Paper

# **Properties of the Multiservice Erlang's Ideal Gradings**

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Abstract—The design and optimization process of modern telecommunications networks is supported by a range of appropriate analytical models. A number of these models are based on the Erlang's Ideal Grading (EIG) model, which is a particular case of non-full-availability groups. A possibility of the application of the EIG model results from the fact that telecommunications systems show properties and features distinctive to non-full-availability systems. No detailed studies that would decisively help determine appropriate conditions for the application of the EIG model for modeling of other non-full-availability groups, that would be models corresponding to real telecommunications systems, have been performed. Therefore, this article attempts to find an answer to the following question: what are the prerequisite conditions for the application of the EIG model and when the model can be reliably used?

Keywords—Erlang's Ideal Grading, multiservice systems, traffic engineering.

# 1. Introduction

For the past number of years, we have been witnessing an exponential growth in the development of wired and wireless telecommunications networks [1], [2]. The constantly decreasing access services prices have made the number of network users (devices that make use of data transmission) growing rapidly. This, in turn, have effected in the increase in the amount of data sent over networks, particularly in wireless networks. Transmitting such a data poses an enormous challenge to telecommunications and computer networks and telcos. In order to use network resources in the best possible way operators are forced to implement advanced traffic management mechanisms, such as reservation [3]–[5], compression [6]–[12], priorities [13]–[15] or traffic overflow [16]-[19]. Those mechanisms influence advantageously the parameters of sent data streams and, in this way, make all resources of a network available in optimal way. The resource optimization process and network design are supported by and benefited from analytical modeling that allows characteristics of telecommunications systems to be determined on the basis of appropriate mathematical dependencies. The bulk of the models of telecommunications systems addressed in the literature of the subject uses either multiservice models of the full-availability group [20], [21] or limited-availability group [22]. An alternative solution for these groups of models, however, are models that make use of non-full-availability group models, i.e. Erlang's Ideal Grading (EIG). EIG is a particular (ideal) case of a non-full-availability group, since in this group a uniform load of group resources is assumed (which results from an appropriate number of load groups), despite the fact that individual traffic sources have no access to all resources of the group, but only to a part of it. The adoption of such assumption has made it possible to develop a simple analytical model of this group [23]. A. K. Erlang, who developed the structure of the EIG group and its analytical model for single-service traffic, noticed that this particular model could also be applied to approximate other non-fullavailability systems (those with non-uniform loads). It is worthwhile to add that the EIG group model has been successfully used for modeling single-service switching networks [24], [25]. Regrettably, the developments in technology and the subsequent appearance of multiservice systems caused the EIG model to be abandoned and left out in the early 1980s, since a multiservice model of EIG was nonexistent at the time. This unfavorable situation for the non-full-availability group was changed, however, when the model presented in [26], and derivations thereof, were proposed for multiservice traffic with differentiated availabilities, including non-integer availabilities [5], [27].

Present-day telecommunications systems can be viewed as non-full-availability systems. This assumption is confirmed by models of systems that use the EIG group model described in [18], [28], [29]. However, no available publications provide key information that would, in an unambiguous way, determine the range of versatile application possibilities for EIG in modeling other non-full-availability systems. The present article is an attempt to provide an answer to these questions.

The remaining part of the article has been structured as follows. Section 2 presents the issues related to nonfull-availability systems and an analytical model of the EIG group. Exemplary results are provided in Section 3, whereas Section 4 sums up the article.

# 2. Non-full-availability Systems

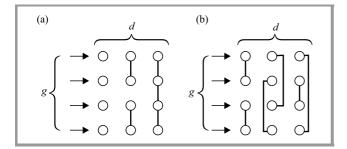
Non-full-availability systems are characterized by the fact that individual traffic sources do not have access to all resources of the system (expressed in  $BBU^1$ ), but only to a part of them. A good example of these systems are

<sup>&</sup>lt;sup>1</sup>The BBU is defined as the greatest common divisor of equivalent bandwidths of all call streams offered to the system [30].

