Paper

Optimal Spectrum Sensor Assignment in Multi-channel Multi-user Cognitive Radio Networks

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Abstract—Accurate detection of spectrum holes is the most important and critical task in any cognitive radio (CR) communication system. When a single spectrum sensor is assigned to detect a specific primary channel, then the detection may be unreliable because of noise, random multipath fading and shadowing. Also, even when the primary channel is invisible at the CR transmitter, it may be visible at the CR receiver (the hidden primary channel problem). With a single sensor per channel, a high and consistently uniform level of sensitivity is required for reliable detection. These problems are solved by deploying multiple heterogeneous sensors at distributed locations. The proposed spectrum hole detection method uses cooperative sensing, where the challenge is to properly assign sensors to different primary channels in order to achieve the best reliability, a minimum error rate and high efficiency. Existing methods use particle swarm optimization, the ant colony system, the binary firefly algorithm, genetic algorithms and non-linear mixed integer programming. These methods are complex and require substantial pre-processing. The aim of this paper is to provide a simpler solution by using simpler binary integer programming for optimal assignment. Optimal assignment minimizes the probability of interference which is a non-linear function of decision variables. We present an approach used to linearize the objective function. Since multiple spectrum sensors are used, the optimal constrained assignment minimizes the maximum of interferences. While performing the optimization, the proposed method also takes care of the topological layout concerned with channel accessibility. The proposed algorithm is easily scalable and flexible enough to adapt to different practical scenarios.

Keywords—channel accessibility matrix, cognitive radio network, optimal assignment, probability of interference, secondary users, spectrum sensing.

1. Introduction

Cognitive Radio Networks (CRNs) have become attractive solutions that are suitable for different applications [1]–[4]. At present, with a few exceptions, almost the entire radio spectrum available is regulated and allocated exclusively to licensed users who are called primary users (PUs), i.e. TV broadcasters, mobile communication service providers. In general, the radio spectrum is not fully utilized by its PUs. The unused spectrum bands with respect to time, space and frequency, are called spectrum holes [5]. These spectrum holes are utilized by secondary users (SUs) for communication-related purposes.

In a CRN, SUs have to detect the presence or the absence of a primary radio transmission within a specified channel, in order to identify spectrum holes. Efficient sensing of spectrum holes is an important prerequisite in a CRN. Spectrum hole sensing by measuring the received energy level over a certain time interval is a well-known basic method [6]. The received signal strength depends on existing noise levels, which are random and may vary considerably. Therefore, statistical methods for efficient spectrum sensing are used here.

When cooperative sensing is used to sense multiple primary channels, optimal matching of secondary sensors to primary channels is an optimization problem encountered in the permutation space. Several articles have already been published in this regard. In [7], binary particle swarm optimization (BPSO) is used. This method provides channel assignment that maximizes the total bandwidth utilization by SUs. BPSO is basically an iterative algorithm that may take relatively more time to achieve convergence. The binary firefly (BF) algorithm is used in [8]. Here, both bandwidth utilization and fairness among SUs are fitness functions that are maximized. In BF, authors use the coding method to reduce the search space. This is also an iterative algorithm. The ant colony system is described in [9]. The genetic algorithm is used in [10]–[11]. Non-linear mixed integer programming is used in [12]. A survey of various mathematical programming methods for solving channel selection in CRNs is given in [13].

2. Basic CRN Model

Consider a multi-channel multi-user CRN, over a certain geographical area, with M licensed primary transmitters (PTs) designated as PT(1), PT(2), ..., PT(M). The non-overlapping radio frequency channels used by these M PTs are respectively designated as ch(1), ch(2), ..., ch(M).

Each transmitter uses a single primary channel. Each PT covers a certain spatial region designated as its coverage area as shown in Fig. 1. When PT(j) covers area(j) using ch(j), it means that the primary and secondary receivers within this area can receive the transmitted signal with a sufficient signal-to-noise ratio (SNR) for successful reception. In this context, when the radio signal from PT(j) covers area(j), we also say that ch(j) covers area(j) for $j = 1$ to M.
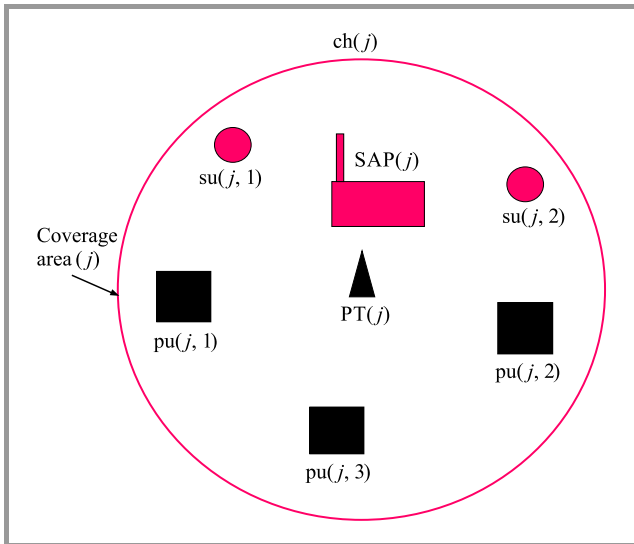

Fig. 1. Layout of a primary wireless channel ch(j) with SUs and SAP(j).

