Paper

Path Diversity Protection in Two-Layer Networks

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Abstract— The paper addresses an optimization problem related to dimensioning links in a resilient two-layer network. A particular version of the problem which assumes that links of the upper layer are supported by unique paths in the lower layer is considered. Two mixed-integer programming formulations of this problem are presented and discussed. Direct resolving of these formulations requires pre-selection of "good" candidate paths in the upper layer of the network. Thus, the paper presents an alternative approach which is based on decomposing the resolution process into two phases, resolved iteratively. The first phase subproblem is related to designing lower layer path flows that provide the capacities for the logical links of the upper layer. The second phase is related to designing the flow patterns in the upper layer with protection assured through diversity of paths. In this phase we take into account the failures of the logical links that result from the failures of the lower layer links (so called *shared risk link groups*).

Keywords— link dimensioning, path diversity, resilient routing, two-layer network optimization.

1. Introduction

One of the most important internet architectures is based on IP-over-WDM (wavelength division multiplexing) networks. IP-over-WDM refers to a complex network model which uses a layered structure of resources, operated according to two distinct network protocols. Resources of a layered network form a hierarchical structure with each layer constituting a proper network.

In the considered network model the lower layer is an WDM network composed of a set of fibers connecting WDM cross-connects. The upper layer is an IP network composed of a given set of logical connections between routers (see [1] and [2]). The logical IP links are supported by paths composed of the WDM links (WDM paths).

In this paper we address an optimization problem related to dimensioning capacity of the WDM fibers. Throughout the paper we assume that each IP link is supported by an unique path in the WDM layer (uniqueness property). Dimensioning cost for a given realization of the upper layer links is calculated as a sum of capacity costs of distinct links of the lower layer. For a given set of traffic requirements (demands) the considered problem consists in determining a set of IP layer paths (IP paths) and their realizations in the WDM layer for which the total dimensioning cost is minimized.

In the paper we consider two-layer network model for which the WDM links are subject to failures. We assume that during a failure one of the WDM links becomes unavailable. Failure of an WDM link causes that all IP links associated with affected paths are unavailable too. Thus, a failure of single WDM link may cause unavailability of multiple IP links. In the literature such failure model is called shared risk resource group (see [3]).

Suppose that flows assigned to a specific traffic demand are bifurcated and diversified (recall that the uniqueness property refers only to the WDM layer). Diversification of the flows constitutes a means for protecting the IP-over-WDM network against failures. We refer to this protection type as path diversity (see [4]).

In the general two-layer network model the set of lower layer nodes is wider than the set of the upper layer nodes, i.e., only selected sites comprise either WDM cross-connects and IP routers, while the others comprise only WDM cross-connects. Note that when all sites comprise devices of both kinds the problem can be reduced to dimensioning IP links, each related to the corresponding WDM link.

In the following we present a mixed-integer programming (MIP) formulation of the problem related to dimensioning the WDM links in the resilient IP-over-WDM network. As this formulation requires identifying all possible paths in the IP layer, resolving it using general MIP solvers is not efficient. Thus, we propose a dedicated method based on decomposing the resolution process into two iteratively invoked phases.

The paper is organized as follows. In Section 2 we formulate the considered problem as a mixed-integer program. The resolving method is presented in Section 3. The numerical results illustrating the efficiency of the method are discussed in Section 4. The paper is summarized in Section 5, where the conclusions are drawn.

2. Problem Formulation

Let $\mathscr{V}$ and $\mathscr{W}$ be the set of IP nodes and WDM nodes, respectively. We define $\mathscr{E}$ and $\mathscr{F}$ as the link sets associated with upper and lower layer, respectively. The network graphs associated with network layers are denoted by $\mathscr{G}(\mathscr{V},\mathscr{E})$ – the IP layer graph and $\mathscr{H}(\mathscr{W},\mathscr{F})$ – the WDM layer graph. In the balance of this paper we assume that each origin-destination (O-D) pair, constituting the set of demands $d \in \mathscr{D}$, is associated with a specific portion of requested bandwidth h_d. By $\mathscr{P}_d$ we denote a set of candidate paths for demand d.