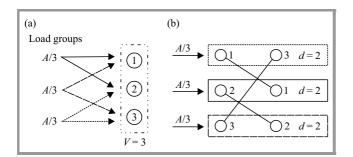
switching networks in which, due to the connecting paths set up in a given state of the network, a connecting path between a given input and output is not possible [24], [26].

Another example of modern telecommunications systems that can be treated as non-full-availability systems is the radio interface in a 3G mobile network. In this particular case, this non-full-availability stems from limitations in available resources of the interface imposed by noise and signal characteristics, e.g. interference from neighboring cells [31]. Yet another examples are the traffic overflow system in which non-full-availability results from limited access to resources to which connections are transferred (overflow) [29] and the VoD system [28].

In traffic engineering non-full-availability systems are modeled by non-full-availability groups. Each group of this type is described by three parameters: capacity V, availability d and the number of load groups g. Availability d defines the amount of resources of the group to which a traffic source has access. Traffic sources that have access to the same BBUs in the system create the so-called load group (component group). Conventionally, non-fullavailability groups are divided into: graded and uniform (homogenous) groups [32]. In graded groups, with an increase in the number of BBUs, the number of load groups that have access to this BBU increases (or remains unchanged). In uniform groups, each BBU is always available to the same number of load groups. Figure 1 shows both examples. A particular case of uniform groups is the Erlang's Ideal Grading - ideally symmetrical non-fullavailability groups. The latter group assumes all resources of the group to be uniformly loaded, while the number of



*Fig. 1.* Non-full-availability group for V = 7 BBUs, g = 4 and d = 3 BBUs: (a) graded group, (b) uniform group.



*Fig. 2.* Erlang's Ideal Grading with single-service traffic for V = 3 BBUs, g = 3 and d = 2 BBUs: (a) offered traffic distribution, (b) idea of availability.

load groups is equal to the number of possible choices *d* of resources, from among *V*:

$$g = \begin{pmatrix} V \\ d \end{pmatrix}.$$
 (1)

In Fig. 2a the example of EIG with single service traffic is presented. The capacity (V) of this grading is equal to 3 BBUs. The availability is equal to 2 BBUs. Figure 2b presents the idea of availability.

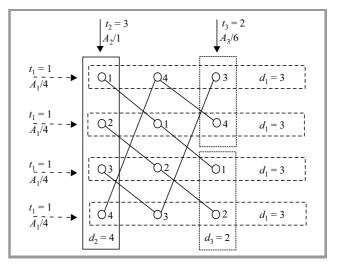
## 2.1. Model of Erlang's Ideal Grading with Various Availabilities

Let us consider Erlang's Ideal Grading [33] with various availabilities that is offered *m* independent Poisson call streams with the intensities  $\lambda_1, ..., \lambda_i, ..., \lambda_m$ . The service time of calls of particular classes has an exponential distribution with the parameters  $\mu_1, ..., \mu_i, ..., \mu_m$ . Therefore, traffic offered  $A_i$  by individual call streams can be determined on the basis following formula:

$$A_i = \frac{\lambda_i}{\mu_i}.$$
 (2)

The calls offered to grading are characterized by different values of demanded BBUs to set up a connection  $t_1, ..., t_i, ..., t_m$  and different availability  $d_1, ..., d_i, ..., d_m$ . This means that each class of calls is related to a different number of load groups:

$$g_i = \begin{pmatrix} V \\ d_i \end{pmatrix}.$$
 (3)



*Fig. 3.* Erlang's Ideal Grading: V = 4, m = 3,  $t_1=1$ ,  $d_1 = 3$ ,  $g_1 = 4$ ,  $t_2=3$ ,  $d_2 = 4$ ,  $g_2 = 1$ ,  $t_3=2$ ,  $d_3 = 2$ ,  $g_3 = 6$ .

Figure 3 presents an example of such grading. This EIG is composed of 4 BBUs (V = 4). The grading services m = 3 class of calls:  $t_1 = 1$ ,  $d_1 = 3$ ,  $t_2 = 3$ ,  $d_2 = 4$ ,  $t_3 = 2$ ,  $d_3 = 2$ . The number of load groups for particular class of call is equal:  $g_1 = 4$ ,  $g_2 = 1$ ,  $g_3 = 6$ .

