Scattering cancellation using dipolar arrays

Charlene M. Woelfel, Neil Jerome Egarguin, M.S., Daniel Onofrei, Ph.D.

ABSTRACT

This study explores a strategy for cancelling the scattered far field of a dipole interrogating a perfectly conducting plane. The interrogating dipole acts as a monostatic radar (i.e., an interrogator collocated with a receiver) and for the scattering cancellation effect we are using a distribution of infinitesimal dipoles located above the target. We assume that the interrogating dipole is located at (0,0,q). The goal is to determine the current feed required for the defending dipoles to cancel the scattered far field on a patch around the interrogating dipole while having a near zero effect on the field elsewhere. We mention that the additional effect of very small fields outside a small patch around the interrogating dipole could lead to other important applications such as field focusing, for example.

We present some numerical simulations of this strategy applied to the scattering cancellation of the interrogator's magnetic far field. We begin with the specific case of a vertical interrogating dipole, in which the cancellation was done using vertical dipoles and their appropriate line currents. We then consider the general case of an arbitrarily-oriented interrogating dipole. Here, cancellation was achieved using dipoles oriented in the three fundamental directions, **i**,**j** and **k**, with appropriate currents.

STATEMENT OF THE PROBLEM

Our goal is to determine the current feed needed for n electric dipoles to cancel the scattered far field around an arbitrarily-oriented interrogating dipole while having a near zero effect on the field elsewhere. The geometry in this problem is shown on the right.

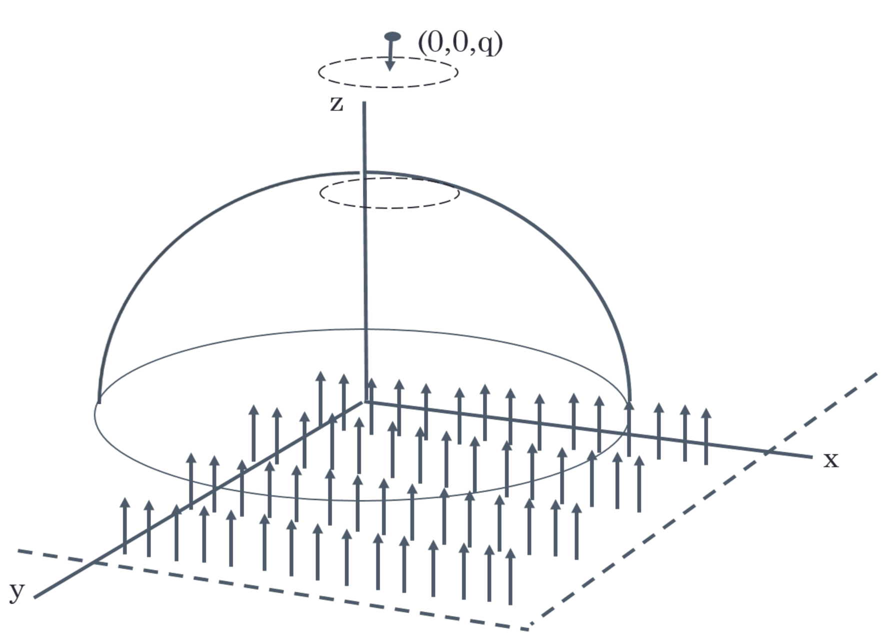

Figure 1. The geometry of the problem.

METHOD OF SOLUTION

We focus here on cancelling the magnetic far field. An electric dipole located in free space at a point **z** with dipole moment I**â** generates the magnetic field $\mathbf{H}_+$, given by

$$\mathbf{H}_+(\boldsymbol{x}) = \mathrm{I}\nabla_x\Phi(\boldsymbol{x},\boldsymbol{z}) \times \hat{\mathbf{a}},$$

where Φ is the fundamental solution to the Helmholtz equation in $\mathbb{R}^3$:

$$\Phi(\boldsymbol{x},\boldsymbol{z}) = \frac{e^{ik|x-z|}}{4\pi|x-z|}, x \neq \mathrm{z}.$$

In the presence of a perfectly conducting plane at z = 0, one should take into account the contribution of the image dipole to the magnetic field. We let $T = \mathrm{diag}(1,1,-1)$. Then the image dipole is located at $T\mathbf{z}$ with dipole moment I(-T**â**) and generates the field

$$\mathbf{H}\_(\boldsymbol{x}) = -\mathrm{I}\nabla_x\Phi(\boldsymbol{x},T\boldsymbol{z}) \times T\hat{\mathbf{a}}.$$

