Paper

Photonic band gaps in complex layered arrays

Sergey L. Prosvirnin, Sergey A. Tretyakov, Tetyana D. Vasilyeva, Arlette Fourrier-Lamer, and Said Zouhdi

Abstract — Reflective and transmitting properties of several layers of double-periodic arrays are studied. In the arrays, elements are conducting inclusions of various shapes. It is shown that in these structures all the phenomena recently found in dense wire grids with periodical defects (so-called photonic band gap structures) can be observed and explained in simple terms of inter-layer and inclusion resonances. Frequency-selective (with two and more stop bands) and polarization transformation properties of these arrays are demonstrated.

Keywords* — *photonic band gap structures, double-periodic arrays, reflective and transmitting properties.

1. Introduction

In recent years, much attention has been given to so-called photonic band gap (PBG) materials. Full band gaps for electromagnetic waves of arbitrary polarization and propagation direction have been found in many 3D periodic structures. These results stimulated renewed interest to 2D and even 1D periodic structures in the microwave regime. Some new applications have been recently proposed [1 – 3].

A very interesting behavior has been observed in a system which is intermediate between bulk 3D periodic media and 2D regular arrays, see [4]. A relatively thin layer of conducting wire mesh has been investigated. Experimental results of [4] lead to the following observations: 1) in a regular periodic structures, the layer is highly reflective at low frequencies and rather transparent at high frequencies; 2) there is a sharp cut-off boundary between the two regimes; 3) if the wires are cut periodically (mesh with defects), transmission peaks appear in the low-frequency band of high reflectivity.

These phenomena were explained in [4] in terms of an effective dielectric constant. The cut-off frequency is identified as that similar to the plasma frequency for electrons flowing in the mesh. Regular 3D wire meshes with cubic cells were considered in papers [5, 6]. Here, the phenomena in regular structures have been explained in terms of the effective mass of electrons moving in conducting wires. However, these simple models fail if the system is studied in a wide frequency range. When the characteristic sizes become comparable to the wavelength (which is the most interesting case where photonic band gaps exist), the system is spatially dispersive. More complicated constitutive relations are needed to model spatial dispersion [7, 8]. We show that similar phenomena can be found in much simpler systems formed of a few parallel planar layers of conducting resonant inclusions, and explained in a very simple and physically clear way.

Let us start from an observation (made already in [4]) that in the low frequency regime the cell structure is not important. And indeed, phenomena 1 and 2 have been also observed in [5, 6] in 3D arrays of parallel conducting wires. Phenomenon 1 can be explained very simply just noticing that at low frequencies when the period of the grid is much smaller than the wavelength, the wire grid behaves as a conductor. On the other hand, when the frequency is high and the period is large compared to the wavelength, the grid is quite transparent for electromagnetic waves. These properties can be modeled for wire grids in terms of the averaged induced current [9]. Sharp cut-off between the two regimes is there because of the finite thickness of the structure and periodicity in the normal direction. Finally, if the wires in each layer are periodically cut (introducing defects), new resonances of high transmission appear. The resonance frequencies correspond to the condition that the length of conducting sections equals one or several half-wavelengths. If the section length is large enough (sparse defects), these resonances appear in the low frequency stopband.

We explore properties of periodical structures of several layers of metal elements of various forms in detail and show that indeed all the properties observed in [4] can be found in these arrays. These structures can find practical applications in polarization-selective filters (very sharp resonances due to screening the internal layers of the structure, polarization sensitivity or polarization transformation due to the inclusion shape).

2. Operator of electromagnetic wave scattering by double-periodic arrays

Let us consider the incidence of a plane electromagnetic wave $\mathbf{E}^i = \mathbf{P}\exp(i\mathbf{k}^i \cdot \mathbf{r})$ on an infinite double-periodic array in the plane $z = 0$. Incident and scattered electromagnetic fields can be conveniently represented using transverse to Oz axis components of TE- and TM-wave sets. Consider in the beginning the incidence of a wave having transverse component of the electric field in the form

$$\mathbf{E}^i_t(\mathbf{r}) = \boldsymbol{\psi}^{(p)}_{m'n'}(\boldsymbol{\rho})\exp(-i\Gamma_{m'n'}z), \qquad (1)$$

where index t denotes transverse to Oz axis components of the electric field, $p = 1$ corresponds to TE-waves, and $p = 2$ corresponds to TM-waves,