According to the model [27], [33], the occupancy distribution P(n) is expressed by the formula:

$$nP(n) = \sum_{i=1}^{m} A_i t_i \big[ 1 - \sigma_i (n - t_i) \big] P(n - t_i) , \qquad (4)$$

where  $A_i$  is traffic offered to the group by a call of class i -Eq. (2) – and  $\sigma_i(n)$  is the conditional probability of transition for a traffic stream of class i in occupancy state n in the group

$$\sigma_i(n) = \frac{1 - \sum_{d-t_i+1}^k \binom{d_i}{x} \binom{V-d_i}{n-x}}{\binom{V}{n}},$$
(5)

where:

It should be stressed that the conditional probability of transition ( $\sigma_i(n)$ ) is combinatorial function of availability and it is independent of offered traffic.

The blocking probability for calls of class i can be determined on the basis of the following formula:

$$E_{i} = \sum_{n=d_{i}-t_{i}+1}^{V} \left[1 - \sigma_{i}(n)\right] P(n) \,. \tag{6}$$

#### 2.2. Non-integer Availability

Presented model in Subsection 2.1 enables authors to determine the values of blocking probabilities in EIG only for integer values of availability parameter. In [27] the model for non-integer value of this parameter was proposed. According to this model the blocking probability is calculated as follows.

Let us assume that for class *i* the availability parameter takes on non-integer values. This class of calls is replaced by two fictitious classes with integer values of availability  $(d_{i1}, d_{i2})$  and offered traffic  $(A_{i1}, A_{i2})$ . Values of these parameters are defined in the following way:

$$d_{i1} = \lfloor d_i \rfloor, \tag{7}$$

$$d_{i2} = \left\lceil d_i \right\rceil. \tag{8}$$

The traffic offered by the new fictitious call classes is respectively equal to:

$$A_{i1} = A_i [1 - (d_i - d_{i1})] = A_i (d_{i2} - d_i),$$
(9)

$$A_{i2} = A_i(d_i - d_{i1}), \tag{10}$$

where the difference  $d_i - d_{i1}$  determines the fractional part of the parameter  $d_i$ . Such a definition of the parameters  $A_{i1}$ ,  $A_{i2}$ ,  $d_{i1}$ ,  $d_{i2}$  means that the values of the fictitious traffic  $A_{i2}$  is directly proportional to the fractional part of the availability parameter, i.e. to  $\Delta_i = d_i - d_{i1}$ , while the value of the fictitious traffic  $A_{i1}$  is directly proportional to the complement  $\Delta_i$ , i.e. to the value  $1 - \Delta_i = 1 - (d_i - d_{i1}) = d_{i2} - d_i$  [27].

After replacing class with two fictitious classes: *i*1, and *i*2, with assigned values of availability and traffic intensity, it is possible to determine, on the basis of Eqs. (4)–(6), the blocking probabilities of all classes of calls, including the blocking probability of new classes of calls. The blocking probability of calls of class *i* for non-integer availability  $d_i$  can be determined in the following way:

$$E_i = \frac{A_{i1}E_{i1} + A_{i2}E_{i2}}{A_i} \,. \tag{11}$$

In the case of a higher number of classes with non-integer availabilities, each class of calls is replaced by two fictitious classes with the parameters determined by Eqs. (7)–(10). The maximum number of fictitious classes is equal to 2m.

## 3. The Results

In order to properly define the scope of the applicability of the EIG model for modeling of non-full-availability groups with a different number of load groups and different load in a single BBU as well as imprecisely estimated availability values, appropriate simulation experiments were carried out. For this purpose, a dedicated simulator was devised and successfully implemented. The simulator makes it possible to perform simulations for EIG groups, non-fullavailability groups as well as other telecommunications systems. The simulator was implemented in the C++ language according to the event scheduling method [34].

