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by

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One of the best ways to hide something is to place it on display everywhere. That, in effect, is why the subject matter we are about to discuss has only recently come to our attention. It was all around us and thus remained hidden.

I am often asked the "meaning" of phase, as if phase were a disembodied property of nature. Phase is not a disembodied property, but is a shared partner in a certain type of description. It is one of two parts. Almost every audio and acoustic measurement has two distinguishable parts. We give these two parts special names, depending on how they show up, such as resistance and reactance, inphase and quadrature, real and imaginary, or amplitude and phase. This bipartition is not an accident, but expresses a fundamental relationship. Understanding this relationship can lead us to a better understanding of the meaning of our measurements. That is why it is worth while thinking about such things, even if it seems to verge on philosophy.

PATTERNS

If we stand back and take an overview of the mathematical descriptions of those parts of nature dealing with something that happens we will begin to notice the emergence of certain patterns. First, we are struck with the number of symbolic analogies which exist between this thing and that thing if we replace this parameter with that parameter. The "wave" equation is an example of this. I place quotes around the word wave because the concept of wave itself is an analogous form whose name appears in many diverse disciplines.

Certainly there are detail distinctions, such as one set of equations may be derived from what is called a scalar potential and the other may be derived from a vector potential. But the pattern is there - even to the pattern of a deriving potential. These are not metaphysical things, but are obvious once we begin to look. But a thing can be obvious only if we take notice of its existence.

A second thing we will notice is patterns of relationships which we call symmetries. Both the analogy and symmetry subjects are far too important and lengthy to discuss at this time. But I can recommend an excellent little book on the subject of symmetry, written by Hermann Weyl (1), for those who wish to pursue the subject further.

A third thing we can notice is what I call the pattern of twos. It overwhelms us in audio. If we measure the electrical impedance of a network or loudspeaker we have two parts, a resistance part and a reactance part. Not one, not three, but two. If we measure the free field frequency response of a loudspeaker the sound pressure has two parts, an amplitude part and a phase part. Again, not one, not three, but two. Even those measurements which seem to have only one part, such as instantaneous voltage as a function of time, are the real part of a two-part entity which Dennis Gabor introduced into communication theory and which we call the analytic signal.

Some length of time ago, in my own audio research, I became quite intrigued with this pattern of twos and set out to find why it existed. I felt that it was something which I ought to understand even though I could find no reference to it in the technical literature. What I found, and published (2), is the subject matter of this little discussion.

Only after my formal schooling was over did I realize that the neat three-step derivations found in textbooks are the result of a lengthy polishing and refinement process which begins after the original discovery. In the process of coming up with a simple formula which students can memorize and use to pass exams, the real meat of the matter - the thought process which led to this formula - is discarded. It is my opinion that this is one reason why some people, who ought to know better, will blindly use an equation well past its limit of applicability and get wrong answers. So what I am about to present is not just the neat result, which can be banged out in a few simple steps, but also the thought process which led to this result.

ENERGY

Suppose we want to describe something. Where do we start? Your opinion may differ from mine, but I prefer to start by establishing a frame of reference in which the description may be formed. I wish to distinguish the concept of coordinate system from that of frame of reference. We first have frame of reference, then we can try to establish ordination within that frame of reference. It may not be possible to establish a coordinate system, but let us ignore that situation for the purpose of this particular discussion.

If there is some coordinate system, even if we do not know the specific details of that coordinate system, then what is the most general thing we can state about descriptions which we may be able to form? That is a puzzlement, but if we are going to describe anything related to the real world (whatever that is), then there is one obvious statement we can make: the total energy of whatever we are describing is finite. It may be very large, but it is finite.

There is really not much else we can state as a general fact without bringing the specifics of the coordinate system into play. We cannot, for example, say that the description has a guaranteed dimensionality, or that it is measured in such-and-such units. We know that we can change a description from one system to another and modify dimensionality and units of measure. But it seems that we are not able to change the total energy of something simply by looking at it in terms of a different coordinate system. We can change the way energy is partitioned, and that is the true significance of a coordinate system, but we cannot change its total amount.

