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Math Active Learning Lab: Math 92 Notebook

Michele Iiams
University of North Dakota, michele.iiams@UND.edu

Gwennie Byron
University of North Dakota, gwennie.byron@UND.edu

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Math 92 Notebook

University of North Dakota

Revised August 30, 2018

Welcome to the MALL

Welcome to UND's Math Active Learning Lab (MALL)! As part of a nationwide movement, the UND Mathematics Department has redesigned our curriculum and pedagogy to reflect the current research on learning math. The MALL is based on the emporium model. The premise of this model is that the best way to learn math is by doing math, not by watching someone else do math. This means that most of your time in this course will be spent doing math, and your instructor will spend little time lecturing. Instructors and tutors are available in the MALL to support your learning during the required lab time. The philosophy of the MALL is well described by H. A. Simon's quote

> "Learning results from what the student does and thinks and ONLY from what the student does and thinks. The teacher can advance learning only by influencing what the student does to learn."

For many of you, this is your first college math course. Quite possibly, this course and our expectations may be quite different from your high school mathematics experiences. We cannot stress too strongly your role in ensuring your success in this class. More than anything else, your choices will determine your success in this course. Attending class regularly, diligently working in ALEKS, studying for exams, and seeking help when you need it will lead to success. Our approach includes cooperative learning. In class your instructor will facilitate group activities and discussion rather than repeating to you content of the text. We will be asking you to use the ALEKS resources and to work in your notebooks before coming to class. There will also be times when you will be expected to learn topics that will not be formally discussed in the classroom.

Instead of sitting in a lecture class for hours each week AND then being expected to do practice problems outside of class, part of your "class time" is spent doing homework in ALEKS. This provides instant feedback and links you to resources as needed. Using ALEKS allows us to individualize the student learning path. Students can move quickly through topics they are familiar with and take the time they need to learn more challenging topics. To help you get the most out of ALEKS, we have created this notebook. If ALEKS and the notebook are still leaving you confused about a topic, we expect you to ask an instructor or tutor for help.
We are excited about this approach to teaching and learning mathematics, and we look forward to learning along with you this semester.

MALL staff

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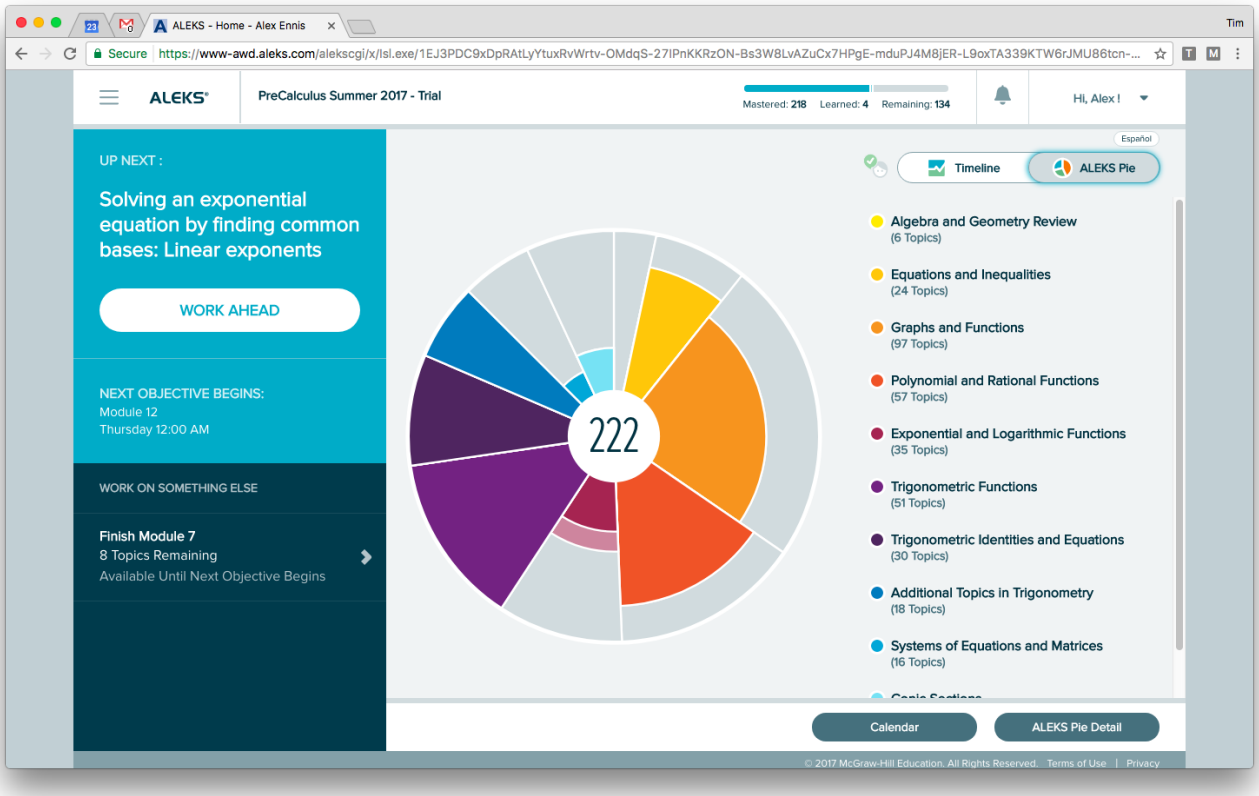
How to use ALEKS

After you complete the Initial Knowledge Check, each time you login to ALEKS, you will see your home screen, which looks like



The big pie in the middle is your ALEKS pie. Your goal for the course is to fill your pie. Each slice of the pie is a "general topic objective", and is made of many sub-topics. Gray areas of the pie are topics that you've not yet learned, lightly colored areas are topics that you've learned but not mastered, and darkly colored areas are topics you've mastered.