The input data for the simulator were the parameters that described the system, i.e. its structure, capacity and the parameters that describe the call stream offered to the system (the number of classes m, demands of individual classes  $t_i$  and availability  $d_i$ ). Additionally, it is also possible to determine the parameters of the simulation experiment itself, such as the total number of simulation series and the length of a single simulation series (expressed in the number of defined events). Results obtained in this way make a determination of 95% confidence intervals possible.

#### 3.1. Erlang's Ideal Grading vs. Full Availability Group

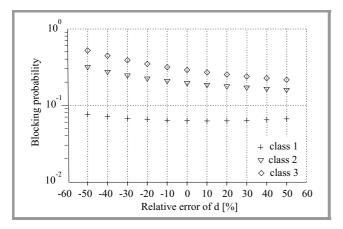
A full-availability group with multiservice traffic is the most frequently used model of telecommunications systems. The occupancy distribution in this group can be determined on the basis of the recurrent dependence known as the Kaufman-Roberts [20], [21] formula:

$$nP(n) = \sum_{i=1}^{m} A_i t_i P(n - t_i) .$$
 (12)

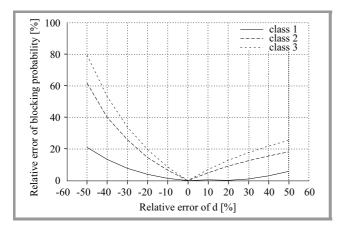
It should be noticed that this group is in fact a particular case of the EIG group, the fact that seems to be notoriously overlooked by researchers studying telecommunications traffic engineering. Observe that in the case where availability of all classes' is equal to the capacity of a considered system, i.e.  $d_i = V$ ,  $(1 \le i \le m)$ , Equation (4) will be simplified to Eq. (12) (parameter  $\sigma_i(n) = 1$ ). The multiservice EIG model, because of its general nature, is thus even a more universal and versatile tool supporting any analysis of modern telecommunication systems.

### 3.2. The Influence of the Evaluation on the Results

In order to use the EIG model to model present-day telecommunications systems it is necessary to determine availability values for all serviced traffic classes. Availability parameters are generally defined by the structure of a modeled system and offered traffic. In most cases, this availability can be determined on the basis of a relatively simple mathematical dependence [22], [35], [29], because the accuracy of obtained results directly derives from and depends on the precision of the evaluation of the value of individual availability parameters. To illustrate this problem, an experiment for an EIG group with the capacity of 30 BBUs servicing m = 3 classes of calls that demanded respectively  $t_1 = 1$ ,  $t_1 = 3$ ,  $t_3 = 5$  BBUs was carried out. The assumption was that the accurate availability values for the system were equal to:  $d_1 = 10$ ,  $d_2 = 15$ ,  $d_3 = 20$  BBUs.

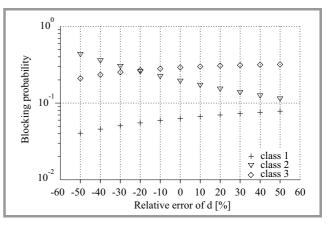


*Fig. 4.* Blocking probability as a function of relative error of availability.

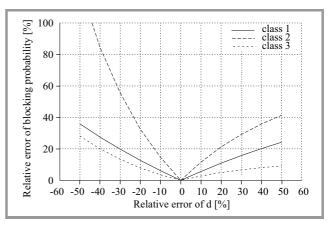


*Fig. 5.* Relative error of blocking probability as a function of relative error of availability.

Figure 4 shows the blocking probability as a function of relative error of availability. If the availability parameter is underestimated (the determined values are lower than the precise values), the blocking probability is higher than in the reference EIG group. If, on the other hand, the values of availability parameters are overestimated, the values of probability are lower than in the reference EIG group (the relationship is least evident in the class demanding the lowest number of BBUs to be served). This occurs regardless of the offered traffic value. In turn, Fig. 5 shows the relative error determined on the basis of the EIG model with the assumption that the availabilities of all classes were not accurately estimated. The identical nature of underestimation was adopted for all classes. In the second case (Figs. 6 and 7) presented here, erroneous estimation of the



*Fig. 6.* Blocking probability as a function of relative error of availability.

