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Transmitter diversity antenna selection techniques for wireless channels utilizing differential space-time block codes

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Abstract— The paper deals with transmitter diversity antenna selection techniques (ASTs) for wireless channels utilizing differential space-time block codes (DSTBCs). The proposed ASTs tend to maximize the signal-to-noise ratio (SNR) of those channels. Particularly, we propose here the so-called *general* $(M,N;K)$ AST/DSTBC scheme for such channels. Then, based on this AST, we propose two modified ASTs which are more amenable to practical implementation, namely the *restricted* $(M,N;K)$ AST/DSTBC scheme and the $(N+\bar{N},N;K)$ AST/DSTBC scheme. The *restricted* $(M,N;K)$ AST/DSTBC scheme provides relatively good bit error performance using only one feedback bit for transmission diversity purpose, while the $(N+\bar{N},N;K)$ AST/DSTBC scheme shortens the time required to process feedback information. These techniques remarkably improve bit error rate (BER) performance of wireless channels using DSTBCs with a limited number (typically 1 or 2) of training symbols per each coherent duration of the channel. Simulations show that the proposed AST/DSTBC schemes outperform the DSTBCs without antenna selection even with only 1 training symbol.

Keywords—differential space-time modulation, differential space-time block codes, diversity antenna selection, MIMO.

1. Introduction

The diversity combination of space-time codes (STCs) and a closed loop antenna selection technique (AST) assisted by a feedback channel to improve the performance of wireless channels in multiple input multiple output (MIMO) systems has been intensively examined in literature for the case of *coherent detection*, such as [5–10]. However, ASTs for channels utilizing differential space-time block codes (DSTBCs) with *differential detection* have not been considered yet. The backgrounds on DSTBCs can be found in [11–17].

In this paper[1], we propose some ASTs which tend to maximize the signal-to-noise ratio (SNR) for the channels using DSTBCs with arbitrary number M of transmit antennas (MTx antennas) and with K receive antennas (KRx antennas). Particularly, we first propose an AST called the *general* $(M,N;K)$ AST/DSTBC where the transmitter selects NTx antennas out of MTx antennas ($M > N$) to maximize the channel SNR. The antenna selection (at the transmitter) is based on the results of the comparison carried out (at the receiver) between the instantaneous powers of signals which are received during the initial transmission. The *general* $(M,N;K)$ AST/DSTBC significantly improves the performance of channels using DSTBCs. However, when M and N grow large, the number of feedback bits required to inform the transmitter also grows large. This drawback impedes the *general* $(M,N;K)$ AST/DSTBC from practical implementation if M and N are large.

The aforementioned drawback can be overcome by either reducing the number of feedback bits or shortening the time required to process feedback information. Based on these observations, we modify the *general* $(M,N;K)$ AST/DSTBC and derive the two following ASTs which are more amenable to practical implementation.

First, we propose the so-called *restricted* $(M,N;K)$ AST/DSTBC, which provides good bit error performance using only 1 feedback bit for transmission diversity purpose.

Then, we describe the so-called $(N+\bar{N},N;K)$ AST/DSTBC which shortens the average time required to process feedback information in comparison with the *general* $(M,N;K)$ AST/DSTBC, where $M = N+\bar{N}$. This AST is first motivated by the $(N+1,N;K)$ AST/STBC which we mentioned in [1] for channels using space-time block codes (STBCs) with *coherent detection*. The background on STBCs can be found in [18–21].

We show that DSTBCs associated with the proposed ASTs provide much better bit error performance than that without antenna selection. The proposed ASTs in this paper are the generalization of our ASTs published in [2, 3]. The content of this paper is also somewhat related to our published papers [1, 4].

Although, the authors propose here the ASTs for a very general case, where the system contains arbitrary numbers of Tx and Rx antennas, it is important having in mind that it is more practical to have diversity antennas installed at the transmitter, e.g., a base station in mobile communication systems, rather than at the hand-held, tiny receiver, such as a mobile phone. It is well known that the installation of more than 2Tx antennas in mobile phones is almost impractical due to the battery life-time and the small size of the phones.

Consequently, by using the term *antenna selection* in this paper, we mean *transmitter diversity* antenna selection, rather than receiver diversity antenna selection, i.e., all KRx antennas are used without selection (although the generalization of the proposed ASTs to receiver diversity antenna selection is straightforward). It should be also

[1] Related to the content in this paper are the published works [1–4].

noted that the term *differential space-time block codes* (DSTBCs) used throughout this paper means *complex, orthogonal* DSTBCs.
This paper is organized as follows.
Section 2 reviews the conventional DSTBCs mentioned in literature and provides some remarks on the time-varying Rayleigh fading channels where DSTBCs can be practically used. In Section 3, we mention some notations and assumptions used throughout this paper. Section 4 starts with the discussion on the criterion of antenna selection in channels using STBCs and then analyzes our modifications to apply to channels using DSTBCs. In Section 5, we propose the *general* $(M,N;K)$ AST/DSTBC. In Section 6.1, we propose the *restricted* $(M,N;K)$ AST/DSTBC. The $(N+\bar{N},N;K)$ AST/DSTBC is proposed in Section 6.2. Section 7 provides the mathematical expression of the relative time reduction gained by the $(N+\bar{N},N;K)$ AST/DSTBC in comparison with the *general* $(M,N;K)$ AST/DSTBC. In Section 8, we give some comments on the spatial diversity order of our proposed ASTs. Simulation results are presented in Section 9 and the paper is concluded by Section 10.

2. Reviews on DSTBCs

In this section, we review the conventional DSTBCs mentioned in literature and provide some remarks on the time-varying Rayleigh fading channels where DSTBCs can be practically used. This section is indispensable in order for the readers to understand what has been modified in the transmission procedures of DSTBCs in our proposed ASTs. It is also vital for the readers to notice the underlying requirement of all conventional DSTBCs that the channel coefficients must be constant during at least two consecutive code blocks. We also show here in which scenarios DSTBCs (differential detection) should be used instead of STBCs (coherent detection).

2.1. Conventional DSTBCs without diversity antenna selection

Differential space-time block codes are the candidate for the channels where fading changes so fast that the transmission of the training signals (eg., a large overhead) is either impractical or uneconomical. DSTBCs have been considered intensively and a number of DSTBCs have been proposed in literature such as [11–17]. In [2, 3], we have proved that all conventional DSTBCs (without antenna selection) provide a full spatial diversity order.
Let us consider the unitary DSTBC proposed by Ganesan *et al.* in [13] as an example. We consider a system with NTx antennas and KRx antennas. Let $\mathbf{R}_t$, $\mathbf{A}$, $\mathbf{N}_t$ be the $(K\times N)$-sized matrices of received signals at time t, channel coefficients between Rx and Tx antennas, and noise at the Rx antennas, respectively. The $\kappa\eta$th element of $\mathbf{A}$, namely $a_{\kappa\eta}$, is the channel coefficient of the path between the ηth Tx antenna and the κth Rx antenna. Channel coefficients are assumed to be identically independently distributed (i.i.d.) complex, zero-mean Gaussian random variables. Noises are assumed to be i.i.d. complex Gaussian random variables with the distribution $\mathcal{CN}(0,\sigma^2)$.
Let $\{s_j\}_{j=1}^p=\{s_j^R+is_j^I\}_{j=1}^p$ (where i^2=−1, s_j^R and s_j^I are the real and imaginary parts of s_j, respectively) be the set of p symbols, which are derived from a unitary power signal constellation S and transmitted in the tth block. Consequently, each symbol has a unitary energy, i.e., $|s_j|^2=1$.
We define a matrix $\mathbf{Z}_t=\frac{1}{\sqrt{p}}\sum_{j=1}^p(\mathbf{X}_j s_j^R+i\mathbf{Y}_j s_j^I)$, where the square, order-N weighting matrices $\{\mathbf{X}_j\}_{j=1}^p$ and $\{\mathbf{Y}_j\}_{j=1}^p$ are orthogonal themselves and they satisfy the permutation property. These weighting matrices are considered as the amicable orthogonal designs (AODs). The backgrounds on AODs can be found in [22]. The coefficient $\frac{1}{\sqrt{p}}$ is to guarantee that $\mathbf{Z}_t$ is a unitary matrix, i.e., $\mathbf{Z}_t\mathbf{Z}_t^H=\mathbf{I}$.
For illustration, the Alamouti DSTBC corresponding to $N=2$ is defined as

$$\mathbf{Z}_t=\frac{1}{\sqrt{2}}\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}. \tag{1}$$

A DSTBC corresponding to $N=4$ is given below:

$$\mathbf{Z}_t=\frac{1}{\sqrt{3}}\begin{bmatrix} s_1 & s_2 & s_3 & 0 \\ -s_2^* & s_1^* & 0 & s_3 \\ -s_3^* & 0 & s_1^* & -s_2 \\ 0 & -s_3^* & s_2^* & s_1 \end{bmatrix}. \tag{2}$$

The transmission starts with an initial, identity, order-N matrix $\mathbf{W}_0=\mathbf{I}_N$ carrying no information. The matrix transmitted at time t $(t=1,2,3\ldots)$ is given by

$$\mathbf{W}_t=\mathbf{W}_{t-1}\mathbf{Z}_t\,. \tag{3}$$

As $\mathbf{Z}_t$ is a unitary matrix, the matrix $\mathbf{W}_t$ is also a unitary one. The model of the channel at time t, for $t=0,1,2\ldots$, ($t=0$ means the transmission of the first block $\mathbf{W}_0$, i.e., the initial transmission) is:

$$\mathbf{R}_t=\mathbf{A}\mathbf{W}_t+\mathbf{N}_t\,. \tag{4}$$

In all propositions of conventional DSTBCs, the channel coefficients must be constant during *at least* two adjacent code blocks[2], i.e., constant during *at least* $2N$ symbol time slots (STSs). It means that if the channel coefficient matrix $\mathbf{A}$ is assumed to be constant over two consecutive blocks $t-1$ and t, the maximum likelihood (ML) detector for the symbols $\{s_j\}_{j=1}^p$ is calculated as follows [13, 23]:

$$\{\hat{s}_j\}_{j=1}^p=\operatorname{Arg}\left\{\max_{\{s_j\},s_j\in S}\operatorname{Re}\{\operatorname{tr}(\mathbf{R}_t^H\mathbf{R}_{t-1}\mathbf{Z}_t)\}\right\}, \tag{5}$$

where Arg{.} denotes the argument operation, tr(.) denotes the trace operation, Re{.} and Im{.} denote the real and the imaginary parts of the argument, respectively.
If we denote T_c to be the average coherent time of the channel which represents the time-varying nature of the channel, then the channel is considered to be constant during

[2] This means that the channel coefficients are constant during each window of at least two consecutive code blocks and windows do not overlap each other.

this time. Therefore, after each duration T_c, the transmitter restarts the transmission and transmits a new initial block $\mathbf{W}_0$ followed by other code blocks $\mathbf{W}_t$ $(t = 1,2,3\ldots)$. These procedures are repeated until all data are transmitted.

Due to the orthogonality of DSTBCs, the transmitted symbols are decoded separately, rather than jointly. Therefore, if we denote:

$$D_j = \mathrm{Re}\big\{\mathrm{tr}(\mathbf{R}_t^H \mathbf{R}_{t-1}\mathbf{X}_j)\big\} + i\mathrm{Re}\big\{\mathrm{tr}(\mathbf{R}_t^H \mathbf{R}_{t-1} i\mathbf{Y}_j)\big\} \quad (6)$$

then the ML detector for the symbol s_j is [2, 3]:

$$\hat{s}_j = \mathrm{Arg}\big\{\max_{s_j \in S} \mathrm{Re}\{D_j^* s_j\}\big\}, \quad (7)$$

where D_j^* is the conjugate of D_j.

Expressions (6) and (7) show that the detection of the symbol s_j is carried out without the knowledge of channel coefficients. Particularly, the symbol s_j can be decoded by using the received signal blocks in the two consecutive transmission times, *provided that the channel coefficients are constant during two consecutive code blocks* (otherwise, we will not have the decoding expressions (5) and (7)).