The way in which energy is expressed in a coordinate system is called energy density. Energy density will be defined for the purpose of this discussion as the way in which energy is distributed among the coordinates of a frame of reference. When we add up all the energy density over the whole system of coordinates, we have the total energy.

In cogitating this matter it seemed that if there was going to be some special relationship in nature which gives rise to this pattern of twos it might be associated with the partitioning of energy density. This did not seem too far fetched, because in the study of mechanics the energy of a system is considered to be composed of two terms: a potential energy and a kinetic energy. This seemed to be more than a casual clue, so I began searching for a general method of describing energy density in terms of a potential energy and a kinetic energy term.

PYTHAGORAS

Most of us are familiar with the Theorem of Pythagoras, which equates the square of the hypotenuse of a right triangle to the sum of the squares of the other two sides. In contemporary mathematical jargon the Pythagorean

Theorem is a relationship found in spaces of finite square measure. This means that if we add up the square of the length of functional values for all of the possible coordinates, the sum will be finite. That sounds temptingly like the relationship we know to be true for total energy density. And we can bring it into the form we want if we can somehow make total energy density proportional to the sum of the square of two physical parameters which are expressed in the system of coordinates. If there are two, and only two, such parameters, and if they always add up to give total energy, then we can possibly equate the squares of these parameters to potential and kinetic components of total energy.

We know that the sum of energy density over all coordinates will be equal to the total energy. Let the letter E stand for energy and s stand for whatever state variables, or coordinates, we choose for our description. Add up all E as a function of s and get the total E which is finite, the math symbolism for which is,

$$\int_{s} E(s)\, ds = E < \infty$$

In order to generate a function of finite square measure we only need to find that function which, when squared, is the energy density, or,

$$\left| \sqrt{E(s)} \right|^2 = E(s)$$

So this little bit of toedancing produces a function which is of finite square measure. The math name for this is that the function of of class L^2. This is not abstract foolishness, because we now have cast our problem into a form in which there has been a great deal of math experience in the past half century.

The next step in this mental process is to find some pattern of twos in L^2 spaces. I found it in a book which, regrettably, now receives little attention (3). Theorem 95, on page 128 of E. C. Titchmarsh's book on the Fourier Integral gives us the relationship. It was not derived for energy, but it is exactly what we need if we redefine our terms a bit.

Suppose I define the complex vector,

$$\sqrt{E(s)} = f(s) + i\, g(s)$$

where i is the operator signifying that the terms on the right are at right angles to each other, thus forming a right triangle relationship, and where the square root identifies that thing which when squared equals the energy density. I will leave off the plus and minus sign in front of the square root.

Titchmarsh proved that if $\sqrt{E(s)}$ is of Class L^2 (that is, you square it, add it all appropriately, and get a finite number), then not only are f and g of class L^2, but under quite reasonable constraints they are Hilbert transforms of each other.

Now let us take the last step. If the relationship is that of a right triangle, then by the Theorem of Pythagoras,

$$E(s) = [f(s)]^2 + [g(s)]^2$$

And from Titchmarsh, Theorem 91, when we add everything up the total energy is made up of one half part due to f and one half part due to g.

of textbooks for many years, but we never heard the cry. Now, starting from the basis that an identifiable scalar quantity, total energy, should be finite, this derivation showed that there should be two terms and that each term will be half of the whole. The association of these terms with our classic concepts of potential and kinetic energy not only makes sense in terms of any particular observation, but brings a universal relationship to bear on all such observations. So I call these two parts by the names: potential energy density and kinetic energy density.

