Paper

Ultra-wideband radar targets discrimination based on discrete E-pulse synthesis

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Abstract—A frequency domain approach to the E-pulse radar target discrimination technique is introduced. This approach allows the interpretation of E-pulse phenomenon via the E-pulse spectrum. The discrete E-pulse and its relation to continuous E-pulse are shown. The addition of extra zeroes to E-pulse structure has been suggested and its influence on the increasing of discrimination accuracy has been proved. The results of discrimination scheme digital simulation by using the characteristic E-pulse parameters for known targets are presented.

Keywords—radar target recognition, signal processing.

1. Introduction

A growing interest in the target discrimination methods using ultra-wideband target response has been arisen recently. Ultra-wideband radar proposes using sharp pulses with duration about units of nanoseconds. These pulses excite electromagnetic oscillations in the target defined by geometric range and form of it. According to the Baum's singularity expansion method (SEM) [1], which provides the necessary mathematical formulation for describing the transient behavior of conducting targets, scattered target response can be represented as a sum of damped oscillations:

$$y(t) = x(t) + w(t)$$

$$= \sum_{k=1}^{K} A_k e^{-\sigma_k t} \cos(\omega_k t + \varphi_k) + w(t), \qquad (1)$$

where $s_k = \sigma_k + j\omega_k$ is the kth aspect-independent natural complex frequency of the target, and A_k and φ_k are the aspect- and excitation-dependent amplitude and phase of the kth target mode, respectively, $w(t)$ is an additive Gaussian band-limited noise. The number of natural resonances K is determined by the finite frequency content of the waveform exciting the target and by the geometrical shape of the object.

The identification methods using in the ultra-wideband radar can be divided into parametric and non-parametric. Parametric methods consist in the estimation of the target specified features based on as its natural frequencies in the measured response [2]. Determined parameters can be compared to the known parameters of the targets included in the database and identification decision can be made relaying on it. The main part of these methods consists – of feature extraction technique: Prony's method, pencil-of-function method or ESPRIT.

Another way of the ultra-wideband target discrimination is the E-pulse method [3]. This method offers to fit special signal (E-pulse) to the target response so the convolution of the response and the signal is minimum (or equal to zero in the ideal case) at the time period determined by the signal parameters. Frequency domain method for the synthesis of the discrete subsectional E-pulse is described in this paper.

The paper is organized as follows. In Section 2, the frequency domain E-pulse method is presented. Section 3 describes the discrete E-pulse synthesis. The model chosen for simulation and the parameter for estimation the identification quality are introduced in Section 4. The results of digital simulation are presented in Section 5. Concluding remarks are drawn in Section 6.

2. Frequency domain E-pulse method

E-pulse is a special waveform which fitted for target response in such a way that its convolution with the response gives zero since the certain moment:

$$c(t) = e(t) \cdot x(t) = 0 \quad t > T_L. \qquad (2)$$

There T_L may be chosen equal to zero or positive value reasonably. Frequency domain approach [4] allows the convolution (2) to be written in the form

$$c(t) = \sum_{k=1}^{K} A_k |E(s_k)| e^{-\sigma_k t} \cos(\omega_k t + \psi_k), \quad t > T_L, \qquad (3)$$

where

$$E(s) = L\{e(t)\} = \int_0^{T_E} e(t) e^{-st} dt \qquad (4)$$

is the Laplace transform of a finite duration ($T_E \leq T_L$) E-pulse waveform, and

$$\psi_k = \varphi_k + \arg\big(E(s_k)\big). \qquad (5)$$

Now, equality $c(t)$ to zero for $t > T_L$ requires:

$$E(s_k) = E(s_k^*) = 0, \quad 1 \leq k \leq K. \qquad (6)$$

The expression written above clears the way to various synthesis possibilities. It is enough to set the desired E-pulse waveform with some unknown parameters and in accordance with chosen T_L. Next step is the equation set composition which solution makes all the parameters determined and the E-pulse will consider to be found.

