correlation factors with regard to each channel in the interference area.

Thus, the main goal of this paper is to present an evaluation of performance of the DS-CDMA system with different pseudo-noise (PN) as well as orthogonal Walsh code families. Bit error rate BER on detector output is the measure of interference influence on the transmission quality. It can be used for SNR_i evaluation on a receiver input for the channel assignment and handover procedures. BER is defined here as:

$$p_e = \lim_{n \to \infty} \frac{n_i}{n}\,, \tag{1}$$

where: n_i – number of errors, n – total number of transmitted bits.

The results of interference analysis for the pseudo-noise sequences (M-sequences, Gold, de'Bruin and Jennings) as well as for the Walsh orthogonal mapping are presented here. The standard Gaussian approximation was used for BER evaluation. BER was estimated for $n = 10^3 \div 10^5$.

2. Multi-access interference in WCDMA with PN spreading

Let consider the simplified CDMA system with BPSK, with the ideal power control and without the synchronisation errors. We assume a radio channel with $n(t)$ – additive white Gaussian noise (AWGN). Let K is a number of users each generating series $b_k(t)$ of binary data with bit duration T (Fig. 1). Each user's data is multiplied by chip that is a binary PN spreading sequence $a_k(t)$ with duration $T_C \ll T$:

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)}\, p_{T_C}(t - jT_C)\,, \tag{2}$$

where $p_T(t) = 1$ for $0 \le t \le T$ or $p_T(t) = 0$ otherwise.

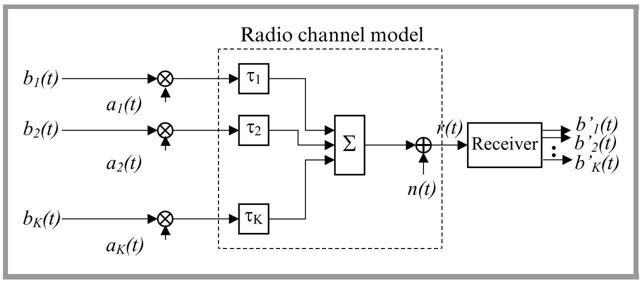

Fig. 1. DS-CDMA model.

We assume that $N = T/T_C$, so we have N chips $a_0^{(k)}, a_1^{(k)}, \ldots, a_{N-1}^{(k)}$ in each data bit.

The signals on the transceiver output and on the receiver input are respectively:

$$s_k(t) = \sqrt{2P} \cdot a_k(t)\, b_k(t) \cos(\omega_C t + \theta_k)\,, \qquad (3)$$

$$r(t) = n(t) + \sum_{k=1}^{K} \sqrt{2P}\, a_k(t-\tau_k) b_k(t-\tau_k) \cdot \cos(\omega_C t + \phi_k)\,, \qquad (4)$$

where: P – power, ω_C – carrier frequency, $\phi_k = \theta_k - \omega_C \tau_k$, θ_k – phase, τ_k – delay of k-th signal.
It can be noticed that $\theta_i = 0$ and $\tau_i = 0$, so $0 < \tau_k < T$ and $0 < \theta_k < 2\pi$ for $k \neq i$.
A signal on the output of the receiver correlator can be expressed as [5]:

$$Z_i = \sqrt{\frac{P}{2}} \left\{ b_{i,0} T + \sum_{\substack{k=1 \\ k \neq i}}^{K} \left[b_{k,-1} R_{k,i}(\tau_k) + \right.\right.$$
$$\left.\left. + b_{k,0} \hat{R}_{k,i}(\tau_k) \right] \cdot \cos\phi_k \right\} + \int_0^T n(t) \cdot a_i(t) \cos \omega_C t dt\,, \qquad (5)$$

where $R_{k,i}$, $\hat{R}_{k,i}$ are the correlation functions. For $0 \leq lT_C \leq \tau \leq (l+1)T_C \leq T$, they are given by:

$$R_{k,i}(\tau) = C_{k,i}(l-N)T_C + \left[C_{k,i}(l+1-N) + \right.$$
$$\left. - C_{k,i}(l-N) \right] \cdot (\tau - lT_C)\,, \qquad (6)$$

$$\hat{R}_{k,i}(\tau) = C_{k,i}(l)T_C + \left[C_{k,i}(l+1) + \right.$$
$$\left. - C_{k,i}(l) \right] \cdot (\tau - lT_C)\,, \qquad (7)$$

where $C_{k,i}$ is the aperiodic cross-correlation function for sequences $(a_j^{(k)})$ and $(a_j^{(i)})$ defined as:

$$C_{k,i}(l) = \begin{cases} \sum_{j=0}^{N-1-l} a_j^{(k)} \cdot a_{j+1}^{(i)} & 0 \leq l \leq N-1 \\ \sum_{j=0}^{N-1+l} a_{j-l}^{(k)} \cdot a_j^{(i)} & 1-N \leq l < 0 \\ 0 & |l| \geq N \end{cases} . \qquad (8)$$