The IP layer links $e \in \mathscr{E}$ are supported by paths in the WDM layer. For each link $e \in \mathscr{E}$ we define a set of such candidate paths $\mathscr{P}_e$. Throughout the paper we assume that each WDM link can be subject to a failure, and only single link $f \in \mathscr{F}$ can fail at a time. Still, a specific realization of the IP links can induce complex failures of multiple links in the IP layer.

The set of failure states is denoted by $\mathscr{S}$. Each element of $\mathscr{S}$ corresponds to a failure of a specific link $f \in \mathscr{F}$, what is determined by values of constants ρ_{fs}. Value zero of ρ_{fs} indicates that the corresponding link f is affected, and cannot be used to transit traffic in failure state s; $\rho_{fs} = 1$, otherwise.

Objective in the considered problem is to find capacities of the WDM links of the minimal cost. The unit cost of link f capacity is given by ξ_f, while the total capacity of this link is determined by variable g_f. The total link flow is calculated as a sum of particular path flows realizing the related demands; they are denoted by z_{eq}. Due to assumed uniqueness property, each candidate path is associated with a binary variable u_{eq}. Thus, the flow associated with a path, selected to support link e ($u_{eq} = 1$), must carry entire flow of this link, determined by value of variable y_e. Binary variable w_{es} constitutes a link-failure incidence factor which determines if IP link e is affected by a failure of the associated WDM path. Subsequently, binary variables r_{dps} determine if IP path p is affected by failure s due to unavailability of at least one WDM links traversed by this path. Finally, we define x_{dps} as a variable representing the traffic flow assigned to IP path p in state s. An MIP formulation of the discussed problem reads:

$$\min \sum_{f\in\mathscr{F}} \xi_f g_f, \tag{1}$$

$$\text{st.} \quad \sum_{p\in\mathscr{P}_d} x_{dps} \geq h_d \qquad d \in \mathscr{D}, s \in \mathscr{S}, \tag{2}$$

$$x_{dps} \leq x_{dp\mathscr{O}} \qquad d \in \mathscr{D}, p \in \mathscr{P}_d, s \in \mathscr{S}, \tag{3}$$

$$x_{dps} \leq (1 - r_{dps})h_d \qquad d \in \mathscr{D}, p \in \mathscr{P}_d, s \in \mathscr{S}, \tag{4}$$

$$\sum_{d\in\mathscr{D}} \sum_{p\in\mathscr{P}_{ed}} x_{dp\mathscr{O}} \leq y_e \qquad e \in \mathscr{E}, \tag{5}$$

$$|\mathscr{E}_p| r_{dps} \geq \sum_{e\in\mathscr{E}_p} w_{es} \qquad d \in \mathscr{D}, p \in \mathscr{P}_d, s \in \mathscr{S}, \tag{6}$$

$$\sum_{q\in\mathscr{Q}_e} z_{eq} = y_e \qquad e \in \mathscr{E}, \tag{7}$$

$$\sum_{e\in\mathscr{E}} \sum_{q\in\mathscr{Q}_{fe}} z_{eq} \leq \rho_{fs} g_f \qquad f \in \mathscr{F}, s \in \mathscr{S}, \tag{8}$$

$$\sum_{q\in\mathscr{Q}_e} u_{eq} \leq 1 \qquad e \in \mathscr{E}, \tag{9}$$

$$z_{eq} \leq u_{eq} M \qquad e \in \mathscr{E}, q \in \mathscr{P}_e \tag{10}$$

$$\sum_{q\in\mathscr{P}_e} \sum_{f\in\mathscr{F}_q} \rho_{fs} u_{eq} \leq |\mathscr{F}| w_{es} \qquad e \in \mathscr{E}, s \in \mathscr{S}. \tag{11}$$