The requirement that the channel coefficients must be constant during at least 2 consecutive code blocks can be relaxed if the linear prediction is used at the receiver. In this scenario, the receiver uses multiple previous received code blocks $\mathbf{R}_{t-1}, \mathbf{R}_{t-2}$, etc., to predict the relation between the current channel coefficient matrix, say $\mathbf{A}_t$, and the previous channel coefficient matrices. This approach has been mentioned in [24]. Certainly, the penalty of this approach is the complexity of the receiver structure.

It has been proved in our paper [2, 3] that all conventional DSTBCs (without ASTs) provide a full diversity of order NK, where N and K are the number of Tx and Rx antennas, respectively. We also can realize this observation in Section 4 of this paper.

2.2. Remarks on the time-varying Rayleigh fading channels

According to the frequency of channel coefficient changes, we distinguish three typical scenarios which are usually examined in practice and present the most common, real propagation conditions (see [17, p. 13] and [25, p. 2]).

1. Channel coefficient matrix $\mathbf{A}$ is random and its entries change randomly at the beginning of each symbol time slot (STS) and are constant during one STS. This scenario is referred to as the *fast* Rayleigh flat fading channel.
2. $\mathbf{A}$ is random and its entries change randomly after a duration containing a number of STSs. This scenario is referred to as the *block* Rayleigh flat fading channel. The example of this scenario will be mentioned later.
3. $\mathbf{A}$ is random but is selected at the beginning of transmission and its entries keep constant all the time. This scenario is referred to as the *slow* or *quasi-static* Rayleigh flat fading channel. Local area networks (LANs) or wide local area networks (WLANs) with a slow fading rate and a high data rate are the examples of the *quasi-static* Rayleigh flat fading channels, where the channel coefficients may be constant during thousands of STSs.

Given the above clarifications, we have the following important note. Owing to the condition that channel coefficients must be *constant during, at least, two consecutive code blocks*, in all conventional DSTBCs mentioned in literature, the channels are considered as block fading channels, although the coherent time of the channels in the case of DSTBCs (with differential detection) are much shorter than that in the case of STBCs (with coherent detection).

To illustrate, for the case of the Alamouti DSTBC, the channel coefficients must be constant during at least 4 STSs. During the first two STSs, the initial, order-2, identity matrix $\mathbf{I}_2$ which carries no information is transmitted. During the next two STSs, the Alamouti code carrying 2 symbols is transmitted. This note clarifies how fast fading channels may change when DSTBCs is utilized. Certainly, a longer coherent duration of the channel results in a more efficient utilization of DSTBCs.

We give 2 examples of block Rayleigh fading channels where coherent STBCs or DSTBCs can be used.

Example 1: We consider the scenario where the Alamouti STBCs with coherent detection can be used for the cellular mobile system with the carrier frequency $F_c = 900$ MHz. Speed of the mobile user is $v = 5$ km/h (walking speed) and the STS is assumed to be $T_s = 0.125$ ms (equivalently, the baud rate is $F_s = 8$ KBd/s. Denote $\mathrm{c} = 3.10^8$ m/s to be light speed. The maximum Doppler frequency is then calculated as

$$f_m = vF_c/\mathrm{c} = 4.17 \text{ Hz}.$$

The average coherent time T_c of the channel is estimated by the following empirical expression [26, p. 204]:

$$T_c = \frac{0.423}{f_m} = 101.52 \text{ ms}.$$

It means that the channel coefficients can be considered to be constant during almost $T_c/T_s \approx 812$ consecutive STSs, i.e., approximately 406 consecutive Alamouti code blocks. In this case, the channel coefficients change so slow that the training signals can be transmitted. In other words, STBCs with coherent detection are preferred than DSTBCs with differential detection.

Example 2: We consider another scenario where the Alamouti DSTBCs with different detection can be used for the cellular mobile system with the carrier frequency $F_c = 900$ MHz. Speed of the mobile user is $v = 60$ km/h (vehicular speed) and the STS is assumed to be $T_s = 0.5$ ms corresponding to the baud rate $F_s = 2$ KBd/s. The maximum Doppler frequency is then calculated as

$$f_m = vF_c/\mathrm{c} = 50 \text{ Hz}.$$

Similarly, the average coherent time T_c of the channel is estimated as [26, p. 204]

$$T_c = \frac{0.423}{f_m} = 8.46 \text{ ms} .$$

It means that the channel coefficients can be considered to be constant during $T_c/T_s \approx 16$ consecutive STSs, i.e., 8 consecutive Alamouti code blocks. The channel is a block Rayleigh fading one where DSTBCs can be employed. In this case, it is either impractical or uneconomical to use STBCs with coherent detection since the coherent time is too short to transmit multiple training symbols in order for the receiver to estimate the channel coefficients.

3. Definitions, notations and assumptions

For ease of exposition, we define some notations as follows.

Definition 1: $\mathcal{F}$ is defined as an order-N operation on M non-negative, real numbers $\{\varepsilon_1, \ldots, \varepsilon_M\}$ where the N indices ($N < M$) corresponding to the N largest values out of M values $\{\varepsilon_1, \ldots, \varepsilon_M\}$ are selected. We denote this operation as $\mathcal{F}_N(\varepsilon_1, \ldots, \varepsilon_M)$. The output of the operation $\mathcal{F}$ is the set of N indices which is denoted by $\hat{\mathcal{I}}_N$.

Example 3: $M = 3$, $N = 2$, $\varepsilon_1 = 10$, $\varepsilon_2 = 20$ and $\varepsilon_3 = 30$. We have:

$$\hat{\mathcal{I}}_2 = \mathcal{F}_2(\varepsilon_1, \varepsilon_2, \varepsilon_3) = \{2, 3\} .$$

The elements of the set $\hat{\mathcal{I}}_2$ are the indices of ε_2 and ε_3, which are in turns the 2 largest values among $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$.

Definition 2: We define the $(M, N; K, L)$ AST/DSTBC scheme to be the transmitter and receiver diversity antenna selection technique for channels using DSTBCs with differential detection where NTx antennas are selected out of MTx antennas ($N < M$), while LTx antennas are selected out of KRx antennas ($L < K$) for transmission.

Given that notation, the $(M,N;K)$ AST/DSTBC scheme refers to as the *transmitter* diversity antenna selection technique for channels using DSTBCs with differential detection where NTx antennas are selected out of MTx antennas ($N < M$) for transmission. All KRx antennas are used without selection. Similarly, the $(M;K,L)$ AST/DSTBC scheme refers to as the *receiver* diversity AST for channels using DSTBCs where LTx antennas are selected out of KRx antennas for transmission, while MTx antennas are used without selection.

In the paper, we mainly focus on the transmitter diversity AST, i.e., the $(M,N;K)$ AST/DSTBC schemes. We sometimes compare the proposed $(M,N;K)$ AST/DSTBC schemes with the respective schemes in channels which use STBCs with coherent detection. Hence, similarly, we use the notation $(M,N;K)$ AST/STBC to refer to the transmitter diversity AST for channels using STBCs with coherent detection.

For example, if $M = 4$, $N = 2$ and $K = 1$, then the (4,2;1) AST/DSTBC is the AST where the 2Tx antennas are selected (depending on certain criteria) from 4Tx antennas for transmission, while the receiver has 1Rx antenna.

Some assumptions considered in the paper are given below.

Assumption 1: The channel coefficients between the transmitter and receiver antennas are assumed to be i.i.d. complex, zero-mean Gaussian random variables. Noises are assumed to be i.i.d. complex Gaussian random variables with the distribution $\mathcal{CN}(0, \sigma^2)$. These assumptions are applicable when the Tx and Rx antennas are sufficiently separated from one another (by a multiple of half of the wavelength) so that the Tx (and Rx) antennas are uncorrelated. The scenario where the antennas are correlated will be examined in our future works.

Assumption 2: Although channels with differential detection change faster than those with coherent detection, so that the transmission of multiple training signals is uneconomical (and, consequently, the utilization of DSTBCs is useful), we make a reasonable assumption that it is possible to transmit a few feedback bits (for each channel coherent duration T_c) from the receiver to the transmitter via a feedback channel *with a certain feedback error rate*. The feedback error rate is typically assumed to be 4% to 10%.

Finally, we want to stress the following important remarks.

Remark 1: Due to the tiny size of the receivers, such as the hand-held mobile phones in the cellular mobile systems, it is well known that employment of more than 2Tx antennas at the receiver is uneconomical. Hence, the receiver diversity antenna selection is not considered in this paper, although the generalization of the proposed techniques for the receiver diversity antenna selection is straightforward.

Remark 2: We use the modified notation $(N + \bar{N}, N; K)$ AST/DSTBC, rather than $(M,N;K)$ AST/DSTBC, where $M = N + \bar{N}$, to refer to our 3th proposed AST/DSTBC scheme in this paper. The main purpose of using this notation is to stress that $\bar{N}$Tx antennas among $(N + \bar{N})$ available Tx antennas are the standby Tx antennas. These standby Tx antennas are only used in certain conditions stipulated by the selection criteria. Those selection criteria will be mentioned in more details later.

4. Basis of transmitter antenna selection for channels using DSTBCs

In our papers [2, 3], we have proved that all conventional DSTBCs mentioned in literature, such as [13–16], provide a full spatial diversity order. This means that, if the channel contains NTx and KRx antennas, then square, order-N DSTBCs provide a full spatial diversity of order NK provided that the DSTBCs have a full rank.

Let us consider the unitary DSTBCs mentioned in Section 2.1 for instance. It is shown in [2, Eq. (11)], [3, Eq. (9)],

[12] and [23, Eq. (5.30)], that the SNR of the statistic D_j in Eq. (6) is approximately:

$$SNR_{diff} \approx \frac{\|\mathbf{A}\|_F^2}{2p\sigma^2} = \frac{\mathrm{tr}(\mathbf{A}^H\mathbf{A})}{2p\sigma^2} = \frac{\sum_{\eta=1}^{N}\left[\sum_{\kappa=1}^{K}|a_{\kappa\eta}|^2\right]}{2p\sigma^2}, \quad (8)$$

where $\|\mathbf{A}\|_F$ is the Frobenius norm of the matrix $\mathbf{A}$. Clearly, SNR has $2NK$ freedom degrees. As a result, the unitary DSTBC considered provides a full spatial diversity of order NK.

Let $\xi_\eta \equiv \sum_{\kappa=1}^{K}|a_{\kappa\eta}|^2$ $(\eta = 1,\ldots,N)$ be the total power of signals received by KRx antennas during each STS. We can rewrite SNR_{diff} as follows:

$$SNR_{diff} \approx \frac{\sum_{\eta=1}^{N}\xi_\eta}{2p\sigma^2}. \quad (9)$$

It is obvious that greater values of ξ_ηs result in a greater SNR_{diff}.

Let us consider a system comprising MTx antennas $(M > N)$ and KRx antennas. We now want to select the N best Tx antennas out of MTx antennas so that SNR_{diff} is maximized. From Eqs. (8) or (9), to maximize SNR_{diff}, we need to maximize $\|\mathbf{A}\|_F^2$. Equivalently, the N *first maximum values* out of M values $\{\xi_1,\xi_2,\ldots,\xi_M\} = \{\sum_{\kappa=1}^{K}|a_{\kappa1}|^2, \sum_{\kappa=1}^{K}|a_{\kappa2}|^2,\ldots,\sum_{\kappa=1}^{K}|a_{\kappa M}|^2\}$ must be selected. In other words, the indices of the N best Tx antennas are selected by the following antenna selection criterion:

$$\hat{\Im}_N = \digamma_N\Big(\xi_1,\ldots,\xi_M\Big) = \digamma_N\left(\sum_{\kappa=1}^{K}|a_{\kappa1}|^2, \sum_{\kappa=1}^{K}|a_{\kappa2}|^2,\ldots,\sum_{\kappa=1}^{K}|a_{\kappa M}|^2\right). \quad (10)$$

Again, note that the transmitter diversity antenna selection, rather than receiver diversity antenna selection, is examined in this paper. All KRx antennas are used without antenna selection.

