Paper

# A general framework of agent-based simulation for analyzing behavior of players in games

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**Abstract—In this paper, we give a general framework of agent-based simulation for analyzing behavior of players in various types of games. In our simulation model, artificial adaptive agents have a mechanism of decision making and learning based on neural networks and genetic algorithms. The synaptic weights and thresholds characterizing the neural network of an artificial agent are revised in order that the artificial agent obtains larger payoffs through a genetic algorithm. The proposed framework is illustrated with two examples, and, by giving some simulation result, we demonstrate availability of the simulation analysis by the proposed framework of agent-based simulation, from which a wide variety of simulation settings can be easily implemented and detailed data and statistics are obtained.**

***Keywords*—*artificial adaptive agents, simulation, games, behavior of players.***

## 1. Introduction

In games with multiple equilibria, it is difficult to predict which equilibrium will be realized because of uncertainty about actions of opponents. Even in games with a unique equilibrium, it is known that in some special games, the prediction of the Nash equilibrium does not always correspond to reality. To examine human behavior in such games, numerous experiments have been accumulated.

Especially, a considerable number of studies have been made on experiments for examining human behavior in coordination games, generalized matching pennies games, ultimatum bargaining games, market entry games and so forth [3, 9, 16, 17, 19, 20, 21, 22]. Although in experimental studies, situations in accordance with game models are formed in laboratories and human subjects are motivated by money, for such experimental environments with human subjects there exist limitations with respect to the number of trials, the number of subjects, variations of parameter settings and so forth.

In most of mathematical models in economics and game theory, it is assumed that players are rational and maximize their payoffs, and they can discriminate between two payoffs with a minute difference. Such optimization approaches are not always appropriate for analyzing human behavior and social phenomena, and models based on adaptive behavior can be alternatives to such optimization models. Recently as complements of conventional mathematical models, a large number of adaptive behavioral models have been proposed [1, 4, 6, 7, 8, 13, 17, 18, 21, 23].

It is natural that actions of artificial agents in simulation systems are described by using adaptive behavioral rules, and simulation can be a promising approach to modeling situations where it is difficult to assume hyperrational behavior of decision makers. We suppose that simulation is a complement to experiments with human subjects because an extensive range of treatments can be easily performed by varying values of the parameters characterizing games in simulation systems while there exist the above mentioned limitations in experiments with human subjects. As concerns such approaches based on adaptive behavioral models, Holland and Miller [12] interpret most of economic systems as complex adaptive systems, and point out that simulation using artificial societies with adaptive agents is effective for analysis of such economic systems. Axelrod [2] insists on the need for simulation analysis in social sciences, and states that purposes of the simulation analysis include prediction, performance, training, entertainment, education, proof and discovery.

In this paper, we give a general framework of agent-based simulation for analyzing behavior of players in various types of games. In our simulation model, the decision mechanism of an artificial agent is based on a neural network with several inputs, and the agent chooses a strategy in accordance with the output of the neural network. The synaptic weights and thresholds characterizing the neural network are revised so that an artificial agent obtains larger payoffs through a genetic algorithm, and then this learning mechanism develops artificial agents with better performance. In Section 2, we describe the agent-based simulation system with decision and learning mechanisms based on neural networks and genetic algorithms. In Section 3, we provide some simulation result of the coordination games to demonstrate availability of the simulation analysis. Finally in Section 4, to conclude this paper, we make some remarks.

## 2. Simulation model

In this section, a general framework of agent-based simulation is presented together with two applications to specific games: the minimum strategy coordination game and the generalized matching pennies game [14, 15]. An artificial adaptive agent in our simulation system has a mechanism of decision making and learning based on neural networks (see, e.g., Hassoun [11]) and genetic algorithms (see, e.g., Goldberg [10]).

### 2.1. Decision making by a neural network

Artificial agents repeatedly play a game; agents obtaining larger payoff are likely to be reproduced in the next period, and conversely agents obtaining only a little payoff are likely to be weeded out. In our model of an artificial genetic system embedded in the agent-based simulation model, the whole population is divided into $m$ game groups, and in each game group the game is played by $n$ agents. The number of agents, $n$, depends on setting of a game.

An artificial agent corresponds to a neural network, which is characterized by synaptic weights between two nodes in the neural network and thresholds which are parameters in the output function of nodes. In our simulation model, an action of an artificial agent is determined by a vector of outputs from a nonlinear function with several input data that the agent can know after playing a stage game. This decision mechanism is implemented by a neural network. The synaptic weights and thresholds characterizing the neural network are revised so that the artificial agent obtains larger payoffs through a genetic algorithm, and then this learning mechanism develops artificial agents with better performance.

