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CHAPTER 6

INTRODUCTION

In math terminology the word "operator" is used to designate a certain type of procedure; a "doing something" to a "something else". When an audio signal is amplified, we can say that the amplifier "operates" on the signal to produce a larger version of that signal. If the amplifier inverts the signal we can say that an operation of inversion was performed. If a network rotates the phase of all components by ninety degrees, then we can say that the signal was operated on by a quadrature operator, because dual application of this procedure (first ninety degrees, then another ninety degrees) produces 180 degrees, or inversion. We give such an operator a special name; we call it the j-operator. The j-operator plays a very important role in audio analysis.

Whenever we "do something" to a signal to produce another version of that signal, we can analyze what we have done by using the math concept of operator. You can readily see that a "distorted" signal can be considered to be the "undistorted" signal which has been operated on by a "distortion operator". In this way we are able to begin approaching that most difficult of all audio subjects: distortion.

There are nice, neat operators that are easy to handle (and so we tend to use them in analyzing audio devices), and there are scary nonlinear operators that begin to model the things that go on in subjective perception. We do not use them. Yet. But we will. The study of operators can lead to an understanding of what happens to an audio signal that causes it to sound different.

COMMUTATION OF OPERATORS

What does it mean when two operators do not commute? If an audio signal is first processed by operator R and then by operator S, why should it happen that we may not get the same result by first processing that signal by operator S and then R?

Somehow, there is a grand separation of operators into (at least) two classes. Those that are painted red will commute with others that are also painted red, but not with those that are painted blue. And blue ones commute with blue ones, or do they? Can there be green ones that don't commute with either red or blue?

On face value, the fact that there is a distinction between operators indicates a form of linkage between such operators. Operators that do not commute must be linked in some way not evident in our ongoing paradigm. I submit that this link is the fact that operators which are alternative procedures under some map, M, cannot commute if applied in the same frame of reference. They know who they are for the simple reason that they are different versions of each other. They are alternatives.

I am a geometry-oriented person, and can better grasp concepts when they can be converted to pictorial displays. Figure 1 is one way I envision the distinction between “map” and “operator”.

Two alternative frames of reference are illustrated, called A and B. System A has coordinates x. System B has coordinates y. Operators are identified by capital letters and maps are identified by upper case script letters. Functions (signals) are identified by lower case boldface letters.

A signal in terms of coordinates x can be changed to another form in coordinates x by operator R. It takes a map to convert a signal in terms of x into a signal in terms of y. There are two ways by which a signal a(x) can be converted to a form d(y): we can first operate with R, then map to B; or we can map directly to B and then operate with S. Operator S is the M-transform alternative to operator R. Conversely, operator R is the N-transform alternative of operator S, where N is the inverse map to M. As an example, if M is the Fourier transform and R is the derivative operator, then S is multiplication times the coordinate and then times the j-operator.

There is a math convention when dealing with sequential procedures. If we have an a(x) and operate on it with R, we write Ra(x). If we then do something, such as S, on the resultant, we write it as SRa(x). When dealing solely with operators and maps, we can remove the functional form a(x) and simply write SR to indicate operation first by R then S.

The paradigm of alternative imposes an additional geometric distinction in the notation. This is illustrated in figure 2. In order to identify which alternative frame of reference we are in it is necessary to stack alternatives vertically. Level A is everything happening in alternative A, same for levels B, C, etc. Functions are nodes in this diagram. Operators move horizontally, since their source and destination forms lie in the same frame of reference. Maps, on the other hand, move vertically between alternatives. The direction of operator and map are shown by the arrows. The direction of inverse operators and maps is contrary to the arrows.

We can immediately see that if R and S are transform alternatives, then MR=SM; and it is not possible that MR=RM. This is an immediate proof that operators and maps cannot commute.

OPERATOR C

We have defined the alternatives as being C-alternatives under the map originally given as relation 8. Looking at figure 1, you see that I have shown the first M-transform alternative of operator R by letter S, but I have shown the second M-transform alternative of R by the same letter with a "hat" over it. This is my not-so-subtle way of indicating that there is a special relationship between double transform alternatives. To see why this is true, we need only rewrite the-double transform relationship as given in relation 20. Note that, the first integral is taken with respect to x, while the second is taken with respect to y. Even if the dimensionality of y is different than x, we will have a dual transfer version which can be interpreted in terms or coordinates x. This means that "R-hat" can be written as some new operator "C" which operates on "R". That is, R hat = CR.

Operator C does something special. Operator C does something to the form of the COORDINATES, but leaves unchanged the form of the function on which it operates. I call it "C" for coordinates, in order to distinguish it from other operators.

The C operator reverses the coordinates under the Fourier map. The C operator multiplies the coordinates by the imaginary unit under the simple quadratic TDS map.