It is easy to see, the correlation performance of the PN codes should be analysed for MAI evaluation. Signal-to-noise ratio on I-th output of correlation receiver can be estimated using Gaussian approximation taking into account that binary data, time delay and phase shift of each signal are independent random variables. So, Z_i is normally distributed with the mean value $\sqrt{P/2} \cdot T$ and covariance:

$$\sigma^2 = \frac{PT^2}{12N^3} \sum_{\substack{k=1 \\ k \neq i}}^{K} r_{k,i} + \frac{N_0 T}{4}\,, \qquad (9)$$

where $r_{k,i}$ can be calculated from [6]:

$$r_{k,i} = \sum_{l=1-N}^{N-1} C_k(l) \left[2C_i(l) + C_i(l+1) \right]. \qquad (10)$$

Here, $C_{k,l}$ is the autocorrelation function.

SNR is defined as:

$$SNR_i = \frac{\sqrt{P/2} \cdot T}{\sigma_i}\,. \qquad (11)$$

Replacing (9) in (11) the SNR on the output of correlator can be calculated as:

$$SNR_i = \frac{1}{\sqrt{\frac{1}{6N^3} \cdot \sum_{\substack{k=1 \\ k \neq i}}^{K} r_{k,i} + \frac{N_0}{2E}}}\,, \qquad (12)$$

where $E = P \cdot T$ is the bit energy.
Thus BER can be estimated from

$$P_e \cong Q(SNR_i)\,, \qquad (13)$$

where $Q(\cdot)$ is the error function defined as follows:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt\,. \qquad (14)$$

3. Multi-access interference in WCDMA with Walsh mapping

One of the main methods used for MAI reduction is the orthogonal mapping of data stream generated by the user in the uplink. Each group of data are replaced by the orthogonal sequence in this method. Most often, Walsh sequences are used in practise [1]. The model of quadrature modulator for system with M-ary mapping is presented in Fig. 2.

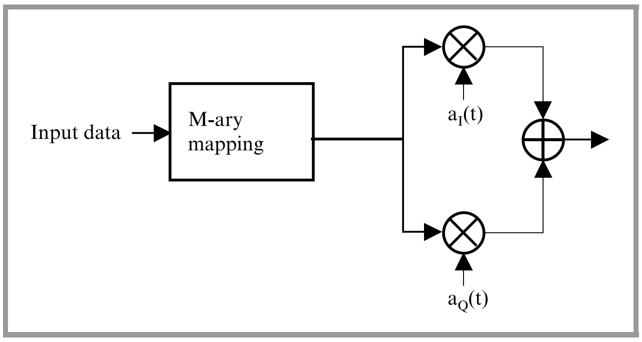

Fig. 2. The quadrature modulator with M-ary mapping.

The orthogonal sequence of k-th user is given by [8]:

$$W^k(t) = \sum_{\nu=-\infty}^{\infty} W_\nu^k(t) \cdot P_T(t - \upsilon T)\,, \qquad (15)$$

where $P_T(t) = 1$ for $0 \leq t \leq T$ or $P_T(t) = 0$ otherwise.

Here, each of m bits is replaced by one of M Walsh functions of length T (Fig. 3). In (15) W_ν^k is the Walsh symbol of k-th user from the set $\{W_0(t), W_1(t), W_2(t), \ldots, W_{M-2}(t), W_{M-1}(t)\}$ corresponding to m bits of input data. Symbol W_ν^k consists of M

Walsh "chips" $w_{i,j}$ $(j = 0, 1, \ldots, M-1)$ of duration T_W $(T = M \cdot T_W)$.