$$\boldsymbol{\psi}_{mn}^{(1)}(\boldsymbol{\rho}) = \frac{1}{\sqrt{Q}} \frac{\boldsymbol{\chi}_{mn} \times \mathbf{e}_z}{\chi_{mn}} e^{i\boldsymbol{\chi}\cdot\boldsymbol{\rho}}, \ \boldsymbol{\psi}_{mn}^{(2)}(\boldsymbol{\rho}) = \frac{1}{\sqrt{Q}} \frac{\boldsymbol{\chi}_{mn}}{\chi_{mn}} e^{i\boldsymbol{\chi}\cdot\boldsymbol{\rho}} \tag{2}$$

are vector harmonics; $\boldsymbol{\chi}_{mn} = \mathbf{e}_x(k_x^i - 2\pi m/d_x) + \mathbf{e}_y(k_y^i + -2\pi n/d_y)$, $\Gamma_{mn} = \sqrt{k^2 - \chi_{mn}^2}$, $\boldsymbol{\rho} = \mathbf{e}_x x + \mathbf{e}_y y$, $Q = d_x d_y$, d_x and d_y are the array periods along Ox and Oy axes, respectively. The corresponding reflected field we write as

$$\mathbf{E}_t^r(\mathbf{r}) = \sum_{q=1}^{2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} a_{mn}^{(pq)}(\boldsymbol{\chi}_{m'n'}) \boldsymbol{\psi}_{mn}^{(q)}(\boldsymbol{\rho}) \exp(i\Gamma_{mn} z). \tag{3}$$

If the set of partial waves

$$\mathbf{E}_t^i(\mathbf{r}) = \sum_{p=1}^{2} \sum_{m'=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} q_{m'n'}^{(p)} \boldsymbol{\psi}_{m'n'}^{(p)}(\boldsymbol{\rho}) \exp(-i\Gamma_{m'n'} z) \tag{4}$$

is incident upon an array, the reflected field may be represented in the form

$$\mathbf{E}_t^r(\mathbf{r}) = \sum_{q=1}^{2} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} b_{mn}^{(q)} \boldsymbol{\psi}_{mn}^{(q)}(\boldsymbol{\rho}) \exp(i\Gamma_{mn} z). \tag{5}$$

Let us define an operator of scattering by a double-periodic grating as an operator which connects coefficients $b_{mn}^{(q)}$ of the reflected field and the coefficients $q_{m'n'}^{(p)}$ of the incident field:

$$b_{mn}^{(q)} = \sum_{p=1}^{2} \sum_{m'=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} a_{mn}^{(pq)}(\boldsymbol{\chi}_{m'n'}) q_{m'n'}^{(p)} \tag{6}$$

or, in short operator notation, $b = rq$, where r is the operator of reflection. Coefficients $a_{mn}^{(pq)}(\boldsymbol{\chi}_{m'n'})$ may be found by using any known method of the analysis of electromagnetic wave scattering by single double-periodic arrays. Method of moments may be used particularly in the cases of wave scattering by arrays of thin strips [10 – 12].

Let us consider the incidence of a plane electromagnetic wave with the frequency that is below the lowest of the so-called sliding frequencies, i.e. the frequency values which divide frequency regions corresponding to propagating or evanescent spatial partial waves. Further, only the amplitudes of propagating waves will be important, i.e., we will consider characteristics of the reflected fields at positions spaced from the array plane so that the influence of non-propagating partial waves can be neglected. Operator of reflection in this case may be defined approximately by expression $b^{(q)} = \sum_{p=1}^{2} a^{(pq)} q^{(p)}$, where the lower indices are omitted for shortness, they are all equal to zero. All the statements formulated above regarding operator of reflection are correct also for operator of transmission t.

3. Operators of reflection and transmission for a system of a finite number of arrays

The structure is assumed to be equidistant and to consist of identical arrays. The field in each gap between planar arrays may be represented in the form of a set of partial TE- and TM-waves. The amplitudes of the transverse components of the partial waves are denoted as following: q for the incident field, $r^{(n)}q$ for the reflected field, $t^{(n)}q$ for the transmitted field, and $A^{(n-1)}$, $B^{(n-1)}$ for the fields in the gap between the next to the last array and the last array of the structure, see Fig. 1.