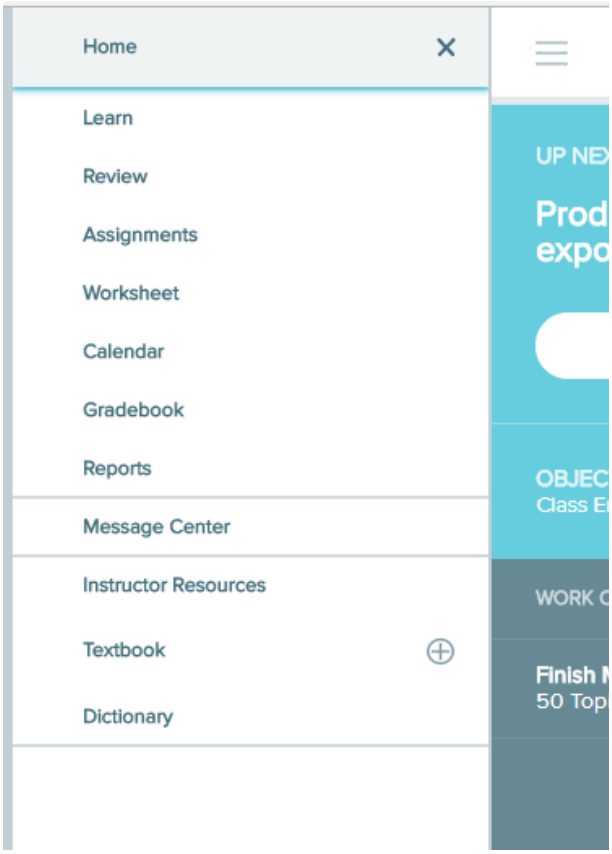
Topics are mastered through **Knowledge Checks**. After learning 20 topics (or spending 5 hours in ALEKS), ALEKS will give you a Knowledge Check. This will focus on your learned topics, but will also ask about previously mastered topics and possibly future topics. Topics you demonstrate an understanding of become mastered and are added to your pie. Topics you don't understand are not added to your pie and may be removed from it if you miss questions on a topic you previously mastered.

Getting Help

ALEKS Technical Support is available at `https://www.aleks.com/support/contact_support` or by phone at (714) 619-7090. They won't help you learn a topic, but will help you if you have trouble accessing your account.

Navigating ALEKS

The blue menu on the left shows your current options for working in ALEKS. Links to additional information can be found under the *hamburger menu* at the top left of your screen; it looks like ☰
The entries in the menu are:



Home Takes you back to the home screen.

Learn Opens the next topic ALEKS has ready for you to learn. You can also filter the topics to focus on others.

Review Opens up topics you have learned or mastered for you to review. But since you've already learned or mastered these topics, they can't help fill your pie as well as learning new topics.

Assignments & Worksheet Shows links to the occasional item posted by your instructor.

Calendar Opens a calendar view of deadlines for weekly objectives, knowledge checks and tests.

Gradebook Shows your grades for ALEKS assignments and exams. The complete and official gradebook is in Blackboard.

Reports Opens a menu of reports that provide additional information about your progress in ALEKS. We encourage you to take a look at these pages.

Message Center You can send an email to your instructor or others in your class.

Textbook This link takes you to the E-Book.

Dictionary This link takes you to a dictionary that is organized by pie slice categories.

How to use this Notebook

This course Notebook, has been designed to help you get the most out of the ALEKS resources and your time.

- Topics in the Notebook are organized by weekly learning module.
- Space for notes from ALEKS learning pages, e-book and videos directs you to essential concepts.
- Examples and "You Try It" problems have been carefully chosen to help you focus on these essential concepts.
- Completed Notebook is an invaluable tool when studying for exams.

When you ask a tutor for assistance, the first thing she/he will ask is to see your Notebook. This is necessary for the tutor to determine how best to respond to your questions. The following icons will appear in the Notebook and on the ALEKS learning pages:

the play icon will show a video about the topic.

the book icon will go to the appropriate section of the e-book.

the dictionary icon will look up terms in the course dictionary.

Testing in ALEKS

To prepare for a test in ALEKS, in your Blackboard course, select "Syllabus & Textbooks" and download and install the "Respondus LockDown Browser".
To take a test, start the "LockDown Browser" application, connecting to the "UND Blackboard Learn" server. Log in to your Blackboard course, navigate to ALEKS, and a tutor will enter the password to start your exam.
An ALEKS test is another Knowledge Check, although it may have a few more questions. As with regular Knowledge Checks, these will ask about topics you've previously mastered (even from the beginning of the course) and possibly future topics. Topics where you show mastery will be added to your pie. Topics where you show that you have not learned the material will be subtracted from your pie.
ALEKS uses your responses to determine how many topics in your pie are mastered. Each test has a target number of topics. If you meet or exceed that number, your grade on the test is 100%. If you fall short, your grade is the percentage of topics that you've mastered out of the target. This means that it's possible ALEKS will say that you have lost a few topics from your pie, but that you're still ahead of the target and therefore earn 100%. On the other hand, it's also possible that you add several topics to your pie, but because you're still below the target, you don't earn as much for a grade.
The target number of topics is the number of topics in the modules on the exam (including the prerequisite topics). You can find the number of topics in each module by looking at ALEKS' syllabus for your course. This means that if you know all of the topics for the modules you've done so far, you'll earn 100% on the exam. It's also possible, however, to master topics from later modules that will take the place of topics from past modules.

Math _____ — _____
Fall 2018

Instructor: _____
Email: _____
Phone: _____
Office: _____
Focus Group: _____

Course prerequisites and content: [See your syllabus on Blackboard]

The Math Active Learning Lab (MALL): The MALL is based on the emporium model, which is based on the premise that the best way to learn math is by doing math, not watching someone else do math. This means that most of your time in this course will be spent doing math, and your instructor will spend little time lecturing. Instructors and tutors are available in the MALL to support your learning during the required MALL time.

All email correspondence will go to your official UND email address.

Outside of each scheduled class meeting (focus group) from _____ to _____ , you must spend at least _____ hours working in the MALL (O'Kelly 33).

- Credit for MALL time is based only on UND ID card swipes.
- Swipe your ID when entering **and** exiting the MALL.
- Swiping another student's ID is academic dishonesty.
- Minutes _____ from one week to another.
- Class time _____ toward your MALL time.

MALL Expectations:

- The MALL is a math classroom. Please be considerate of others by keeping conversations focused on math and at a reasonable volume while in the MALL.
- Food, companions, and using your phone are NOT allowed in the MALL.
- Activities such as socializing, surfing the Internet, _____, doing work for another course, sleeping, etc. are not allowed in the MALL. If these activities are observed, you will be asked to leave the MALL.
- The use of a MALL computer is on a first-come first-serve basis; no reservation can be made.
- Please do not hesitate to ask questions in the MALL. Staff members in the MALL _____.