System of three constraints (2)–(4) assures that at least h_d amount of bandwidth survives in each situation $s \in \mathscr{S}$ (according to the principles of the path diversity protection model). Equivalently, this can be expressed by the following nonlinear inequality:

$$\sum_{p\in\mathscr{P}_d} r_{dps} x_{dp\mathscr{O}} \geq h_d \quad d \in \mathscr{D}, s \in \mathscr{S}. \tag{12}$$

Constraint (5) is a capacity constraint. Constraint (6) indicates if certain path p is affected by specific failure s ($r_{dps} = 1$), i.e., if at least one of the IP links supporting path p is failed. The appropriate values of variables w_{es} are induced by constraint (11) which forces $w_{es} = 1$ when at least one of the links of the path supporting link e fails in state s (i.e., $\rho_{fs} u_{eq} = 1$).

The presented mathematical model corresponds to a classical two-layer dimensioning problem where the capacities of the IP links (given by y_e, $e \in \mathscr{E}$) are supported by the WDM path flows (according to constraint (7)). Constraint (9) is used to assure the uniqueness property. Constraint (8) is a capacity constraint of the WDM links. The considered objective is minimizing the total capacity installation cost associated with these links.

Below, we present an optimization model which does not require the predefined lists of candidate paths $\mathscr{Q}_e$, $e \in \mathscr{E}$. Instead, the so called node-link [2] notation of multicommodity flow optimization is used [2]. The node-link notation implicitly take into account all possible paths. Let $f \in \delta^+(v)$ and $f \in \delta^-(v)$ be the sets of links outgoing from and incoming to node $v \in \mathscr{W}$, respectively, Δ_{ve} be a constant which is equal to 1 if v is the starting node of e, to -1 if v is terminating node of e, and to 0, otherwise, and variables z_{fe} denote the WDM flows associated with the paths supporting the IP links. Accordingly, we define u_{fe} as a binary variable determining if link f is contained in the path supporting e:

$$\min \sum_{f\in\mathscr{F}} \xi_f g_f, \tag{13}$$

st. (2) – (6),

$$\sum_{f\in\delta+(v)} z_{fe} - \sum_{f\in\delta-(v)} z_{fe} = \Delta_{ve} y_e, \qquad v \in \mathscr{V}, e \in \mathscr{E}, \tag{14}$$

$$\sum_{f\in\delta+(v)} u_{fe} \leq 1 \quad v \in \mathscr{V}, e \in \mathscr{E}, \tag{15}$$

$$z_{fe} \leq M u_{fe} \quad e \in \mathscr{E}, f \in \mathscr{F}, \tag{16}$$

$$\sum_{e\in\mathscr{E},} z_{fe} \leq g_f \quad f \in \mathscr{F}. \tag{17}$$

Constraint (14) expresses a flow conservation principle which is characteristic of the node-link notation. The uniqueness property is assured by integrality of u_{fe} and constraint (15). Due to (16) the flows of selected links are non-negative (M is maximal capacity of an IP link). Finally, inequality (17) constitutes a capacity constraint related to the WDM links.

Observe that values of vector $w = (w_{es} : e \in \mathscr{E}, s \in \mathscr{S})$ can be calculated as products of ρ_{fs} for the set of links $f \in \mathscr{F}$ determining the realization of specific link e.

3. Resolution Approach

To the best of our knowledge the problem related to the path diversity protection can be formulated only using the link-path notation (as far as linear constraints and continuous variables are considered). Formulation (2) is such a formulation using variables assigned to each of the candidate paths in the IP layer. Each of these variables represents a portion of the related demand. To effectively use this formulation one has to determine proper set of the candidate paths. Because the number of possible candidate paths grows exponentially with the number of nodes in the network graph, resolving a link-path formulation involving all candidate paths is inefficient. Thus, we develop a method which decomposes the resolving process into two subsequently invoked phases. The proposed method allows to identify the set of the necessary candidate paths in the IP layer using the column generation technique (see [2]), and to design the IP links realizations by resolving appropriate MIP.