Therefore

$$W_i(t) = \sum_{j=0}^{M-1} w_{i,j} \cdot P_{T_W}(t - jT_W) . \qquad (16)$$

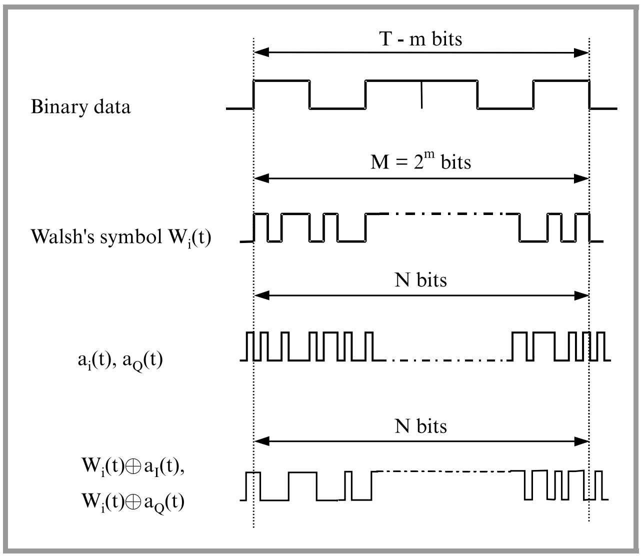

Fig. 3. The sequences in modulator from Fig. 2.

The transmitted signal of k-th user can be expressed as

$$s_k(t) = \sqrt{P} \cdot W^{(k)}(t) \cdot A^{(k)}(t) \cdot \exp(j\theta_k) , \qquad (17)$$

where $A^{(k)}(t)$ is a complex spreading sequence of k-th user that consists two sequences $a_I^{(k)}$ and $a_Q^{(k)}$ in the in-phase I and quadrature Q branches, respectively.

For a system with K users working asynchronously and simultaneously, the receiver input signal is [8]:

$$r(t) = \sum_{k=1}^{K} \sqrt{P} \cdot W^{(k)}(t - \tau_k) \cdot A^{(k)}(t - \tau_k) \cdot \exp(j\phi_k) + n(t) . \qquad (18)$$

The detection could be performed using a set of M matched filters [8].

From [8], the variance of signal on the output of k-th filter is

$$\sigma^2 = \frac{1}{2}\mathrm{var}\{Z_k\} = \frac{PT}{24N^3} \sum_{\substack{k=1 \\ k \neq i}}^{K} \gamma_{k,i} + \frac{N_0}{2} \qquad (19)$$

and the probability of bit error can be obtained from:

$$p_e = \frac{2^{K-1}}{2^K - 1} \left\{ 1 - \sum_{k=0}^{M-1} \frac{(-1)^k}{k+1} \cdot \binom{M-1}{k} \cdot e^{-\frac{s^2 k}{2\sigma^2(k+1)}} \right\} , \qquad (20)$$

where

$$p_c = \frac{1}{2\sigma^2} \int_0^\infty \left(1 - e^{-x/2\sigma^2} \right) \cdot e^{-(x+s^2)/2\sigma^2} \cdot I_0 \left(\frac{s\sqrt{x}}{\sigma^2} \right) dx . \qquad (21)$$

In (21), $s^2 = P \cdot T$ and $I_0(\cdot)$ is the modified zero-order Bessel function.

4. Numerical results

Basing on above analyses, the simulation program was elaborated in C++. The generators of maximal length sequences (M-sequences), Gold sequences, de'Bruin sequences, Jennings sequences as well as Walsh (Walsh-Hadamard) orthogonal sequences were implemented in this program. It enabled to examine the BER as a function of E_b/N_0 for code families mentioned above. Some numerical results are presented below.

Figure 4 shows simulation results for sequences length $N = 63$ (64) and $N = 511$ (512) for $K = 10$ users. The numbers in round brackets concern de'Bruin and Jennings sequences. The length increase causes the BER decrease. In practise, the codes of bigger length are used. For example, the codes with $N = 2^{42} - 1$ and $N = 2^{15} - 1$ are used in IS-95 systems.

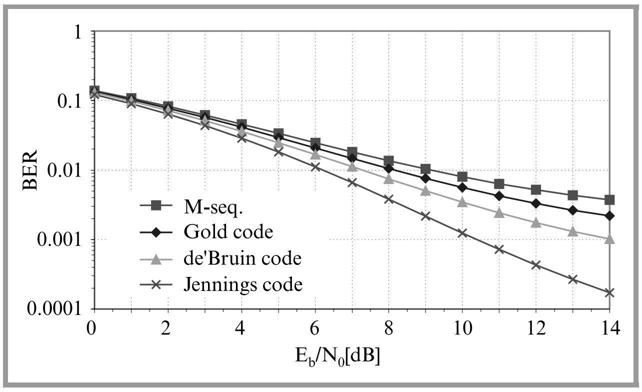

Fig. 4. Simulation results for various codes, $K = 10$, $N = 31$.