Thus, the magnetic field generated by the dipole located at z with dipole moment I**â** is:

$$\mathbf{H}(\boldsymbol{x}) = \mathbf{H}_+(\boldsymbol{x}) + \mathbf{H}\_(\boldsymbol{x}).$$

Moreover, the magnetic far field generated by this dipole is

$$\mathbf{H}_\infty(\hat{\boldsymbol{x}}) = \frac{ikI}{4\pi}\left[e^{-ik\hat{\boldsymbol{x}}\cdot\boldsymbol{z}}(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{a}}) - e^{-ik\hat{\boldsymbol{x}}\cdot T\boldsymbol{z}}(\hat{\boldsymbol{x}}\times T\hat{\boldsymbol{a}})\right],$$

where $\hat{\boldsymbol{x}} = \frac{\boldsymbol{x}}{|\boldsymbol{x}|}$ and $|\boldsymbol{x}| \to \infty$.

METHOD OF SOLUTION

Suppose that the interrogating dipole is located at $\mathbf{q} = (0,0,\tilde{z})$, $\tilde{z} \gg 1$ with dipole moment $\tilde{I}\hat{\boldsymbol{b}}$, where $\hat{\boldsymbol{b}} = \alpha\hat{\boldsymbol{\imath}} + \beta\hat{\boldsymbol{\jmath}} + \gamma\hat{\boldsymbol{k}}$ and $\|\hat{\boldsymbol{b}}\| = 1$. Its image dipole is located at $\tilde{\boldsymbol{q}} = T\boldsymbol{q}$ with orientation $-T\hat{\boldsymbol{b}} = -\alpha\hat{\boldsymbol{\imath}} - \beta\hat{\boldsymbol{\jmath}} + \gamma\hat{\boldsymbol{k}}$. The scattered magnetic far field is given by

$$\widetilde{\mathrm{H}}_\infty(\hat{\boldsymbol{x}}) = \frac{ik\tilde{I}}{4\pi}e^{-ik\hat{\boldsymbol{x}}\cdot\tilde{\boldsymbol{q}}}\left(-\alpha(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\imath}}) - \beta(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\jmath}}) + \gamma(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{k}})\right).$$

To cancel this field, we use three sets of n dipoles oriented along the 3 fundamental directions $\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$. Each set has the far field

$$\mathrm{H}_{\infty,i}(\hat{\boldsymbol{x}}) = \frac{ik}{4\pi}\left(\sum_{l=1}^n I_l^{(i)}\left[e^{-ik\hat{\boldsymbol{x}}\cdot\boldsymbol{z}_l^{(i)}} - e^{-ik\hat{\boldsymbol{x}}\cdot T\boldsymbol{z}_l^{(i)}}\right]\right)(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\imath}}),$$

$$\mathrm{H}_{\infty,j}(\hat{\boldsymbol{x}}) = \frac{ik}{4\pi}\left(\sum_{l=1}^n I_l^{(j)}\left[e^{-ik\hat{\boldsymbol{x}}\cdot\boldsymbol{z}_l^{(j)}} - e^{-ik\hat{\boldsymbol{x}}\cdot T\boldsymbol{z}_l^{(j)}}\right]\right)(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\jmath}}),$$

and

$$\mathrm{H}_{\infty,k}(\hat{\boldsymbol{x}}) = \frac{ik}{4\pi}\left(\sum_{l=1}^n I_l^{(k)}\left[e^{-ik\hat{\boldsymbol{x}}\cdot\boldsymbol{z}_l^{(k)}} - e^{-ik\hat{\boldsymbol{x}}\cdot T\boldsymbol{z}_l^{(k)}}\right]\right)(\hat{\boldsymbol{x}}\times\hat{\boldsymbol{k}}).$$

By superposition, this array of dipoles generates the magnetic far field

$$\mathrm{H}_\infty(\hat{\boldsymbol{x}}) = \mathrm{H}_{\infty,i}(\hat{\boldsymbol{x}}) + \mathrm{H}_{\infty,j}(\hat{\boldsymbol{x}}) + \mathrm{H}_{\infty,k}(\hat{\boldsymbol{x}}).$$

We want to find the current $I_l^{(\cdot)}$ for each of the $3n$ dipoles to cancel the field on a patch P around the north pole of the unit sphere $\mathrm{B}_1(\mathbf{0})$, that is solving

$$\mathrm{H}_\infty(\hat{\boldsymbol{x}}) = \begin{cases} -\widetilde{\mathrm{H}}_\infty(\hat{\boldsymbol{x}}), & \hat{\boldsymbol{x}} \in P \\ 0, & \text{elsewhere} \end{cases}.$$

Our strategy is to do term-by-term matching, i.e., the $\hat{\boldsymbol{\imath}}$ dipoles cancelling the scattered field's $\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\imath}}$ component, and so on.

