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# Numerical simulation of capillary dynamics of thin liquid layers at high-frequency oscillations of deformable substrate

A V Alexandrov<sup>1</sup>, I M Kuzmin<sup>1</sup> and L E Tonkov<sup>2</sup>

<sup>1</sup>Institute of Mechanics, Udmurt Federal Research Center, Ural Branch of the Russian Academy of Sciences, 34 T. Baramzinoy, Izhevsk, Russia, 426067

<sup>2</sup>Department of Computational Mechanics, Udmurt State University, 1 Universitetskaya Street, Izhevsk, Russia, 426034

E-mail: i.m.kuzmin@gmail.com

**Abstract.** The numerical simulation of fluid-structure interaction problems is topical both from an applied point of view (in connection with the need to create various technical devices) and theoretical ones (it's caused by the development of methods, algorithms, and HPC systems). Fluid-structure interaction is complex and requires a solution of both the elastodynamic and the hydrodynamics equations. The partitioned approach to solving fluid-interaction problems is one of the most common. Its allows solving each of the physical problems independently, using specific numerical schemes and a proprietary parallelism model. However, in this approach, a procedure for coupling solutions on the interface boundary is necessary to carry out. This article is considered the numerical simulation of forced oscillation of the liquid layer located on the surface of a bending plate. Forced vibrations of the cantilevered plate are excited by the piezoelectric element. At the excitation of vibrations, the thin layer of viscous liquids has flowed to antinodes of vibrations of the oscillated plate.

## 1. Introduction

The problem of forced vibrations of a liquid located on a vibrating substrate is of interest in view of the wide spreading of droplet dynamics in various processes and devices. It is of interest to investigate the interaction of the liquid with elastic bodies when bending vibrations are excited. In this case, it is possible to obtain a large amplitude in comparison with the amplitude of the longitudinal oscillations. Usually, for investigating the parametric (Faraday) instability in a liquid, undeformed plates are used, which vibrate with the same amplitude along the entire area of the contact with a liquid. The vibrations of bodies such as beams are bending vibrations with distributed amplitude. At high frequencies of the bending vibrations of beams, the length of the bending waves in them is comparable to the sizes of the region of the contact with a liquid layer and distributed vibrations can appreciably influence the liquid behavior. In our studies [1 – 4], we investigated the interaction of a thin plate that performs bending vibrations and liquids at the interphase boundary.

The problems of numerical simulation of the interaction between a deformed solid and a fluid are due to the development of methods, algorithms and computer systems. Fluid-structure interaction (FSI) problems are complex, as they require a solving both the equations of solid dynamics and the equations fluid dynamics. Partition approach [5] of the solving FSI problems allows each problem to solve independently by using a coupling on the interface boundary.



## 2. Mathematical model and numerical method

The vibrations of a liquid with a free surface lead to the oscillation of the surface of the liquid and to the parametric instability of the liquid surface and the formation of gravity-capillary waves. Two-dimensional standing waves create a periodic structure in the form of Faraday ripples.

Let us consider the problem of the dynamics of a layer of a viscous fluid lying in the field of gravity in the air on a horizontal deformable hydrophobic substrate. The substrate undergoes vertical oscillations under the action of an applied force varying in harmonic law  $z = A \sin(2\pi ft)$  where the amplitude  $A$  and frequency  $f$  are determined. One of the widespread approaches to solve the investigating fluid dynamic problem is representation a bulk as an immiscible incompressible two-phase mixture described by Navier-Stokes equations with the dynamic equilibrium condition at the interface and subsequent application the Volume of Fluid Method [6]. Let us denote by subscript  $i = 1$  the liquid phase and by subscript  $i = 2$  the air phase. The motion of the  $i$ -th phase is described by the Navier-Stokes equations:

$$\nabla \cdot u_i = 0, \quad (1)$$

$$\frac{\partial \rho_i u_i}{\partial t} + u_i \cdot \nabla \rho_i u_i = -\nabla p_i + \nabla \cdot \tau_i + \rho_i g, \quad (2)$$