ALEKS Access & Notebook: An ALEKS access code can be purchased from `https://www.aleks.com/` or the UND Bookstore. The course Notebook is only available at the Bookstore. You will be expected to bring the Notebook to your Focus Group meetings and the MALL. Graded Notebook checks will occur weekly.

Tests: There will be _____ along with the final exam. Notes, the book, calculators, and other electronic devices will not be allowed on any of the exams. Each test will have two parts.

- Paper-pencil portion will be given during the Focus Group meeting.
- Scheduled Knowledge Checks in ALEKS must be completed in the MALL testing area the _____ the paper-pencil test.

Exam Dates: _____
_____
_____
_____

Test Rules:

- Scheduled Knowledge Checks (tests and final exam) in ALEKS will be taken in the MALL.
- Do not wait until the last minute to take your ALEKS exams. You will not be allowed to start a test if the MALL is scheduled to close before the end of your full allotted time.
- Bring your ID and pencils with you. The MALL Testing Proctor will check your ID, give you scratch paper, and direct you to your seat. Once you have started the Lockdown Browser the proctor will input the test password. When you are finished, bring all your papers to the Testing Proctor
- Absolutely NO _____ (this includes cell phones) may be active in the testing area. Use of any electronic device during a test will be treated as academic dishonesty.
- Cellphones and other smart devices must be turned completely off and and placed on the testing table.
- You may not share any test information with anyone who hasn't taken the test. Violators will be charged with academic dishonesty.
- You may not leave your table during a test without permission. This includes getting water and using the restroom. Cell phones must be left with your belongings in the testing area.

Grading: Your course grade will be a weighted average of the following:

Tests	____%
Final Exam	____%
MALL Time & Focus Group Activities*	15%
Module Completion	15%

*Your lowest Focus Group score will be dropped. This will take into account any unexcused absences.

Try Score: Your Try Score reflects your effort in this course. The Try Score is composed of:

- focus group participation,
- notebook completion,
- attempting every exam and retaking when your first attempt is less than 80%,
- spending at least _____ hours per week working in the MALL, and
- completing the module or spending sufficient time working in ALEKS.

This is **not** included in your course grade, but will be shared with your academic advisor.

Working in ALEKS at home: You can work in ALEKS anywhere you have internet access. This does NOT count toward your _____. Work well ahead of deadlines to be safe. Deadlines will NOT be extended because of home computer/internet issues.

Attendance & Participation:

- Students who do not attend the first class meeting, or contact the instructor the first week, will be dropped from the course.
- Students who do not complete their Initial Knowledge Check within two full days of their first class meeting will be dropped from the course.
- Assignments given during the Focus Group meetings will be completed in small groups and will require your full attention.
  - Regular and on-time attendance _____. Repeated absences or late arrivals will significantly impact your Focus Group grade.
  - Unless required for the Focus Group activity, cell-phone or computer use will result in a zero for the day.
  - Absences will usually be excused if due to serious emergency. An emergency serious enough to cause an absence from a Focus Group activity or test is also serious enough to _____ documentation.
  - Students with valid excuse approved prior to or within _____ of a test will be able to make up **one** test on reading and review day.
  - Students anticipating absences due to athletic commitments (or any other type of university sanctioned commitment) must document their need to be absent from class the _____ prior to the absence.

Absences will be dealt with on a case-by-case basis; however, two situations occur commonly enough to merit mention here. Travel plans _____ cause for an excused absence. *In particular, having bought a plane ticket is not sufficient reason to reschedule a student's final exam.* Also, an activity related to social functions (including those that involve a students' residence hall, apartment complex, sorority or fraternity) is never sufficient for an excused absence.

Disability Accommodations: Contact me to request disability accommodations, discuss medical information, or plan for an emergency evacuation. To get confidential guidance and support for disability accommodation requests, students are expected to register with DSS at `http://und.edu/disability-services/`, 190 McCannel Hall, or 701.777.3425.

Academic Honesty: All students in attendance at the University of North Dakota are expected to be honorable and to observe standards of conduct appropriate to a community of scholars. Academic misconduct includes all acts of dishonesty in any academically related matter and any knowing or intentional help or attempt to help, or conspiracy to help, another student. The UND Academic Dishonesty Policy will be followed in the event of academic dishonesty.

Module 1

Word problem with addition or subtraction of integers

YOU TRY IT:

1. The table gives the average high temperature for a week in January in St. John's, Newfoundland.

Day	High Temp ($^\circ$C)
Mon	-2
Tues	-6
Wed	-7
Thurs	0
Fri	1
Sat	8
Sun	10

How much higher was the average temperature on Sunday than on Wednesday?

Addition and subtraction of 3 fractions involving signs

Watch *Video 6: Practice with Subtraction of Real Numbers* to complete the following.

PROCEDURE Subtracting Real Numbers

If a and b are real numbers, then _____.

Complete ONLY part a.
Subtract.
a.

EXAMPLE:
Add or subtract. $\frac{1}{8} - \frac{3}{16} - \left(-\frac{3}{4}\right)$

First we find the least common denominator. Here the LCD is 16.

$$\begin{aligned}\frac{1}{8} - \frac{3}{16} - \left(-\frac{3}{4}\right) &= \frac{2}{2}\cdot\frac{1}{8} - \frac{3}{16} - \frac{4}{4}\cdot\left(-\frac{3}{4}\right)\\ &= \frac{2}{16} - \frac{3}{16} - \left(-\frac{12}{16}\right)\\ &= \frac{2-3-(-12)}{16}\\ &= \frac{2-3+12}{16}\\ &= \frac{11}{16}\end{aligned}$$

YOU TRY IT:
Add or subtract.

2. $\frac{3}{7} - \frac{2}{3} - \frac{2}{21}$

Signed fraction division

Watch *Video 9: Dividing Real Numbers Involving Fractions* to complete the following.

Simplify.

a.

b.

EXAMPLE:
Divide $-\frac{8}{9} \div \frac{3}{7}$.

We multiply by the reciprocal.

$$\begin{aligned}-\frac{8}{9} \div \frac{3}{7} &= -\frac{8}{9}\cdot\frac{7}{3}\\ &= \frac{-8\cdot 7}{9\cdot 3}\\ &= -\frac{56}{27}\end{aligned}$$

YOU TRY IT:

3. Divide $-\frac{9}{2} \div \frac{6}{7}$.

Exponents and integers: Problem type 2

Watch the video *Exercise: Evaluating an exponential expression* to complete the following.