Because a structure of neural networks is determined by the number of layers and the number of nodes in each layer, an artificial agent is prescribed by the fixed number of parameters if the numbers of layers and nodes are fixed. In our model, we form a string compound of these parameters which is identified with an artificial agent, and the string is treated as a chromosome in an artificial genetic system embedded in the simulation model.

### 2.2. Evolutionary learning through a genetic algorithm

In a simulation system, the game is played by $n$ artificial agents in each of $m$ game groups. Therefore, there are $m$ agents for each type of players. There are $s$ alternative strategies, and each of the agents chooses one among them. Mixed strategies can be implemented by some simple devises if necessary. The payoffs of artificial agents are determined by outcomes of the game. Repeatedly playing the game, agents obtaining larger payoffs are likely to survive; if this is not the case, such agents are culled out in time. In our simulation model, genetic algorithms are employed as an evolutionary learning mechanism. Because a fitness in the artificial genetic system is calculated by the obtained payoffs, agents obtaining larger payoffs are likely to survive. The general structure of simulation model is shown in Fig. 1.

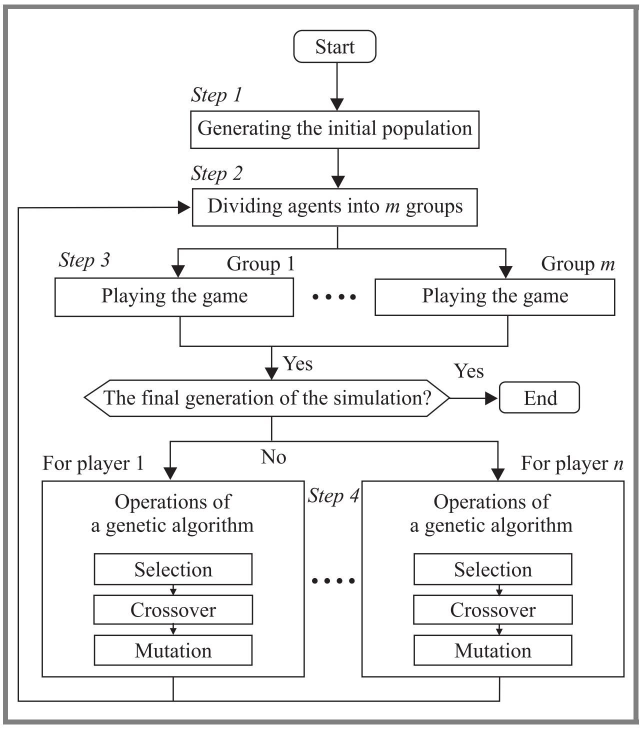


***Fig. 1.*** Flow of the simulation.

The procedure of the simulation is summarized as follows.

*Step 1*: **Generating the initial population**. Let the number of players in the game and the number of groups for playing the game be $n$ and $m$, respectively. Then, the whole population of $mn$ artificial agents is initialized by assigning random numbers in the interval $[-1, 1]$ to the parameters of the synaptic weights and the thresholds characterizing the neural network.

*Step 2*: **Forming groups for playing the game**. The whole population with $mn$ agents is divided into $m$ groups for playing the game.

*Step 3*: **Playing the game**. Each agent chooses a strategy in accordance with the output of the neural network, and the game is played in each of the groups. The strategy of each agent is determined by the output of the neural network; an agent selects the strategy corresponding to the node with the largest output in the neural network. If the number of generations in the genetic algorithm reaches the final period of the simulation, the procedure stops.

*Step 4*: **Performing genetic operations**. The $i$th subpopulation is formed by gathering the $i$th players (agents) from the $m$ groups; there are $n$ subpopulations. The genetic operations are separately executed to each subpopulation consisting of $m$ agents.

*Step 4-1*: **Reproduction**. Let $\pi_i$ denote a payoff of agent $i$ in the present period. The fitness of agent $i$ is calculated as a function of $\pi_i$. As a reproduction operator, a certain method such as the roulette wheel selection is adopted. If the roulette wheel selection, by a roulette wheel with slots sized by the probability $p_i^{selection} = f_i / \sum_{i=1}^{mn} f_i$, each chromosome is selected into the next generation.

*Step 4-2*: **Crossover**. A certain crossover method such as the single-point crossover operator is applied to any pair of chromosomes with the probability of crossover $p_c$. If the single-point crossover operator is employed, a point of crossover on the chromosomes is randomly selected and then two new chromosomes are created by swapping subchromosomes which are the right side parts of the selected point of crossover on the original chromosomes. A new population is formed by exchanging the population in which the crossover operation is executed for the present generation with a given probability $G$. The probability $G$ is called the generation gap. An agent keeps the history of obtaining payoffs in the past games, and the payoffs are divided between two offsprings in the proportion of sizes of the swapped subchromosomes.