where  $\rho_i$  – density of  $i$ -th phase,  $u_i$  – velocity field of  $i$ -th phase,  $p_i$  – pressure field of  $i$ -th phase,  $\tau_i = \mu_i(\nabla u_i + \nabla u_i^T)$  is viscous stress tensor,  $\mu_i$  – dynamic viscosity,  $g$  – acceleration of gravity. In the VOF method, the indicator function  $\alpha$  is evolved with an advection equation of the form:

$$\frac{\partial \alpha}{\partial t} + \nabla(\alpha u) = 0. \quad (3)$$

On the interface boundary  $\Gamma_0$ , the conditions of dynamic equilibrium are satisfied:

$$\begin{aligned} (\tau_1 - \tau_2)e &= (p_1 - p_2 + \sigma K)e, \\ u_1 &= u_2, \end{aligned} \quad (4)$$

where  $e$  – unit vector of the normal to the surface  $\Gamma_0$ ,  $K$  – curvature of the surface  $\Gamma_0$ ,  $\sigma$  – coefficient of surface tension. The wetting angle at the triple contact point is taken into account when calculating the curvature in its vicinity.

The capillary forces dominate in the considered flow. The algorithm for computing capillary term of the volume forces is proposed including iterative regularization when gradient and divergence discrete operators are calculated. An approximation of differential operators is performed by the finite volume method with the front artificial compression method, which used to calculate advection equation of indicator function  $\alpha$ .

The thin layers and droplets of the liquid on the vibrating flexible plate are bounded by the free surface due to the surface tension forces. Because of the small weight of the liquid, the surface oscillations in the liquid layers and droplets are mainly due to capillary oscillations, and the influence of the gravity force can be neglected.

The equations of motion of elastodynamic problem in the Lagrangian formulation, in the general case, take the form:

$$\begin{aligned} \rho \frac{\partial^2 u}{\partial t^2} &= \nabla \cdot (F \cdot S) + \rho f, \\ u(t_0) &= u^0, \\ \dot{u}(t_0) &= \dot{u}^0, \\ n \cdot (F \cdot S) &= p, \end{aligned} \quad (5)$$

where  $u$  – displacement vector,  $\rho, \nabla$  – density and divergence operator in the reference configuration,  $f, p$  – vector of mass and surface forces,  $F$  – deformation gradient,  $S = F^{-1} \cdot \sigma \cdot F^{-T} \det F$  – symmetrical stress tensor of Piola-Kirchhoff,  $\sigma$  – Cauchy stress tensor.

The elastodynamic problem taking into account geometric and physical nonlinearity is solved by the finite element method. Integration of elastodynamic problem equations is performed by the explicit scheme takes into account the dissipative properties of the system [7].

The taking into account of the influence of the liquid mass distribution on the plate's vibrations is based on the weak coupling algorithm. In this case, the coupling of solutions between the two problems is performed at the interface boundary between fluid and structure.

### 3. Results and discussion

The considered mathematical model makes it possible to reproduce the characteristic features of the liquid layer distribution on the plate surface.



**Figure 1.** The droplet of vacuum oil on the vibrating plate in the experiment described in [4] (a) and calculated (b), representation longitudinal bending of the vibrating plate, at a scale of 20:1.

The numerical results show that the liquid is distributed to the regions corresponding to the antinodes of the excited oscillation mode. Also, the proposed model is allowed us to evaluate the effect of viscosity on the formation of surface waves in the Faraday ripple type.

The considered algorithms for implicit coupling were used to numerical simulation of the physical experimental investigation of the interaction of the vibrating console plate with a layer of viscous liquid deposited on its surface [2]. Forced vibrations of a plate with a frequency of 4.5 kHz are excited by a piezoelectric element, with a cantilevered plate.

The figure 1 (a) shows the result of the experiment performed for the vacuum oil with and figure 1 (b) shows the result of numerical simulation. At the excitation of vibrations, viscous liquids applied as a thin layer on the plate surface initially flow to the plate surface areas with the antinodes of vibrations taking a convex form.