The selection criterion in Eq. (10) is applicable only when the channel coefficients are perfectly known at the receiver. This scenario is realistic when the channel changes so slowly that the multiple training signals can be transmitted. This scenario is commonly examined in channels using STBCs with coherent detection. The ASTs are referred to as the $(M,N;K)$ AST/STBC schemes which have been intensively considered in literature [5–10].

As oppose to coherent detection, in channels using DSTBCs with differential detection, channel coefficients change faster so that the transmission of multiple training signals is either impractical or uneconomical, and consequently, the channel coefficients are unknown at the receiver.

Therefore, the antenna selection criterion in Eq. (10) cannot be directly applied to channels using DSTBCs with differential detection. However, we will show that this criterion can be modified to apply to channels using DSTBCs with differential detection.

Particularly, we will prove later in this paper that, at high SNRs, the statistical properties, i.e., means and variances, of the received signals $r_{0\kappa\eta}$s – the elements of the matrix $\mathbf{R}_0$ received during the initial transmission – are similar to those of the channel coefficients $a_{\kappa\eta}$s. As a result, at high SNRs, maximizing $\|\mathbf{R}_0\|_F^2$ tends to be the same as maximizing $\|\mathbf{A}\|_F^2$.

Based on this observation, we propose the modified antenna selection scheme for channels using DSTBCs. The transmitter selects Tx antennas on the basis of the comparison, which is carried out once per each channel coherent duration T_c at the receiver, between the power of the signals which are received by all KRx antennas during the initial transmission (the first block $\mathbf{W}_0$).

If we denote $\hat{\Im}_N$ to be the set of the N indices of the Tx antennas which should be selected, then the modified antenna selection criterion for channels using DSTBCs is:

$$\hat{\Im}_N = \digamma_N\Big(\chi_1,\ldots,\chi_M\Big) = \digamma_N\left(\sum_{\kappa=1}^{K}|r_{0\kappa1}|^2, \sum_{\kappa=1}^{K}|r_{0\kappa2}|^2,\ldots,\sum_{\kappa=1}^{K}|r_{0\kappa M}|^2\right).$$

This modified selection criterion is mentioned in more details in the so-called *general* $(M,N;K)$ AST/DSTBC scheme proposed as below.

5. The general $(M,N;K)$ AST/DSTBC for channels utilizing DSTBCs

In this section, we generalize our AST/DSTBC proposed in [2, 3] for channels using DSTBCs with arbitrary numbers of Tx and Rx antennas.

Let us consider a system containing MTx antennas and KRx antennas using the unitary, square, order-N DSTBCs $(N < M)$ proposed by Ganesan *et al.* [13, 27]. Note that the proposed ASTs are also applicable to any conventional DSTBC regardless of being unitary or not.

In the following analysis, the normal, lower case letters denote scalars, the bold, lower case letters denote vectors, while the bold upper case letters denote matrices. For simplicity, we omit the superscripts indicating the different coherent durations T_cs of the channel when a certain coherent duration is being considered. The superscripts are only used when we consider different coherent durations T_cs simultaneously.

The *general* $(M,N;K)$ AST/DSTBC is proposed as follows:

- At the beginning of transmission, the transmitter sends an initial block $\tilde{\mathbf{W}}_0 = \mathbf{I}_M$ via MTx antennas, rather than sending an initial block $\mathbf{W}_0 = \mathbf{I}_N$ via

NTx antennas like in all conventional DSTBCs. This transmission is referred to as the initial transmission.

We note the change in the size of matrices compared to Eq. (4) by using the tilde mark for matrices as below:

$$\begin{aligned}\tilde{\mathbf{W}}_0 &= \mathbf{I}_M ,\\ \tilde{\mathbf{A}} &= \left[\, \mathbf{a_1}\ \mathbf{a_2}\ \ldots\ \mathbf{a_M} \,\right] ,\\ \tilde{\mathbf{N}}_0 &= \left[\, \mathbf{n_{01}}\ \mathbf{n_{02}}\ \ldots\ \mathbf{n_{0M}} \,\right] ,\end{aligned}$$

where $\mathbf{a_j}$ $(j = 1 \ldots M)$ is the column vector of the channel coefficients a_{ij} $(i = 1 \ldots K)$ corresponding to the channel from the jth Tx antenna to the ith Rx antenna, i.e, $\mathbf{a_j}$=$[a_{1j},\ldots,a_{Kj}]^T$, and $\mathbf{n_{0j}}$ is the noise affecting these channels during the initial transmission, i.e., $\mathbf{n_{0j}} = [n_{01j},\ldots,n_{0Kj}]^T$. Here, the superscript T denotes the transposition operation.

- The receiver determines the matrix $\tilde{\mathbf{R}}_0$ of received signals during the initial transmission as given below:

$$\begin{aligned}\tilde{\mathbf{R}}_0 &= \tilde{\mathbf{A}}\tilde{\mathbf{W}}_0 + \tilde{\mathbf{N}}_0\\ &= \tilde{\mathbf{A}}\mathbf{I}_M + \tilde{\mathbf{N}}_0\\ &= \left[\, \mathbf{r_{01}}\ \mathbf{r_{02}}\ \ldots\ \mathbf{r_{0M}} \,\right] \qquad (11)\\ &= \left[\, \mathbf{a_1}+\mathbf{n_{01}}\ \ \mathbf{a_2}+\mathbf{n_{02}}\ \ldots\ \mathbf{a_M}+\mathbf{n_{0M}} \,\right] ,\end{aligned}$$

where

$$\begin{aligned}\mathbf{r_{0j}} &= \mathbf{a_j} + \mathbf{n_{0j}}\\ &= [a_{1j}+n_{01j},\ldots,a_{Kj}+n_{0Kj}]^T \quad j = 1\ldots M .\end{aligned}$$

- From the initial received matrix $\tilde{\mathbf{R}}_0$, the receiver determines semiblindly the N best channels based on the initial, received matrix $\tilde{R}_0$ by comparing M terms χ_j=$\|\mathbf{r_{0j}}\|_F^2$, for $j = 1\ldots M$, i.e., comparing the total power of the signals received by all KRx antennas from the jth Tx antenna during the jth STS:

$$\chi_j = \sum_{i=1}^{K} |r_{0ij}|^2 = \sum_{i=1}^{K} |a_{ij} + n_{0ij}|^2 \qquad (12)$$

to search for the first N maximum values. In other words, the antenna selection criterion is:

$$\begin{aligned}\hat{\mathfrak{I}}_N &= \mathcal{F}_N(\chi_1,\ldots,\chi_M)\\ &= \mathcal{F}_N\left(\sum_{i=1}^{K}|r_{0i1}|^2, \sum_{i=1}^{K}|r_{0i2}|^2,\ldots,\sum_{i=1}^{K}|r_{0iM}|^2\right)\\ &= \mathcal{F}_N\left(\sum_{i=1}^{K}|a_{i1}+n_{0i1}|^2, \sum_{i=1}^{K}|a_{i2}+n_{0i2}|^2,\ldots,\right.\\ &\qquad \left.\sum_{i=1}^{K}|a_{iM}+n_{0iM}|^2\right) , \qquad (13)\end{aligned}$$

where $\hat{\mathfrak{I}}_N$ denotes the set of N indices of the Tx antennas which should be selected.

Without loss of generality, we assume here that these maximum values are corresponding to the first N elements in the matrix $\tilde{\mathbf{R}}_0$, i.e.,

$$\hat{\mathfrak{I}}_N = \{1,2,\ldots,N\} .$$

Then, the receiver carries out the two following tasks:

1. The receiver informs the transmitter via a feedback channel to select the first NTx antennas to transmit data.
2. The receiver generates the matrix $\mathbf{R}_0$, which is used to decode the next code blocks, by taking the first N elements of the matrix $\tilde{\mathbf{R}}_0$, corresponding to the first N maximum values, i.e., $\mathbf{R}_0$=$\left[\, \mathbf{a_1}+\mathbf{n_{01}}\ \ \mathbf{a_2}+\mathbf{n_{02}}\ \ldots\ \mathbf{a_N}+\mathbf{n_{0N}} \,\right]$.

- The transmitter selects the NTx antennas indicated by the feedback information. In this case, the first NTx antennas are selected to transmit data. The transmission is now exactly the same as that in the system using the N first Tx antennas only.

If T_c is the average coherent time of the channel, then after each duration T_c, the transmitter restarts the transmission and transmits a new initial block $\tilde{\mathbf{W}}_0$ followed by other code blocks $\mathbf{W}_t$ $(t = 1,2,3\ldots)$. The above procedures are repeated until all data are transmitted.
The transmission procedure is shown in Fig. 1. The superscripts are used to indicate the different coherent durations T_cs of the channel. The code blocks $\tilde{\mathbf{W}}_0$ are transmitted via MTx antennas in M STSs and the following blocks via NTx antennas in N STSs.

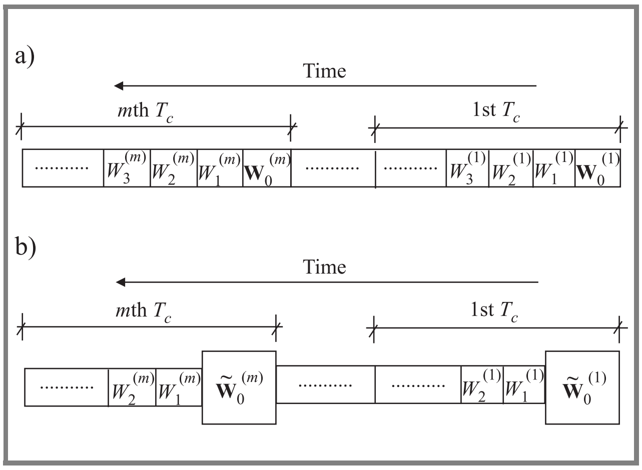

Fig. 1. Transmission of DSTBCs without (a) and with (b) the antenna selection technique.

From the aforementioned algorithm, we have following remarks.

Remark 3: At the transmitter, after the initial matrix $\tilde{\mathbf{W}}_0$ is transmitted, the next matrices $\mathbf{W}_t$ $(t = 1,2,3,\ldots)$ can be calculated by using a tacit default matrix $\mathbf{W}_0 = \mathbf{I}_N$ in Eq. (3). We use the term *tacit default matrix* to refer to the fact that the matrix $\mathbf{W}_0 = \mathbf{I}_N$ is tacitly used at the transmitter to generate the next code blocks $\mathbf{W}_t$ by Eq. (3), rather than being actually transmitted. Owing to this fact, it is also important to note that the generation of the matrices $\mathbf{W}_t$ does not necessarily take place after the transmitter obtains the feedback information. Instead, the next code blocks $\mathbf{W}_t$ are automatically generated by multiplying the previous block $\mathbf{W}_{t-1}$ with the tacit default matrix $\mathbf{W}_0 = \mathbf{I}_N$ following Eq. (3).

Remark 4: The above proposed AST is carried out with only $\mathcal{N}_{training} = (M - N)$ training symbols for each coherent duration T_c. The typical values of $\mathcal{N}_{training}$ are 1 or 2 symbols.

Remark 5: The number of feedback bits required to inform the transmitter about the best channels in the *general* $(M,N;K)$ AST/DSTBC is:

$$\mathcal{N} = \left\lceil \log_2 \binom{M}{N} \right\rceil, \tag{14}$$

where $\lceil . \rceil$ is the ceiling function.

Remark 6: In all conventional DSTBCs, the initial matrix $\mathbf{W}_0 = \mathbf{I}_N$ is only used to initialize the transmission. Particularly, $\mathbf{W}_0$ is used to calculate the next transmitted matrices following Eq. (3), and to generate the initial, received matrix $\mathbf{R}_0$ *directly*, which is combined with the next receiving matrix $\mathbf{R}_1$ to decode transmitted symbols.