Evaluate the expression.

Exponents and signed fractions

Watch *Video 10: Simplifying Expressions Involving Exponents* to complete the following.

DEFINITION Definition of b^n

Suppose that b is a _____ and n is a _____. Then,

$$b^n = _____$$

Simplify.

a.

b.

c.

d.

YOU TRY IT:
Simplify.

4. $-(-5)^3$

5. $(-\frac{1}{5})^3$

Order of operations with integers and exponents

We must follow the rules for **order of operations**. Here is that order.

1.

2.

3.

4.

YOU TRY IT:
Simplify.

6. $4^2 - (5-2)^2 \cdot 3$

7. $\dfrac{1^2 - (-3)^2 + 2}{1 - 2^2}$

Evaluating a linear expression: Signed fraction multiplication with addition or subtraction

EXAMPLE:
Evaluate $3x - 2y$ when $x = -\frac{1}{5}$ and $y = \frac{2}{3}$.

We replace x with $-\frac{1}{5}$ and y with $-\frac{2}{3}$, then follow the order of operations.

$$\begin{aligned} 3x - 2y &= 3\left(-\frac{1}{5}\right) - 2\left(\frac{2}{3}\right) \\ &= -\frac{3}{5} - \frac{4}{3} \\ &= -\frac{9}{15} - \frac{20}{15} \\ &= \frac{-9-20}{15} \\ &= -\frac{29}{15} \end{aligned}$$

YOU TRY IT:

8. Evaluate $4x + y$ when $x = \frac{1}{7}$ and $y = -\frac{3}{4}$.

Properties of real numbers

Complete the following chart. For these properties, we suppose that x, y, and z are real numbers.

	Properties of Addition	**Properties of Multiplication**
Commutative		
Associative		
Identity		
Inverse		
Distributive Property		
and		
Multiplication Property of Zero		
and		

YOU TRY IT:
State the Property of Real Numbers that is used.

9. $3 + (6 + x) = (3 + 6) + x$

10. $4 \cdot (y + 3) = 4 \cdot y + 4 \cdot 3$

11. $7 \cdot x = x \cdot 7$

Distributive property: Integer coeffiecients

Watch *Video 5: Practice Applying the Distributive Property* to complete the following.

Apply the distributive property of multiplication over addition.

1.

2.

3.

YOU TRY IT:
Simplify.

12. $-3(4x-5)$

13. $(3x+2)7$

Evaluating a quadratic expression: Integers

EXAMPLE:
Evaluate y^2-5y+4 when $y=-3$.

We substitute -3 for y, then follow the order of operations.

$$\begin{aligned} y^2-5y+4 &= (-3)^2-5(-3)+4 \\ &= 9-5(-3)+4 \\ &= 9+15+4 \\ &= 28 \end{aligned}$$

YOU TRY IT:

14. Evaluate b^2+7b-5 when $b=2$.

Additive property of equality with signed fractions

Addition and Subtraction Properties of Equality

Let a, b, and c represent real numbers.

Addition property of equality: If $a = b$, then _____.

Subtraction property of equality: If $a = b$, then _____.

EXAMPLE:

Solve $x + \frac{4}{5} = \frac{2}{3}$ for x.

Subtract $\frac{4}{5}$ from both sides of the equation.

$$\begin{aligned} x + \frac{4}{5} &= \frac{2}{3} \\ x + \frac{4}{5} - \frac{4}{5} &= \frac{2}{3} - \frac{4}{5} \\ x + 0 &= \frac{10}{15} - \frac{12}{15} \\ x &= \frac{10 - 12}{15} \\ x &= -\frac{2}{15} \end{aligned}$$

Check:

$$\begin{aligned} x + \frac{4}{5} &= \frac{2}{3} \\ -\frac{2}{15} + \frac{4}{5} &\stackrel{?}{=} \frac{2}{3} \\ -\frac{2}{15} + \frac{12}{15} &\stackrel{?}{=} \frac{2}{3} \\ \frac{10}{15} &\stackrel{?}{=} \frac{2}{3} \\ \frac{\cancel{10}^{2}}{\cancel{15}_{3}} &= \frac{2}{3} \checkmark \end{aligned}$$

YOU TRY IT:

Solve for y.

15. $y - \frac{3}{5} = \frac{2}{7}$

Check:

Multiplicative property of equality with signed fractions

Watch *Video 3: Multiplication and Division Properties of Equality* to complete the following.

PROPERTY Multiplication and Division Properties of Equality
Let a, b, and c represent real numbers. Then

- Multiplication property of equality: If $a = b$

  then _____

- Division property of equality: If $a = b$,

  then _____ provided _____

1.

Check:

2.

Check:

3.

Check:

YOU TRY IT:

16. Solve $\frac{4}{3}y = -5$ for y.

Check:

Using distribution with double negation and combining like terms to simplify: Multivariate

Watch *Video 9: Simplifying an Expression with Nested Parentheses* to complete the following.

Simplify the expressions by clearing parentheses and combining like terms.

1.

2.

YOU TRY IT:
Simplify the expressions by clearing parentheses and combining like terms.

17. $-3(4x-5y)+2(7x+y)$

18. $2(3x+4y)-(x+5y)-3x$

Additional Notes:

Module 2

Solving a two-step equation with integers

Watch *Video 8: Solving a Linear Equation Requiring Multiple Steps* and complete the following.

Solve the equation.

Check:

YOU TRY IT: Solve.

19. $-3y + 4 = 10$

20. $\frac{x}{3} - 5 = 2$

Solving an equation to find the value of an expression

EXAMPLE:
Find the value of $y + 3$ given that $8y + 9 = -7$.

First solve the equation $8y + 9 = -7$ for y.

$$\begin{aligned} 8y + 9 &= -7 \\ 8y &= -16 \\ y &= -2 \end{aligned}$$

Now find the value of $y + 3$ for $y = -2$.

$$\begin{aligned} y + 3 &= -2 + 3 \\ &= 1 \end{aligned}$$

YOU TRY IT:

21. Find the value of $x - 4$ given that $3x + 2 = -13$.

Least common multiple of 2 numbers

Least Common Multiple (LCM): The LCM is the _____

_____ given numbers.

EXAMPLE:
Find the LCM of 10 and 8.