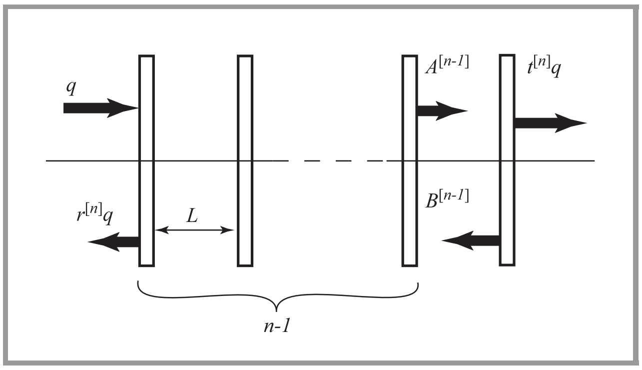

Fig. 1. Layered array: amplitudes of the reflected, transmitted, and partial waves.

Let us assume operators r, t for a single array to be known, as well as $r^{(n-1)}$, $t^{(n-1)}$ for the system of $(n-1)$ arrays, and show that the operators for the whole system can then be found recursively. The amplitudes of the partial waves satisfy equations

$$A^{(n-1)} = t^{(n-1)}q + r^{(n-1)}eB^{(n-1)},$$

$$B^{(n-1)} = reA^{(n-1)},$$

$$r^{(n)}q = r^{(n-1)}q + t^{(n-1)}eB^{(n-1)},$$

$$t^{(n)}q = teA^{(n-1)}, \tag{7}$$

where e is the plane-wave propagator operator from the plane of one array to the next array plane. After elimination of vectors $A^{(n-1)}$ and $B^{(n-1)}$ one obtains recurrent expressions which allow to find operators $r^{(n)}$ and $t^{(n)}$ in the form

$$r^{(n)} = r^{(n-1)} + t^{(n-1)}ere(I - r^{(n-1)}ere)^{-1}t^{(n-1)}, \tag{8}$$

$$t^{(n)} = te(I - r^{(n-1)}ere)^{-1}t^{(n-1)}. \tag{9}$$

4. Numerical results and discussion

For shortness, we will call straight strips I-shaped inclusions, the shape of open loops we associate with the shape of letter C, and an Ω-shaped conductive inclusion we call the omega particle. The frequency dependence of the transmission and reflection coefficients of an array of I-shaped elements has a resonant behavior. The resonance appears when the length of an element approximately equals to one half of the incident wavelength. Naturally, an increase of the element length leads to a decrease of the resonant frequency. Frequency dependence of the reflection coefficient of an array of infinitely long strips (an array without defects as periodic cutting of strips) has no resonances.

Further reduction of the resonance frequency is possible by changing the element shape so that it is more compact but has a longer stretched length. In the frequency dependencies for an array of C-shaped elements there are two resonances. The second resonance appears when the length of an element is close to one and one half of the wavelength. Knowing the resonant behavior of layered structures as dependent on the shape, length of strips, periods of arrays, elements and distance between the layers we can create systems of layers with interesting and useful properties. A 4-layer structure of I-shaped strips exhibits the same behavior as the 3D wire mesh with defects described in [4].

A system with two zones of full reflection can be made using a 4-layer structure ($L = 5/6d_x$) with C-shaped elements ($a = 5/12d_x$, $\phi_1 = \pi/18$, $2w = 1/30d_x$, $d_x = d_y$). Its frequency characteristics are shown in Fig. 2. The first reflection zone is in the low frequency area. It is the first resonance (polarization along axis Oy) depending on the element length ($S = 2.47d_x$, $d_x = d_y$) with the resonance between the first and the fourth layers ($d_x/\lambda \approx 0.2$). The second zone is the second length resonance with resonances between layers: layer 1 and layer 2 ($L_{12} \approx \lambda/2$), layer 1 and layer 3 ($L_{13} \approx \lambda$), layer 1 and layer 4 ($L_{14} \approx 3\lambda/2$), when $d_x/\lambda \approx 0.6$.

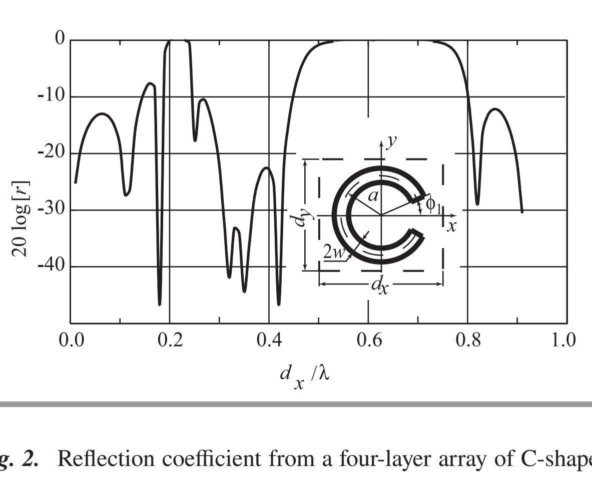

Fig. 2. Reflection coefficient from a four-layer array of C-shaped inclusions.