Unlike the conventional DSTBCs without ASTs, in the proposed technique, the initial identity matrix $\tilde{\mathbf{W}}_0 = \mathbf{I}_M$ is transmitted. This matrix has two main roles. It enables the receiver to generate the initial, received matrix $\mathbf{R}_0$ *indirectly* (from the received matrix $\tilde{\mathbf{R}}_0$). Simultaneously, in some sense, it also plays a role of training signals, which assist the receiver to determine semiblindly the best channels. This is the main difference between the differential space-time coding with our AST and the one without AST.

Remark 7: Similarly to the conventional DSTBCs without ASTs mentioned in Section 3, in our proposed technique, channel coefficients are required to be constant during at least two consecutive code blocks. Therefore, the channels must be constant during, at least, $(M+N)$ STSs in our proposed AST, while they must be unchanged during at least $2N$ STSs in all conventional DSTBC techniques without the proposed ASTs if the delay of transmitting feedback information from the receiver to the transmitter is not considered. In the case when the delay is considered, the channel coefficients must stay longer.

Remark 8: The procedures of the proposed *general* $(M,N;K)$ AST/DSTBC is more explicitly presented in Fig. 2. Steps 1a, 1b, 4 and 5 are carried out at the transmitter, while the remaining steps are carried out at the receiver. As stated earlier, Step 1b is not necessarily carried out after Step 3a finishes. In other words, the transmitter can perform Step 1b right after finishing Step 1a. Similarly, because the matrix $\mathbf{R}_0$ is created straightforwardly from the matrix $\tilde{\mathbf{R}}_0$, the receiver can perform Step 3b right after finishing Step 3a. These properties reduce unnecessary delays during transmission.

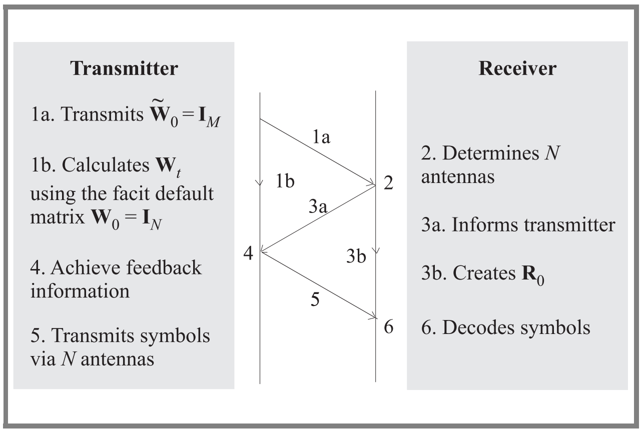

Fig. 2. The *general* $(M,N;K)$ AST/DSTBC for the system using DSTBCs.

6. The restricted $(M,N;K)$ AST/DSTBC and the $(N+\bar{N},N;K)$ AST/DSTBC

As mentioned in Eq. (14), the number of feedback bits required in the *general* $(M,N;K)$ AST/DSTBC is:

$$\mathcal{N} = \left\lceil \log_2 \binom{M}{N} \right\rceil .$$

It is easy to realize that, $\mathcal{N}$ is large for large values of M and N. For instance, in the *general* (6,4;K) AST/DSTBC (K is arbitrary), we have $\mathcal{N} = 4$. Therefore, it is either impractical or uneconomical to employ the *general* $(M,N;K)$ AST/DSTBC for large values of M and N, except when either the number of feedback bits or the time required to process feedback information is reduced.

Motivated by this observation, we derive here the two AST/DSTBC schemes which are the modifications of the aforementioned, *general* $(M,N;K)$ AST/DSTBC scheme. We refer those ASTs to as the *restricted* $(M,N;K)$ AST/DSTBC and the $(N+\bar{N},N;K)$ AST/DSTBC. The two modified ASTs are more amenable to practical implementation in channels using DSTBCs than the *general* $(M,N;K)$ AST/DSTBC.

The *restricted* $(M,N;K)$ AST/DSTBC requires only 1 feedback bit, while providing a relatively good bit error performance. Meanwhile, the $(N+\bar{N},N;K)$ AST/DSTBC requires at most an equal number of feedback bits as the *general* $(M,N;K)$ AST/DSTBC where $M = N + \bar{N}$, while shortening the time required to process feedback information. Especially, when $\bar{N} = 1$, the $(N+1,N;K)$ AST/DSTBC scheme provides the *same* bit error performance as the *general* $(M,N;K)$ AST/DSTBC scheme, where $M = N+1$, while shortening the processing time for feedback information. For $\bar{N} > 1$, there exists a degradation of the bit error performance of the $(N+\bar{N},N;K)$ AST/DSTBC scheme, compared to the *general* $(M,N;K)$ AST/DSTBC scheme where $M = N+\bar{N}$. Therefore, the $(N+1,N;K)$ AST/DSTBC scheme is of our particular interest in this paper.

6.1. The restricted (M,N;K) AST/DSTBC

In the scenario where the capacity limitation of the feedback channel, especially in the uplink channels of the 3G mobile communication systems, needs to be considered, the number of feedback bits is as small as possible. More importantly, limiting the number of feedback bits is necessary when fading changes fast. Based on the *general* $(M,N;K)$ AST/DSTBC mentioned in Section 5, we propose here the *restricted* $(M,N;K)$ AST/DSTBC for channels using DSTBCs, where only 1 feedback bit is required for each channel coherent duration T_c to inform the transmitter.

In the *restricted* $(M,N;K)$ AST/DSTBC, the set of MTx antennas is divided into two subsets. Each subset includes NTx ($N < M$) antennas. Subsets may partially overlap each other. Figure 3 presents 3 cases for illustration. In Fig. 3a,

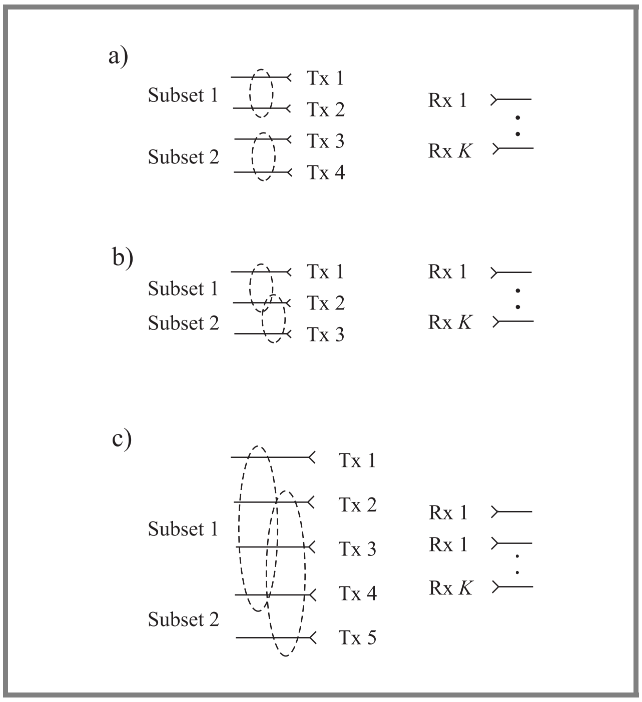

Fig. 3. Some examples of the transmitter antenna grouping for (a) the *restricted* (4,2;K) AST/DSTBC; (b) the *restricted* (3,2;K) AST/DSTBC; (c) the *restricted* (5,4;K) AST/DSTBC.

we give an example where 4Tx antennas are divided into 2 subsets including 2Tx antennas each, while in Fig. 3b, 3Tx antennas are divided into 2 subsets containing 2Tx each. These 2 cases can be applied, for instance, to the Alamouti DSTBC with the *restricted* (4,2;K) AST/DSTBC and with the *restricted* (3,2;K) AST/DSTBC, respectively. Figure 3c, we derive other example where 5Tx antennas are divided into 2 subsets which partially overlap one another and include 4Tx antennas each. This case can be applied, for instance, to the order-4 DSTBC with the *restricted* (5,4;K) AST/DSTBC.

Let Ψ and Φ be the sets of indices indicating the order of the Tx antennas in each subset, respectively. The selection criterion for the restricted $(M,N;K)$ AST/DSTBC is as follows.

During each coherent duration T_c of the channel, the receiver compares:

$$\sum_{j\in\Psi} \chi_j = \sum_{j\in\Psi} \left[\sum_{i=1}^{K} |r_{0ij}|^2 \right]$$

and

$$\sum_{j\in\Phi} \chi_j = \sum_{j\in\Phi} \left[\sum_{i=1}^{K} |r_{0ij}|^2 \right],$$

ie., the receiver compares the total power of the signals received by all KRx antennas during the initial transmission from two subsets of Tx antennas, and then informs the transmitter to select the subset providing the greater total power. If $\sum_{j\in\Psi} \chi_j$ is lager, then the receiver, via a feedback loop, informs the transmitter to select the Tx antennas corresponding to the set of indices Ψ. Otherwise, the Tx antennas corresponding to the set of indices Φ should be selected. These procedures are repeated for different coherent durations T_cs of the channel until the transmission of data is completed.

It is obvious that only one feedback bit per each coherent time T_c is required for transmission diversity purpose.

6.2. The $(N+\bar{N},N;K)$ AST/DSTBC

In this section, we consider a system containing $M = (N+\bar{N})$Tx antennas and KRx antennas and transmitting square, order-N DSTBCs. Among MTx antennas, NTx antennas are called default Tx antennas which are normally used to transmit signals, and $\bar{N}$ remaining Tx antennas are the standby ones which are only used when the selection criterion is satisfactory. The diagram of the system in this technique is shown in Fig. 4.

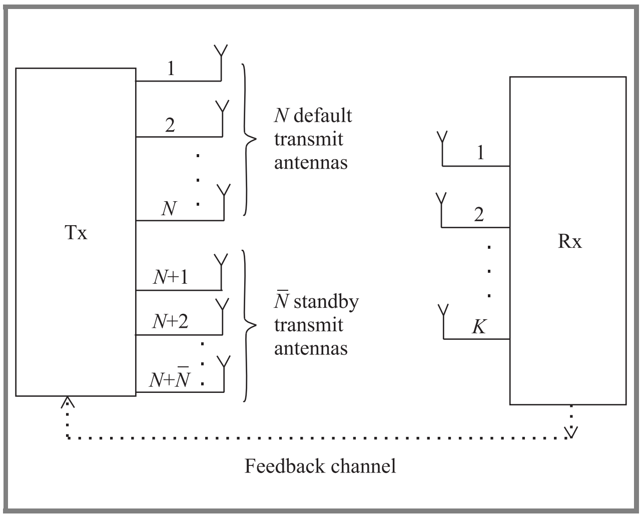

Fig. 4. The diagram of the $(N+\bar{N},N;K)$ AST/DSTBC.

We propose here a modified AST/DSTBC scheme for this structure of the system which is referred to as the $(N+\bar{N},N;K)$ AST/DSTBC. This AST shortens the time required to process feedback information in comparison with the *general* (M,N,K) AST/DSTBC where $M=N+\bar{N}$.

Note that $\bar{N}$ is *strictly smaller* than N, i.e., $\bar{N}<N$. It will be shown later that when $\bar{N}=N$, the $(N+\bar{N},N;K)$ AST/DSTBC turns into the *restricted* $(M,N;K)$ AST/DSTBC where $M=N+\bar{N}$.

Without loss of generality, we number $(N+\bar{N})$Tx antennas by indices from 1 to $(N+\bar{N})$, and assume that the N default Tx antennas are indexed from 1 to N while the $\bar{N}$ standby Tx antennas are indexed from $(N+1)$ to $(N+\bar{N})$.