RESULTS AND PLOTS

In the following numerical simulations, we assume the interrogating dipole to be at q = (0,0,1000) with current $\tilde{I} = 1$ and orientation $\hat{\boldsymbol{b}} = \frac{1}{\sqrt{3}}\langle 1,1,-1\rangle$. We discretize the unit hemisphere H into 10,000 points, then add 600 more points on the patch P = {(x,y,z) ∈ H : $x^2 + y^2 \leq 0.05^2$}. We arrange 30,000 dipoles (10,000 per orientation) on the square {(x, y, 0.05) : $-25 \leq x, y \leq 25$}.

We use Matlab to find the least squares solution of the three overdetermined systems. To make our solution less sensitive to noise, we apply Tichonov regularization.

The three sets of plots below show the results for each component of the field. The top row in each set shows a visual comparison of the real parts of the desired and generated fields. The bottom row depicts the pointwise error measurements: relative error on the patch and the absolute error elsewhere.

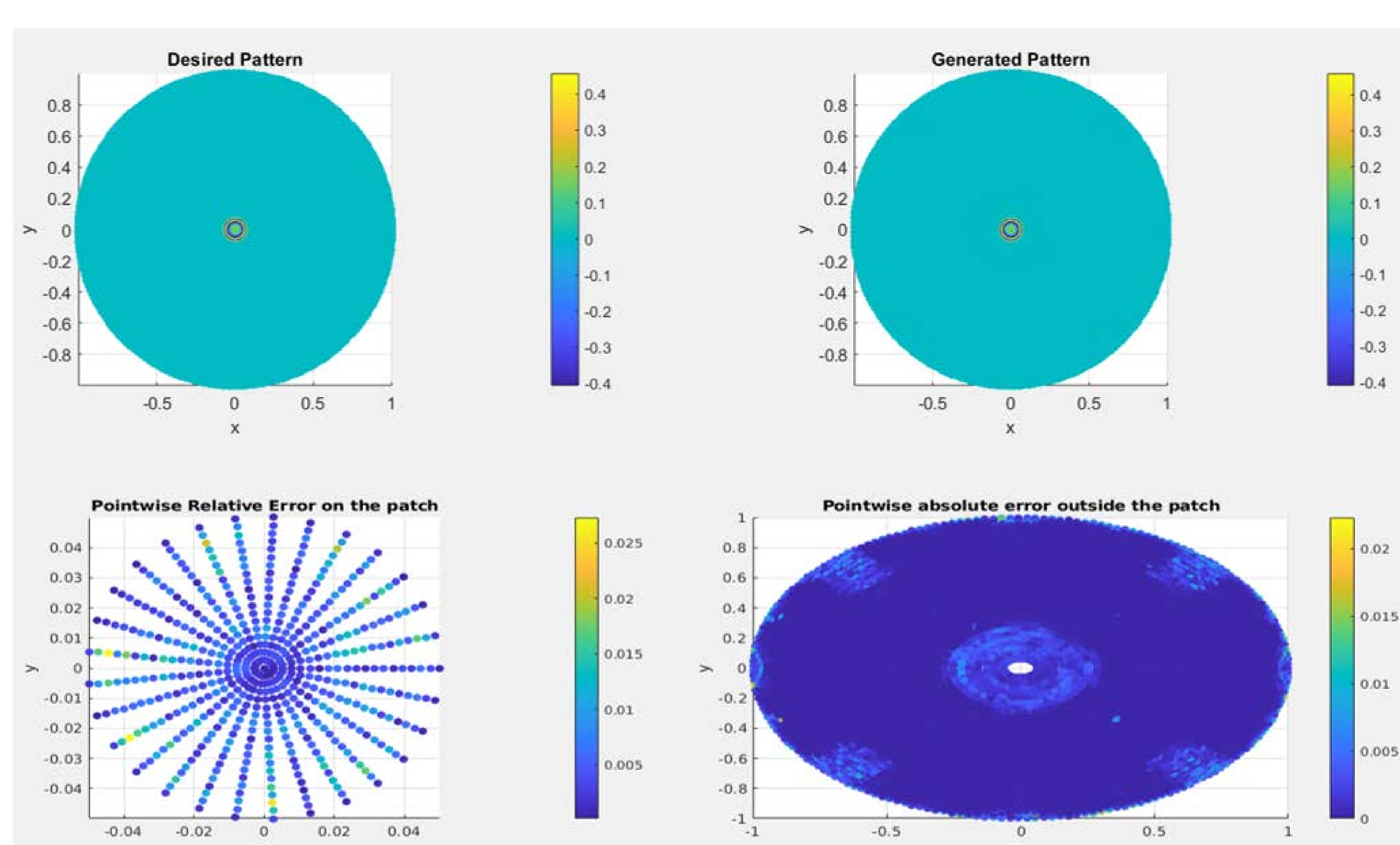

Figure 2. Results for the $\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\imath}}$ component.