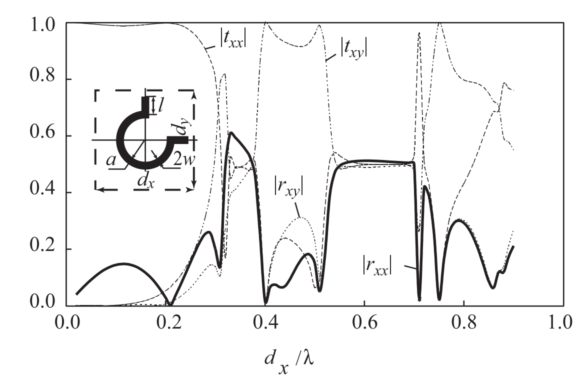

Fig. 3. Reflection and transmission coefficients from a four-layer array of Ω-shaped inclusions.

Similar behavior of the frequency dependence of the reflection coefficient we can see in Fig. 3 in the case of a 4-layer structure ($L = 0.5d_x$) with Ω-shaped elements ($a = 0.25d_x$, $l = 0.35d_x$, $2w = 0.019d_x$, $d_x = d_y$). Because we have chosen in this case symmetrical displacements of elements with respect to the diagonal of the array cells, cross-polarized field components exist in the transmitted and reflected field. There are no frequencies of full reflection in this case because of the polarization transformation. Different responses of the array on different polarizations allow to use such structures not only as frequency selective filters but also as polarization-sensitive filters.

5. Conclusion

There are three reasons for resonant behavior of the frequency dependencies of the reflection coefficient for such structures. The first reason is interference phenomena between layers. The second reason is introduced by defects (cut strips, in our case) that leads to finite length of array elements and, as a consequence, to additional resonances. The third reason is interference between layers on frequencies near to the resonant frequencies of elements in a single array. In the last case new resonances can appear because of strong dispersion of the phase of the transmission coefficient for a single array.

Regular structures of conducting elements without defects are strongly reflecting in the low frequency region. Layered structures of arrays of finite-length elements are transparent in the low frequency region and have properties of PBG structures in the frequency region where the main sizes of the elements and the whole array thickness are approximately equal to the wavelength.

Acknowledgement

This work has been supported in part by the NATO Science Programme, linkage grant HTECH.LG 974668.

References

[1] E. R. Braun, C. D. Parker, and E. Yablonovich, "Radiation properties of a planar antenna on a photonic-crystal substrate", *J. Opt. Soc. Am. B,* vol. 10, no. 2, pp. 404–407, 1993.

[2] H. Y. D. Yang, N. G. Alexopoulos, and E. Yablonovitch, "Photonic bandgap materials for high-gain printed circuit antennas", *IEEE Trans. Anten. Propag.,* vol. 45, no. 1, pp. 185–187, 1997.

[3] F.-R. Yang, K.-P. Ma, Y. Qian, and T. Itoh, "A uniplanar compact photonic-bandgap (UC-PBG) structure and its applications for microwave circuits", *IEEE Trans. Microw. Theory Techn.,* vol. 47, no. 8, pp. 1509–1514, 1999.

[4] D. F. Sievenpiper, M. E. Sickmiller, and E. Yablonovitch, "3D wire mesh photonic crystals", *Phys. Rev. Lett.,* vol. 76, no. 14, pp. 2480–2483, 1996.

[5] J. B. Pendry, A. J. Holden, W. J. Stewart, and I. Yongs, "Extremely low frequency plasmons in metallic mesostructures", *Phys. Rev. Lett.,* vol. 76, no. 25, pp. 4773–4776, 1996 (comment by S. A. Mikhailov and author's reply: vol. 78, no. 21, pp. 4135–4136, 1997).

[6] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Low frequency plasmons in thin-wire structures", *J. Phys.: Condens. Matter,* vol. 10, pp. 4785–4809, 1998.