The approach is based on the assumption that realizations of the IP links are known during the first phase. It means that values of variables r_{dps} are fixed and given. Thus, technique called column generation (see [2]) can be used to resolve the problem related to designing capacities of the IP links. The problem can be defined by the system of constraints (5), (12) and objective function (18):

$$\min \sum_{e \in \mathscr{E}} \zeta_e y_e, \tag{18}$$

where ζ_e is a specific capacity unit cost associated with current realization of the links in the lower layer, i.e., each ζ_e is calculated as a sum of ξ_f along paths supporting link e. The problem, referred to as master problem in the context of path generation, is denoted by $\mathbf{M}$.

In the column (path) generation algorithm [5] not all the columns of the constraints matrix are stored. Instead, only a subset of the variables (columns) that can be seen as an approximation (restriction) of the original problem is kept. The column generation algorithm iteratively modifies the subset of variables by introducing new variables in a way that improves the current optimal solution. At the end, the set contains all the variables (paths) necessary to construct the overall optimal solution which can use all possible paths in the graph.

Let $(\lambda_d^s)^*$ $(d \in \mathscr{D}, s \in \mathscr{S})$ be current optimal dual variables associated with constraint (12) and $\Lambda_d^* = \sum_{s \in \mathscr{S}} (\lambda_d^s)^*$ $(d \in \mathscr{D})$ be current optimal auxiliary dual variables. At each iteration we are interested in generating path p for which the reduced price $\sum_{e \in p} \xi_e + \sum_{s \in \bar{\mathscr{S}}_p} (\lambda_d^s)^* - \Lambda_d^*$ has the smallest and negative value, as we can expect this will improve the current optimal solution to the greatest possible extent.

The pricing problem stated above can be approached in a way described in [6]. The basic idea is to compute the dual length $\langle p \rangle = \sum_{e \in p} \xi_e + \sum_{s \in \bar{\mathscr{S}}_p} (\lambda_d^s)^*$ of each path p, skiping, however, the computations for many paths for which apply some domination rules as proposed in [6]. The set of all non-dominated paths can be generated by means of a label-setting algorithm for shortest-path problems with resource constraints (SPPRC) [7].

As an extension of the SPPRC algorithm, we have also introduced path length limitation – an important contribution to the reduction of the size of the set of non-dominated paths. The extension is based on the observation that excessively long paths are useless as they cannot improve the current solution, or the solutions they represent are known to be worse than some already known solutions. For example, a simple path length restriction may be expressed as follows: $\sum_{e \in p} \xi_e < \Lambda_d^*$. Also, knowledge of some path p' representing a feasible solution can help to tighten the path length restriction. In such a case we are only interested in finding a path p satisfying $\sum_{e \in p} \xi_e < \sum_{e \in p'} \xi_e + \sum_{s \in \bar{\mathscr{S}}_{p'}} (\lambda_d^s)^*$. Applying path length limitation results in significant reduction of the pricing time.

Let $y^* = (y_e : e \in \mathscr{E})$ be the vector of link capacities of the IP links, obtained as the optimal solution of $\mathbf{M}$. In the proposed approach y^* is an input for the second phase which adjusts the realizations of the IP links to better fit conditions of the IP layer. The resulting vector of the link-failure incidence factors $w = (w_{es} : e \in \mathscr{E}, s \in \mathscr{S})$ determines new values of $r = (r_{dps} : d \in \mathscr{D}, p \in \mathscr{P}_d, s \in \mathscr{S})$ for the next iteration of the procedure. The subproblem of the second phase, given by the system of constraints (14)–(17) and objective function (19), is denoted by $\mathbf{R}$:

$$\min \sum_{e \in \mathscr{E}} \sum_{f \in \mathscr{F}} u_{fe}. \tag{19}$$

The general idea of the proposed approach is presented by Algorithm 1. Note that $\mathbf{M}$ can be formulated equivalently