RESULTS AND PLOTS

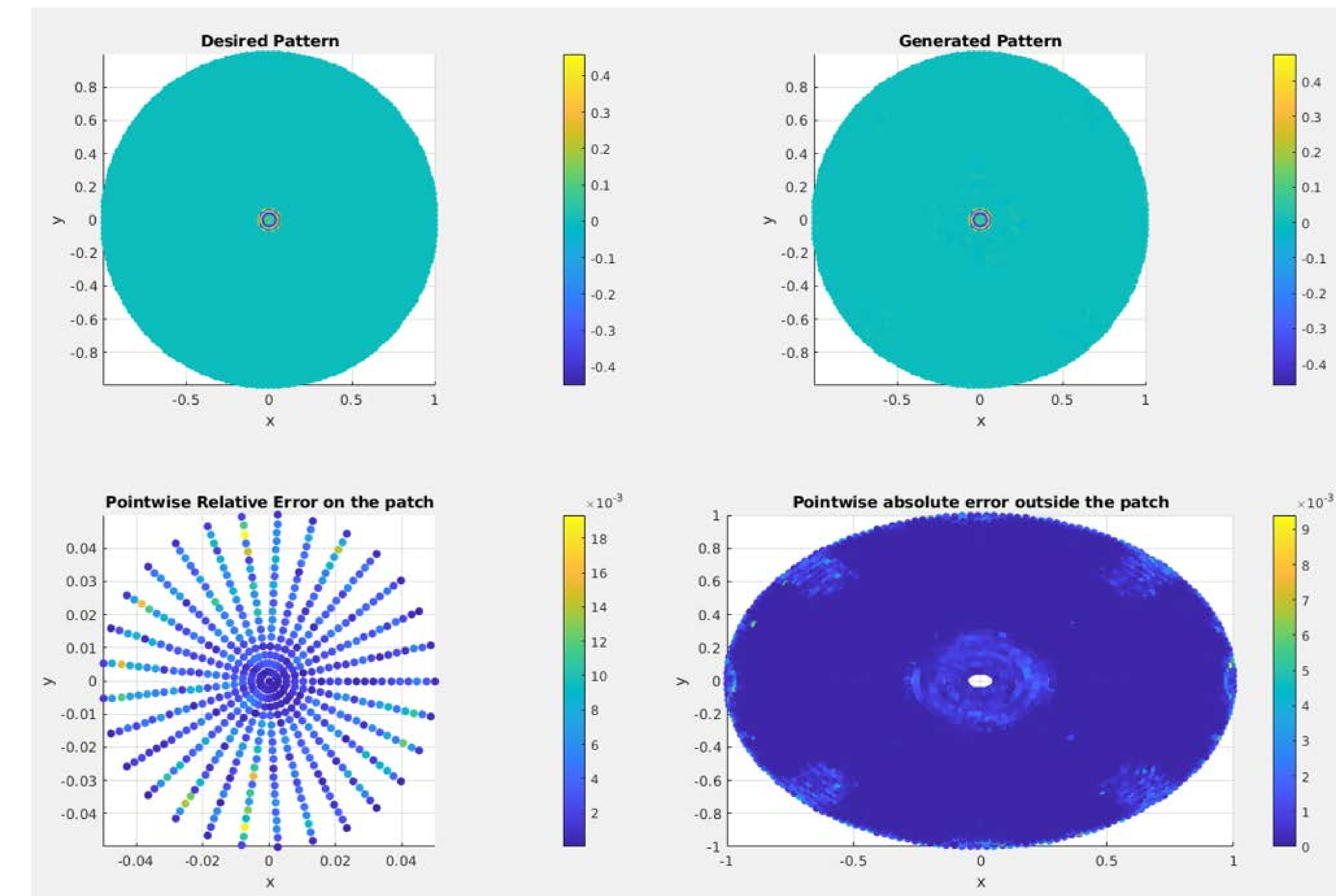

Figure 3. Results for the $\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\jmath}}$ component.