We've tried to analyze (6) itself to state the necessary condition for all synthesized waveform to be E-pulse. Understanding of the E-pulse as a finite duration waveform means that its Laplace transform doesn't consist of any poles and can be generally represented as a polynomial of s. Therefore the values of s making the polynomial $E(s)$ vanish are its roots by definition. Consequently E-pulse indispensable condition is zero arrangement on the s-plane providing its matching with the poles of the target response E-pulse is constructed for:

$$E_{nec}(s) = \prod_{k=1}^{K} (s - s_k)(s - s_k^*). \qquad (7)$$

But the attempt of utilizing the waveform which Laplace transform includes only the necessary zeroes has no sense because of the inverse transformation of such polynomial (7) contains high order derivatives of delta-function. This case agrees with $T_L = 0$ consequently the application of E-pulse waveform providing $T_L > 0$ is evident. Hence it requires the adding of extra zeroes covered by the specified waveform synthesis scheme chosen to create E-pulse. Different schemes assign different rules for zero addition and make its placing on the s-plane various.

Generally E-pulse can be defined as an arbitrary waveform. However the E-pulse proposes to be expressed as a sum

$$e(t) = \sum_{m=0}^{M} f_m(\alpha_m, t), \qquad (8)$$

where $f_m(\alpha_m, t)$ is the mth basis function, α_m describes the function parameters that can vary in order to provide (2), and a number of basis function used to construct the extinction component is defined by M. A very useful application results from using subsectional basis function [8]:

$$f_m(\alpha_m, t) = \begin{cases} g(\alpha_m, t - m\Delta), & m\Delta \leq t \leq (m+1)\Delta \\ 0, & \text{elswhere}, \end{cases} \qquad (9)$$

where $g(t)$ is a Laplace transformable function and Δ is the section width. If $g(t)$ assumes to be delta-function $\delta(t)$ the E-pulse will be written:

$$e_d(t) = \sum_{m=0}^{M} \alpha_m \delta(t - m\Delta). \qquad (10)$$

This case corresponds to the degenerate E-pulse that can be considered as a specific model for phenomenon explanation.

For considered poles model (1) the zeroes of $e_d(t)$ can be found:

$$s_{01r} = \sigma_1 + j\frac{2\pi}{\Delta}r, \quad r = 0, \pm 1, \pm 2, \ldots \qquad (11)$$

$$s_{0kr} = \sigma_k + j\left(\frac{2\pi}{\Delta}r \pm \omega_k\right), \quad k = \overline{2, K} \qquad (12)$$

$$\Delta = \frac{p\pi}{\omega_1}, \quad p = 1, 2, \ldots \qquad (13)$$

where ω_1 is the maximum of ω_k. This case writes the natural E-pulse described in [5]. Otherwise section width Δ is chosen freely the zeroes are located by

$$s_{0kr} = \sigma_k + j\left(\frac{2\pi}{\Delta}r \pm \omega_k\right), \quad k = \overline{1, K}, \quad r \in Z \qquad (14)$$

and such E-pulse is called forced. Natural E-pulse only exists for discrete range of section width Δ while forced E-pulse being doesn't depend on Δ.

The example applied often is the rectangular E-pulse that can be obtained by integrating $e_d(t)$. This case suggests the constant function $g(t) = \alpha$. Then

$$f_m(t) = \begin{cases} \alpha_m, & m\Delta \leq t \leq (m+1)\Delta \\ 0, & \text{elsewhere}. \end{cases} \qquad (15)$$

Since the basis function are defined the equation set can be composed to find the unknown parameters α_m and after it's solved E-pulse will construct.

3. Discrete E-pulse synthesis

The advantage of digital signal processing can be successfully applied for E-pulse technique. Since the sampling procedure leads to mapping s-plane zeroes and poles onto z-plane via

$$z = \exp(pT_s), \qquad (16)$$

where T_s is a period of sampling. Now the section width should obviously be equal to integer number of periods T_s. Natural E-pulse zeroes allocation on z-plane can be received by applying the transformation (16) to the expressions (11–13):

$$z_{01r} = \exp(\sigma_1 T_s)\exp\left(j\frac{2\pi}{N_\Delta}r\right), \quad r = 0, 1, \ldots, N_\Delta - 1, \qquad (17)$$

$$z_{0kr} = \exp(\sigma_k T_s)\exp\left(j\frac{2\pi}{N_\Delta}r\right)\exp(\pm j\omega_k T_s),$$
$$k = \overline{2, K}, \quad r = 0, 1, \ldots, N_\Delta - 1, \qquad (18)$$

where N_Δ is the width of section in samples.