Algorithm 1: The decomposed iteractive procedure

Step 1:	For each link $e \in \mathscr{E}$ find the cheapest realization with respect to link costs ξ_f. Denote the obtained vector of upper link capacities by y^0, link realization by u^0, and link-failure incidence by r^0.
Step 2:	For fixed $r = r^0$ solve $\mathbf{M}$ using the path generation, and denote the obtained capacity vector by y^0.
Step 3:	Put $y^0 \equiv 0$. Solve $\mathbf{R}$ for fixed vector $y = y^0$, and denote the obtained link realization by u^0. For each $e \in \mathscr{E}$ calculate a vector of link-failure incidence vector r^0. If stopping criterion is not met, return to Step 1.

as a system of constraints (2)–(4) with objective function (18). Let $\lambda = (\lambda_{dps} : d \in \mathscr{D}, p \in \mathscr{P}_d, s \in \mathscr{S})$, $\beta = (\beta_{dps} : d \in \mathscr{D}, p \in \mathscr{P}_d, s \in \mathscr{S})$, and $\gamma = (\gamma_{dps} : d \in \mathscr{D}, p \in \mathscr{P}_d, s \in \mathscr{S})$ be the vectors of Lagrangean multipliers associated with constraints (2), (3), and (4), respectively. The dual corresponding to the considered formulation of **M** reads:

$$\max \sum_{d\in\mathscr{D}}\sum_{s\in\mathscr{S}} h_d\lambda_{ds} - \sum_{d\in\mathscr{D}}\sum_{p\in\mathscr{P}_d}\sum_{s\in\mathscr{S}}(1-r^*_{dps})h_d\beta_{dps}, \tag{20}$$

$$\text{st.} \sum_{s\in\mathscr{S}}\alpha_{dps} \le \sum_{e\in\mathscr{E}_p}\pi_e \quad d\in\mathscr{D},\ p\in\mathscr{P}_d, \tag{21}$$

$$\lambda_{ds} \le \alpha_{dps}+\beta_{dps} \quad d\in\mathscr{D},\ p\in\mathscr{P}_d,\ s\in\mathscr{S}, \tag{22}$$

$$\pi_e \le \xi_e \quad e\in\mathscr{E}. \tag{23}$$

Let $(\lambda^0, \alpha^0, \beta^0, \pi^0)$ be an optimal solution of problem (20)–(23). Consider path p and state s for which $r^*_{dps}=1$ and $\beta^0_{dps}>0$. Suppose that the value of r^*_{dps} is decreased to 0, and problem (3) is re-optimized. It can be shown that the value of λ_{dps} can be smaller than λ^0_{dps} due to new value of β_{dps} which is equal to zero. Thus, we conclude that it can be advantageous to set r_{dps} to zero in the next step of the procedure because we can potentially decrease the optimal value of the primal objective function, i.e., decrease the dimensioning cost.

Similarly, setting r^*_{dps} to zero for path p and state s for which $\alpha^0_{dps}>0$ can also lead to decrease of the optimal value of objective function (20). Due to these observations, in the following we consider a slightly different form of the objective function of **R**. We define $\mathscr{I}_a$ and $\mathscr{I}_b$ as sets of triplets (d,p,s) for which $\alpha_{dps}>0$ and $\beta_{dps}>0$, i.e., $\mathscr{I}_a=\{(d,p,s):\ \alpha_{dps}>0\}$ and $\mathscr{I}_b=\{(d,p,s):\ \beta_{dps}>0\}$,

$$\max \sum_{(d,p,s)\in\mathscr{I}_b}\beta^0_{dps}(1-r_{dps}) + \sum_{(d,p,s)\in\mathscr{I}_a}\alpha^0_{dps}(1-r_{dps}). \tag{24}$$