Method 1.

- List the multiples of the largest number: Multiples of 10 are $10, 20, 30, 40, 50, 60, 70, 80, 90, ...$
- Find the first multiple that is also divisible by the second number: From this list 40 and 80 are common multiples of 10 AND 8.
- Thus, the **smallest (least)** common multiple of both 10 and 8 is 40.

Method 2. Prime factorization

- Find the prime factorization of both numbers: $8 = 2 \times 2 \times 2 = 2^3$ and $10 = 2 \times 5$.
- The LCM must include the greatest number of each prime factor: The LCM of 10 and 8 must include 2^3 and 5^1 as factors.
- This makes the LCM $2^3 \cdot 5 = 8 \cdot 5 = 40$.

YOU TRY IT:

22. Find the LCM of 6 and 20

23. Find the LCM of 18 and 14

Solving a two-step equation with signed fractions

Watch the video *Solving an Equation by First Clearing Fractions* to complete the following.

Solve the equation.

YOU TRY IT:

24. Solve. $\frac{2}{7}x + \frac{1}{14} = 2$

Solving a multi-step equation given in fractional form

EXAMPLE:

Solve $\dfrac{2-3x}{5} = 2$ for x.

First multiply both sides by 5.

$$\frac{2-3x}{5} = 2$$
$$5 \cdot \frac{2-3x}{5} = 2 \cdot 5$$
$$\frac{\cancel{5}^{1}}{1} \cdot \frac{2-3x}{\cancel{5}_{1}} = 10$$
$$2 - 3x = 10$$
$$-3x = 8$$
$$x = -\frac{8}{3}$$

YOU TRY IT:

25. Solve $\dfrac{3y-4}{2} = -5$ for y.

Solving a linear equation with several occurrences of the variable: Variables on both sides and distribution

Watch *Video 5: Guidelines for Solving a Linear Equation in One Variable* and complete the following.

PROCEDURE Solving Linear Equations in One Variable

1. Simplify _____ of the equation.
2. Collect all _____ on _____ of the equation.
3. Collect all _____ on the _____ of the equation.
4. Use the multiplication or division property of equality to obtain a _____ for the variable.
5. Check the potential _____ in the _____ equation.

Solve the equation.

YOU TRY IT:

26. Solve. $3(y-6)=2y-5$

Solving a linear equation with several occurrences of the variable: Variables on both sides and two distributions

Watch *Video 6: Solving a Linear Equation in One Variable* and complete the following.

Solve the equation.

YOU TRY IT:

27. Solve. $13 + 4x = -5(-x - 6) + 2(x + 1)$

Solving a linear equation with several occurrences of the variable: Variables on both sides and fractional coefficients

Watch the video *Clearing Fractions*.

Solve the equation.

YOU TRY IT:

28. Solve. $\frac{2}{3}y - \frac{5}{6} - 3 = \frac{1}{2}y - 5$

Solving a linear equation with several occurrences of the variable: Fractional forms with binomial numerators

Watch *Video 9: Practice Clearing Fractions Numerator has two Terms* to complete the following.

Solve the equation.

YOU TRY IT:

29. Solve. $\dfrac{2}{3} - \dfrac{w+2}{6} = \dfrac{5w-2}{2}$

Solving for a variable in terms of other variables using addition or subtraction: Basic

Supplementary Resources: Watch *Video 8: Solving an Equation for a Given Variable*.

Solve for P.

Solving for a variable in terms of other variables using addition or subtraction: Advanced

YOU TRY IT:

30. Solve for x. $A + y - x = 12$

Converting between temperatures in Fahrenheit and Celsius

The formulas to convert between Fahrenheit and Celsius are:

$C =$ $F =$

YOU TRY IT:

31. Convert $25^{\circ}C$ to Fahrenheit.

Writing a one-step expression for a real-world situation

EXAMPLE:
Yesterday Sef read n pages of his history book. Today he read 56 pages of his history book. Using n write an expression for the total number pages, of his history book, he read on these two days.

The total number of pages Sef read is

$$n + 56 \text{ or } 56 + n$$

.

EXAMPLE:
Last week Missy spent 84 hours gaming. This week she spent g hours gaming. Using g write an expression for how many more hours Missy spent gaming last week than this week.

Missy spent $84 - g$ more hours gaming last week than this week.

YOU TRY IT:

32. Fred watched only 2 episodes of "Game of Thrones" and George binge watched t episodes. Using t write an expression for how many more episodes George watched than Fred.

Translating a sentence into a multi-step equation

EXAMPLE:
Twice the difference of x and 3 is 18.

- "Twice" means 2 times
- "difference of x and 3" is $(x - 3)$.
- "is 18" means $= 18$

The equation is $2(x - 3) = 18$.

EXAMPLE:
3 is the same as 5 less than the quotient of 16 and a number m.

- "is the same as 3" means $3 =$.
- the "quotient of 16 and a number m" is written $\frac{16}{m}$
- "5 less than" is $\frac{16}{m} - 5$

The equation is $3 = \frac{16}{m} - 5$ or $\frac{16}{m} - 5 = 3$

YOU TRY IT:

33. 7 more than the quotient of a number d and 6 is 9.

Additional Notes:

Module 3

Additive property of inequality with integers

Addition and Subtraction Properties of Inequality

Let a, b, and c represent real numbers.

*Addition property of Inequality: If $a < b$, then _____.

*Subtraction property of Inequality: If $a < b$, then _____.

*These properties may also be stated for $a \leq b$, $a > b$, and $a \geq b$.

Additive property of inequality with signed fractions

See **Addition and Subtraction Properties of Inequality** above.

YOU TRY IT: Solve.

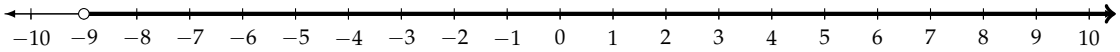
34. $x + 4 < 9$

35. $-\frac{3}{4} + x \geq \frac{1}{8}$

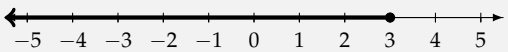
Graphing a linear inequality on the number line

To graph $a > -9$, we show all number _____.

Numbers _____ are to the _____ on the number line.

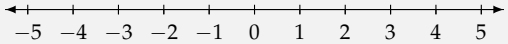


EXAMPLE: Graph $x \leq 3$ on a number line.