The expression (17) being demands the sampling frequency to be divisible by natural frequency of any poles:

$$\omega_1 T_s = 2\pi n, \quad n = 1, 2, 3, \ldots \qquad (19)$$

So the usage of natural E-pulse in discrete form has the notable difficulties. The sampling frequency can't be surely taken to provide the appropriate section width. Therefore only forced E-pulse should be applied. The expression (16) maps its *s*-plane zeroes onto *z*-plane via:

$$z_{0kr} = \exp(\sigma_k T_s) \exp\left(j\frac{2\pi}{N_\Delta} r\right) \exp(\pm j\omega_k T_s),$$

$$k = \overline{1, K}, \quad r = 0, 1, \ldots, N_\Delta - 1. \tag{20}$$

Figure 1 shows *z*-plane poles allocation for example while forced E-pulse is constructed for the response that contains only one pair of poles. The section width has chosen to be 3 samples. As follows all zeroes belong to the same pole component lie on the circle symmetrically around the lines divided *z*-plane into equal parts.
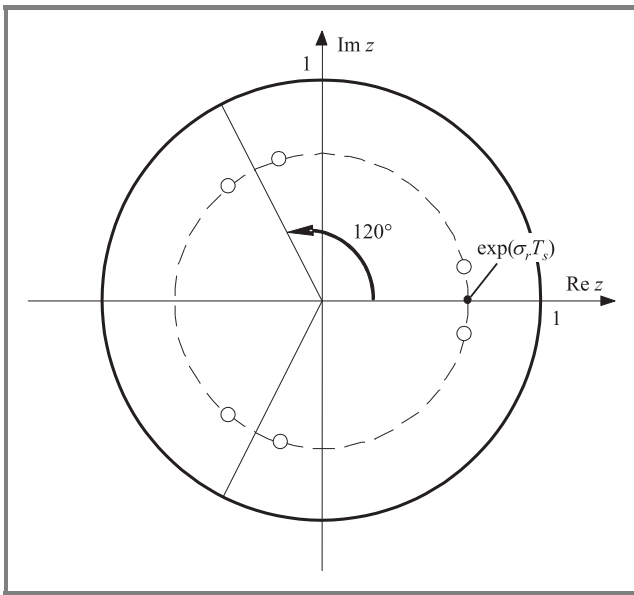
Fig. 1. Discrete E-pulse zeroes allocation on *z*-plane for one pole discrimination and section width equal to 3 samples.

E-pulse can be represented via *z*-transform technique directly. Z-transform of the sampled target response $x[n]$ can be written as

$$\widetilde{X}(z) = \frac{L(z)}{P(z)}, \tag{21}$$

where $L(z)$ and $P(z)$ are polynomials of *z*. Carrying out expression (2) the signal $c[n]$ (as sampled $c(t)$) must be finite, so its *z*-transform must be free from poles. On the other hand the E-pulse itself is the finite waveform and its *z*-transform contains no poles:

$$\widetilde{C}(z) = \widetilde{X}(z) \cdot \widetilde{E}(z) = \frac{L(z) \cdot E(z)}{P(z)} = C(z), \tag{22}$$

where $\widetilde{X}(z)$, $\widetilde{E}(z)$, $\widetilde{C}(z)$ are *z*-transforms of the target response, the E-pulse, fitted to the response, and their con-

volution correspondingly, and $L(z)$, $E(z)$, $P(z)$, $C(z)$ are *z*-power polynomials.

It's obviously that expression (21) requires E-pulse to meet the condition:

$$E(z) = P(z) \cdot D(z), \tag{23}$$

where polynomial $P(z)$ coincides with the denominator of *z*-transform $\widetilde{X}(z)$, and $D(z)$ is a *z*-power polynomial.