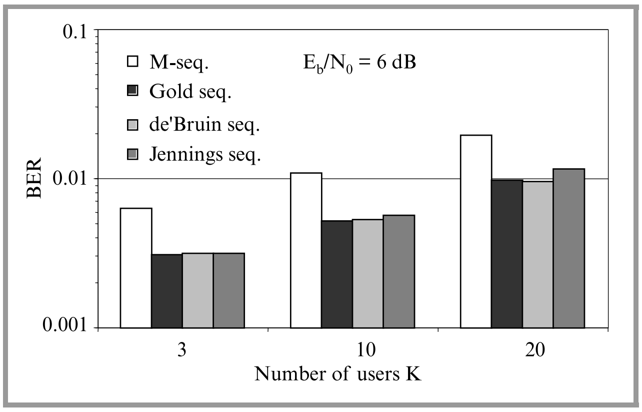

Fig. 5. BER versus users number, $N = 127$, $E_b/N_0 = 6$ dB.

Figures 5 and 6 show the BER for $K = 3$, 10 and 20 users for code sequences length $N = 127$ (128) and two values of E_b/N_0 equal 6 and 12 dB respectively. It can be observed the differences between results obtained for various codes (except M-sequences) decrease with E_b/N_0 increasing.

Figures 7, 8 and 9 show the results obtained for Walsh mapping with additional PN spreading by Gold and M-sequences. The PN length equals $N = 2^n$, here one 0 bit is added to the sequence for 0 and 1 balancing [8].

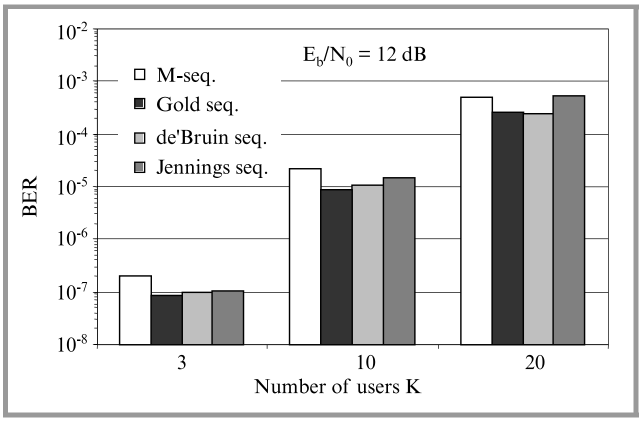

Fig. 6. BER versus users number, $N = 127$, $E_b/N_0 = 12$ dB.

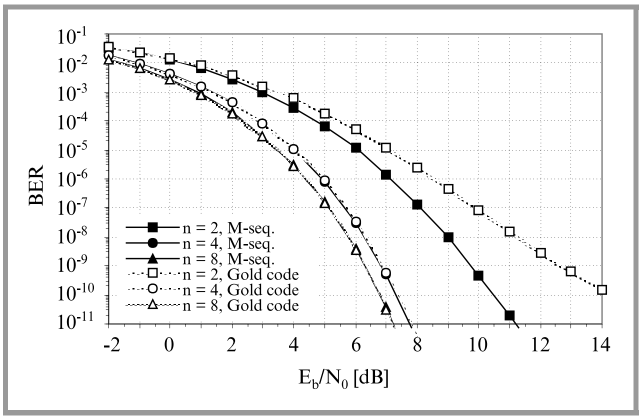

Fig. 7. Simulation results for Walsh mapping, $M = 64$, $K = 10$, added M- and Gold sequences spreading.

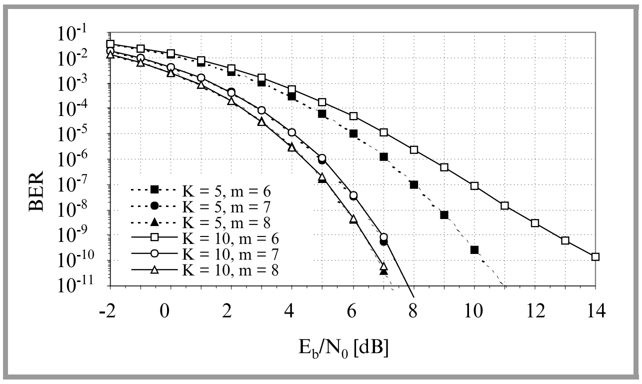

Fig. 8. Simulation results for Walsh sequences, $K = 5, 10$, added Gold sequences spreading.

In Fig. 7, n is the number of PN bits added to one Walsh chip. Here $m = 6$, so one of Walsh 64 bits sequence corresponds to 6 bits of input data. In IS-95, $n = 4$ and $m = 6$. Figure 8 shows the impact of number mapping bits m for two numbers of users $K = 5$ and 10. Here, the $127 + 1$ bits Gold sequence is used for spreading. The m increase decreases the level of MAI.