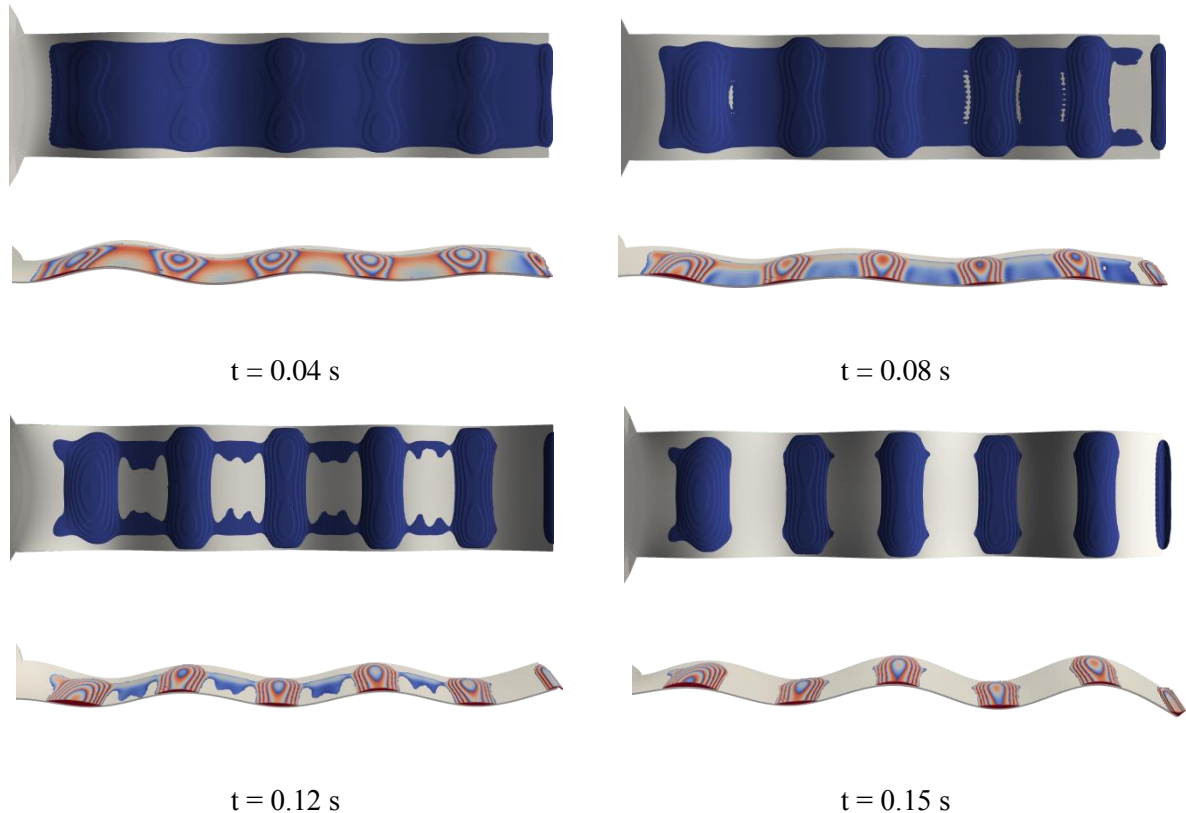
The coupled solution of the problems is carried out on hexahedral non-matching meshes with a size of 1300000 cells for the fluid dynamics problem and 23000 cells for the elastodynamic problem. The point-concentrated force is applied at the center of the piezoelectric element. It is important to note that the vibrations of a thin plate in the form of the superposition of longitudinal (see figure 2)) and transverse waves allow obtaining stable droplet patterns (see figure 2,  $t = 0.15$ ) which cannot be formed on an undeformed substrate.

Compared with the experiment in numerical simulation, the destruction of a thin liquid film between droplets formed at antinodes occurs more slowly. This is a feature of the numerical solution of the indicator function advection equation near the wall.

The study showed that the topological features of the distribution of the fluid are determined by the peculiarities of the bending vibrations of the plate.

The comparison of the results of numerical simulation with the experimental data allows us to

conclude that the numerical methods and algorithms used to describe the processes of interaction between the liquid layer and the vibrating plate quite accurately.



**Figure 2.** The distribution of liquid over the surface of the plate and the longitudinal bending of the plate at different time intervals in numerical simulation.

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