In Fig. 1, the coverage area(j) is indicated by the enclosing circle designated as ch(j). SUs within coverage area(j) are su(j, 1), su(j, 2). pu(j, 1), pu(j, 2) and pu(j, 3) are the PUs. We assume that the SUs within ch(j) are served by their own secondary access point SAP(j), for $j = 1$ to M.
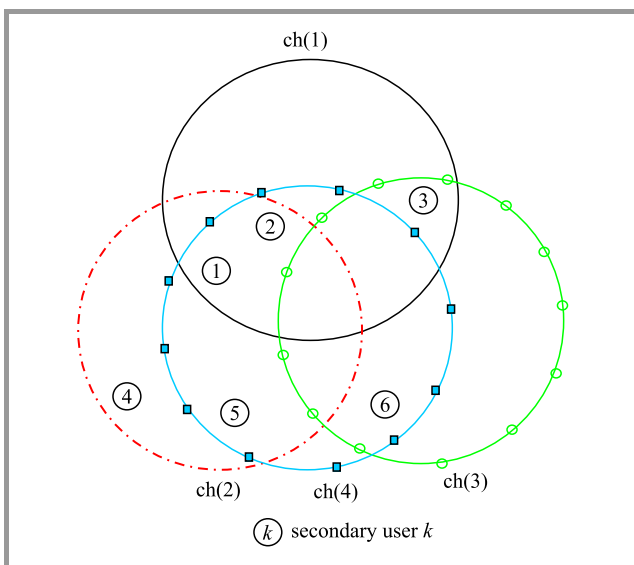

Fig. 2. Primary channels covering the SUs. (For color pictures visit www.nit.eu/publications/journal-jtit)

2.1. Multi-channel Multi-user Layout

A multi-channel multi-user layout is shown in Fig. 2. The areas covered by individual channels are overlapping, while their frequency bands are non-overlapping. The SUs (also called nodes) are globally identified by 1, 2, ..., N and is simply the integer set $\{1 : N\}$. We use su(k) to denote the SU whose ID is k. The SUs are assumed to be static. In Fig. 2, the number of primary wireless channels is $M = 4$, and the number of secondary users is $N = 6$. The secondary access points are not shown here.

The subset of SUs covered by channel ch(j) is denoted by SS(j) for $j = 1$ to M. The subsets of SUs covered by different channels (subnets) are shown in Table 1. An SU can belong to more than one channel. Thus, in Fig. 2 SUs 1 and 2 belong to ch(1), ch(2) and ch(4).

Table 1
Channels and their corresponding subsets

Channel	Boundary color	Subset of SUs
ch(1)	Black ——	SS(1) = {1, 2, 3}
ch(2)	Red —·—·—	SS(2) = {1, 2, 4, 5}
ch(3)	Green —○—	SS(3) = {3, 6}
ch(4)	Blue —■—	SS(1) = {1, 2, 5, 6}

2.2. Channel Model

A specific wireless communication channel is a radio frequency band with center frequency f_c and a bandwidth BW. Each wireless transmitter has an isotropic antenna. We assume that the primary transmission power and its antenna gain are known directly or indirectly, so that the coverage area of each transmitter is made available to the secondary network. The propagation delay is neglected. Received power is calculated using the simplified two-ray ground propagation loss model [14] as:

$$Pr(d) = Pr(1) \cdot d^{-4} , \qquad (1)$$

where $Pr(0)$ is the received power in watts at a distance of 1 m from the source, d is the distance from the source to the receiver in meters, and $Pr(d)$ is the received power in watts. We assume that the SNR is above the minimum threshold of the receivers within the coverage area. For simplicity, we assume that all devices are static and the parameters of the system do not change with time.

2.3. Secondary Users or Cognitive User Nodes

These nodes are equipped with Software Defined Radio (SDR) [15], where transceivers can dynamically adjust their radio communication parameters. Each secondary node is fitted with special spectrum sensing circuits (spectrum sensors) to detect the presence or the absence of available primary channels.

2.4. Channel Accessibility Matrix

At a specified time t, an SU may have access to several primary channels. This information is represented by the Channel Accessibility Matrix $\mathbf{CA}(t)$ [16], sized $M \times N$. The rows represent the channels, while the columns represent the SUs. $\mathbf{CA}(t)$ is a binary matrix and its element $ca(j, k, t)$ at row j and column k at time t is defined as:

$$ca(j, k, t) = \begin{cases} 1 & \text{if } su(k) \text{ is within the communication} \\ & \text{range of } ch(j) \\ 0 & \text{otherwise} \end{cases} \quad . \quad (2)$$

For the configuration shown in Fig. 2, matrix $\mathbf{CA}(t)$ is given by the values as shown in Table 2. The element $\mathbf{CA}(t)$ depends on the geographical locations of SUs and on the locations of primary base stations. For non-mobile SUs, $\mathbf{CA}(t)$ is independent of t. Then, we use the symbol $\mathbf{CA}$ instead of $\mathbf{CA}(t)$. Matrix $\mathbf{CA}$ represents the topological information.