*Step 4-3*: **Mutation**. With a given small probability of mutation $p_m$, each gene which represents a synaptic weight or a threshold in a chromosome is randomly changed. The selected gene is replaced by a random number in $[-1, 1]$.

### *2.3. Applications*

So far, we have given the general framework of agent-based simulation for analyzing behavior of players in variety types of games. In this subsection, the proposed framework is illustrated with two examples: the minimum strategy coordination game and the generalized matching pennies game [14, 15].

**Minimum strategy coordination game**. Based on the proposed framework, agent-based simulation systems can be developed for a wide variety of games, and we can perform an extensive range of treatments of the corresponding simulation by using the system. For instance, we develop an agent-based simulation system [14] for analyzing the coordination game treated in the experimental investigation by Van Huyck *et al.* [22]. Because this coordination game is characterized by the minimum values of the strategies selected by players, we refer to it as the minimum strategy coordination game.

Before describing the specific structure of the neural network for artificial agents, we give the outline of the minimum strategy coordination game. Let the set of players be $N = \{1, \ldots, n\}$. All the players have the common set of strategies: $S = \{1, \ldots, \bar{s}\}$. Let $x_i \in S$ denote a strategy of player $i$. Then, the payoff function of player $i$ is represented by

$$\pi(x_i, \underline{x}_i) = a \min(x_i, \underline{x}_i) - b x_i + c,$$
$$\underline{x}_i = \min(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n), \ a > b > 0, \ c > 0. \quad (1)$$

The payoff of player $i$ decreases with a strategy $x_i$ of self, and increases with the minimum $\underline{x}_i$ among strategies of the others. To guarantee positive payoffs, the constant $c$ is added.

An artificial agent as a player in the minimum strategy coordination game can be implemented by a neural network which is characterized by synaptic weights and thresholds. The structure of the neural network is depicted in Fig. 2.

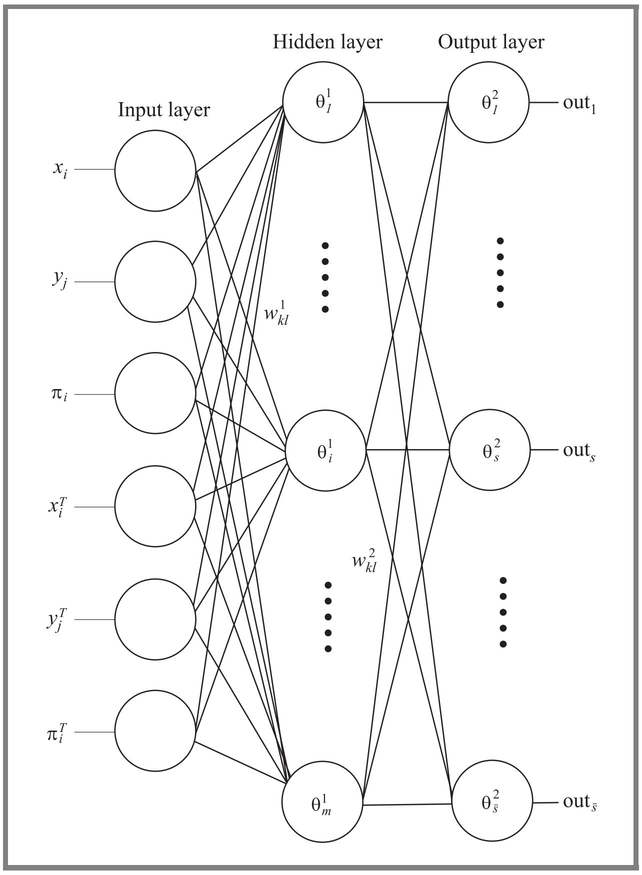


***Fig. 2.*** The structure of the neural network for the minimum strategy coordination game.

There are six inputs in the neural network. In the following information of the inputs, the subscript $i$, $i = 1, \ldots, n$ means player $i$ and the subscript $j$, $j = 1, \ldots, m$ means game group $j$. Thus, the subscript $ij$ identifies a particular agent in the agent-based simulation system. For inputs 1 and 2, because human subjects in the experiment are informed of the minimum strategy at the last game, and it is supposed that they remember the strategies selected by themselves, the strategy $x_{ij}$ of agent $ij$ and the minimum strategy $y_j$ in game group $j$ are given as inputs of the neural network; the payoff $\pi_{ij}$ obtained by agent $ij$ at the last period is also given as input 3. Supposing that a player does not remember an exact history of strategies in the past periods, but the player remembers at least the most frequent strategy in the past periods, we provide the weighted most frequent strategy $x_{ij}^T$ in the last $T$ periods as input 4 to the neural network. In the definition of $x_{ij}^T$, assuming that old memory is apt to decay, the discount factor $w$, $0 < w < 1$ is introduced. Similarly, as inputs 5 and 6, the weighted most frequent minimum strategy $y_j^T$ and the weighted sum of obtained payoffs in the last $T$ periods are also given.