The modified **R** shall also involve an appropriate set of constraints (6) related to triples contained in $\mathscr{I}_a\cup\mathscr{I}_b$. In practical implementations of the discussed approach it can be advantageous to consider a combined objective function of **R**:

$$\min\ \varepsilon\Big(\sum_{e\in\mathscr{E}}\sum_{f\in\mathscr{F}}u_{fe}\Big)+(1-\varepsilon) \times\Big(\sum_{(d,p,s)\in\mathscr{I}_b}\beta^0_{dps}r_{dps}+\sum_{(d,p,s)\in\mathscr{I}_a}\alpha^0_{dps}r_{dps}\Big), \tag{25}$$

where ε is an optimization parameter.

Notice that for given link realizations **M** can be infeasible due to empty set of allowable candidate paths, i.e., at least one path cannot be affected in any failure state. Because of that, in the following, we consider specific inequalities which can be used to exclude the infeasible link realizations from the solution space of **R**. The basic form of the inequalities refers to a cut set in graph $\mathscr{H}(\mathscr{W},\mathscr{F})$. Let $\delta(\mathscr{W}')$ be a cut set associated with subset of nodes $\mathscr{W}'$:

$$\sum_{e\in\delta(\mathscr{W}')} w_{es}\ge 1 \quad s\in\mathscr{S}. \tag{26}$$

Inequality (26) assures that at least one IP link must be available for given cut set $\delta(\mathscr{W}')$ in $\mathscr{H}(\mathscr{W},\mathscr{F})$. In particular, (26) is valid for the set of links outgoing from one specific node, i.e., $\delta^+(v)$. Still, the number of potential cut sets grows exponentially with the number of nodes. Thus, we assume that only specific cut sets, related to one, two or three nodes could be examined in the practical implementations.

It may still appear that the feasible solution space of **M** is empty for a specific realization of the IP links, and we must exclude the current solution from the feasible solution space of **R**. For this purpose we use simple inequality which excludes binary vectors related to non-feasible link realizations. Let $\mathscr{U}_0$ and $\mathscr{U}_1$ be the sets of (e,f) pairs for which u_{ef} is equal to zero and one in the excluded realization, respectively. The discussed inequality reads:

$$\sum_{(e,f)\in\mathscr{U}_0}u_{ef}+\sum_{(e,f)\in\mathscr{U}_1}(1-u_{ef})\ge 1. \tag{27}$$

Inequalities above are introduced into the formulation of **R** each time **M** is infeasible and a cut set which assures feasibility of **M** cannot be identified.

4. Numerical Results

The main goal of our computational experiments was to assess the efficiency of the two phase algorithm considered in the paper. For this purpose we performed a series of tests which, first, could tell us how the parameter ε in function (25) influences the algorithm efficiency and, finally, how fast the value of the generated solutions improves during the method execution. Aiming at this we implemented Algorithm 1 supported by the linear solver of CPLEX 10.0 which was used to resolve problems **R** and **M**. The computations were conducted on a PC equipped with P4 Quad Core processor and 4 GB memory.

We used two network instances from Survivable Network Design Data Library: *pdh* (11 nodes, 34 links, 24 demands) and *newyork* (16 nodes, 49 links, 240 demands). The topologies of these networks defined the topologies of the lower layers. We assumed that the nodes of the lower layer were also the nodes of the upper layer, i.e., having both IP and WDM switching capabilities. The graph of the upper layer was assumed to be fully-connected.

Using the two example networks we investigated the influence of the value of parameter ε. For this purpose we run our two phase algorithm using different values of this parameter. In the computations we assumed every single link failure in the lower layer. The time limit was set to 2 hours.

In Tables 1 and 2 we present the values of the objective function (25) of the best solution found within the assumed time limit.