Table 2
Channel accessibility matrix elements for the topology shown in Fig. 1

	su(1)	su(2)	su(3)	su(4)	su(5)	su(6)
ch(1)	1	1	1	0	0	0
ch(2)	1	1	0	1	1	0
ch(3)	0	0	1	0	0	1
ch(4)	1	1	0	0	1	1

2.5. Channel Sensing Model

For a given primary channel, we introduce two mutually exclusive and exhaustive hypotheses H(0) and H(1) as:

- H(0) – the channel is not occupied by the PU and it is available for SUs,

- H(1) – the channel is being engaged by the PU and is not available for SUs.

The received signal sample at the specific SU is given by [6]:

$$y(i) = \begin{cases} \eta(i) & : \text{H}(0) \\ s(i) + \eta(i) & : \text{H}(1) \end{cases} \quad . \quad (3)$$

Here, $\eta(i)$ is the noise sample, $s(i)$ is the received signal sample and $y(i)$ is the resultant, for $i = 1, 2, \ldots, n$, where n is the number of signal samples. The decision variable W at the sensing receiver after normalization of $y(i)$'s is given by [6]:

$$W = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{y(i)}{\sigma} \right)^2 \quad . \quad (4)$$

Here, σ is the standard deviation of the noise samples. Let T be the detection threshold, which is assumed to be known. If $W > T$, then the SU declares the presence of the primary channel, else its absence. Because of the dynamic nature of the radio propagation environment and the ambient noise, $W > T$ is a probabilistic event.

2.6. Probability of Detection and Probability of Interference

The conditional probability of detection under H(1) is defined as:

$$P_D = \text{prob}\big(W > T | H(1)\big) \quad . \quad (5)$$

The conditional probability of interference with the primary channel [6] is:

$$P_I = \text{prob}\big(W \leq T | H(1)\big) \quad . \quad (6)$$

P_I is the probability that the spectrum sensor at that SU decides that the primary channel is free while it is actually occupied. P_I is same as the probability of misdetection [17]. Note that $P_D + P_I = 1$. The ideal case is $P_D = 1$ and $P_I = 0$. A large value of P_I means more interference with the PU. The P_I value of a specific SU with respect to a specific PTs depends on the statistical properties of W, which depend on several factors, such as distance between the secondary sensor and the transmitter, transmitter power, radio environment, etc.

2.7. Probability of Interferences in Multi-channel Multi-user Layout

Our multi-channel multi-user optimization problem has M primary channels (transmitters) and N SUs. In general, $N \geq M$. The spatial layout is such that multiple SUs can access signals from multiple PTs, as shown in Fig. 3.
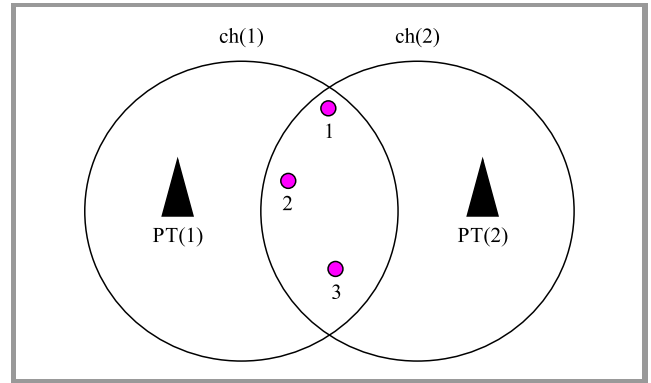

Fig. 3. Multi-channel multi-user layout ($M = 2$, $N = 3$).

Let $P_I(j, k)$ represent the probability of interference with the PT(j) by SU(k). This occurs when the SU(k) fails to detect ch(j) under H(1). Therefore, from Eqs. (5) and (6):

$$P_I(j, k) = 1 - P_D(j, k) \quad . \quad (7)$$

We assume that the $P_I(j, k)$'s values are known (either by calculations or estimation) for $j = 1$ to M and $k = 1$ to N. The collection of $P_I(j, k)$'s in a matrix format is denoted by the $\mathbf{PI}$ matrix, where j is the row and k is the column.

Example 1. As an example, $P_I(j, k)$ values for $M = 2$ and $N = 3$ are shown in a matrix form in Table 3.
In this example, all sensors can access all primary channels, and all $\mathbf{CA}$ elements are ones.