An algorithm for evolutionary learning through the genetic algorithm is modified if necessary. In the experiment conducted by Van Huyck *et al.* [22], subjects understand the payoff table defined by the payoff function (1), and it is not true that they start to play the game without any prior knowledge of the game. Therefore it is natural for artificial agents in our system to have some knowledge of the game before playing it. To do so, by using the error back propagation algorithm (see, e.g., Hassoun [11]) with the teacher signals, the parameters of the synaptic weights and the thresholds in the neural network are adjusted ahead.

**Generalized matching pennies game**. We provide another application of the proposed framework of agent-based simulation to the generalized matching pennies game treated in the experiment by Ochs [16]. The payoff table of the generalized matching pennies game is shown in Table 1. In this game, the row player has the two choices $U$ and $D$ and the column player also has the two choices $L$ and $R$. When an outcome is $(U,L)$ or $(D,R)$, the row player receives a positive payoff of $a$ or 1, respectively. When an outcome is $(U,R)$ or $(D,L)$, the column player receives a positive payoff of 1. For the payoff $a$ of the row player with respect to the outcome $(U,L)$, we assume that $a \geq 1$. When $a = 1$, the game is symmetric, and when $a > 1$, it is asymmetric. It is known that, in the generalized matching pennies game, there does not exist any Nash equilibrium with pure strategies but there exists only a unique Nash equilibrium with strict mixed strategies.

Table 1
A generalized matching pennies game

| Row player | Column player | |
|---|---|---|
| | $L$ | $R$ |
| $U$ | $(a,0)$ | $(0,1)$ |
| $D$ | $(0,1)$ | $(1,0)$ |

Let $p$ denote a probability of choosing strategy $U$ for the row player, and let $q$ denote a probability of choosing strategy $L$ for the column player. Then, expected payoffs $\pi_R$ and $\pi_C$ of the row player and the column player, respectively, are represented by

$$\pi_R = apq + (1-p)(1-q), \tag{2}$$

$$\pi_C = (1-p)q + p(1-q), \tag{3}$$

and the corresponding Nash equilibrium is $(p^{\mathrm{Nash}}, q^{\mathrm{Nash}}) = (1/2, 1/(a+1))$. Because this game is not a zero-sum game, the maximin strategy is different from the Nash equilibrium. The maximin strategies of the row player and the column player are given by $\arg\max_p \min_q \pi_R(p,q)$ and $\arg\max_q \min_p \pi_C(p,q)$, respectively, and therefore the pair of the maximin strategies is $(p^S, q^S) = (1/(a+1), 1/2)$.

Artificial agents playing the generalized matching pennies game can be implemented by a neural network in a way similar to in the previous application of the minimum strategy coordination game. The structure of the neural network is depicted in Fig. 3.

The inputs of the neural network include not only payoffs of self obtained in the past periods but also payoffs of an opponent, and the set of inputs consists of five values. Let $x_i(j)$ be a payoff of player $i$ at period $j$. Then, the total payoff of player $i$ at period $t$ is represented by