YOU TRY IT: Graph $b \geq 1$ on a number line.

36.



Graphing a compound inequality on the number line

Compound Inequality

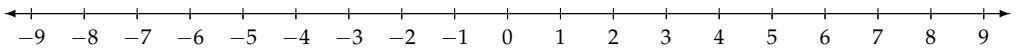
Two inequalities joined by the word _____ or the the word _____ form a _____.

Solutions to compound inequalities:

- If the two inequalities are joined by _____, the solution is made up of values that

  _____.

  - $x \geq 0$ **and** $x < 3$
    The solution is made up of all the numbers _____.

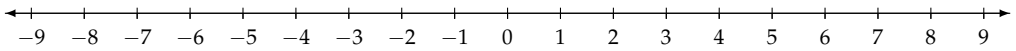


- If the two inequalities are joined by _____, the the solution is made up of values that

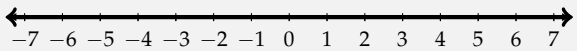
  _____.

  - $x \leq 0$ **or** $x > 3$
    The solution is made up of all the numbers _____

    _____.

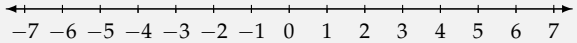


EXAMPLE:
Graph $x > 1$ or $x < 4$ on a number line.
The solution is made up of all the numbers.



For any number x, **at least one** of these two inequalities will be true.

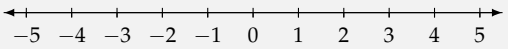
EXAMPLE:
Graph $x < 3$ and $x > 5$ on a number line.
The solution is the empty set.



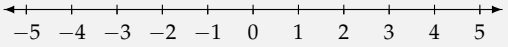
No number can be **both** smaller than 3 **and** greater than 5.

YOU TRY IT:

37. Graph $b \leq 1$ or $b \geq 2$ on a number line.

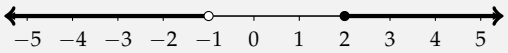


38. Graph $b \leq -2$ or $b \geq -3$ on a number line.



Writing a compound inequality given a graph on the number line

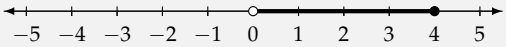
EXAMPLE:
Write a compound inequality for the graph shown below.



The shaded regions include all numbers less than -1 or all numbers greater than or equal to 2 $\implies$ $x < -1$ or $x \geq 2$.

YOU TRY IT:

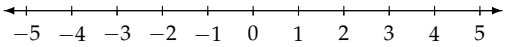
39. Write a compound inequality for the graph shown below.



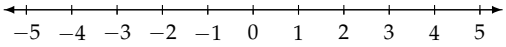
Set-builder and interval notation

Watch *Video 5: Introduction to Interval Notation* to complete the following.

$\{\quad\}$



$\{\quad\}$



Use _____ to exclude an endpoint.

Use _____ to include an endpoint.

NOTE:

The _____ and _____ both indicate that a point **IS NOT** included in a set.

The _____ and _____ both indicate that a point **IS** included in a set.

Union and intersection of finite sets

Watch the video *Exercise: Union and Intersection of Sets* to complete the following.

Given the sets $M = \{$ $\}$ and $N = \{$ $\}$.

List the elements of the following sets:

a.

b.

YOU TRY IT:
Given $A = \{a, b, c, 2, 4, 6\}$ and $B = \{a, c, d, 1, 2, 4\}$, find the following. Write your answer as a set.

40. $A \cap B$

41. $A \cup B$

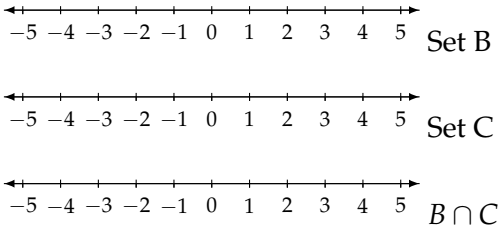
Union and intersection of intervals

Watch *Video 2: Determining the Union and Intersection of Two Sets* to complete the following.

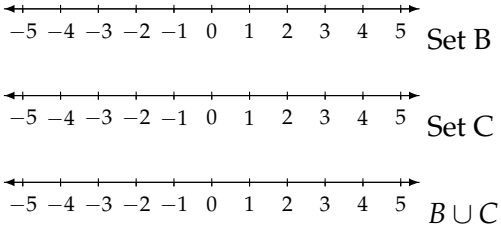
Given the sets B and C, determine the union or intersection as indicated.

$B =$ $C =$

1. $B \cap C =$



2. $B \cup C =$



YOU TRY IT:
Given $A = \{x|x \leq 4\}$ and $B = \{x|x > -2\}$, find the following. Write your answer in interval notation.

42. $A \cap B$

43. $A \cup B$

Multiplicative property of inequality with signed fractions

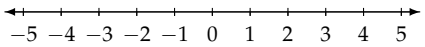
Watch *Video 5: Multiplication and Division Properties of Inequality* to complete the following.

PROPERTY **Multiplication and Division Properties of Inequality**
Let a, b, and c represent real numbers.

*If c is **positive** and $a < b$, then _____.

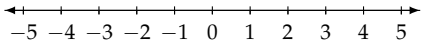
*If c is **negative** and $a < b$, then _____.

*These properties may also be stated for $a \leq b$, $a > b$, and $a \geq b$.



If you multiply or divide both sides of an inequality by a _____, you must _____ the inequality sign.

Solve the inequality. Express the solution set in set-builder notation and in interval notation.



YOU TRY IT: Solve.

44. $\frac{1}{7}x < -9$

45. $-\frac{3}{4}x \geq 8$

Solving a two-step linear inequality: Problem type 2

EXAMPLE:
Solve $15 < 21 - 2x$.

$15 - 21 < 21 - 2x - 21$ isolate the variable term

$-6 < -2x$

$\frac{-6}{-2} > \frac{-2x}{-2}$ divide both sides by -2 and reverse the inequality

$3 > x$

$(-\infty, 3)$

YOU TRY IT:

46. Solve $12 \geq 6 + 3y$

Solving a two-step linear inequality with a fractional coefficient

EXAMPLE:
Solve $-12 \geq \frac{2}{3}x - 4$.