Thus the zeroes of $\widetilde{E}(z)$ should be placed in the same point of the complex plane where the poles of the target response $\widetilde{X}(z)$ lie. However in addition to them E-pulse can also consist extra zeroes, all of them are described by the polynomial $D(z)$. The quality of target discrimination can be improved by optimal allocation of these extra zeroes.

Inverse *z*-transform of the polynomial $P(z)$ represents the minimal duration basic E-pulse $e_{base}[n]$. In contrast to the continuous this E-pulse is practically realized and can find practical application if discriminated targets poles lie far from each other only. In the case of close allocated target poles discriminating possibilities of this E-pulse is extremely low, so extra zeroes addition is required.

The discussed zeroes allocation (20) meets to the necessary condition while $k = 0$. Inverse *z*-transform of such the polynomial constructed by these zeroes gives

$$e[n] = \sum_{k=0}^{2K} \alpha_k \delta\left[n - kN_\Delta\right], \tag{24}$$

where K describes the pairs of poles number in target response model, $\delta[n]$ is a discrete delta-function.

More sophisticated E-pulse structure can be reach by using high order polynomial section. In the case of rectangular section the E-pulse determined by the expression (24) should be convolved with rectangular pulse of the N_Δ samples duration. For the application of the higher then the first order base function it's reasonable to use the continuity condition to section border for the waveform and its derivatives [8].

4. Target model and estimation parameter

To make the simulation considerable it's important to choose appropriate target model. We use the model of two aircrafts F-4 and MIG-27 based on three main natural frequencies. The target poles are shown in Fig. 2.

The choice of three main resonances was made on the assumption that they contain the most part of the object response energy. Actually the number of significant poles may exceed several dozens and depends on the target geometry complication.

E-pulse discrimination ratio (EDR) is used for the estimation of discrimination algorithm quality. This param-

eter allows creating the automated target discrimination scheme [7, 8]:

$$EDR_p \ [\text{dB}] = 10 \lg \left(\frac{EDN_p}{\min_p(EDN)} \right), \qquad (25)$$

where p describes the number of channel tuned for specified expected target, EDN means E-pulse discrimination number calculated as:

$$EDN_p = \sum_{n=N_L}^{N_L+N_W} c_p^2[n] \Big/ \sum_{n=0}^{N_E} e_p^2[n], \qquad (26)$$

where N_L is the time in samples showing the beginning of the late-time part of the convolution received target response with E-pulse and N_W is the measuring interval width, N_E is the E-pulse duration. A priori minimum EDN
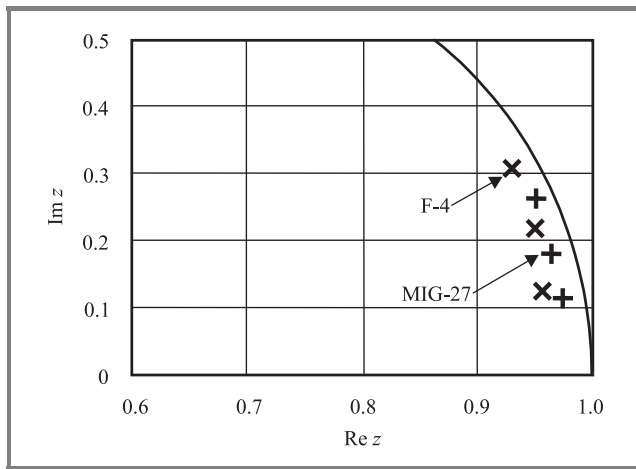
Fig. 2. Complex z-plane plot of aircraft models.

supposes to be in the channel fitted to the expected target since EDR for expected target is identically zero [dB] and called "baseline". The unexpected targets EDRs differ from baseline and the greater exceeding of the baseline demonstrate the better identification possibilities.