As it can be seen in Fig. 9, the increase of users number does not significant decrease of the BER for longer Walsh

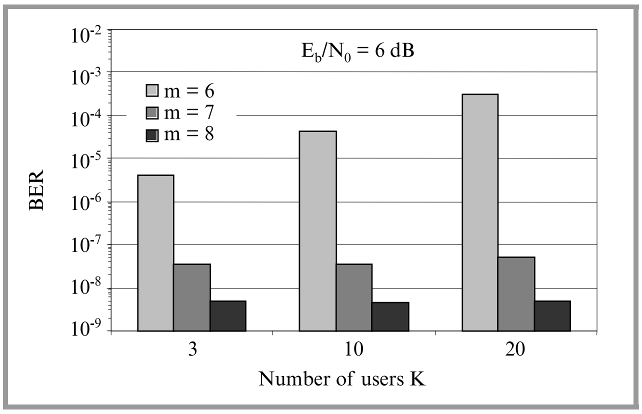

Fig. 9. Simulation results for Walsh sequences, $M = 64$, $K = 3, 10, 20$, $E_b/N_0 = 6$ dB, added Gold sequences $N = 128$.

sequences. It is caused by BER reduction companying the increasing of mapped bits that increases the spreading code length.

5. Conclusions

The multi-access is the main source of interference in WCDMA systems.

Taking into account the simulation results it can be concluded that:

- M-sequences have the worst correlation properties comparing to the other considered PN codes; moreover, the number of different sequences in code family is very limited, so the main goal of such codes analysis is they are basis for many other PN codes;
- the very good results was obtained for Gold sequences; it explains these codes usage in real systems;
- the code length has a significant impact on MAI level, so the longer codes are preferred to use in WCDMA systems; that is no problem to generate the long codes, but it can cause an increasing of synchronising time;
- the number of users increasing causes the MAI increasing that limits the system capacity;
- the best results was achieved for Gold and de'Bruin sequences that prefer their usage in WCDMA; but, the performances of Jennings sequences (large number of codes in family, not poor correlation properties, 0–1 symbols symmetry) make these codes very interesting for using in military systems;
- in Walsh mapping, the mapped bits number increase causes the significant reduction of BER even a few ranges;
- the very good results in BER decreasing are obtained using Walsh mapping companying to PN spreading;

– the PN code type has a less influence on MAI level than a number of mapped bits.

It must be noticed, that presented analysis and results was obtained while some simplified assumptions had been done. It especially concerns the power control in up- and down-links, users mobility, absence of the noise from narrowband systems, the call traffic performances, etc.
However the results of this analysis can be used in channel access as well as in handover procedures.

References

[1] E. Dahlman, P. Beming, J. Knutsson, F. Ovesjo, M. Petrsson, and Ch. Roobol, "WCDMA – the radio interface for future mobile multimedia communications", *IEEE Trans. Veh. Technol.*, vol. 47, no. 4, 1998.

[2] J. Buczyński, P. Gajewski, and J. Krygier, "Modelling of the third generation mobile system", in *Proc. Africon'99*, Sept. 1999.

[3] P. Gajewski, J. Krygier, J. Łopatka, and J. Buczyński, "Performance of a DS-CDMA system with dynamic channel allocation and soft handover", in *Proc. ISSSTA'98*, Sept. 1998.

[4] M. Amanowicz and P. Gajewski, "Compatibility criteria for digital radio systems in nonlinear channels", in *Proc. Africon'96*, Stellenbosch, Oct. 1996.

[5] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple access communication". Part I: "Systems analysis", *IEEE Trans. Commun.*, vol. COM-25, no. 8, pp. 795–799, 1977.

[6] M. B. Pursley and D. V. Sarwate, "Performance evaluation for phase-coded spread-spectrum multiple access communication". Part II: "Code sequence analysis" , *IEEE Trans. Commun.*, vol. COM-25, no. 8, pp. 800–803, 1977.

[7] M. B. Pursley and H. F. A. Roefs, "Numerical evaluation of correlation parameters for optional phases of binary shift-register sequences", *IEEE Trans. Commun.*, vol. COM-27, no. 10, 1979.

[8] E. K. Hong, K. J. Kim, and K. C. Whang, "Performance evaluation of DS-CDMA system with M-ary orthogonal signaling", *IEEE Trans. Veh. Technol.*, vol. 45, no. 1, pp. 57–63, 1996.

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