Similarly to the *general* (M, N, K) AST/DSTBC, in the $(N+\bar{N},N;K)$ AST/DSTBC, the transmitter starts the transmission by transmitting an identity, order-M matrices $\tilde{\mathbf{W}}_0=\mathbf{I}_M=\mathbf{I}_{N+\bar{N}}$ during each channel coherent time T_c. Let $\tilde{\mathbf{R}}_0$ be the initial, received matrix $\tilde{\mathbf{R}}_0$ during the initial transmission, i.e., the time when the initial matrix $\tilde{\mathbf{W}}_0$ is transmitted. Similarly to Eq. (11), we have:

$$\tilde{\mathbf{R}}_0=\left[\,\mathbf{r_{01}}\ \ \mathbf{r_{02}}\ \ \ldots\ \ \mathbf{r_{0N}}\ \ \mathbf{r_{0N+1}}\ \ \ldots\ \ \mathbf{r_{0N+\bar{N}}}\,\right].$$

In this expression, $\mathbf{r_{0j}}$ is the column vector of the signals received by all KRx antennas during the jth STS from the jth Tx antenna. Let χ_j=$\|\mathbf{r_{0j}}\|_F^2$ which is the total power received by all KRx antennas from the jth Tx antenna $(j=1,\ldots,N+\bar{N})$.

We denote φ_k to be the set of $\bar{N}$ indices of the $\bar{N}$ default Tx antennas which are arbitrarily taken from N default Tx antennas. There are total $q=\binom{N}{\bar{N}}$ such sets. Furthermore, for $k=1,\ldots,q$, we denote:

$$\begin{aligned}\alpha_k &= \sum_{j\in\varphi_k}\chi_j\\ &= \sum_{j\in\varphi_k}\|\mathbf{r_{0j}}\|_F^2\\ &= \sum_{j\in\varphi_k}\left[\sum_{i=1}^{K}|r_{0ij}|^2\right].\end{aligned}$$

The proposed $(N+\bar{N},N;K)$ AST/DSTBC is as follows. On the one hand, the receiver searches for the minimum value among q values $\{\alpha_1,\ldots,\alpha_q\}$. Let α be this minimum value and $\hat{\mathfrak{I}}_{\bar{N}}$ be the set of indices of the corresponding default Tx antennas. This action can be mathematically presented by

$$\alpha=\min\ \left\{\alpha_1,\ldots,\alpha_q\right\}.$$

On the other hand, the receiver calculates the total power of the received signals value which are received by all KRx antennas during the initial transmission from $\bar{N}$ standby Tx antennas. If we denote this total power to be β, then this action can be expressed as:

$$\begin{aligned}\beta &= \sum_{j=(N+1)}^{(N+\bar{N})}\chi_j\\ &= \sum_{j=(N+1)}^{(N+\bar{N})}\|\mathbf{r_{0j}}\|_F^2\\ &= \sum_{j=(N+1)}^{(N+\bar{N})}\left[\sum_{i=1}^{K}|r_{0ij}|^2\right].\end{aligned}$$

If $\alpha\geq\beta$, then the Tx antennas which the transmitter should select are all default Tx antennas $\{1,\ldots,N\}$.

If $\alpha<\beta$, the $\bar{N}$ default Tx antennas which have the indices listed in the set $\hat{\mathfrak{I}}_{\bar{N}}$ will be replaced by the standby antennas. To illustrate, we assume that $\hat{\mathfrak{I}}_{\bar{N}}=\{1,2,\ldots,\bar{N}\}$, i.e., the first $\bar{N}$ default Tx antennas provide the minimum value α. If $\alpha\geq\beta$, then the Tx antennas are $\{1,2,\ldots,N\}$. Otherwise, the first $\bar{N}$ default Tx antennas are replaced by the $\bar{N}$ standby Tx antennas. Consequently, the NTx antennas which should be selected are $\{\bar{N}+1,\ldots,N-1,N, N+1,\ldots,N+\bar{N}\}$.

The antenna selection mechanism for this example is presented more clearly by the flowchart in Fig. 5.

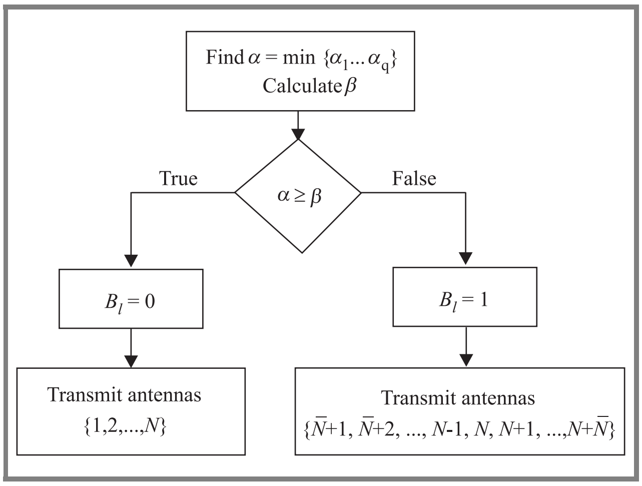

Fig. 5. The flow chart of the $(N+\bar{N},N;K)$ AST/DSTBC .

Associated with this antenna selection mechanism, we propose the structure of the feedback information as presented in Fig. 6. The bit B_l is used to indicate whether the trans-

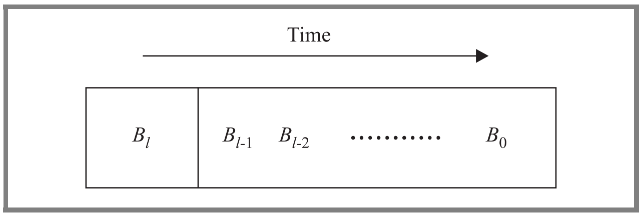

Fig. 6. The proposed structure of the feedback information for channels using DSTBCs.

mitter has to replace $\bar{N}$ default antennas with the standby ones. The bit B_l is zero if the answer is no, i.e., $\alpha \geq \beta$, and B_l is unity otherwise. The l following bits indicate which $\bar{N}$ antennas among N default antennas should be replaced by the standby ones.

It is easy to realize that $l = \lceil \log_2 \binom{N}{\bar{N}} \rceil$. With this structure, the transmitter first considers the bit B_l. As soon as it realizes that $B_l = 0$, the rest of the feedback information is not necessarily processed. The transmitter will transmit signals via the default Tx antennas $\{1, 2, \ldots, N\}$. If $B_l = 1$, the transmitter uses the l following bits $B_{l-1}, \ldots, B_0$ to recognize which default antennas should be replaced by the standby ones.

Therefore, the number of feedback bits required to be transmitted in the $(N+\bar{N},N;K)$ AST/DSTBC is at most equal to:

$$\mathcal{N}_1 = l + 1 = 1 + \left\lceil \log_2 \binom{N}{\bar{N}} \right\rceil. \tag{15}$$

We want to stress that, theoretically, there is no need to transmit l bits $B_{l-1}, \ldots, B_0$ in the case $B_l = 0$. If so, a single feedback bit (bit B_l) is required to be transmitted.

Note that the number of feedback bits required to be transmitted and processed in the *general* $(M,N;K)$ AST/DSTBC where $M = N+\bar{N}$ is always:

$$\mathcal{N}_2 = \left\lceil \log_2 \binom{M}{N} \right\rceil = \left\lceil \log_2 \binom{N+\bar{N}}{N} \right\rceil.$$

It is easy to realize that if N is a power of 2, we have $\mathcal{N}_1 \leq \mathcal{N}_2$. For instance, for $N = 2$ and $\bar{N} = 1$, we have $\mathcal{N}_1 = \mathcal{N}_2 = 2$. For $N = 4$ and $\bar{N} = 2$, we have $\mathcal{N}_1 = 3$ and $\mathcal{N}_2 = 4$.

Therefore, if N is the power of 2, the number of feedback bits required to be transmitted in the $(N+\bar{N},N;K)$ AST/DSTBC is almost equal to that required in the *general* $(M,N;K)$ AST/DSTBC ($M = N+\bar{N}$). The number of feedback bits required to be processed in the $(N+\bar{N},N;K)$ AST/DSTBC is either $(l+1)$, which is equal to the number of transmitted feedback bits $\mathcal{N}_1$, or only 1 (smaller than $\mathcal{N}_1$) depending on the bit B_l. The smaller number of feedback bits required to be transmitted and to be processed in the $(N+\bar{N},N;K)$ AST/DSTBC shortens the time required to process feedback information in the $(N+\bar{N},N;K)$ AST/DSTBC in comparison with the time required in the *general* $(M,N;K)$ AST/DSTBC. The quantitative estimation of this time reduction will be mentioned later.

From the aforementioned algorithm, we have the following remarks on the $(N+\bar{N},N;K)$ AST/DSTBC.

Remark 9: Theoretically, it is not necessary to transmit l bits $B_{l-1}, \ldots, B_0$ in the case $B_l = 0$. Only one feedback bit B_l is required to be transmitted (and processed) in this case. This observation may further shortens the time for feeding information back.

Remark 10: The N default Tx antennas are always used for transmission whenever $\beta \leq \alpha$, i.e., the set of $\bar{N}$ standby antennas is not better[3] than the worst set of $\bar{N}$ default Tx antennas among N default Tx antennas.

When $\beta > \alpha$, i.e, the set of $\bar{N}$ standby antennas is better than the worst set of $\bar{N}$ default Tx antennas among N default Tx antennas, these $\bar{N}$ standby antennas are used to replace the $\bar{N}$ default antennas.

Remark 11: If $\bar{N} = N$, not only the antenna selection criterion of the $(N+\bar{N},N;K)$ AST/DSTBC is exactly the same as that of the *restricted* $(M,N;K)$ AST/DSTBC where $M = N+\bar{N}$, but the required numbers of feedback bits of both ASTs are also the same (only 1 feedback bit is required). Therefore, the $(N+\bar{N},N;K)$ AST/DSTBC turns into the *restricted* $(M,N;K)$ AST/DSTBC. Owing to this reason, $\bar{N}$ must be strictly smaller than N in the $(N+\bar{N},N;K)$ AST/DSTBC.

Remark 12: If $2 \leq \bar{N} < N$, the $(N+\bar{N},N;K)$ AST/DSTBC is suboptimal as the set containing the N best Tx antennas among $(N+\bar{N})$Tx antennas is not always selected for transmission, and consequently, it provides a worse BER performance than the *general* $(M,N;K)$ AST/DSTBC where $M = N+\bar{N}$. In return for this disadvantage, the $(N+\bar{N},N;K)$ AST/DSTBC shortens the time required to process feedback information in comparison with the *general* $(M,N;K)$ AST/DSTBC.

Remark 13: If $\bar{N} = 1$, the antenna selection criterion of the $(N+\bar{N},N;K)$ AST/DSTBC turns into the selection criterion of the *general* $(M,N;K)$ AST/DSTBC where $M = N+\bar{N} = N+1$. Intuitively, both the $(N+1,N;K)$ AST/DSTBC and the *general* $(M,N;K)$ AST/DSTBC select the N optimal Tx antennas out of $(N+1)$Tx antennas. Consequently, the BER performance of the $(N+1, N; K)$ AST/DSTBC is the same as that of the *general* $(M,N;K)$ AST/DSTBC $(M = N+1)$.

The main advantage of the $(N+1, N; K)$ AST/DSTBC over the *general* $(M,N;K)$ AST/DSTBC is that the time required to process feedback information in the former is shorter than that in the later. This advantage will be mentioned in more details in the next section in which the quantitative estimation of the time reduction gained by the $(N+1, N; K)$ AST/DSTBC in comparison with the *general* $(M,N;K)$ AST/DSTBC is derived.

Owing to these reasons, the $(N+\bar{N},N;K)$ AST/DSTBC with $\bar{N} = 1$, i.e., the $(N+1, N; K)$ AST/DSTBC, is of our particular interest in this paper.

Let $\chi_j = \sum_{i=1}^{K} |r_{0ij}|^2$ for $j = 1, \ldots, (N+1)$. The $(N+1, N; K)$ AST/DSTBC scheme can be slightly modified from the $(N+\bar{N},N;K)$ AST/DSTBC and stated as follows.

The receiver searches for the minimum value $\chi_{\min}$ among $(N+1)$ values $\{\chi_1, \ldots, \chi_{N+1}\}$, i.e.:

$$\chi_{\min} = \min\{\chi_1, \ldots, \chi_{N+1}\}.$$

We assume that $\chi_{\min} \equiv \chi_n$ where $n = 1, \ldots, (N+1)$.