Table 3
Example of a **PI** matrix

(j, k)	1	2	3
1	0.45	0.1	0.2
2	0.35	0.3	0.4

To improve channel detection capability, one or more sensors can be assigned to detect a single channel [1]. Let $SS(j)$ be a set of sensors simultaneously assigned to sense $ch(j)$, same as $PT(j)$. Here, $SS(j)$ is a subset of SUs. Theoretically, all possible simultaneous assignments will form the power set of $\{1 : N\}$, that is, $\mathcal{P}(\{1 : N\})$. The number of subsets in a power set is 2^N. Therefore, $SS(j)$ can be any one of the possible subsets out of 2^N, except for the empty set. These $SS(j)$ sensors send their sensed information to a fusion centre (FC). The FC determines the overall probability of detection of $ch(j)$ based on *hard decision combining*. There are several combining rules, such as AND, OR, the majority rule [1], etc. In the presented scheme, we use the OR combining rule.

We assume that sensor detection events are statistically independent. Therefore, the probability that all simultaneously assigned sensors fail to detect $ch(j)$, denoted by the *overall probability of interference*, represented by $OPI(j)$, is the joint probability due to the SUs that belong to the subset $SS(j)$. Hence, $OPI(j)$ can be represented using the multiplication rule for independent events as:

$$OPI(j) = PI\big[j, u(1)\big] \cdot PI\big[j, u(2)\big] \cdot \ldots \cdot PI\big[j, u(|SS(j)|)\big] , \quad (8)$$

where $u(1), u(2), \ldots, u(|SS(j)|)$ are the members of set $SS(j)$, and $|SS(j)|$ is the size of $SS(j)$ or the number of SUs in $SS(j)$.

Since $0 \leq PI(j, k) \leq 1$ for all j's and k's, the product term $OPI(j)$ also lies between 0 and 1. Thus, $OPI(j)$ is non-negative.

Equation (8) can be concisely expressed as:

$$OPI(j) = \prod_{u(i) \in SS(j)} PI\big[j, u(i)\big] . \quad (9)$$

3. Basic Objective and Constraints

The objective is to minimize the worst case interference at PTs due to SUs. The worst case scenario occurs when $OPI(j)$ is at its maximum, and the optimal assignment is one that minimizes the maximum of OPI. This is a minimax assignment problem [17]–[21].

From Eq. (9) one can see that the $OPI(j)$ value depends on the composition of $SS(j)$. The objective is to choose the $SS(j)$ set optimally, so that the maximum of $OPI(j)$ over j is minimized. Therefore, the objective function to be minimized can be expressed as:

$$OF_1 = \max_{j \in \{1:M\}} OPI(j) . \quad (10)$$

3.1. Assignment Matrix **X**

We introduce the assignment matrix **X** of size $M \times N$ with $N > M$. Its elements $x(j, k)$ are defined for $j = 1$ to M and $k = 1$ to N as:

$$x(j, k) = \begin{cases} 1 & \text{if } su(k) \text{ is assigned to } PT(j) \\ 0 & \text{otherwise} \end{cases} . \quad (11)$$

The objective is to determine **X** to minimize the objective function given by Eq. (10).

3.2. Constraints on **X**

There are several constraints on $x(j, k)$.

Removal of unassignable SUs: When $ca(j, k) = 0$, that assignment is impossible because $su(k)$ is outside the coverage area of $PT(j)$. To take care of this condition, we make $x(j, k) = 0$ (no assignment) when $ca(j, k) = 0$ and $x(j, k) = 0$ or 1 (can take any value) when $ca(j, k) = 1$. This can be expressed by the constraint as:

$$\big[1 - ca(j, k)\big] \cdot x(j, k) = 0 , \quad (12)$$

for $j = 1$ to M and $k = 1$ to N.

Equation (12) can be expressed in a matrix notation as:

$$(1 - \mathbf{CA}). * \mathbf{X} == 0 . \quad (13)$$

Here ". *" represents the element-wise multiplication in the Matlab notation.

Row sum of X: The number of 1s in row j of **X** represents the number of SUs assigned to $PT(j)$. We have to assign at least one SU (sensor) to each j, otherwise we cannot monitor that channel. Of course, we can assign more than one to reduce the probability of interference. Therefore, the row-sum of **X** should be greater than or equal to one:

$$\sum_{k=1}^{N} x(j, k) \geq 1 \quad \text{for} \quad j = 1 \text{ to } M . \quad (14)$$

This constraint can be expressed, using the matrices, as:

$$\mathbf{X} * \mathbf{E}_{N \times 1} \geq \mathbf{F}_{M \times 1} , \quad (15)$$

where $\mathbf{E}_{N \times 1}$ and $\mathbf{F}_{M \times 1}$ are column vectors of all 1s of size $N \times 1$ and $M \times 1$. The corresponding transposes are:

$$(\mathbf{E}_{N \times 1})^T = [1, 1, \ldots, 1] ,$$

$$(\mathbf{F}_{M \times 1})^T = [1, 1, \ldots, 1] ,$$

where N number of 1's, and M number of 1's.