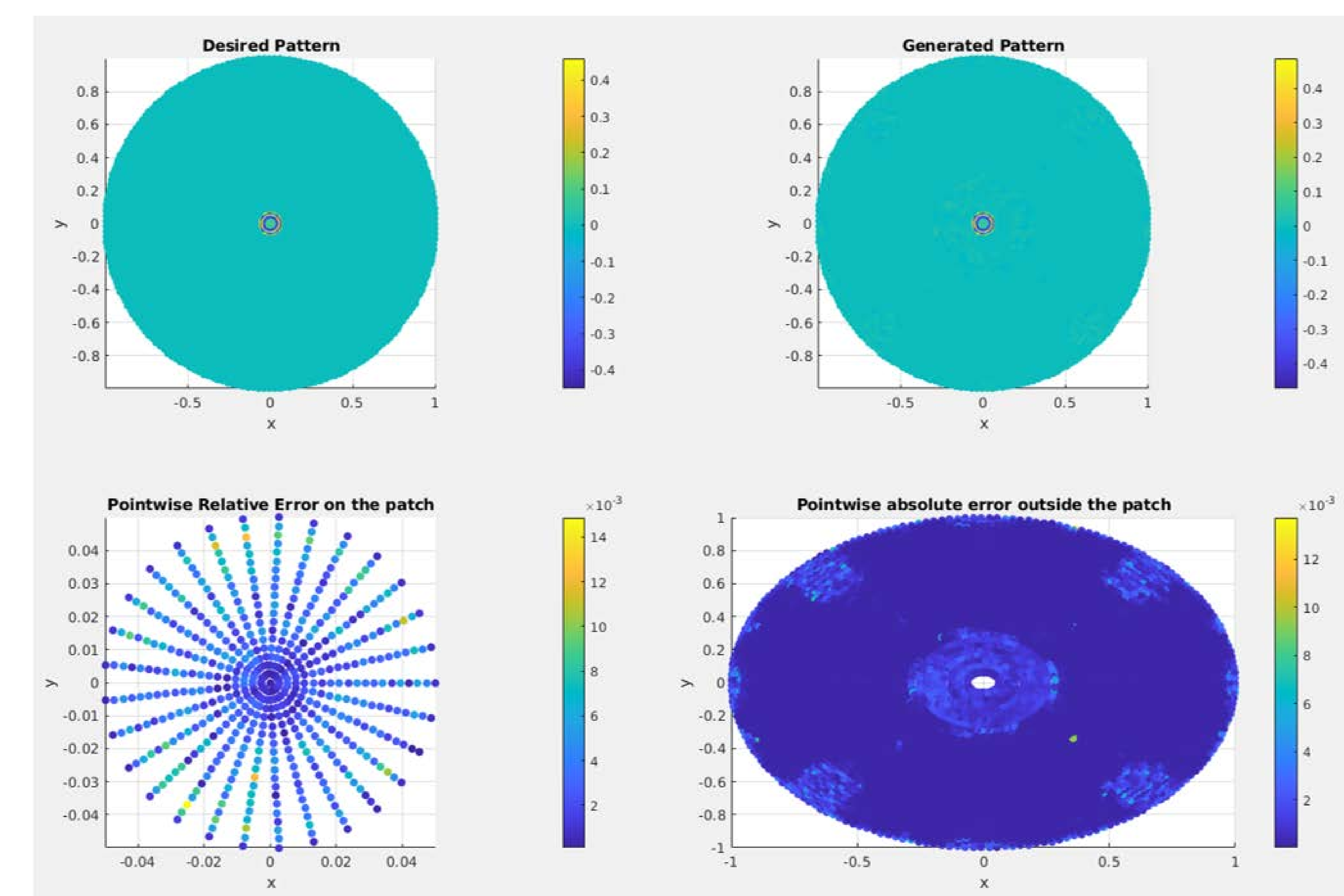

Figure 4. Results for the $\hat{\boldsymbol{x}}\times\hat{\boldsymbol{k}}$ component.

Some global error measures for each component are shown in the table below.

Component	L^2-relative error	Mean pointwise absolute error
$\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\imath}}$	0.018287	0.0010478
$\hat{\boldsymbol{x}}\times\hat{\boldsymbol{\jmath}}$	0.058947	0.0038582
$\hat{\boldsymbol{x}}\times\hat{\boldsymbol{k}}$	0.080255	0.0060090

POSSIBLE FUTURE WORK

There is vast potential for future work in this line of research, including:

- scattering cancellation on more complicated geometries including the case of a perfectly conducting sphere,
- scattering cancellation using techniques other than the method of images,
- field focusing, and
- determining the location, current, and orientation of the interrogating dipole.

Reference: T.S. Angell ad A. Kitsch, Optimization Methods in Electromagnetic Radiation. Springer, 2004.