$-12 + 4 \geq \frac{2}{3}x - 4 + 4$ isolate the variable term

$-8 \geq \frac{2}{3}x$

$\frac{3}{2} \cdot (-8) \geq \frac{3}{2} \cdot \frac{2}{3}x$ multiply both sides by $\frac{3}{2}$ the inequality stays the same

$-12 \geq 1x$

$-12 \geq x$

$(-\infty, -12]$

YOU TRY IT:

47. Solve $-\frac{3}{5}y + 4 < -2$

Solving a linear inequality with multiple occurrences of the variable: Problem type 2

Watch the video *Exercise: Solving linear inequalities with parentheses* to complete the following.

Solve the inequality and graph the solution set. Write the solution set in (a) set-builder notation and (b) interval notation.

YOU TRY IT:

48. Solve $4 - 4(x - 2) \leq -5x + 6$

Solving a linear inequality with multiple occurrences of the variable: Problem type 3

Watch the video *Solving a Linear Inequality Containing Fractions* to complete the following.

Solve the inequality. Express the solution set in set-builder notation and in interval notation.

YOU TRY IT:

49. Solve $\frac{1}{2}x - \frac{2}{3} \leq 2$

Solving inequalities with no solution or all real numbers as solutions

EXAMPLE:
Solving $4x + 3 < 4x - 5$

$$4x - 4x + 3 < 4x - 4x - 5$$
$$3 < -5$$

results in a false inequality, since 3 is NOT less than -5. This implies that $4x + 3 < 4x - 5$ has NO solution. There are no values for x that will make this inequality true.

EXAMPLE:
Solving $4x + 3 > 4x - 5$

$$4x - 4x + 3 > 4x - 4x - 5$$
$$3 > -5$$

results in an inequality that is true for all possible values of x, since 3 is ALWAYS greater than -5. This implies that the solution set for $4x + 3 < 4x - 5$ is ALL reals numbers.

EXAMPLE:
Solving $5x + 3 > 4x - 5$

$$5x - 4x + 3 > 4x - 4x - 5$$
$$x + 3 - 3 > -5 - 3$$
$$x > -8$$

results in an inequality that is true only for values of x that are greater than -8.

YOU TRY IT:

50. Determine whether the inequality has no solution or all real numbers as solutions.

$$3(4 - x) + 20 \geq -3(x + 7)$$

Solving a compound linear inequality: Graph solution, basic

Watch *Video 7: Solving an Inequality of the Form* $a < x < b$ to complete the following.

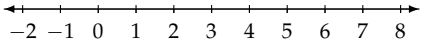
DEFINITION **Inequalities of the form** $a < x < b$**.**

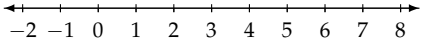
An inequality of the form _____ is equivalent to the compound inequality

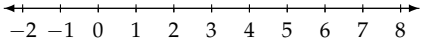
_____.

$$a \quad < \quad x \quad < \quad b$$

Solve the inequality.







Show the "easier" way to solve this inequality:

YOU TRY IT:

51. Solve $-3x - 5 \geq 4$ or $4 - x < 6$.

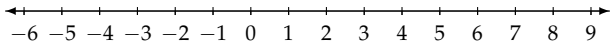
Solving a compound linear inequality: Interval notation

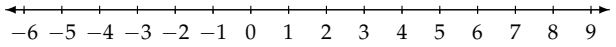
Watch *Video 9: Solving a Compound Inequality Joined by "Or"* to complete the following.

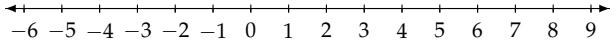
PROCEDURE **Solving Compound Inequalities**

- The solution to two inequalities joined by the word **AND** is the _____ of their solution sets.
- The solution to two inequalities joined by the word **OR** is the _____ of their solution sets.

Solve the compound inequality.







YOU TRY IT:

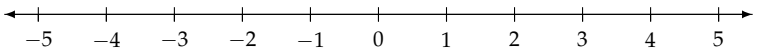
52. Solve $-3x - 5 \geq 4$ or $4 - x < 6$.

Introduction to solving an absolute value equation

Watch the video *Introduction to Absolute Value Equations* to complete the following.

Solve the equation.

1.



"We can interpret the absolute value of x as _____. So geometrically,

if the _____, then x must either be _____".

Definition **Absolute Value**

$$|x| = \begin{cases} x & \text{if } _____ \\ -x & \text{if } _____ \end{cases}$$

PROCEDURE **Solutions to Absolute Value Equations**

If a is a _____ real number, then the equation $|x| = a$ is equivalent to

_____ or _____.

Solve the equation.

2. 3. 4.

YOU TRY IT:

53. Solve $|x| = 3$

Solving an absolute value equation: Problem type 1

Watch the video *Exercise: Solving Absolute Value Equations* to complete the following.

Solve the absolute value equation.

YOU TRY IT:

54. Solve $|x| + 4 = 11$.

Solving an absolute value equation: Problem type 4

Watch ***Video 3****: Solving an Absolute Value equation* to complete the following. *NOTE:* This may not be the first video that pops up. Select it from the video list.

PROCEDURE **Solving an Equation Involving One Absolute Value**

1. Isolate the _____.
2. The equation is now in the form $|x| = a$.
   - If _____ then there is _____.
   - If _____ then write the absolute value equation as an _____ of equations _____.
3. Solve the pair of equations from step 2 and write the solution set.

Solve.

YOU TRY IT:

55. Solve $-|2x-3|+5=-8$.

Additional Notes:

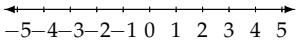
Module 4

Solving an absolute value inequality: Problem type 1

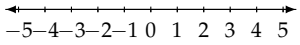
Watch *Video 1: Introduction to Absolute Value Inequalities* to complete the following.

Write the solution set.

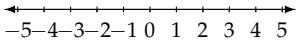
1.



2.



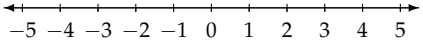
3.



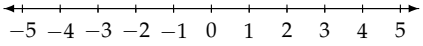
DEFINITION **Absolute Value Inequality**

Suppose that a represents a nonnegative real number. Then,

1. *The inequality $|x| < a$ is equivalent to the compound inequality: _____



2. *The inequality $|x| > a$ is equivalent to the compound inequality: _____



*Both cases can also be stated using the inequality symbols $\leq$ and $\geq$.