As an example of the C operator under the Fourier map, consider the case where R is the derivative operator. The operator "R hat" is the derivtive with respect to a reversed coordinate; so that if "R" is d/dx then "R hat" is d/d(-x). The operator "S", on the other hand, would be ix in coordinates x, while the double transform version "S hat" would be i(-x). We can easily see that if the original function is a(x), the resultant function d(z) can be expressed as d/dz[f(z)] where z is -x.

Because the C operator is like a map in the sense that it only changes coordinates, C cannot commute with any operator R which does more than alter coordinates. This is shown in relation 21.

NONCOMMUTING OPERATORS

The problem of non-commuting operators is shown in figure 3. In A, a functional form a(x) is carried, through two operational procedures, R and S, into a functional form b(x). The question to be addressed is: if a(x) is sequentially operated on by procedure S and then R (the second path shown in system A), then under what conditions will this result in the same functional b(x)? It is clear that there is absolutely no clue to be had solely within frame of reference A.

Let me now set up two assumed conditions. First, operator R and operator S are M-transform alternatives as discussed earlier. Second, the node reached by SR coincides with the node reached by RS, so that R and S commute.

The operator algebra of figure 3 shows that in order for these conditions to be met, operator C must commute with operator S. But it cannot. With the meaning that any R and S which are related by a map M cannot themselves commute.

What prevents R and S from commuting is the condition that S and R are different versions of each other under map M. This is the same reason why a(x) and c(y) cannot be codetermined with-indefinite precision.

HOW MANY COMMUTING OPERATORS?

There are an infinite number of operators which we can use in any frame of reference. Are there more non-commuting operators than commuting ones, or are they split about equal?

Figure 4 illustrates one approach to answering that question. If we have two commuting operators, call then T and U, then we can always generate two more which will commute with either T or U. Operator V takes the intermediate form of T into the intermediate form of U, such that,

VT = U and TV = U,

so that V now commutes with T under the relation,

VT = TV.

Clearly, the inverse of V also commutes with U. Since V and T commute, we cn generate another, call it W, as shown in figure 5. And so on.

Starting with one commuting pair, we can generate a countable infinity of commuting pairs. That is, we can call TU pair number one, VW pair number two, and continue on a one-to-one basis through all countable numbers.

How about non-commuting? How big is their set? Surprisingly, it may be uncountably large. We can start with any operator;, such as V, and generate a (possibly countable) infinity of new operators that will not commute with it by mapping to alternative spaces. I don't know the answer yet, but a possible approach would be to show that if operators R and S do not commute, then each R produces an infinite number that also do not commute with S, and each of these produce a new infinite set that also do not commute. And so on and so on. Since commuting pair number one generates an infinity of non-commuting pair by that method, and pair number two generates another infinity of noncommuting, etc, it is possible that we could not put the noncommuting operators on a one to one basis with the countable numbers.

DISCUSSION

The impact of this prediction (that transform related operators cannot commute) is much deeper than I can go into in these brief discussions. I can only outline a few of the results, and then must go on to pure audio related matters.

First, we can write down, by inspection, whether many important operators will or will not commute. For example, we now know that any Fourier transform related alternative operators cannot commute. This includes everything of importance in contemporary quantum theory, since they are grounded in the use of the Fourier transform. Some operational procedures that can and those that cannot commute are outlined in figure 4.

Second, we can see the reason why non-commutation is intimately associated with the uncertainty relation; namely, they are both manifestations of alternatives under some map.

Third, and possibly the most significant, if we find two operational procedures that do not commute then we can suspect that they are transform alternatives under some as yet undiscovered map. Imagine that we are Flatlanders, yearning to discover a higher-dimensional perspective. Even though we live in Flatland, the existence of a higher-dimensional alternative is made known to us through the fact that, in Flatland, two procedures do not commute. It might be possible that we can find that map from a detail analysis of these Flatland consequences. (I say might because all we just demonstrated was the necessity, not sufficiency).

Before leaving this particular matter, and getting back to mainstream audio, I want to make a few points clear.

First, I use certain symbolic language in oder to call up new ways of thinking about scientific matters and to break the hold which our contemporary paradigm has on our imagination. I do not mean to impy that there are multi-dimensional beings "among" us. I do mean, as Pogo would say, that "They is us". We can "look" in other dimensions than the four which we commonly consider, namely space-time.

Second, I am deliberately facing head on the most difficult scientific problem of this century in order to demonstrate that we are dealing in a brand new ballgame with entirely new equipment. I am using a baseball bat to get the attention of my fellow technologists and, in effect, am saying "look, if we can come up with entirely new solutions to purely objective technical problems, then do not back away when we get to the really difficult part of using these tools to try and model subjective perception". If I merely waltzed into an operational expansion of multi-dimensional subjective perception, few persons might pay any attention; assuming, pehaps, that it was an interesting toy but had little "practical" use. Believe me, subjective audio is a lot harder to analyze than quantum mechanics, and things are going to get a lot more difficult to handle as we progress in these brief discussions.

Next time, we'll return to circuit theory and discuss the Delay Plane.

END OF CHAPTER 6