5. Digital simulation

In our opinion using the predefined poles model of two aircraft serves the good example for exposure the features of the E-pulse technique.
Power spectra of target responses are shown in Fig. 3. It's obviously that both aircraft response spectra take the wide frequency band.
Pulse response based on constructed E-pulse to annul the scattered signal of F-4 aircraft is shown in Fig. 4.
The basis function has been chosen rectangular and each section duration is equal to 8 samples.
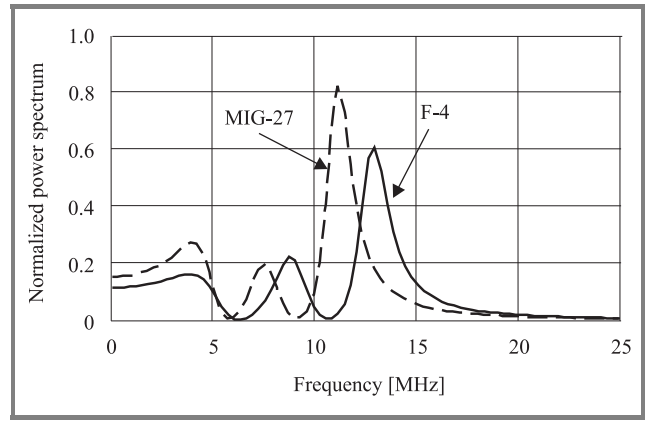Figure 5 displays the convolution of the F-4 E-pulse with restored responses of the expected target and an unex

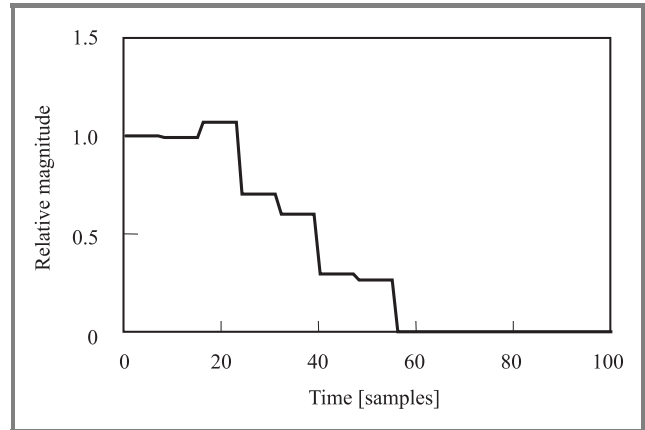**Fig. 3.** Power spectrum of aircraft models.

Fig. 4. E-pulse constructed for F-4 aircraft model, the section width is equal to 8 samples.
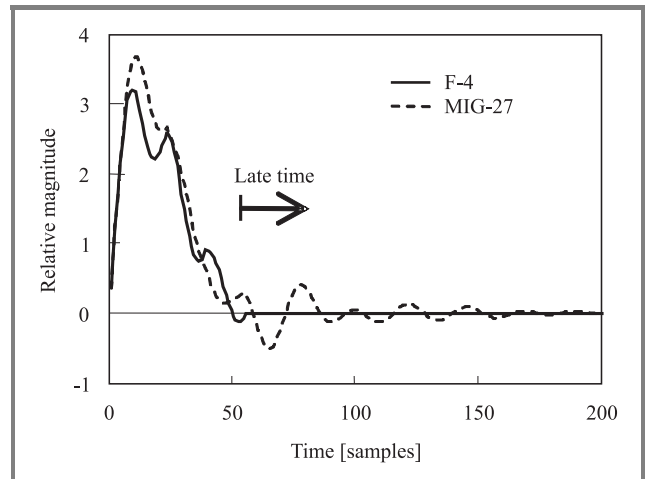

Fig. 5. Convolution of the E-pulse with responses of the expected and an unexpected targets.