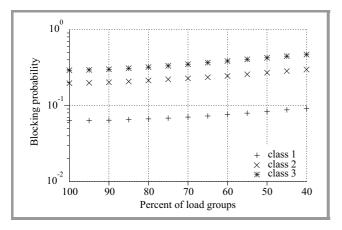


*Fig.* 7. Relative error of blocking probability as a function of relative error of availability.

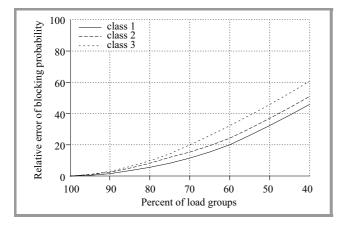
value of the availability parameter was to be found for only one class (class 2). As it is easy to observe, an erroneous estimation of availability parameters has a detrimental and negative influence on the correctness of results to be obtained. The results of blocking probability are better when the values of availability parameters are overestimated. For the group under investigation, acceptable results are obtained when it does not exceed about 20%. Presented re-

## 3.3. Other No-full-availability Groups

When considering real systems as non-full-availability systems, the fact that the number of load groups in such a system is lower than the number of groups in the EIG group should be taken into consideration. The next step then is to examine what influence the structure of the approximated system has upon the accuracy of obtained results. Figures 8–11 show the results for a non-full-availability

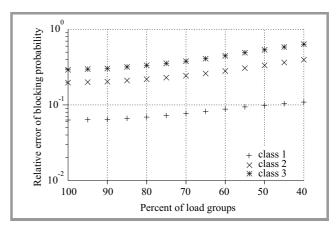


*Fig. 8.* Blocking probability as a function of number of load groups.

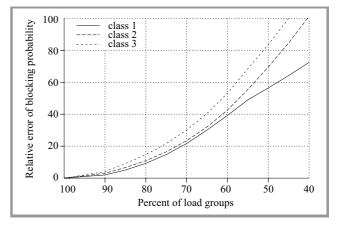


*Fig. 9.* Relative error of blocking probability as a function of number of load groups.

group with the capacity V = 20 BBUs that services two classes of calls demanding  $t_1 = 1$  and  $t_2 = 3$  BBUs, respectively. The availability is equal to  $d_1 = 10$  BBUs and  $d_2 = 15$  BBUs. The assumption is that presented real system has a structure of a homogenous group (Fig. 1a). The adoption of this assumption introduces the possibility that, despite a decreasing number of load groups, the load in each BBU is uniform. Hence, even when this decrease in the number of load groups is significantly high (in the considered case, acceptable results are still obtained



*Fig. 10.* Blocking probability as a function of relative error of availability.



*Fig. 11.* Relative error of blocking probability as a function of relative error of availability.

with the number of groups lower by even 40%), Figs. 8 and 9, the load in individual groups remains equal. A different situation, however, is to be observed with the case of a system that has a structure of a grading group (Fig. 1b). In this case, even a 30% change in the number of load groups results in a significant impact on the obtained results (Figs. 10 and 11). This phenomenon results from the occurrence of the uneven load of BBU in a group.

## 4. Summary

This article presents the results of an investigation into a broad range of potential applications of the EIG group model for modeling of telecommunications systems. Even though only a small excerpt of the case study is presented here, the results are robust enough to make a conclusion that the EIG group and its model are indeed ideal tools for modeling telecommunications systems, provided a proper evaluation (with a certain degree of accuracy) of the value of availability parameters can be executed. It has to be stressed that the number of load groups in a system has a lower influence on obtained results than an error in the estimation of availability parameters. As yet the authors have managed to find simple dependencies between the structure of a real system and the availability that characterizes particular classes of calls in the system. The only exception is the system with reservation. For this particular case, however, an algorithm has been developed that makes a precise evaluation of values of these parameters possible [27].