$$x_i^{\mathrm{total}}(t) = \sum_{j=1}^{t} \phi_i^{t-j} x_i(j). \tag{4}$$

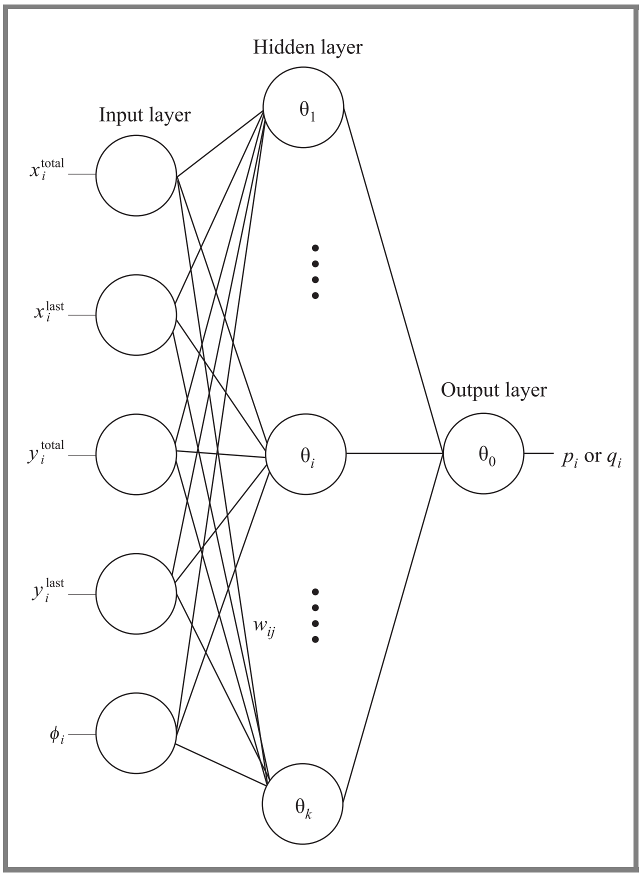


***Fig. 3.*** The structure of the neural network for the generalized matching pennies game.

On the assumption that a decision of the artificial agent is affected by the payoff obtained before, though the previous payoff is reduced by $(1-\phi_i)$, the total payoff $x_i^{\text{total}}$ is used as an input 1 of the neural network. In the extended reinforcement model by Erev and Roth [7], a similar parameter is incorporated. In the experiment by Ochs [16], it is found that there exist some players who take an outcome of the previous game into account and make a decision. From this viewpoint, we employ the payoff $x_i^{\text{last}}$ obtained at the last game as input 2 of the neural network. Furthermore, Duffy and Feltovich [5] claim that choices of players are influenced by the behavior or payoff of the others, and therefore in our model the payoffs $y_i^{\text{total}}$ and $y_i^{\text{last}}$ of an opponent as well as the payoffs of self are incorporated as inputs 3 and 4 of the neural network. The value of $(1-\phi_i)$ can be interpreted as the rate of forgetting. On our model, the value of $\phi_i$ as input 4 to the neural network is fixed for each agent and it is randomly assigned to each agent at the beginning of the simulation.

The output of the neural network is a probability that the artificial agent chooses strategy $U$ if the agent is the row player or strategy $L$ if the agent is the column player.

Because the generalized matching pennies game is a two-person game, the number of players is $n = 2$. For development of an agent-based simulation system to this game, we slightly modify the procedure of the simulation to allow artificial agents to use mixed strategies. Furthermore, to examine effect of error in decisions and risk attitude of players, the fitness of an artificial agent is defined as a function of the payoff and the parameters of error and risk attitude in the genetic algorithm embedded in the simulation system.

Although we have shown only two examples, our general framework of agent-based simulation can be applicable to various games and economic situations such as the ultimatum bargaining game, the market entry game, and so forth.

# 3. Analysis of simulation data

In this section, by giving a part of the simulation result of the minimum strategy coordination game, we demonstrate availability and effectiveness of the simulation analysis by the proposed framework of agent-based simulation, from which a wide variety of simulation settings can be easily implemented and detailed data and statistics are obtained.

In this example, a variety of treatments are performed by varying values of some parameters characterizing the game; the three different simulations are arranged: *simulations coefficients*, *information*, and *size*.

If the coefficient $b$ of the second term in the payoff function (1) is positive, it follows that players who select larger strategies than the minimum strategy pay the penalty. If the coefficient $b$ is equal to zero, the coordination problem such as coordination failure and disequilibrium is eliminated. In the experiment by Van Huyck *et al.* [22], when $b = 0$, the payoff dominant equilibrium is observed; when $b = 0.1$, the subjects avoid risky strategies and choices of the subjects settle into the secure equilibrium. From this result, it is reasoned that by making the value of $b$ larger from zero, outcomes of the game shift from the payoff dominant equilibrium to the secure equilibrium. In *simulation coefficient*, two treatments are performed, varying the values of the coefficients $b$ and $a$. Moreover, after putting artificial agents in experiencing the payoff dominant equilibrium in case of $b = 0$, the agents play the games with $b \neq 0$. By examining the choices of agents and the realization rate of equilibria in this simulation, we investigate the relation between the penalty and the behavior of artificial agents.

In the experiment with human subjects by Van Huyck *et al.* [22], two types of treatments of information on outcomes of the game are performed: one treatment where the subjects are informed only of the minimum strategy, and the other treatment where the subjects are informed of the distribution of strategies selected by all the players. Comparing the two treatments, they conclude that informing the subjects of the distribution of the strategies accelerates the convergence of behavior of the subjects. In *simulation information*, artificial agents are provided three types of information on outcomes of the game: the minimum strategy, the minimum and the maximum strategies, and the distribution of strategies. We examine the effect of information given to artificial agents on the choices of them and the realization rate of equilibria.