[3]A better set provides a larger total power which is received by all KRx antennas during the initial transmission.

If $n \equiv (N+1)$, then all N default Tx antennas are used to transmit signals. In this case, bit $B_l = 0$. Otherwise, the indexed-n default Tx antenna is replaced by the standby Tx antenna (the $(N+1)$th Tx antenna). This standby antenna is combined with the $(N-1)$Tx antennas to transmit signals. In this case, bit $B_l = 1$.

7. Relative reduction of the average processing time of the $(N+\bar{N},N;K)$ AST/DSTBC

In order to estimate the time reduction obtained by the $(N+\bar{N},N;K)$ AST/DSTBC, we compare the average time required to process feedback information in this AST and that required in the *general* $(M,N;K)$ AST/DSTBC ($M = N+\bar{N}$) in Section 5.

Although, there is a fact that the time required to process the feedback information does not necessarily increase linearly with the number of feedback bits, it is easier to calculate the time benefit of the proposed technique when the average processing time is assumed to increase linearly with the number of feedback bits. Obviously, the result we derive as follows is only aimed at providing the readers with the lower bound of the relative reduction of the average processing time obtained by the $(N+\bar{N},N;K)$ AST/DSTBC in comparison with that of the *general* $(M,N;K)$ AST/DSTBC.

Let P_0 be the probability of the event that the set of $\bar{N}$ standby Tx antennas is not used in the $(N+\bar{N},N;K)$ AST/DSTBC. In other words, P_0 is the probability of the event that $\beta \le \alpha$, i.e. $P_0 = P(\beta \le \alpha)$. Similarly, let P_1 be the probability of the event that the $\bar{N}$ standby Tx antennas are used for transmission, i.e, $P_1 = P(\beta > \alpha)$. Clearly, we have $P_1 = (1-P_0)$.

We now calculate P_0 in the two following cases which are different in the underlying essences.

- When $\bar{N} = 1$, as mentioned earlier in Remark 13, the default Tx antenna is only used when it is the worst Tx antenna among $(N+1)$Tx antennas. We make a reasonable assumption that the event where a certain Tx antenna (either default or standby antenna) is the worst antenna among $(N+1)$Tx antennas is equiprobable. Then we have:

$$P_0 = P(\beta \le \alpha) = \frac{1}{\binom{N+1}{1}} = \frac{1}{(N+1)} . \quad (16)$$

- When $\bar{N} \ge 2$, we make a reasonable assumption that the event in which a set containing the certain $\bar{N}$ default Tx antennas selected from the N available default Tx antennas is the worst set, is equiprobable. This means that:

$$P(\alpha \equiv \alpha_1) = \cdots = P(\alpha \equiv \alpha_q) = \frac{1}{\binom{N}{\bar{N}}} = \frac{1}{q} .$$

We also assume the following conditional probability:

$$P(\beta \le \alpha | \alpha \equiv \alpha_k) = 0.5$$

for $k = 1,\ldots,q$. As a result, we have:

$$\begin{aligned} P_0 &= P(\beta \le \alpha) \\ &= \sum_{k=1}^{q} P(\beta \le \alpha|\alpha \equiv \alpha_k) P(\alpha \equiv \alpha_k) \\ &= \sum_{k=1}^{q} 0.5 \cdot \frac{1}{q} \\ &= 0.5 . \end{aligned} \quad (17)$$

Let ϑ be the average processing time for 1 feedback bit. Because the transmitter has to process 1 feedback bit (bit B_l) only if $\beta \le \alpha$ and has to process $\mathcal{N}_1 = \left(1 + \left\lceil \log_2 \binom{N}{\bar{N}} \right\rceil\right)$ feedback bits if $\beta > \alpha$, the average time required to process feedback information in the $(N+\bar{N},N;K)$ AST/DSTBC is:

$$\begin{aligned} \tau_1 &= P_0 \vartheta + P_1 \mathcal{N}_1 \vartheta \\ &= P_0 \vartheta + (1-P_0)\left(1 + \left\lceil \log_2 \binom{N}{\bar{N}} \right\rceil\right) \vartheta . \end{aligned}$$

On the other hand, in the *general* $(M,N;K)$ AST/DSTBC where $M = N+\bar{N}$, the transmitter always has to process $\mathcal{N}_2 = \left\lceil \log_2 \binom{N+\bar{N}}{N} \right\rceil$ feedback bits. Therefore, the average processing time is:

$$\tau_2 = \mathcal{N}_2 \vartheta = \left\lceil \log_2 \binom{N+\bar{N}}{N} \right\rceil \vartheta .$$

Hence, the relative reduction of the average processing time between two techniques is:

$$\begin{aligned} \frac{\triangle\tau}{\tau_2} &\triangleq \frac{\tau_2 - \tau_1}{\tau_2} \\ &= 1 - \frac{1 + (1-P_0)\left\lceil \log_2 \binom{N}{\bar{N}} \right\rceil}{\left\lceil \log_2 \binom{N+\bar{N}}{\bar{N}} \right\rceil} . \end{aligned} \quad (18)$$

For $\bar{N} = 1$, from Eqs. (16) and (18), we have:

$$\frac{\triangle\tau}{\tau_2} = 1 - \frac{1 + (1 - \frac{1}{N+1})\left\lceil \log_2 N \right\rceil}{\left\lceil \log_2 (N+1) \right\rceil} .$$

For $\bar{N} \ge 2$, from Eqs. (17) and (18), we have:

$$\frac{\triangle\tau}{\tau_2} = 1 - \frac{1 + 0.5\left\lceil \log_2 \binom{N}{\bar{N}} \right\rceil}{\left\lceil \log_2 \binom{N+\bar{N}}{\bar{N}} \right\rceil} .$$

The relative time reduction $\frac{\triangle\tau}{\tau_2}$ [%] for some particular values of N and $\bar{N}$ is presented by the table in Fig. 7. We only need to calculate the time reduction for the pair of N and $\bar{N}$ satisfying $\bar{N} < N$.

From this table, we realize that the average processing time reduction is considerable even for $\bar{N} = 1$. In this case, the average processing time reduction for $N = 2$, 4 and 8 is 16.67, 13.33 and 8.33%, respectively. To illustrate, the $(2+1, 2; 1)$ AST/DSTBC in the system using the Alamouti DSTBC with 2 default Tx antennas, 1 standby Tx antenna and 1Rx antenna gains the relative time reduction of 16.67%.

$\bar{N}$ \ N	2	4	8
1	16.67	13.33	8.33
2		37.5	41.67
3		66.67	50
4			50

Fig. 7. Relative time reduction [%] of the $(N+\bar{N}, N; K)$ AST/DSTBC compared to the *general* $(M, N; K)$ AST/DSTBC where $M = N + \bar{N}$.

It is worth to stress that the time reduction is probably much greater than the above figures if we take its non-linear proportionality with the number of feedback bits into consideration.

8. Some comments on spatial diversity order of the proposed ASTs

In this section, we consider the spatial diversity order of the ASTs proposed for channels using DSTBCs with differential detection. To do that, at first, we review the same issue for channels using STBCs with coherent detection, to provide the readers with the state of the art of this issue.

The spatial diversity order of the ASTs for channels using space-time codes with coherent detection has been somewhat examined in a few papers, such as [8–10, 28–31]. Particularly, in [28] and [30], the authors considered the combination of the transmitter antenna selection and space-time trellis codes (STTCs) and proved that the (M,2;1) AST/STTC and (M,2;2) AST/STTC schemes provide a full spatial diversity order when SNR is very large (see Eqs. (26) and (27) in [28]) as long as the STTCs have a full rank. In [31], the authors considered the receiver (not transmitter) diversity selection associated with the use of STCs (either STBCs or STTCs) in MIMO systems over the quasi-static (slow) Rayleigh fading channels. The author proved there that the $(M; K, L)$ AST/STC schemes (where MTx antennas are used without selection, while the L best Tx antennas are selected out of KRx antennas) provide a full spatial diversity order of MK, provided that the STCs have a full rank (see Eq. (10) in [31]).

It is noted that, in this paper, we consider the *transmitter* (*not receiver*) diversity antenna selection and the use of DSTBCs which have *orthogonal* structures. Therefore, it is useful to review the spatial diversity order of the ASTs associated with STBCs only (not STTCs or other STCs). Having this note in mind, we realize that there are very few works, such as [10] and [29], have mentioned the spatial diversity order of transmitter diversity ASTs for channels using STBCs. In [10] and [29], the authors limited themselves to consider the Alamouti STBC modulated by a binary phase shift keying (BPSK) signal constellation in the (M,2;1) AST/STBC and (M,2;2) AST/STBC schemes only. Those studies are far from the exhaustive research.

In other words, the exhaustive research on the spatial diversity order of transmitter diversity ASTs is still missing even for space-time coded systems with coherent detection. For space-time coded systems with non-coherent detection, such as the systems using DSTBCs, the study on the spatial diversity order of AST/DSTBC schemes has not been examined yet. Due to this reason, in this paper, we do not have ambition to examine this issue for all cases, which certainly requires a lot of studies in future.

Instead, we show that the problem of finding the spatial diversity order of the ASTs proposed for channels using DSTBCs with *differential detection* is the same as that problem for the case of *coherent detection* when SNR$\gg$1. Once this has been shown, we consider the (M,2;1) AST/DSTBC and (M,2;2) AST/DSTBC schemes. Since the respective (M,2;1) AST/STBC and (M,2;2) AST/STBC schemes for channels using STBCs provide a full spatial diversity order [10, 29], then the (M,2;1) AST/DSTBC and (M,2;2) AST/DSTBC schemes for channels using DSTBCs also provide a full spatial diversity order as if all Tx and Rx antennas were used.

We restrict ourselves to consider only the *general* $(M, N; K)$ AST/DSTBC scheme for illustration. Other schemes, such as the *restricted* $(M, N; K)$ AST/DSTBC scheme or the $(N+\bar{N}, N; K)$ AST/DSTBC scheme are similarly analyzed.

To begin with, we review some crucial discussions mentioned in [10] on the spatial diversity order achieved by the (M,2;K) AST/STBC schemes for channels using STBCs with *coherent detection*. We use the superscript l ($l = 1, 2, 3, \ldots$) to indicate the different coherent durations of the channel. Since the coherent detection is being considered, the channel coefficients between Tx and Rx antennas denoted by $\bar{a}_{ij}^{(l)}$, for $i = 1, \ldots, K$ and $j = 1, \ldots, M$, are assumed to be perfectly known at the receiver and partially known at the transmitter through a feedback channel. Let $\bar{\xi}_j^{(l)} = \sum_{i=1}^{K} |\bar{a}_{ij}^{(l)}|^2$. We assume that $\bar{a}_{ij}^{(l)}$s are i.i.d. complex Gaussian random variables with the distribution $\mathcal{CN}(0, \sigma_a)$.

With the notation mentioned in Section 3 of this paper, we rewrite the Tx antenna selection criterion, which was mentioned by Eq. (1) in [10], for the (M,2;K) AST/STBC scheme during the lth coherent duration as

$$\begin{aligned} \mathfrak{J}_2^{(l)} &= \mathcal{F}_2\left(\bar{\xi}_1^{(l)}, \bar{\xi}_2^{(l)}, \ldots, \bar{\xi}_M^{(l)}\right) \\ &= \mathcal{F}_2\left(\sum_{i=1}^{K} |\bar{a}_{i1}^{(l)}|^2, \sum_{i=1}^{K} |\bar{a}_{i2}^{(l)}|^2, \ldots, \sum_{i=1}^{K} |\bar{a}_{iM}^{(l)}|^2\right). \quad (19) \end{aligned}$$

Denote $\gamma = \frac{E_b}{N_0}$ to be the SNR per bit. It has been shown in [10], the BER expression, say $P_{2,1}$, of the (M,2;1) AST/STBC, where there is only 1Rx antenna, in flat Rayleigh fading channels for binary phase shift keying modulation asymptotically approaches (see Eq. (7) in [10]):

$$P_{2,1} \approx \frac{(2M-1)!}{2^{2M-1}(M-1)!}\left(\frac{1}{\gamma}\right)^M$$

when $\gamma \to \infty$. This equation shows that a full diversity order of M is achieved asymptotically for the (M,2;1) AST/STBC when $\gamma \to \infty$.