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## References

- [1] "Ericsson Mobility Report", Tech. Rep., Ericsson, 2014.
- [2] "Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update 2014-2019 – white paper", Tech. Rep., Cisco, 2015.
- [3] J. Roberts, "Teletraffic models for the Telcom 1 integrated services network", in *Proc. 10th Int. Teletraff. Congr. ITC 83*, Montreal, Canada, 1983, p. 1.1.2.
- [4] P. Tran-Gia and F. Hübner, "An analysis of trunk reservation and grade of service balancing mechanisms in multiservice broadband networks", in *Modelling and Evaluation of ATM Networks*, H. G. Perros, G. Pujolle, and Y. Takahashi, Eds., *IFIP Trans.*, vol. C-15, pp. 83–97. Amsterdam: North-Holland, 1993.
- [5] M. Stasiak and S. Hanczewski, "Approximation for multi-service systems with reservation by systems with limited-availability", in *5th European Performance Engineering Workshop*, N. Thomas and C. Juiz, Eds., *LNCS*, vol. 5261, pp. 257–267. Palma de Mallorca, Spain: Springer, 2008.
- [6] M. Głąbowski, A. Kaliszan, and M. Stasiak, "Modeling productform state-dependent systems with BPP traffic", J. Perform. Eval., vol. 67, no. 3, pp. 174–197, 2010.
- [7] I. D. Moscholios, J. S. Vardakas, M. D. Logothetis, and A. C. Boucouvalas, "Congestion probabilities in a batched poisson multirate loss model supporting elastic and adaptive traffic", *Annales des Télécommun.*, vol. 68, no. 5-6, pp. 327–344, 2013.
- [8] J. Kaufman, "Blocking with retrials in a completly shared recource environment", J. Perform. Eval., vol. 15, no. 2, pp. 99–113, 1992.
- [9] S. Rácz, B. P. Gerö, and G. Fodor, "Flow level performance analysis of a multi-service system supporting elastic and adaptive services", *Perform. Eval.*, vol. 49, no. 1-4, pp. 451–469, 2002.
- [10] M. Sobieraj, M. Stasiak, J. Weissenberg, and P. Zwierzykowski, "Analytical model of the single threshold mechanism with hysteresis for multi-service networks", *IEICE Trans. Commun.*, vol. E95-B, no. 1, pp. 120–132, 2012.
- [11] I. Moscholios, M. Logothetis, and G. Kokkinakis, "Connectiondependent threshold model: a generalization of the Erlang multiple rate loss model", *Perform. Eval.*, vol. 48, no. 1-4, pp. 177–200, 2002.
- [12] M. Stasiak, M. Głąbowski, A. Wiśniewski, and P. Zwierzykowski, Modeling and Dimensioning of Mobile Networks. Wiley, 2011.
- [13] M. Stasiak, P. Zwierzykowski, J. Wiewióra, and D. Parniewicz, "An approximate model of the WCDMA interface servicing a mixture of multi-rate traffic streams with priorities", in *Computer Performance Engineering*, D. Parniewicz, M. Stasiak, J. Wiewióra, and P. Zwierzykowski, Eds. *LNCS*, vol. 5261, pp. 168–180. Springer, 2008 (doi: 10.1007/978-3-540-87412).
- [14] L. Katzschner and R. Scheller, "Probability of loss of data traffics with different bit rates hunting one common PCM-channel", in *Proc.* 8th Int. Teletraff. Congr., Melbourne, Australia, 1976, pp. 525/1–8.