In the experiment, it is also observed that when the number of players is two, in comparison with the case of 14 or 16 subjects, it is likely to realize the payoff dominant equilibrium. In *simulation size*, varying the number of artificial agents as well as the value of $b$ representing the degree of the penalty, we investigate influence of the number of agents on their behavior in the game and outcomes of the game.

In this paper, as an example, we provide detailed analysis of only one treatment of *simulation coefficients*. In general as the value of $b$ is made larger and the risk of paying the penalty increases, the payoff of an artificial agent selecting a large strategy such as the payoff dominant strategy 7 becomes a small value, and therefore it is likely to fail in coordination. However, the risk-free game with $b = 0$ is not the case. In this treatment, fixing the value of $a$ at $a = 0.2$, the value of the penalty coefficient $b$ is varied; it is set at $b = 0.0, 0.005, 0.006, 0.007, 0.008, 0.009, 0.01, 0.02, 0.03, 0.04, 0.05, 0.1$. From the data observed in the treatment, we investigate transitions and steady states of the choice rate of each strategy, the realization rate of each individual equilibrium, and so forth.

Figures 4, 5, 6, and 7 show the choice rate of each strategy, the minimum strategy rate of each strategy, the means of selected strategies and the minimum strategies, and the normalized average payoff, respectively. For comparison, in Figs. 4 and 5, the data from the experiment with human subjects by Van Huyck *et al.* [22] are provided by outline symbols. Moreover, the realization rate of each individual equilibrium and the gross realization rate of equilibria are given in Figs. 8 and 9, respectively.

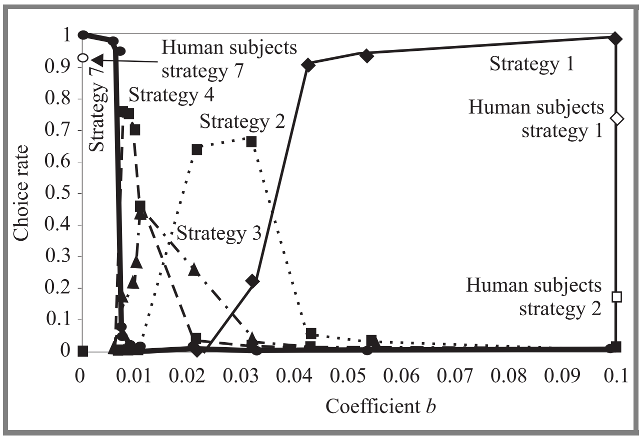


*Fig. 4.* Choice rate of each strategy in treatment $b$.

In Fig. 4, the choice rate of each strategy at the steady state is given. When the penalty is relatively large, i.e., $b \geq 0.04$, the secure strategy 1 is likely to be selected. Namely, most of the artificial agents avoid the risk of paying a large penalty and select the most secure strategy. As the value of $b$ decreases and therefore the penalty becomes small, the modal strategy, which is the most frequently selected strategy, grows large from the strategy 1 to the strategy 4 one by one. When $b$ is smaller than around 0.006, the modal strategy jumps straight to the strategy 7, and strategies 5 and 6 do not become modal.

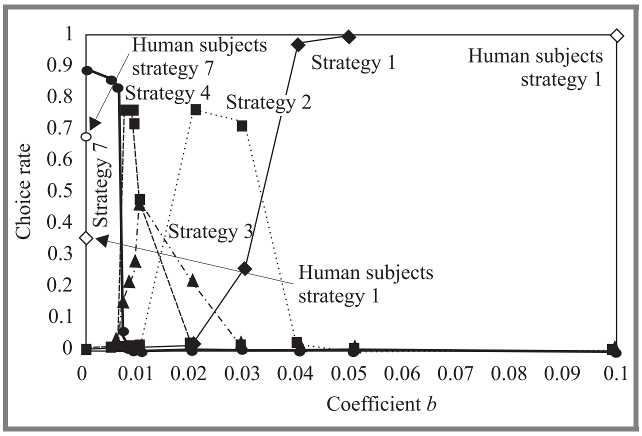


*Fig. 5.* Minimal strategy rate of each strategy in treatment $b$.