The BER expression, say $P_{2,2}$, of the (M,2;2) AST/STBC, where there are 2Tx antennas, in flat Rayleigh fading channels for BPSK modulation asymptotically approaches (see Eq. (8) in [10]):

$$P_{2,2} \approx \frac{M(4M-1)!}{2^{5M-2}(2M-1)(2M-1)!}\left(\frac{1}{\gamma}\right)^{2M}$$

when $\gamma \to \infty$. This equation shows that a full diversity order of $2M$ is achieved asymptotically for the (M,2;2) AST/STBC when $\gamma \to \infty$.

The cases for $K \geq 3$ are not practically significant since it is difficult to employ more than 2Tx antennas at the mobile set in mobile communication downlinks. Due to this reason, the cases for $K \geq 3$ were not presented in [10].

Now we return to consider our proposed, *general* (M,2;K) AST/DSTBC for channels using DSTBCs with *differential detection*. The superscript k ($k = 1,2,3,\ldots,m$) is used to indicate the different coherent durations of the channel (see Fig. 1). Since the differential detection is considered, the channel coefficients between Tx and Rx antennas $a_{ij}^{(k)}$, for $i = 1,\ldots,K,\ j = 1,\ldots,M,\ k = 1,\ldots,m$, are unknown at either the receiver or the transmitter.

As mentioned in Eq. (13) in Section 5, the selection criterion for the *general* (M,2;K) AST/DSTBC during the kth coherent duration is:

$$\begin{aligned}\hat{\mathfrak{I}}_2^{(k)} &= \mathcal{F}_2\left(\chi_1^{(k)},\ldots,\chi_M^{(k)}\right)\\ &= \mathcal{F}_2\left(\sum_{i=1}^{K}|r_{0i1}^{(k)}|^2,\sum_{i=1}^{K}|r_{0i2}^{(k)}|^2,\ldots,\sum_{i=1}^{K}|r_{0iM}^{(k)}|^2\right)\\ &= \mathcal{F}_2\left(\sum_{i=1}^{K}|a_{i1}^{(k)}+n_{0i1}^{(k)}|^2,\sum_{i=1}^{K}|a_{i2}^{(k)}+n_{0i2}^{(k)}|^2,\ldots,\right.\\ &\qquad\left.\sum_{i=1}^{K}|a_{iM}^{(k)}+n_{0iM}^{(k)}|^2\right). \end{aligned} \tag{20}$$

We assume that the channel coefficients $a_{ij}^{(k)}$s and noise $n_{0ij}^{(k)}$s are i.i.d. complex Gaussian random variables with the distribution $\mathcal{CN}(0,\sigma_a)$ and $\mathcal{CN}(0,\sigma)$, respectively. We consider the mean and the variance of the following term:

$$\mu_{ij}^{(k)} \triangleq |a_{ij}^{(k)} + n_{0ij}^{(k)}|^2$$

for $i = 1,\ldots,K,\ j = 1,\ldots,M$ and $k = 1,\ldots,m$.

Since $a_{ij}^{(k)}$ and $n_{0ij}^{(k)}$ are the i.i.d. zero-mean, complex Gaussian random variables, $(a_{ij}^{(k)} + n_{0ij}^{(k)})$ are the i.i.d., complex Gaussian random variables with the distribution $\mathcal{CN}(0,\rho)$ where $\rho = \sigma_a + \sigma$. Therefore, $\mu_{ij}^{(k)}$ are the i.i.d, central chi-squared random variables with $n = 2$ degrees of freedom and with the following mean and variance [32, p. 42]:

$$\begin{aligned}E\{\mu_{ij}^{(k)}\} &= n\frac{\rho}{2} = \rho\,,\\ \sigma_{\mu_{ij}^{(k)}} &= 2n\left(\frac{\rho}{2}\right)^2 = \rho^2\,.\end{aligned}$$

We investigate the case in which the channel SNR $\gg 1$. Equivalently, the variances of noise terms $n_{0ij}^{(k)}$s are very small in comparison with the variances of $a_{ij}^{(k)}$s, and therefore, $\rho \approx \sigma_a$. As a result, the means and the variances of $\mu_{ij}^{(k)}$ are approximately:

$$\begin{aligned}E\{\mu_{ij}^{(k)}\} &\approx \sigma_a\\ \sigma_{\mu_{ij}^{(k)}} &\approx \sigma_a^2\,,\end{aligned} \tag{21}$$

when $SNR \gg 1$.

On the other hand, we consider the following term:

$$\theta_{ij}^{(k)} = |a_{ij}^{(k)}|^2$$

for $i = 1,\ldots,K,\ j = 1,\ldots,M$ and $k = 1,\ldots,m$.

Similarly analyzed, $\theta_{ij}^{(k)}$ are the i.i.d, central chi-squared random variables having $n = 2$ degrees of freedom with the following mean and variance [32, p. 42]:

$$\begin{aligned}E\{\theta_{ij}^{(k)}\} &= \sigma_a\,,\\ \sigma_{\mu_{ij}^{(k)}} &= \sigma_a^2\,.\end{aligned} \tag{22}$$

From Eqs. (21) and (22), we realize that, $\mu_{ij}^{(k)}$s and $\theta_{ij}^{(k)}$s have the same statistical properties, i.e., means and variances when $SNR \gg 1$. We can rewrite the antenna selection criterion of the $(M,N;K)$ AST/DSTBC in Eq. (20) as

$$\hat{\mathfrak{I}}_2^{(k)} \approx \mathcal{F}_2\left(\sum_{i=1}^{K}|a_{i1}^{(k)}|^2,\sum_{i=1}^{K}|a_{i2}^{(k)}|^2,\ldots,\sum_{i=1}^{K}|a_{iM}^{(k)}|^2\right) \tag{23}$$

when $SNR \gg 1$.

Clearly, the antenna selection criterion for the (M,2;K) AST/DSTBC scheme now tends to be the same as the criterion mentioned in Eq. (19) for the (M,2;K) AST/STBC scheme.

We may conclude that, if the channel $SNR \to \infty$, the behavior of the (M,2;K) AST/DSTBC scheme proposed for channels using DSTBCs with differential detection tends to be the same as that of the (M,2;K) AST/STBC scheme mentioned in literature for channels using STBCs with coherent detection, although the (M,2;K) AST/DSTBC scheme is inferior by 3 dB compared to the (M,2;K) AST/STBC scheme due to the fact that the channel coefficients are not known at either transmitter or receiver. As a result, because the (M,2;1) AST/STBC and (M,2;2) AST/STBC schemes achieve a full spatial diversity [10, 29], then so do the (M,2;1) AST/DSTBC and (M,2;2) AST/DSTBC schemes, provided that the channel SNR is very large.

9. Simulation results

In this section, we run some Monte-Carlo simulations to solidify our proposed AST/DSTBC schemes. We consider a wireless link comprising $K = 1$Rx antenna. The channel SNR is defined to be the ratio between the total average power of the received signals and the average power of noise at the Rx antenna during each STS. Note that the numbers of feedback bits which are required for the *general* $(M,N;K)$ AST/DSTBC and the $(N+\bar{N},N;K)$ AST/DSTBC examined in the simulations are calculated by Eqs. (14) and (15), respectively. The number of feedback bit required for the *restricted* $(M,N;K)$ AST/DSTBC is always 1. In simulations, DSTBCs are modulated by a QPSK signal constellation in simulations.

First, the Alamouti DSTBC in Eq. (1) corresponding to $N = 2$ is simulated. We consider 4 following scenarios:

– Alamouti DSTBC without ASTs;

– Alamouti DSTBC with the *general* (3,2;1) AST/DSTBC (2 feedback bits);

– Alamouti DSTBC with the *restricted* (3,2;1) AST/DSTBC (1 feedback bit);

– Alamouti DSTBC with the (2+1,2;1) AST/DSTBC ($N = 2$, $\bar{N} = 1$, 2 feedback bits).

However, as noted earlier in Remark 13 of Section 6.2, the (2+1,2;1) AST/DSTBC has the same BER performance as the *general* (3,2;1) AST/DSTBC, although the time required to process feedback information in the former is shorter than that in the later. For this reason, we do not need to plot the BER performance of the (2+1,2;1) AST/DSTBC scheme.

Furthermore, in each AST/DSTBC scheme, we examine 2 cases where the feedback error rates are assumed to be 4% and 10%. Transmit antennas in the *restricted* (3,2;1) AST/DSTBC are grouped by the scheme mentioned in Fig. 3b.

Note that it would be better if we can compare the performances here with the performance of a DSTBC without ASTs which provides the same spatial diversity order as the diversity order (equal to 3) provided by the proposed AST/DSTBC schemes, i.e., the *general* (3,2;1) AST/DSTBC, the *restricted* (3,2;1) AST/DSTBC and the (2+1,2;1) AST/DSTBC. This means that we should compare the performance of the Alamouti DSTBC (associated with the proposed ASTs) with that of an order-3 DSTBC (without ASTs). However, while the Alamouti DSTBC has a full rate, it is well known that DSTBCs of an order being greater than 2 with a full rate do not exist. For this reason, it is unfair to compare the Alamouti DSTBC with an order-3 DSTBC, because they have different code rates, and consequently, we do not plot the performance of any order-3 DSTBC in the simulation.

As analyzed earlier, channel coefficients must be constant during at least two adjacent code blocks. If T_c denotes the coherent time of the channel, then it is required that:

- $T_c \geq 4$ STSs for the Alamouti DSTBC without ASTs;
- $T_c \geq 5$ STSs for the Alamouti DSTBC with the *general* (3,2;1) AST/DSTBC, with the *restricted* (3,2;1) AST/DSTBC, or with the (2+1,2;1) AST/DSTBC.

Therefore, to compare fairly the performance of the Alamouti DSTBC with different ASTs, the simulation is run for T_c which is not less than 5 STSs. Example 2 in Section 2.2 is one of such practical scenarios.

The performance of the Alamouti DSTBC with and without ASTs is shown in Fig. 8. It can be seen from Fig. 8 that the proposed ASTs significantly improve the BER performance of the channel. Again, the BER performances of the (2+1,2;1) AST/DSTBC is exactly the same as that of the *general* (3,2;1) AST/DSTBC. The main advantage of the (2+1,2;1) AST/DSTBC over the *general* (3,2;1) AST/DSTBC is that the time required to process feedback information is shortened by 16.67% (see Fig. 7). The SNR reductions [dB] gained by our proposed ASTs to achieve the same BER $= 10^{-3}$ as the Alamouti DSTBC without ASTs are given in Table 1.

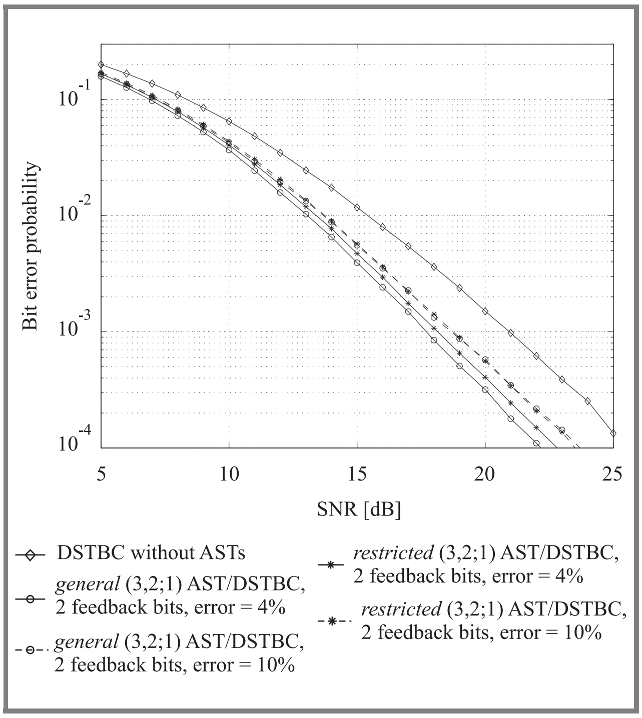

Fig. 8. The Alamouti DSTBC with the *general* (3,2;1) AST/DSTBC and the *restricted* (3,2;1) AST/DSTBC.