What this boils down to can be stated thus: if we have a coordinate system, then the way in which the total energy is partitioned in terms of our system of coordinates is such that there are always two coordinate-dependent parts. These parts represent the partitioning into potential energy density and kinetic energy density. If the total energy is finite, then these two parts are never independent, but are such that they are respective squares of two terms which are Hilbert transforms of each other. The shape of one of these uniquely determines the shape of the other. This relationship was derived without regard for the specific coordinate system we may use and hence the partitioning is true for any valid frame of reference and for any dimensionality of representation.

PROPER MODELS

Before using this energy theorem in audio, there are several matters which need to be tidied up. One of these is that we must have finite total energy before the relationship is true. That does not seem to be an unreasonable assumption, but we must be very careful when using some of the signal concepts which we have been accustomed to for many years, since many of these are not of finite energy. The sine wave, for example. It is a great math model, easy to write and easier to manipulate. It just does not happen to be of finite energy in the L^2 sense. But we cavalierly apply it in our math because, for the duration of any measurement, it can be approximated arbitrarily closely by waveforms which we can generate in the lab. In order to use the math properly we must include terms which lop off the beginning and end of the non-realizeable sine wave and convert it into something that looks like the output of our audio generator from the time we close the switch until the time we turn the generator off. Common sense intrudes on the mathematically profane. These things we do so casually sometimes keep mathematicians busy for years, as in the case of Heaviside's operator notation and Dirac's pathological delta.

The point is that we use certain idealized models for physical observations, and these models serve us very well so long as we confine our observations to reasonable periods of time or conditions of measurement.

But these models may not represent realizeable conditions when we allow certain parameters, such as frequency, to go to zero or infinity. So we must add conditions which lop off the offending regions and bring the model closer to reality. This will be particularly true when we consider the relationship between resistance and reactance.

OTHER FORMS

Another matter to be considered is that of the other forms which the energy relationships may take. There will always be two parts. But there are a number of forms we can choose. The fundamental form will be that of the so-called real and imaginary description,

$$C = A + iB$$

This is the form which we conventionally use to describe impedance and admittance (for acoustic, mechanical, and electrical systems), and the analytic signal (for optics, mechanical, and electrical systems, particle velocity, sound pressure, and current). When in this form we know from what we have just derived that the two parts will be uniquely related to each other, at least within a constant term which does not have coordinate dependence.

It is possible to recombine the A and B to produce another two part form in which there is a magnitude part and angle part. The magnitude part is the positive square root of the sums of the squares of A and B and the ratio of B to C is the tangent of the angle. Because we have mixed up B and C, it does not necessarily follow that there is any unique magnitude and angle relationship as there is for A and B.

For convenience, there is a third way of expressing the two part energy relationship - a way which is familiar to those of us in audio technology. This form is a variation of the magnitude and angle representation in which exponential properties are used. The two parts thus produced are called the amplitude part and the phase part. The reasons for using the names amplitude and phase derive from the first use of this form and not, regrettably, from any thoughtful consideration of the more general properties which we now use. The exponential base which is commonly used is e, giving us the magnitude, α, and phase, φ, notation as follows,

$$c = e^{\alpha + i\varphi} = e^{\alpha} e^{i\varphi}$$

You will observe that I am deliberately not using coordinate dependence (such as amplitude and phase as a function of frequency) because we are now dealing in a general sense with relationships which transcend any particular set of coordinates.

The base e is a natural one to use for our intuitive concept of angle because when φ goes through 2π radians it comes back on itself. Unfortunately, our industry used the other half of this form, the amplitude part, long before it worried about angles, and developed a whole technology geared to the use of the base 10 and expressed amplitude in decibels. I will bow to convention and use decibels for α, instead of the more proper neper.