In Fig. 5, the minimum strategy rate of the strategy $s$ means the rate that the strategy $s$ is the minimum in the game. From the fact that Fig. 5 is highly similar to Fig. 4, it follows that the modal strategy in the steady state is almost the same as the minimum strategy. For the strategy 1, when $b \geq 0.04$, although the choice rate of the strategy 1 shown in Fig. 4 decreases little by little as the value of $b$ becomes small, the minimum strategy rate of the strategy 1 shown in Fig. 5 is almost 1.0. Contrary to the strategy 1, when $b = 0$, the choice rate of the strategy 7 is almost 1.0, but the minimum strategy rate falls below 0.9 because the other strategies are selected on rare occasions.

Compared with the result of the experiment with human subjects, when $b = 0.1$, the choice rate of the secure strategy 1 of the artificial agents, 0.99, is larger than that of the human subjects, 0.72; for the minimum strategy rate, both of them get the highest rate, 1.0. For the case of $b = 0$, in the experiment with human subjects, the value of the gain coefficient $a$ is set at $a = 0.1$, which is slightly different from the setting of the simulation. The choice rate of the payoff dominant strategy 7 of the artificial agents is 0.99 which is close to the result of the human subjects, 0.956; the minimum strategy rate of the artificial agents, 0.891, is larger than that of the human subjects, 0.667. All in all, the result of the simulation is similar to that of the experiment with human subjects, and therefore the result of the simulation supports that of the experiment with human subjects. To be more precise, in the both results, the secure strategy is dominant when the risk of paying a large penalty is high; in the absence of such risk, the payoff dominant strategy is likely to be chosen. From the other perspective on this similarity, the simulation system successfully emulate the human behavior in the game.

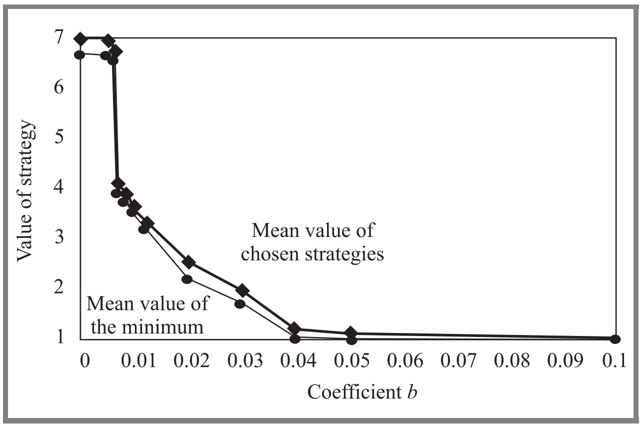


*Fig. 6.* Means of selected strategies and the minimal strategies in treatment $b$.

In Fig. 6, the means of the chosen strategies and the minimal strategies are shown; it can be found that these values are very similar. This fact means that at the steady state, most of the artificial agents choose the minimal strategies.

From Fig. 7, the payoff obtained by an agent decreases as the value of $b$ increases from $b = 0$. At the point of $b = 0.04$, the payoff is equal to the payoff of the secure strategy 1. Because when $b \geq 0.04$, the payoff of the secure strategy 1 grows large with the value of $b$, we can understand that most of the artificial agents choose strategy 1 in such a situation.

The realization rate of each individual equilibrium is given in Fig. 8. When $b = 0.1$, the secure equilibrium $(1, \ldots, 1)$ is realized at the rate of 0.89 in the steady state. Although as the value of $b$ decreases, the realization rate of the secure equilibrium decreases, it should be noted that in the interval $0.04 \leq b \leq 0.1$, only the secure equilibrium $(1, \ldots, 1)$ is realized. As the value of $b$ still decreases over 0.04,