pected target (MIG-27). Late-time region of the MIG-27 response convolution is visibly distinct from the same region of F-4. However the presence of noise makes this difference unclear like Fig. 6 shows.
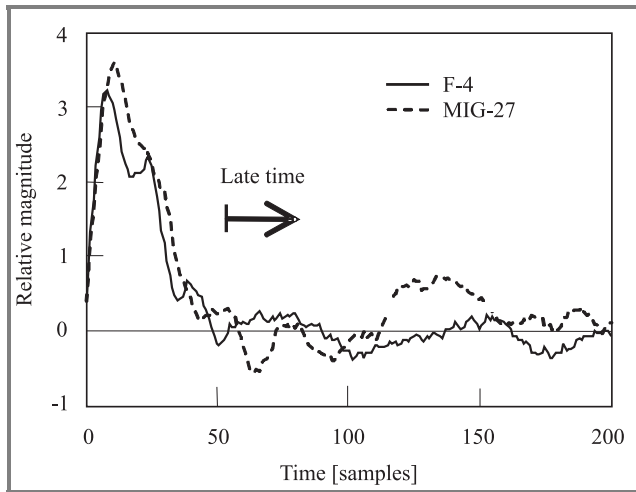
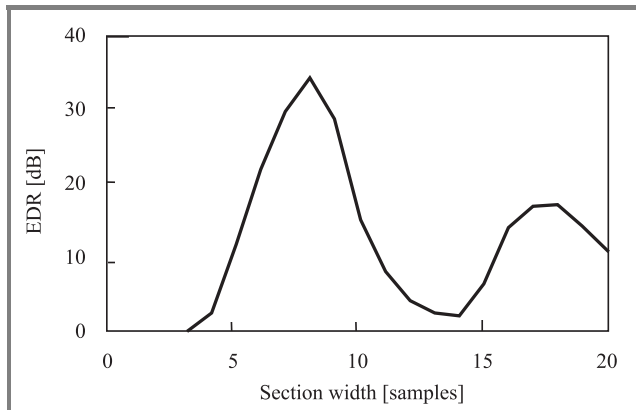**Fig. 6.** Convolution of the E-pulse with responses of the expected and an unexpected targets while SNR = 5 dB.

Fig. 7. EDR for rectangular E-pulse against its section width in samples in the presence of noise, SNR = 30 dB.
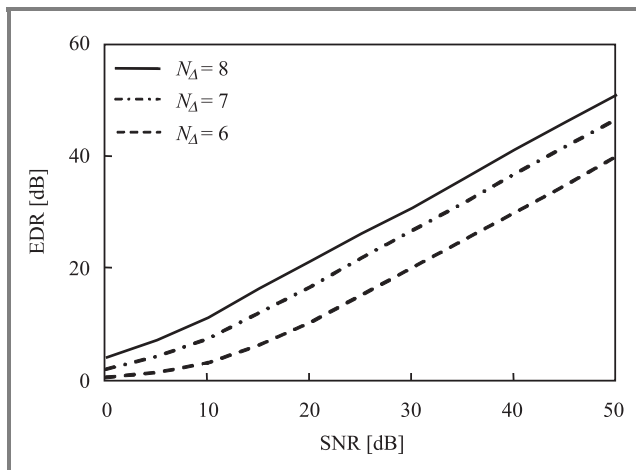

Fig. 8. EDR against SNR for better discrimination capability E-pulses.

The discrete E-pulse application certainly requires determining its duration. Since it can be expressed through the width of the section it's important to choose the width that provides possibly maximum EDR in comparison with others. The results of EDR evaluated for E-pulses which section are measured in samples are shown in Fig. 7. The simulation has been made for additive band-limited Gaussian noise while SNR = 30 dB.

Figure 7 can help to make the decision of E-pulse section width. Obviously that function EDR (N_Δ) has the maximum that corresponds to the section width of 8 samples. The knowledge about the E-pulse section width giving the maximum EDR is able to synthesize the E-pulse having better discrimination capability.

Figure 8 shows the EDRs for the E-pulses with different section width for comparison.

6. Conclusion

Frequency domain approach to E-pulse synthesis can be successfully applied for the resonant model of the complex targets in ultra wideband radar. Based on the known target poles the specified waveform can be constructed for its discrimination. The necessary condition requiring the particular zero allocation for the waveform to be the E-pulse is noticed in the paper.

An attempt to make the proper description of the discrete E-pulse was made. The directly dependence between z-plane zeroes arrangement of discrete E-pulse and s-plane zeroes arrangement of continuous E-pulse was shown. It described the difficulties of the natural E-pulse usage in discrete time and makes the forced E-pulse to be applied.

Digital simulation was carried out by the example of scaled models F-4 and MIG-27 aircrafts. For the E-pulse effectiveness estimation the EDR parameter was engaged. As its result the width of the E-pulse providing better determination capability in comparison with other was found.

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