GLOBAL PERSPECTIVES

On a more thoughtful level, suppose we take a global perspective of what this energy theorem shows us. It seems to provide a rationale for the pattern of twos which we often encounter in our observations of natural processes. It implies that it is we, ourselves, who supply this pattern through our establishment of a coordinate system and systemmatic observations made within the framework of that coordinate system. There will be two parts to our observation because any exchange relationship involving energy density will include two things: density distribution with respect to the configuration of the coordinate system we chose, and density distribution changes in the neighborhood of each part of our coordinate system caused by the overall distribution of energy density, which indicates equilibrating flows of energy within our system.

There is another global consideration which falls out of this analysis. A transform is a recipe for converting one form of description into another form of description. It is the conversion between two different ways of describing the same thing. Often a transform is used to take a description from one type of coordinate and convert it to a description in another type of coordinate. But the Hilbert transform is a bit different in that it takes a description and puts it back into the same coordinate system. This is done by gathering up what happens everywhere, with the major contribution due to the gradient of what is happening at each point, and decreasing weight given to what the gradient is away from that point, then folding the whole mess back into a value for each coordinate location. This means that the thing I called potential energy density is an alternate description of the thing I called kinetic energy density. In terms of the names I have given these parameters: kinetic energy density is the way tendencies for dynamic changes are expressed in a potential energy density frame of reference. A complete description of a process will use both terms.

As an example of what this means, suppose we want to specify, as completely as possible, the description of a signal coming out of a power amplifier and developed across a load. We can establish a coordinate system which we call time and set up apparatus to measure the voltage drop across that load as a function of time. Suppose this apparatus is an oscilloscope. We will observe the wiggles on the oscilloscope screen and say that this is a time representation of the signal.

We are watching something related to potential energy density; voltage squared is proportional to the work which can be done, or is being done, at that place in our coordinate system (at that moment in time). There will be moments when the voltage is zero, as observed in this potential energy density frame of reference. Does this mean that the total energy density corresponding to these moments is zero? Not necessarily, it only means that the potential energy density, as expressed in this particular coordinate system, is zero at these places. It may happen that the total energy density is not zero, in which case all of the energy density at those moments is carried in the kinetic energy density term. The computation of this kinetic energy density is accomplished by taking the Hilbert transform of the voltage signal.

The pendulum can serve as a mechanical analogy of this situation. Potential energy density is related to the angle of the pendulum relative to local vertical, while kinetic energy density is related to its angular speed. If we observe the pendulum in a coordinate system involving angular position as a function of time, much as we observe amplifier voltage as a function of time, then there will be many moments when the pendulum bob in in its lowest potential energy position. If the bob is moving, the energy density at these moments will all be contained in the kinetic form, while the potential form is null. If the bob is not moving, then both potential and kinetic terms are at their minimum value, and that minimum value is zero.

But if we are looking at the pendulum in terms of angular position as a function of time, how can we establish whether the pendulum is moving at any moment? Easy, we find out what the position is for moments in time before and after the moment of our concern. And that is exactly what the Hilbert transform is doing for us. Except that the Hilbert transform solves the problem by knowing the answer. It does this by virtue of the fact that the Hilbert transform is a global-to-local map. It looks at everything that has happened in the past, is happening now, and that will happen in the future and folds that global view of energy density back into a number that tells us what the kinetic energy density component must be at that particular moment of concern in our observation in order to account for the whole energy pie. As I said, kinetic energy is potential energy, it is just that in order to know how much of the energy pie we are looking at when we measure the potential energy density, we need to take everything into account.

So if we want to have a more complete description of our amplifier voltage as a function of time, we must consider both the waveform we see on the scope plus a quadrature (right angle) term which is its Hilbert transform. This is called the analytic signal and has the property that its magnitude is the true envelope of the waveform we observe and its phase shows the exchange rate in the partitioning of energy density.

KNOWING THE FUTURE

But wait a minute. If the Hilbert transform extends forward as well as back, does this mean that we must know the future in order to specify the present?

That one will keep you awake nights if you are the kind who worries about details. See what happens when we begin to look at the reasoning behind our equations?