Column sum of X: An SU assigned to, say $PT(j)$, has to monitor $PT(j)$ regularly. It cannot monitor other channels. Therefore, not more than one $ch(j)$ or $PT(j)$ can be assigned to each SU. In addition, when $N > M$, some SUs may be left out of assignment because of some other con-

siderations. Therefore, a sensor may be assigned either to one primary channel or to none, as:

$$\sum_{j=1}^{M} x(j,k) \leq 1 \quad \text{for} \quad k = 1 \text{ to } N > M \ . \tag{16}$$

This means that any column cannot contain more than one 1, i.e. the sum of columns is either 0 or 1. The constraint represented by inequality (16) can be expressed in the matrix notation as:

$$(\mathbf{F}_{M\times 1})^T * \mathbf{X} \geq (\mathbf{E}_{N\times 1})^T \ . \tag{17}$$

Linearization of the objective function: The objective function given by Eq. (10) contains OPI(j) which is a product term given by Eq. (8). Therefore, it is non-linear and its optimization is complex and may not converge. Hence, we convert the non-linear objective function into an equivalent linear format by taking the logarithm of OPI(j). From Eq. (8):

$$\log\big[\text{OPI}(j)\big] = \log\big[\text{PI}\big(j, u(1)\big)\big] + \log\big[\text{PI}\big(j, u(2)\big)\big] \\ + \log\big[\text{PI}\big(j, u(|\text{SS}(j)|)\big)\big] \ . \tag{18}$$

Since OPI(j) is non-negative, $\log\big[\text{OPI}(j)\big]$ is a monotonically increasing function of OPI(j). Therefore, maximization or minimization of OPI(j) is the same as maximization or minimization of $\log\big[\text{OPI}(j)\big]$, respectively. That is, if V is a vector of real numbers, $\log\big[\max(V)\big] = \max\big[\log(V)\big]$. Hence, the objective function given by Eq. (10) can be reformulated as:

$$\text{OF}_2 = \max_{j \in \{1:M\}} \big[\log\big(\text{OPI}(j)\big)\big] \ . \tag{19}$$

In the light of Eq. (10), Eq. (19) can be expressed as,

$$\text{OF}_2 = \log(\text{OF}_1) \ . \tag{20}$$

Obviously, minimization of OF$_2$ implies the minimization of OF$_1$.

Objective function in terms of X: Consider row j of matrix **X**. The ones represent the SUs assigned to that PT(j). If $x(j,k) = 0$ for certain k, then that PI(j,k) term is absent on the right hand side (RHS) of Eq. (18). On the other hand, if $x(j,k) = 1$, then, PI(j,k) term is present on the RHS. Therefore, Eq. (18) can be expressed as:

$$\log\big[\text{OPI}(j)\big] = \sum_{k=1}^{N} x(j,k) \cdot \log\big[\text{PI}(j,k)\big] \ , \tag{21}$$

for $i = 1$ to M.

Let us introduce matrix **Y** of size $M \times N$ whose elements are $y(j,k)$ to represent the product term on the RHS of Eq. (21) as:

$$y(j,k) = x(j,k)\log\big[\text{PI}(j,k)\big] \ , \tag{22}$$

for $j = 1$ to M and $k = 1$ to N. We can see that the RHS of Eq. (22) is an element by element product of matrices **X** and log(**PI**). Therefore Eq. (22) implies:

$$Y = X. * \log(\mathbf{PI}) \ . \tag{23}$$

Substituting Eq. (22) in Eq. (21) gives:

$$\log\big(\text{OPI}(j)\big) = \sum_{k=1}^{N} y(j,k) \ , \tag{24}$$

for $j = 1$ to M.

The RHS of Eq. (24) is the sum of row j of matrix **Y**. Therefore it can be expressed as:

$$\sum_{k=1}^{N} y(j,k) = Y(j,:) * \mathbf{E}_{N\times 1} \ . \tag{25}$$

In Eq. (24), $\mathbf{Y}(j,:)$ represents row j of **Y** in Matlab notation.
From Eqs. (25) and (24):

$$\log\big(\text{OPI}(j)\big) = Y(j,:) * \mathbf{E}_{N\times 1} \tag{26}$$

for $j = 1$ to M. Let *OPI* be the column vector made up of OPI(j)'s for $j = 1$ to M. Then Eq. (26) can be rewritten as:

$$\log(OPI) = Y * \mathbf{E}_{N\times 1} \ . \tag{27}$$

Now, $\max_{j\in\{1:M\}}\big[\log\big(\text{OPI}(j)\big)\big]$ is the same as the maximum of the column vector $\log(OPI)$. Therefore, Eq. (20) can be expressed as:

$$\text{OF}_2 = \max\big[\log\big(OPI\big)\big] \ . \tag{28}$$

From Eqs. (26) and (27):

$$\text{OF}_2 = \max\big(\mathbf{Y} * \mathbf{E}_{N\times 1}\big) \ . \tag{29}$$

Substituting **Y** from Eq. (23) in (29), we get OF$_2$ in terms of **X** as:

$$\text{OF}_2 = \max\big[\big(\mathbf{X}. * \log(\mathbf{PI})\big) * \mathbf{E}_{N\times 1}\big] \ . \tag{30}$$