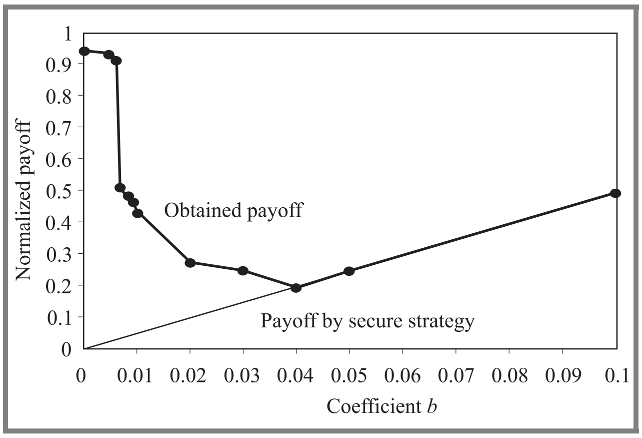


***Fig. 7.*** Normalized average payoff in treatment $b$.

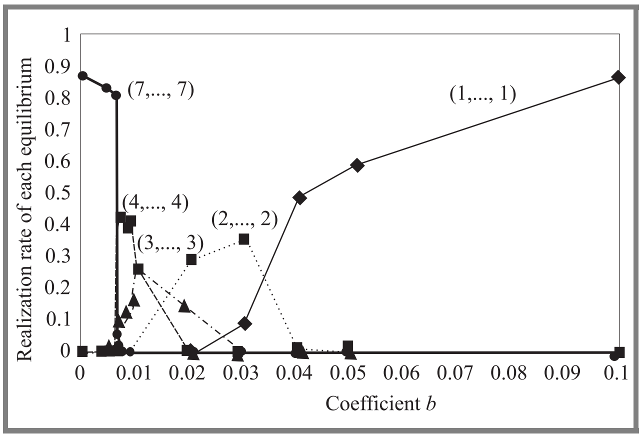


***Fig. 8.*** Realization rate of each individual equilibrium in treatment $b$.

the consecutive equilibria, $(2,\ldots,2)$, $(3,\ldots,3)$, and $(4,\ldots,4)$, can be found, but the realization rates of these equilibria do not exceed 0.5. When $b \leq 0.006$, the payoff dominant equilibrium $(7,\ldots,7)$ is realized at the rate larger than 0.8.

The gross realization rate of equilibria is shown in Fig. 9; it is found that at both ends of the horizontal axis, $b = 0$ and $b = 0.1$, the equilibria are likely to be realized. In the intermediate cases where effectiveness of the risk of paying

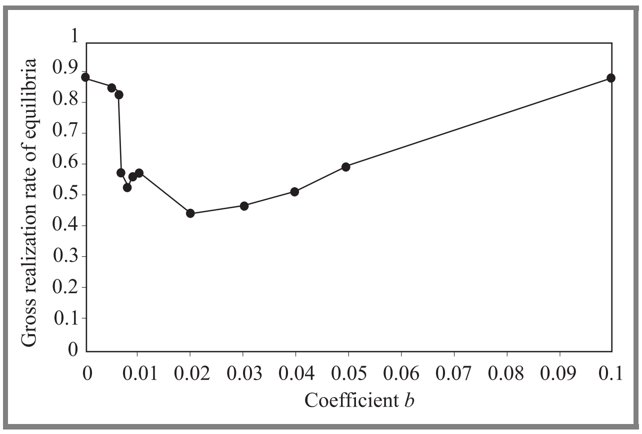


***Fig. 9.*** Gross realization rate of equilibria in treatment $b$.

the penalty is not clear, it becomes difficult for artificial agents to coordinate their strategies, and therefore the gross realization rate of equilibria descends, compared with the cases of $b = 0$ and $b = 0.1$.

As described above we have examined the result of the treatment on change of the coefficient $b$, and several characteristics of behavior of agents in the game can be found through the agent-based simulation based on the proposed general framework. Although only the two cases of $b = 0$ and $b = 0.1$ are performed in the experiment with human subjects, we conduct various runs of the treatment in the agent-based simulation and we obtain the following observations and findings.

1. In the games without the risk of paying any penalty, the artificial agents successfully coordinate their strategies and the payoff dominant equilibrium is realized.
2. In the games with the risk of paying a substantial penalty, coordination among the artificial agents is failed, but they suitably predict strategies of the opponents and the secure equilibrium forms.
3. The games with the risk of paying the intermediate penalty are likely to bring outcomes of disequilibria.
4. As the value of $b$ decreases, artificial agents shift choices of strategies stepwise from the secure strategy 1 to the payoff dominant strategy 7.
5. While the payoff dominant equilibrium is sensitive to increase of the value of $b$, the secure equilibrium is not so sensitive to decrease of the value of $b$.

# 4. Conclusions

In this paper, we have given a general framework of agent-based simulation for analyzing behavior of players in various types of games and economic situations. In our simulation model, the decision mechanism of an artificial agent is based on a neural network with several input data that the agent can know after playing a stage game, and the artificial agent chooses a strategy in accordance with the output of the neural network. The synaptic weights and thresholds characterizing the neural network of an artificial agent are revised so that the artificial agent obtains larger payoffs through a genetic algorithm, and then this learning mechanism develops agents with better performance. Finally, by giving a part of the simulation result of the minimum strategy coordination game, we demonstrate availability and effectiveness of the simulation analysis by the proposed framework of agent-based simulation.

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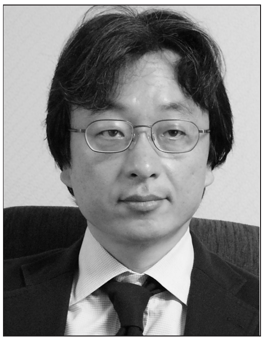

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