It is all too easy to write out an equation and assert that it is a thing called the analytic signal. Then I could have come in with my three step derivation that identifies the square of the terms of the analytic signal with total energy density and its partitioning into potential and kinetic energy density and laid it on you as though it is the equation which nature must solve. And, sure enough, if you plug numbers into the equation you will get answers which agree with observations of natural processes. But if that is all I did I would have performed a disservice, because it would be no better than another wobbly crutch for us to lean on, and possibly break when we need it the most. We have quite enough wobbly crutches in audio without creating another. So what I am striving to do in this little discussion is to get us to think about the basis for the equations which we use in audio.

What is happening is this: if we take into account all the agents of energetic stimulation of an audio system, and all the observed reactions of that system to this stimulation, then there are patterns of behavior in those reactions. It is we who, through our choice of frame of reference, establish the type of pattern that will be observed in that reference system. If we consider the entire range of that frame of reference, one distinct pattern that emerges is a pattern of twos for those dependent parameters which provide a description of the distribution of energy density in that frame of reference. Dynamism - things that change, alter, move - is the manifestation of energy. There will be a particular shape-relationship between the two parts of our observation if we limit our consideration to three conditions: total energy is finite, things are linear, and we use the whole frame of reference. It is this latter condition - we use the whole frame of reference - that gets us out of the bind of believing that our equations predict the future. They do not predict anything; they depend upon using the whole time axis, or frequency axis, or whatever coordinate we set up. If we apply the Hilbert transform relationship as a model to what has already happened, then we will get results which make sense. But as I pointed out in an earlier discussion the coordinate we call time and signify with the symbol t is a mathematical abstraction and is not the time of our ongoing experience.

In order that there be no misunderstanding, let me point out that we are considering energy density, which is the way energetic relationships are expressed in terms of a system of coordinates. Energy which is not uniquely expressible in a system of coordinates is not covered under this energy theorem. If we impress an electrical signal across a resistor the resistor heats up. Energy is passing from an electrical form into a thermal form. If the electrical signal was derived from a simple resonance circuit which had no external source of electrical energy, then the extraction of energy into heat will cause the available total energy of the resonance circuit, as expressed in the system of electrical coordinates of that resonance circuit, to drop at a uniform rate, and this is exactly what the Hilbert transform energy density relationship will show.

ENTER THE FOURIER TRANSFORM

Not surprisingly, there is a conceptual tie between the Hilbert transform and our old friend the Fourier transform. If you remember from our earlier discussion on this matter, the Fourier transform is that map, or recipe, which allows us to convert a description from one frame of reference to another in such a way as to keep the same dimensionality and coordinate orientation, but invert the units of measurement. It is that global-to-local map which takes each place of one frame of reference and associates it with a unique distribution we call a wave over the whole of a special alternative frame of reference. And, if you remember our discussion, we considered failure to recognize this, in conjunction the property of the Fourier transform to preserve dimensionality and orientation of alternative coordinates such that we can think our description is in one alternative when it is in the other, to be the root cause for the confusion which often arises when a description can take on the aspects of either a wave or a position depending on how we set up our descriptive terminology, but never precisely both a wave and a position in the same description.

If the real and imaginary parts of a description are related by Hilbert transform, then the value of the alternative Fourier transformed description will be zero for negative values of the coordinates of that Fourier description. Since time is the alternative to frequency, and negative values of time should have no signal components if that signal arises from a causal process (output after an input, not before; clocks run forward), then the frequency response corresponding to that signal must have real and imaginary parts which are Hilbert transforms of each other.