- [15] K. Subramaniam and A. A. Nilsson, "An analytical model for adaptive call admission control scheme in a heterogeneous UMTS-WCDMA system", in *Proc. IEEE Int. Conf. Commun. ICC 2005*, Seoul, South Korea, 2005, vol. 5, pp. 3334–3338.
- [16] A. Fredericks, "Congestion in blocking systems a simple approximation technique", *Bell System Tech. J.*, vol. 59, no. 6, pp. 805–827, 1980.
- [17] M. Głąbowski, K. Kubasik, and M. Stasiak, "Modeling of systems with overflow multi-rate traffic", *Telecommun. Syst.*, vol. 37, no. 1–3, pp. 85–96, 2008.
- [18] M. Głąbowski, S. Hanczewski, and M. Stasiak, "Erlang's ideal grading in diffserv modelling", in *Proc. IEEE Africon 2011*, Livingstone, Zambia, 2011, pp. 1–6.
- [19] M. Głąbowski, A. Kaliszan, and M. Stasiak, "Two-dimensional convolution algorithm for modelling multiservice networks with overflow traffic", *Mathem. Problems in Engin.*, vol. 2013, p. 18, 2013 (article ID 852082).
- [20] J. Kaufman, "Blocking in a shared resource environment", *IEEE Trans. Commun.*, vol. 29, no. 10, pp. 1474–1481, 1981.
- [21] J. Roberts, "A service system with heterogeneous user requirements – application to multi-service telecommunications systems", in *Performance of Data Communications Systems and their Applications*, G. Pujolle, Ed. Amsterdam: North Holland, 1981, pp. 423–431.
- [22] M. Stasiak, "Blocking probability in a limited-availability group carrying mixture of different multichannel traffic streams", *Annales des Télécommun.*, vol. 48, no. 1–2, pp. 71–76, 1993.
- [23] E. Brockmeyer, H. Halstrom, and A. Jensen, "The life and works of A. K. Erlang", *Acta Polytechnika Scandinavia*, vol. 6, no. 287, 1960.
- [24] V. Eršov, "Rasčët komutacionnych sistem metodom rasdel'nyh poter", in Sistemy massovogo obsluživaniâ i kommutacii, Moscow: Nauka, 1974, pp. 54–66 (in Russian).
- [25] V. Eršov, "Srednââ dostupnost' i rekurrentnyj rasčët poter' v kommutacionnych sistemah", in *Sistemy upravleniâ setâmi*, Moscow: Nauka, 1980, pp. 121–126 (in Russian).
- [26] M. Stasiak, "An approximate model of a switching network carrying mixture of different multichannel traffic streams", *IEEE Trans. Commun.*, vol. 41, no. 6, pp. 836–840, 1993.
- [27] M. Stasiak and S. Hanczewski, "A model of WCDMA radio interface with reservation", in *Proc. Int. Symp. Inform. Theory and Its Appl. ISITA 2008*, Auckland, New Zeland, 2008.
- [28] S. Hanczewski and M. Stasiak, "Performance modelling of video-ondemand systems", in *Proc. 17th Asia-Pacific Conf. Commun. APCC* 2011, Kuala Lumpur, Malaysia, 2011, pp. 784 –788.
- [29] M. Głąbowski, S. Hanczewski, and M. Stasiak, "Modelling of cellular networks with traffic overflow", *Mathem. Problems in Engin.*, vol. 2015, p. 158, 2015 (article ID 286490).
- [30] Broadband Network Teletraffic, Final Report of Action COST 242, J. Roberts, V. Mocci, and I. Virtamo, Eds. Berlin: Springer, 1996.
- [31] M. Stasiak, M. Głąbowski, and S. Hanczewski, "The application of the Erlang's Ideal Grading for modelling of UMTS cells", in 8th Int. Symp. Commun. Syst., Netw. Digit. Sig. Process. CSNDSP 2012, Poznan, Poland, 2012, pp. 1–6.
- [32] A. Jajszczyk, Wstęp do Telekomutacji. Wydawnictwa Naukowo-Techniczne, 1998 (in Polish).
- [33] M. Głąbowski, S. Hanczewski, M. Stasiak, and J. Weissenberg, "Modeling Erlang's Ideal Grading with multirate BPP traffic", *Mathem. Problems in Engin.*, vol. 2012, p. 35, 2012 (article ID 456910).
- [34] J. Tyszer, Object-Oriented Computer Simulation of Discrete-Event Systems. Kluwer, 1999.
- [35] S. Hanczewski and M. Stasiak, "Point-to-group blocking in 3-stage switching networks with multicast traffic streams", in *Proceedings* of First International Workshop (SAPIR 2004), P. Dini, P. Lorenz, and J. N. de Souza, Eds. LNCS, vol. 3126, pp. 219–230. Springer, 2004.



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