Now, the optimization problem can be stated.
Minimize the objective function over the binary assignment matrix **X**, where the objective function given by Eq. (30) is:

$$\text{OF}_2 = \max\big[\big(\mathbf{X}. * \log(\mathbf{PI})\big) * \mathbf{E}_{N\times 1}\big]$$

Minimize OF$_2$

subjected to: $(1-\mathbf{CA}). * \mathbf{X} == 0$, as given by Eq. (13),

$\mathbf{X} * \mathbf{E}_{N\times 1} \geq \mathbf{F}_{M\times 1}$, as given by Eq. (15),

$(\mathbf{F}_{M\times 1})^T * \mathbf{X} \leq (\mathbf{E}_{N\times 1})^T$, as given by Eq. (17) .

This mini-max optimization problem can be solved using different methods mentioned in Section 1. Here, the binary integer programming is applied using the *intlinprog*() function from the optimization tool box in Matlab [22].

Once the optimal $\mathbf{X}_{opt}$ which minimizes OF$_2$ is obtained, the best assignment is found based on Eq. (11) as:

$$\left.\begin{array}{ll} su(k) \text{ is assigned to } PT(j), & \text{if } x_{opt}(j,k) = 1 \\ \text{No assignment} & , \text{ if } x_{opt}(j,k) = 0 \end{array}\right\} \ . \quad (31)$$

Once $\mathbf{X}_{opt}$ is found, the optimal value of $\mathbf{OPI}$ from Eqs. (27) and (23) can be obtained:

$$\log(\mathbf{OPI}_{opt}) = \mathbf{Y}_{opt} \cdot \mathbf{E}_{N\times 1} = \left[\mathbf{X}_{opt}.*\log(\mathbf{PI})\right]\cdot\mathbf{E}_{N\times 1} \ . \quad (32)$$

Then the optimized $\mathbf{OPI}_{opt}$ is:

$$\mathbf{OPI}_{opt} = \exp\left[(\mathbf{X}_{opt}.*\log(\mathbf{PI}))\cdot\mathbf{E}_{N\times 1}\right] \ . \quad (33)$$

The proposed algorithm called multi assignment is given as Algorithm 1 and is shown by examples 2 and 3.

Algorithm 1: Multi assignment

Input: CRN layout. Values of M and N with $N \leq M$, channel accessibility matrix $\mathbf{CA}$ and the probability interference matrix $\mathbf{PI}$.

Output: Optimal assignment matrix $\mathbf{X}_{opt}$ and the minimized maximum overall probability of interference $\mathbf{OPI}_{opt}$.

1. Formulate the multi-assign problem using Eqs. (30), (13), (15), and (17).
2. Solve multi-assign problem to get $\mathbf{X}_{opt}$ using binary integer non-linear programming provided by YALMIP.
3. Get the optimal assignment, using Eq. (31).
4. Get the optimal overall probability of interference, $\mathbf{OPI}_{opt}$ using Eq. (33).
5. Done

In example 2, $M = 4$ and $N = 6$. The $\mathbf{CA}$ matrix is taken as all ones. $\mathbf{PI}$ matrix of size $M \times N$ is taken (assumed to be given) as shown in Table 4.

Table 4
$\mathbf{PI}$ values of example 2

(j,k)	1	2	3	4	5	6
1	0.424	0.209	0.499	0.393	0.103	0.431
2	0.322	0.279	0.309	0.203	0.229	0.235
3	0.045	0.288	0.372	0.228	0.012	0.338
4	0.198	0.364	0.318	0.294	0.372	0.141

Here, $(\mathbf{E}_{N\times 1})^T = [1, 1, 1, 1, 1, 1]$ and $(\mathbf{F}_{M\times 1})^T = [1, 1, 1, 1]$. On solving the multi-assign problem for this $\mathbf{PI}$, using YALMIP we get the optimal assignment matrix $\mathbf{X}_{opt}$ whose elements are found to be:

$$\mathbf{X}_{opt} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \ .$$

The optimal assignment for ch(j) is given by the locations of 1's in row j of matrix $\mathbf{X}_{opt}$. Thus, the optimal

assignment is $\left\{(PT(1)\leftarrow 5), (PT(2)\leftarrow 2, 3), (PT(3)\leftarrow 1), (PT(4) \leftarrow 4, 6)\right\}$. The value of $\mathbf{OPI}_{opt}$ is found to be:

$$\mathbf{OPI}_{opt} = \begin{bmatrix} \mathbf{0.103} \\ 0.087 \\ 0.045 \\ 0.042 \end{bmatrix} \ .$$

Here the minimized max value of the objective function is 0.103. Any other combination would result in a higher value for the overall interference.