[7] H. F. Contopanagos, C. A. Kyriazidou, W. M. Merrill, and N. G. Alexopoulos, "Effective response functions for photonic bandgap materials", *J. Opt. Soc. Am. A,* vol. 16, no. 7, pp. 1682–1699, 1999.

[8] S. I. Maslovski, S. A. Tretyakov, and C. R. Simovski, "Electromagnetic modelling of composite media with second-order spatial dispersion", in *XXVIth URSI General Assembly,* University of Toronto, Canada, 1999, p. 91.

[9] M. I. Kontorovich, M. I. Astrakhan, V. P. Akimov, and G. A. Fersman, *Electrodynamics of Grid Structures*. Moscow: Radio i Sviaz, 1987 (in Russian).

[10] S. L. Prosvirnin, "Analysis of electromagnetic wave scattering by plane periodical array of chiral strip elements", in *Proc. 7th Int. Conf. Complex Media Bianisotropics'98*, Technische Universität Braunschweig, Germany, 3–6 June 1998, pp. 185–188.

[11] T. D. Vasilyeva and S. L. Prosvirnin, "Electromagnetic wave diffraction by the plane array of chiral strip elements of complex shape", *Phys. Wave Proc. Radio Syst.*, vol. 1, no. 4, pp. 5–9, 1998 (in Russian).

[12] S. Zouhdi, G. E. Couenon, and A. Fourrier-Lamer, "Scattering from a periodic array of thin planar chiral structures – calculations and measurements", *IEEE Trans. Anten. Propag.*, vol. 47, no. 6, pp. 1061–1065, 1999.

Sergey L. Prosvirnin graduated from Kharkov Institute of Radio Electronics in 1971, and obtained the degrees of Ph.D. and D.Sc. in radio physics, and Professor Rank from Kharkov State University, Ukraine in 1974, 1986, and 1992, respectively. From 1974 to 1985, he was with the Institute of Radio Physics and Electronics. Since 1985, he has been with the Institute of Radio Astronomy (IRA), Academy of Sciences of Ukraine. He has been Head of Department of Computational Mathematics, IRA since 1989, Professor of Kharkov National University since 1987, Professor of Kharkov Institute of Air Forces of Ukraine since 1997. His main scientific interests are analytical-numerical methods in the theory of scattering and radiation of electromagnetic waves, complex media electromagnetics, and microwave engineering. He is a Senior Member of the IEEE.
e-mail: prosvirn@rian.ira.kharkov.ua
Institute of Radio Astronomy
Chervonopraporna st 4
61002 Kharkov, Ukraine

Sergey A. Tretyakov received the Dipl. Engineer-Physicist, the Ph.D., and the Doctor of Sciences degrees (all in radiophysics) from the St. Petersburg State Technical University (Russia), in 1980, 1987, and 1995, respectively. From 1980 to 2000 he was with the Radiophysics Department of the St. Petersburg State Technical University. Presently, he is Professor of radio engineering in the Radio Laboratory, Helsinki University of Technology. His main scientific interests are electromagnetic field theory, complex media electromagnetics and microwave engineering. Prof. Tretyakov served as Chairman of the St. Petersburg IEEE ED/MTT/AP Chapter from 1995 to 1998.
e-mail: sergei.tretyakov@hut.fi
Radio Laboratory
Helsinki University of Technology
P.O. Box 3000, FIN-02015 HUT, Finland

Tetyana D. Vasilyeva received the Dipl. in electronics engineering from Kharkov University of Radio Electronics, Ukraine, the Ph.D. in radiophysics from Kharkov State University, Ukraine in 1981, and 1992 respectively. She was with the Institute of Radio Physics & Electronics from 1981 to 1985, and with the Institute of Radio Astronomy (IRA), Academy of Sciences of Ukraine from 1985 to 2000. Presently, she is application analyst in HiTech Network Inc., Toronto, ON, Canada. Her main scientific interests are numerical modeling and analysis in the electromagnetic theory of complex media, and in microwave techniques.
e-mail: tetyana.vasilyeva@sympatico.ca
2500 Keele st, #602
Toronto ON, M6L 2N5, Canada

Arlette Fourrier-Lamer
afl@ccr.jussieu.fr
Laboratoire de Genie Electrique de Paris LGEP
Supelec, France

Said Zouhdi
sz@ccr.jussieu.fr
Laboratoire de Genie Electrique de Paris LGEP
Supelec, France