Next, we consider the *general* (4,2;1) AST/DSTBC (3 feedback bits) and the *restricted* (4,2;1) AST/DSTBC (1 feedback bit) in which the transmitter selects $N = 2$Tx antennas out of $M = 4$Tx antennas. Clearly, in this case, we have $\bar{N} = M - N = 2$, i.e., $\bar{N} = N$. In Remark 11, we have stated that the (2+2,2;1) AST/DSTBC reduces to the *restricted* (4,2;1) AST/DSTBC. Therefore, we do not plot the performance of the (2+2,2;1) AST/DSTBC here. Transmitter antennas in the *restricted* (4,2;1) AST/DSTBC are grouped by the scheme mentioned in Fig. 3a.

Table 1
SNR reductions [dB] of the *general* (3,2;1) AST/DSTBC, the *restricted* (3,2;1) AST/DSTBC and the (2+1,2;1) AST/DSTBC in the channel using Alamouti DSTBC

Error [%]	*General* (3,2;1) AST/DSTBC	(2+1,2;1) AST/DSTBC	*Restricted* (3,2;1) AST/DSTBC
4	3.25	3.25	2.9
10	2.25	2.25	2.25

Similarly, it is required that:

- $T_c \geq 4$ STSs for the Alamouti DSTBC without ASTs;
- $T_c \geq 6$ STSs for the Alamouti DSTBC with the *general* (4,2;1) AST/DSTBC or with the *restricted* (4,2;1) AST/DSTBC.

To compare fairly the performance of Alamouti DSTBC with different ASTs, the simulation is run for T_c which is not less than 6 STSs. Example 2 mentioned in Section 2.2 is one of such practical scenarios.

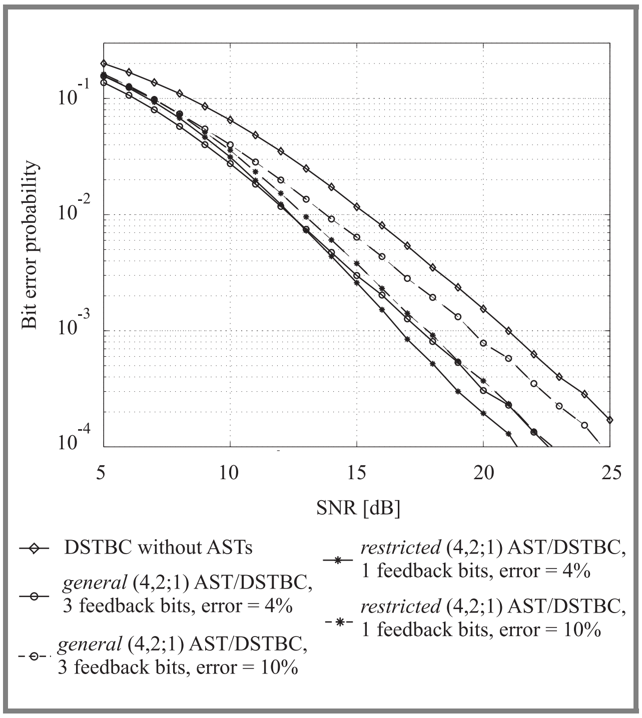

Fig. 9. The Alamouti DSTBC with the *general* (4,2;1) AST/DSTBC and the *restricted* (4,2;1) AST/DSTBC schemes.

The performance of the proposed AST/DSTBC schemes is presented in Fig. 9. The SNR reductions [dB] achieved by our proposed ASTs to have the same BER $= 10^{-3}$ as the DSTBC without ASTs are given in Table 2.

Table 2
SNR reductions [dB] of the *general* (4,2;1) AST/DSTBC and the *restricted* (4,2;1) AST/DSTBC in the channel using Alamouti DSTBC

Error [%]	*General* (4,2;1) AST/DSTBC	*Restricted* (4,2;1) AST/DSTBC
4	3.5	4.3
10	1.5	3.25

Finally, we examine the square, order-4, unitary DSTBC in Eq. (2) corresponding to $N = 4$ and the code rate 3/4. We consider the following 4 scenarios:

- DSTBC without ASTs;
- DSTBC with the *general* (5,4;1) AST/DSTBC (3 feedback bits);
- DSTBC with the *restricted* (5,4;1) AST/DSTBC (1 feedback bit);
- DSTBC with the (4+1,4;1) AST/DSTBC ($N = 4$, $\bar{N} = 1$, 3 feedback bits). Similarly, the BER performance of the (4+1,4;1) AST/DSTBC is exactly the same as that of the *general* (5,4;1) AST/DSTBC, and therefore, we do not need to plot the BER performance of the (4+1,4;1) AST/DSTBC in the simulation.

In each AST, we also consider 2 cases where the feedback error rates are assumed to be 4% and 10%. Transmitter antennas in the *restricted* (5,4;1) AST/DSTBC are grouped by the scheme mentioned in Fig. 3c.
It is required that:

- $T_c \geq 8$ STSs for DSTM without ASTs;
- $T_c \geq 9$ STSs for DSTM with the *general* (5,4;1) AST/DSTBC, with the *restricted* (5,4;1) AST/DSTBC or with the (4+1,4;1) AST/DSTBC.

Therefore, the simulation is run for T_c which is not less than 9 STSs. Example 2 in Section 2.2 is still valid for this scenario.
The performance of the proposed AST/DSTBC schemes is presented in Fig. 10. It is noted that the (4+1,4;1) AST/DSTBC provides the same BER performance as that of the *general* (5,4;1) AST/DSTBC (see Remark 13 in Section 6.2), while shortening the time which is required to process feedback information by 13.33% (see Fig. 7) compared to the *general* (5,4;1) AST/DSTBC.
The SNR reductions [dB] achieved by our proposed ASTs to have the same BER $= 10^{-3}$ as the DSTBC without ASTs are given in Table 3.
From all the above simulations, we realize that the proposed ASTs significantly improve the performance of wireless channels using DSTBCs. Also, we realize that the *restricted*

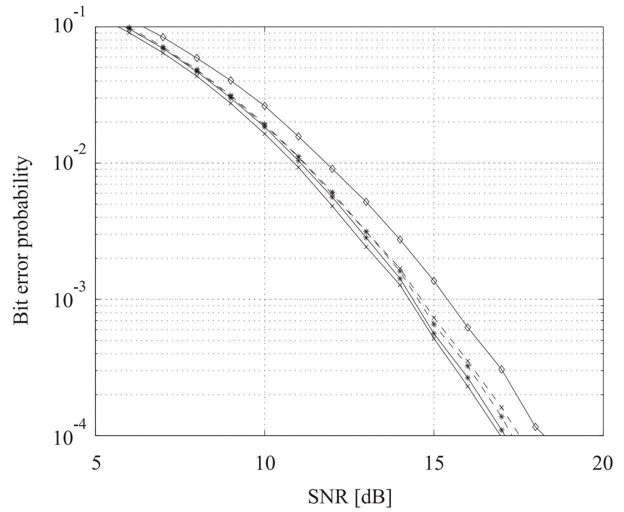

Fig. 10. Square, order-4, unitary DSTBC with the *general* (5,4;1) AST/DSTBC and the *restricted* (5,4;1) AST/DSTBC schemes.

$(M,N;K)$ AST/DSTBC provide a relatively good BER performance compared to the *general* $(M,N;K)$ AST/DSTBC and the $(N+\bar{N},N;K)$ AST/DSTBC, while requiring only 1 feedback bit. More importantly, the *restricted* $(M,N;K)$ AST/DSTBC may perform even better than the *general* $(M,N;K)$ AST/DSTBC and the $(N+\bar{N},N;K)$ AST/DSTBC when the feedback error rate grows large. Intuitively, this

Table 3
SNR reductions [dB] of the proposed (5,4;1) AST/DSTBCs in the channel using square, order-4, unitary DSTBC

Error [%]	*General* (5,4;1) AST/DSTBC	(4+1,4;1) AST/DSTBC	*Restricted* (5,4;1) AST/DSTBC
4	1.2	1.2	1
10	0.8	0.8	0.85

is interpreted by the fact that the *restricted* AST requires only 1 feedback bit while the remaining ASTs require multiple feedback bits. Therefore, when the feedback error rate grows large, the feedback information in the *restricted* ASTs is less likely erroneous than that in the other ASTs. As a result, the *restricted* ASTs are the practical candidates for the channels where fading changes fast.

10. Discussions and conclusion

In this paper, we propose three ASTs referred to as the *general* $(M,N;K)$ AST/DSTBC, the *restricted* $(M,N;K)$ AST/DSTBC, and the $(N+\bar{N},N;K)$ AST/DSTBC for the channels using DSTBCs with arbitrary number of Tx and Rx antennas.

Since the *general* $(M,N;K)$ AST/DSTBC scheme requires a large number of feedback bits when M, N and K are large, it is either impractical or uneconomical for implementation in such cases. The *restricted* $(M,N;K)$ AST/DSTBC and the $(N+\bar{N},N;K)$ AST/DSTBC schemes overcome this shortcoming.

Particularly, the *restricted* $(M,N;K)$ AST/DSTBC is an attractive technique, which provides relatively good bit error performance, compared to the *general* $(M,N;K)$ AST/DSTBC, while requiring only 1 feedback bit. This advantage is very important in the case where the capacity limitation of the feedback channel, such as in the uplink channels of the 3G mobile communication systems, is considered. This advantage is also very beneficial in the channels where fading changes fast and/or the feedback error rate in the feedback channel grows large.

Unlike the restricted AST/DSTBC schemes, where we try to reduce the number of feedback bits, in the $(N+\bar{N},N;K)$ AST/DSTBC schemes, we reduce the average time required to process feedback information. These techniques use at most the same number of feedback bits and provide the same BER performance (if $\bar{N}=1$) as that of the *general* $(M,N;K)$ AST/DSTBC schemes ($M=N+\bar{N}$), but remarkably reduce the average time required to process feedback information.

Simulation show that all three proposed ASTs with a limited number (typically, 1 or 2) of training symbols per each coherent duration of the channel noticeably improve the BER performance of wireless channels utilizing DSTBCs. The improvement is significant even for the case of 1 training symbol, i.e., in the *general* $(M,N;K)$ AST/DSTBC where $M=(N+1)$; in the *restricted* $(M,N;K)$ AST/DSTBC where $M=(N+1)$; or in the $(N+1,N;K)$ AST/DSTBC schemes.

The *restricted* $(M,N;K)$ AST/DSTBC may provide a better BER performance over the *general* $(M,N;K)$ AST/DSTBC and the $(N+\bar{N},N;K)$ AST/DSTBC when the feedback error rate is large. Hence, the restricted AST/DSTBC schemes are a good choice for the channels where fading changes fast and/or the feedback error rate is large.

It is noted that, in this paper, we assume that the carrier phase/frequency is perfectly recovered at the receiver. In fact, phase/frequency recovery errors may exist, which degrade the performance of the proposed ASTs. Those errors may occur due to the difference between the frequency of the local oscillators at the transmitter and the receiver, and/or due to the Doppler frequency-shift effect. The effect of imperfect carrier recovery on the performance of the proposed ASTs in wireless channels utilizing DSTBCs has been examined in our paper [4]. Readers may refer to [4] for more details.

Also, in this paper, the delay of feedback information has not been considered. In reality, the delay of feedback information may somewhat degrade the overall performance of the proposed ASTs. This issue will be mentioned in our other works. Finally, as mentioned earlier, the exhaustive research on the spatial diversity order of the ASTs proposed for channel using DSTBCs has not been derived yet and it must be fully examined in the future work.

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