In example 3 the values are same as in example 2, except $\mathbf{CA}$, taken as shown in Table 2. Now, $\mathbf{X}_{opt}$ and the corresponding value of $\mathbf{OPI}_{opt}$ are found to be:

$$\mathbf{X}_{opt} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{OPI}_{opt} = \begin{bmatrix} \mathbf{0.109} \\ 0.103 \\ 0.126 \\ 0.074 \end{bmatrix} \ .$$

Here, the minimized max value of objective function is 0.209.

4. Alternative Objectives and Constraints

4.1. Minimizing the Total Number of Assigned SUs

Consider a case where the aim is to minimize the total number of SUs assigned, subjected to the condition that max($\mathbf{OPI}$) should be less than a certain given upper bound, say OPIUB. This condition can be expressed as:

$$\max(\mathbf{OPI}) \leq (\text{OPIUB}) \ . \quad (34)$$

Taking logarithm on both sides, Eq. (33) is rewritten as:

$$\log\left[\max(\mathbf{OPI})\right] \leq \log(\text{OPIUB}) \ . \quad (35)$$

Since log is a monotonic function, Eq. (27) can be expressed as:

$$\text{OF}_2 = \max\left[\log(\mathbf{OPI})\right] = \log\left[\max(\mathbf{OPI})\right] \ . \quad (36)$$

From Eqs. (36) and (35):

$$\text{OF}_2 \leq \log(\text{OPIUB}) \ . \quad (37)$$

From Eqs. (30) and (37):

$$\max\left[(\mathbf{X}.*\log(\mathbf{PI}))*\mathbf{E}_{N\times 1}\right] \leq \log(\text{OPIUB}) \ , \quad (38)$$

where constraint (38) represents (34) in terms of $\mathbf{X}$.

Now, the optimization problem is to minimize OF$_3$ where:

$$\text{OF}_3 = (\mathbf{F}_{M\times 1})^T * \mathbf{X} * (\mathbf{E}_{N\times 1}) \quad (39)$$

subjected to: $(1-\mathbf{CA}).*\mathbf{X} == 0$, as given by Eq. (13),
$\mathbf{X}*\mathbf{E}_{N\times 1} \geq \mathbf{F}_{M\times 1}$, as given by Eq. (15),
$(\mathbf{F}_{M\times 1})^T*\mathbf{X} \leq (\mathbf{E}_{N\times 1})^T$, as given by Eq. (17),
$\max\left[\mathbf{X}.*\log(\mathbf{PI})\right] \leq \log(\text{OPIUB})$, as given by Eq. (38) .

In Eq. (39), OF_3 simply gives the total number of 1's in $\mathbf{X}$ which is the total number of SUs *assigned* to monitor the primary channels (note that the total number of *available* SUs is N). Here, the minimization of OF_3 is called the min OF_3 method. Minimization of OF_3 also means the minimization of total energy consumption by the SUs.

In example 4, the $M = 4$, $N = 11$, $\mathbf{CA} =$ all ones. OPIUB is taken as 0.02. Then, the solution for $\mathbf{X}$ is found to be:

$$\mathbf{X}_{\text{opt}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

The actual optimized max(**OPI**) is found to be 0.0192. The minimum number of SUs assigned is found to be 8. As the OPIUB value is increased (relaxed), the minimum number of SUs required to achieve that target decreases.

In example 5, the $M = 4$, $N = 17$, $\mathbf{CA} =$ all ones. OPIUB is varied from 0.01 to 0.10 in steps of 0.01. The minimum number of SUs needed, designated as min SUs, is calculated using our method min OF_3 and the lazy method. In the lazy method, the SUs are assigned as they are available in the order su(1), su(2), and so on.

Table 5
Min SUs for different values of OPIUB

OPIUB		.01	.02	.03	.04	.05	.06	.07	.08	.09
Min SUs	Min OF_3	10	8	8	7	6	6	6	6	6
	Lazy method	16	14	13	13	11	11	9	9	7

The relationship between the min SUs needed and the OPIUB values is shown in Table 5 and the corresponding bar graph is shown in Fig. 4.
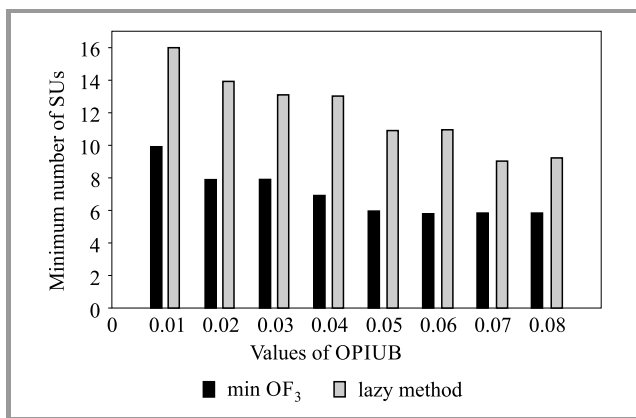

Fig. 4. Minimum number of SU's versus OPIUB.