The shoe can be put on the other foot. If we define the time signal as a complex quantity, with real and imaginary parts related by Hilbert transform, then the frequency representation will only have positive frequency components. Historically, negative frequency components have bothered many persons. Gabor reached into quantum mechanics when he introduced the analytic signal into communication theory (5) because he was bothered by the fact that the average spectral frequency of a sine wave was zero, being composed of a positive frequency component and a negative frequency component. When the time dependence is represented by the analytic signal, the negative frequency component disappears and the average frequency now corresponds to the signal's angular frequency, which made things much more sensible. And in quantum mechanics itself, the negative frequency components of a wave representation caused problems in attempting to understand the significance of negative energy (energy is Planck's constant times the frequency). The great physicist Dirac met the matter head on when developing the relativistic theory of the electron and accepted such solutions as representing positrons, which were subsequently verified. So if any of us in audio become a bit perplexed at times about the meaning of negative frequency in our equations, we are not alone, the problem has been thrashed out many times before. Just remember, nature knows what it is doing. It is we who trip ourselves up in establishing simple math models and then haggling about the interpretation of selected parts of our own equations.

The vanishing of negative time components demands that the real and imaginary parts of the frequency response of any causal network be related by Hilbert transform. That is how it is conventionally approached in textbooks on network theory. When we consider the energy relationship presented in this discussion we can restate the same thing in a different way: namely, the requirement that descriptions of energy density be partitioned into two components which are related by Hilbert transformation demands that the Fourier transformed alternative have no negative spectrum values. Same thing, but now a bit more powerful because we do not have to assume that time cannot run backward; it is forced on us by the energy condition. Also, not being limited to any specific coordinate system, this keeps us on track for other ways of describing things involving energy.

THUS FAR, NO FARTHER

The importance of the Hilbert transform in audio is due to the fact that it is not limited to any particular set of coordinates and that it expresses an essential economy of form where energy distributions are concerned. It is a "thus far, no farther" signpost for energy density concentrations. Any attempt to concentrate the "work producing" part of a signal to its smallest spread in terms of any particular coordinate system can proceed only to the place where the relationship between the two appropriate parts is expressed by the Hilbert transform. We have seen that for any finite energy system the relationship between potential and kinetic parts is always at this most economic status. If we rejuggle our two parts to represent amplitude and phase of the total energy density, then the amplitude can be spread out in a less than optimum fashion with respect to the exchange relationship between the potential and kinetic components (phase), and thus the amplitude and phase need not be related by Hilbert transform. However, if we "scrunch" up the total energy density distribution, without changing the phase, there will eventually be a minimum concentration we can achieve for that partitioning, and then these two parts will be related by Hilbert transform. That is a general rule, and in audio we most often see it in the measurement of the frequency response, where such a condition is referred to as minimum phase.

Minimum phase, when used as a descriptor for the frequency response, refers to the condition where the net phase offset is the smallest possible for any network with that particular gain characteristic. Because the associated time response of a minimum phase network will also have the minimum spread for those signals which are sent through it, this class of network has also been given the name minimum delay by Robinson (6).

Minimum phase, in a frequency response measurement, means that the signal energy has the greatest concentration that is possible for the associated phase. This means that the equivalent time response of such a system has the least amount of time smear consistent with that frequency distribution of power spectral density. Thus far, no farther; if we want a frequency response flat within such and so, from this to that frequency cutoff, then the least amount of time smear will occur for such a response only when the phase response corresponds to the minimum phase condition for that amplitude. That is the physical meaning of minimum phase and is best understood, in my opinion, from considering the energy "pattern of twos".

ENERGY-TIME

Minimum phaseness, of course, is a general property and not limited to frequency response. When the time response of a system has minimum phase characteristics, the amplitude and phase of the analytic signal are related by Hilbert transform. This signifies that the distribution of time dependent total energy density has its greatest concentration consistent with the ongoing exchange rate between kinetic and potential components. The equivalent frequency spectral distribution will have the least amount of bandspread that is possible for any signal which has that time dependent total available energy density.