Table 1
Two phase algorithm: objective values for different values of ε for the *pdh* network

ε	$0.0-0.8$	0.85, 0.88	0.9	0.95	1.0
Objective	98622.7	93323.8	93217.6	93526.2	93632.4
Optimum	91937.5				

According to the results presented in Tables 1 and 2 we conclude that the objective function (25) used when resolving the lower layer problem strongly influences the efficiency of the two phase algorithm. It appeared that the approach based on weighting both components of (25) was the most efficient one. Neither dual based indicators, nor shortest path lengths when used as standalone ($\varepsilon = 1$ or $\varepsilon = 0$) could provide the solutions of the same quality. The optimum value of the objective function was computed as an optimal solution of the single layer path diversity design problem (taking into account every single link failure) with the network topology and the unit capacity link costs as in input networks *pdh* and *newyork*. Observe that this value of the objective function is also an optimal value of the two layer counterpart (with a full graph in the upper layer) only if all nodes of the lower layer are also the nodes in the upper layer – what is in fact true in our case. This value allowed to asses the quality of obtained solutions, which appeared to by only less than 1% worse than the optimum.

Table 2
Two phase algorithm: objective values for different values of ε for the *newyork* network

ε	$0.0-0.9$	0.93	0.98	0.99	1.0
Objective	26012.9	25472.7	25639.7	25831.2	25474.6
Optimum	24453.7				

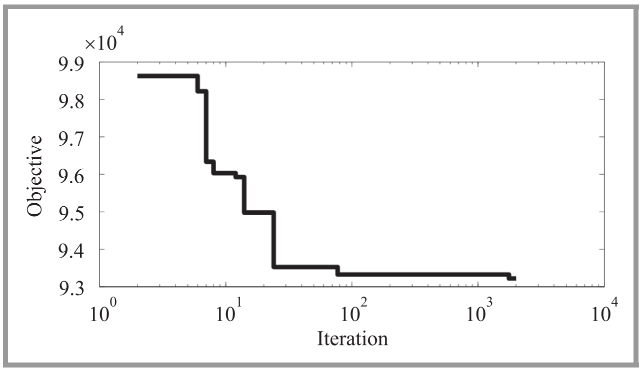

Fig. 1. Two phase algorithm: improvement of the objective function for *pdh* and $\varepsilon = 0.93$.

Analyzing Fig. 1, representing the algorithm convergence, we can conclude that for a proper value of ε the algorithm quickly finds a good quality solution which is only slightly improved until the algorithm terminates.

5. Concluding Remarks

In the paper we investigated an optimization problem related to dimensioning WDM links in a resilient two-layer IP-over-WDM network. In the considered problem the capacities of WDM links must be large enough to accommodate flows associated with selected realization of the IP links. Since the WDM links are subject to failures, we assumed that protection of traffic flows was assured by path diversity. In the paper we proposed a dedicated method for resolving this problem. The method was based on iterative resolving two subproblems, each related to optimizing flows in a distinct network layer. In our numerical experiments we tested the efficiency of the method for different settings of ε in objective function (25). The experiments revealed that neither dual based indicators, nor shortest path lengths when used as standalone could provide the solutions of the best quality.

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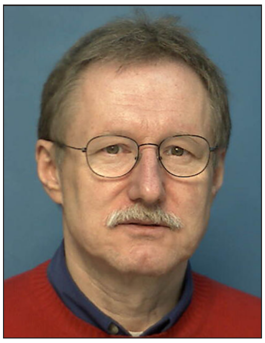

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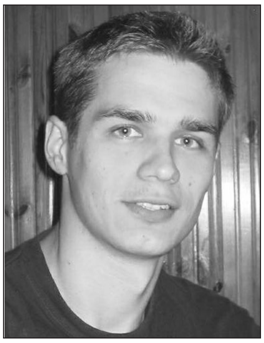

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