In the case of min OF_3, the decrease in the minimum number of SUs could not fall below 6 because of other constraints (15) and (17). From Table 5, we see that min OF_3

is much better compared to the lazy method, in terms of the min SUs needed to meet the target.

4.2. Minimization of OF_2

In the next step, the minimization of OF_2 with pre-fixed number of SU's for each primary channel is analyzed.
In this case, constraint (15) which fixes the number of SUs for each channel can be reframed as:

$$\mathbf{X} * \mathbf{E}_{N \times 1} = \left[g(1), g(2), \ldots, g(M) \right]^T .* \mathbf{F}_{M \times 1} . \qquad (40)$$

Here, $g(j)$ is the number of SUs assigned to the primary transmitter $PT(j)$, for $j = 1$ to M. Obviously, to satisfy Eq. (39), sum of $g(i)$'s should be less than or equal to N. Other conditions and objective functions are same as (13), (17) and (30).

5. Comparison with other Methods

Minimization OF_2, as given by Eq. (30) can be solved by the greedy method [23], [24]. We compare the greedy method method with our multi-assign method. Here, $M = 4$ and $\mathbf{CA} =$ all ones. The number of available SUs N is varied from 10 to 17. The calculated values of minimized OF_2 (minimized max(**OPI**)) for these N's are shown in Table 6 and the corresponding values are shown in Fig. 5.

Table 6
Variation of Min OF_2 as N varies from 10 to 17

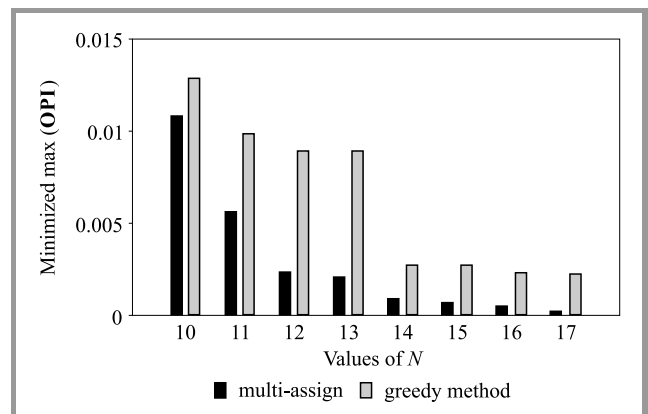| N	Min OF_2	
	Multi assign	Greedy method
10	0.01088	0.01292
11	0.00566	0.00992
12	0.00239	0.00896
13	0.00208	0.00896
14	0.00093	0.00277
15	0.00071	0.00277
16	0.00052	0.00233
17	0.00023	0.00227

Fig. 5. Minimum number of SUs versus OPIUB.

From Fig. 5 we see that our multi-assign method gives lower interference and is much better than the greedy method.

Next, the multi-assign algorithm is compared with the particle swarm optimization (PSO) method and the genetic algorithm (GA) method with reference to the time taken to solve the optimization problem. We took $M = 4$ and N is varied from 6 to 15. The result is shown in Fig. 6.
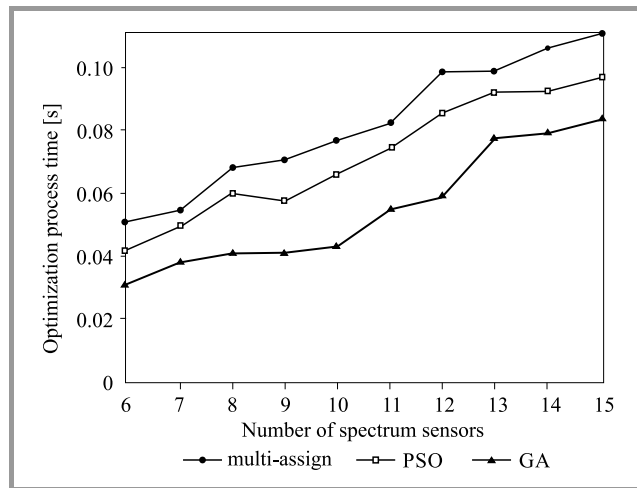

Fig. 6. Optimization process time vs. number of spectrum sensors.

From the result of Fig. 6, we find that the multi-assign algorithm is substantially faster compared to the PSO and GA algorithms. For a relatively large number of densely populated nodes, our method has not much advantage over other methods, because the complexity of our method depends on finding the best permutation of M sensors out of N.

6. Conclusion

A new method of optimal assignment, in a co-operative spectrum sensing CRN where multiple SUs and multiple primary channels are present, is described. The main contribution of this work is to convert the product term in the objective function to linear format by using the logarithm of product terms. Compared to the average of PSO and GA methods, the proposed method takes, on the average, 70% less time to calculate the optimal assignment when the number of SUs is relatively small. The minimized maximum interference value is lower in the presented method compared to the greedy algorithm or lazy methods.

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