That is a minimization problem in modulation theory which can also be employed with some profit in the analytis of audio signals. In fact we use it for our loudspeaker measurements. The measurement I call the energy-time response is a direct plot of the logarithmic amplitude of the analytic signal response for an applied signal having a precisely controlled rectangular bandwidth from dc to 20 kHz. The amount of time smear is thus directly displayed and it is easy to spot what portion of the time smear is due to acoustic reflections, what portion is due to poorly damped resonance situations, etc. From the considerations presented in this discussion it is clear why this measurement is referred to as an energy measurement.

The other part of the two-part energy measurement is the phase of the analytic signal as a function of time. This is a measure of the exchange rate of energy density partitioning and can be used to infer the region of the equivalent frequency response where the energy of each delayed response aberration may be found. We do not plot this phase response in the loudspeaker reviews simply because it is a technically complicated display which requires some experience for interpretation.

Beside representing a long-winded explanation for the math basis behind the loudspeaker energy-time measurement, the energy theorem turns out to be very helpful in the general analysis of audio systems. In future discussions I intend to show examples of this.

THE IMPORTANCE OF PATTERN

There is obviously a tie between economy of energy form, as expressed in terms of frequency, and economy of energy form as expressed in terms of time. This is easy to understand because "timeness" and "frequencyness" bear a special form relationship to each other with regard to dimensionality, orientation, and units of expression. But when we convert a description upward in dimensionality it is no longer obvious what patterns of economy we can expect the energy to take within the coordinates of these higher-dimensional alternatives. There are, as Horatio was cautioned, things undreamed of when we expand our mental horizons to consider those higher-dimensional spaces which might serve as models for certain subjective perceptual processes.

Until very recently, we who perform objective tests on audio systems have played an overly simplified game. Mostly because we did not realize there was anything else, we set up tests and observations in either of two very highly specialized lower-dimensional coordinate systems: time and frequency. It did not bother most of us that the math we used to model audio processes in those specialized frames of reference was linear mathematics. We went ahead and applied this specialized math to everything, including situations involving distortion. Our narrow technological view has been, in the main, concentrated on specialized parameters, such as volts, or amps, or sound pressure, or mechanical force. These, after all, are the things important to a particular piece of audio equipment, such as a phono cartridge or amplifier. These are the local details of audio reproduction. But in a global sense I believe we may have missed the forest for the trees. The inexcusable use of linear math for nonlinear analysis aside (we will discuss that later), it is apparent, once we begin to look, that certain common patterns tend to emerge in our technical observations no matter whether those observations are of mechanical things or electrical things.

If the observations of something expressed in volts as a function of time has a characteristic form startlingly similar to observations of another thing expressed as mechanical force as a function of displacement, and if these patterns seem to emerge whenever we are dealing with natural processes, then perhaps, just perhaps, any other observation of natural processes, such as the perception of sound, may also contain patterns. And quite possibly, rather than getting hung up with the value of a particular parameter at a particular coordinate location, such as the spectral value of voltage at a certain frequency, we might look beyond such things to the relationships among parameters which could indicate a pattern of behavior. Look not at the trees but at the forest.

I do not mean to imply that the type of pattern that may exist in a higher-dimensional alternative will be like that which we can recognize in the lower-dimensional representations now used in objective analysis. But I do suggest that the search for patterns, and the physical reason why such patterns should exist, is a direction we might take in trying to understand the relationship between subjective perception and our observations of the ingredients which go to make up that perception. Pattern, after all, is quite important to music, language, and aesthetics. Why should it not be just as important in the processes leading toward those perceptions?

What I have attempted to do in this brief discussion is present one such pattern which I have been lucky enough to identify, along with my own thought processes that led me to this particular energy relationship. My intent in doing this is not to lay some esoteric math on you, but to stimulate thought about the physical basis of observation and perception. The perception of sound is not a spectator sport. We are all participants. Those who measure cannot separate themselves from those who listen. We need to think about all aspects of this business of audio (and everything which goes to make up our perceptions of audio - which is everything). This particular discussion on pattern does not end at this point; it has just barely begun.