YOU TRY IT:

56. Graph the solution to $|x| \leq 4$ on the number line.

Solving an absolute value inequality: Problem type 4

Watch *Video 2: Solving an Absolute Value Inequality* to complete the following.

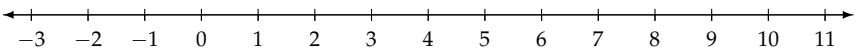
PROCEDURE **Solving Absolute Value Inequalities**

Suppose that a represents a _____ real number.

1. Isolate the _____.
2. Write the absolute value inequality as an equivalent compound inequality.
   - *The inequality $|x| < a$ is equivalent to: _____.
   - *The inequality $|x| > a$ is equivalent to: _____.
3. Solve the inequality from step 2 and write the solution set.

*Both cases can also be stated using the inequality symbols $\leq$ and $\geq$.

Write the solution set.



Solving an absolute value inequality: Problem type 5

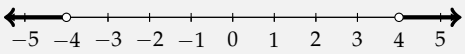
YOU TRY IT:

57. Graph the solution set to $4 \leq 2|x+1| - 4$.

Writing an absolute value inequality given a graph on the number line

The **absolute value** of a number is its _____.

EXAMPLE:
Write an absolute value inequality for the graph below. Use x for your variable.

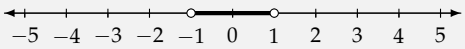


Our graph is made up of all numbers x whose distance from 0 is greater than 4. We write this as

$$|x| > 4$$

YOU TRY IT:

58. Write an absolute value inequality for the graph below. Use x for your variable.



Plotting a point in the Coordinate Plane

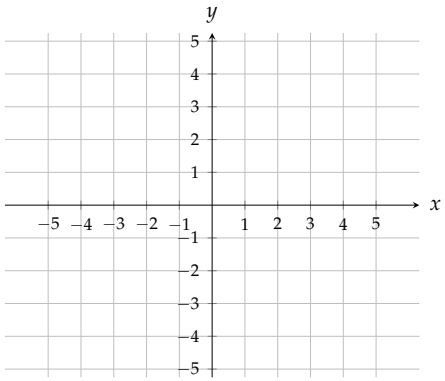
Watch the video *Plotting Points* to complete the following.

Plot the points.

a.

b.

c.



Pause the video and try these yourself.

d. e. f.

Play the video and check your answers.

Identifying solutions to a linear equation in two variables

Watch the video *Solutions to Linear Equations in Two Variables* to complete the following.

DEFINITION Linear Equation in Two Variables

Let A, B, and C be real numbers such that A and B are _____. A **linear equation in two variables** is an equation that can be written in the form:

A **solution** to a linear equation in two variables is an _____ that satisfies the equation.

Determine if the ordered pair is a solution to the equation $4x - 2y = 6$.

a. b. c.

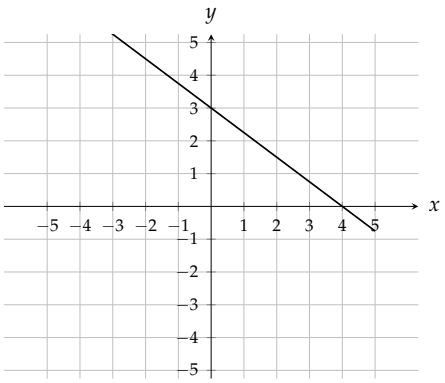
YOU TRY IT:
Determine if the ordered pair is a solution to the equation $3x + 5y = -7$.

59. $(3, -2)$ **60.** $(-4, 6)$ **61.** $(1, -2)$

Finding x- and y-intercepts given the graph of a line on a grid

Watch *Video 8: Introduction to x and y-Intercepts* to complete the following.

1.



DEFINITION x- and y-intercepts

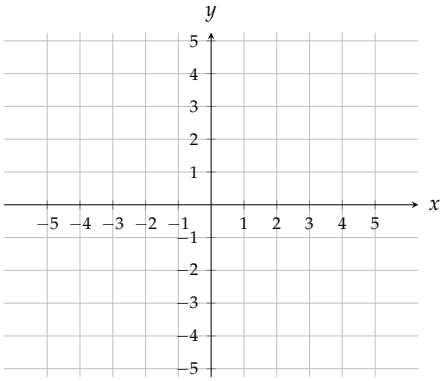
An **x-intercept** is a point _____

where a graph intersects the _____.

A **y-intercept** is a point _____

where a graph intersects the _____.

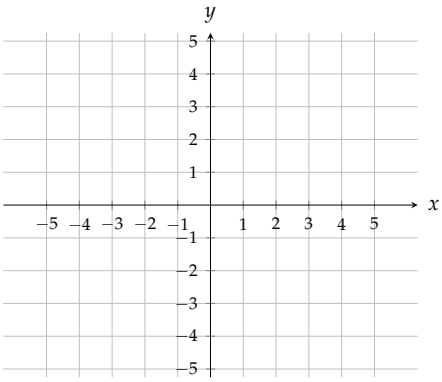
2.



x-intercept:

y-intercept:

3.



x-intercept:

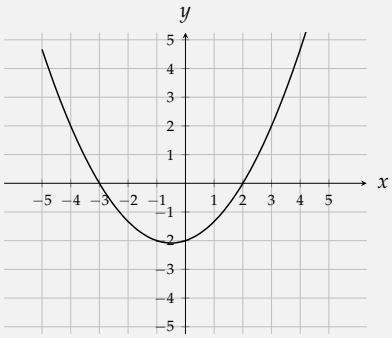
y-intercept:

From the graphs in #4 and #5 we learn that a

- horizontal line has a _____ but no _____.
- vertical line has an _____ but no _____.

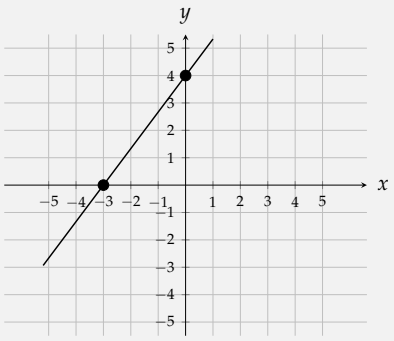
YOU TRY IT:

62. Find the x and y-intercepts of the graph.



Graphing a line given its x- and y-intercepts

EXAMPLE:
Graph the line whose x-intercept is -3 and whose y-intercept is 4.

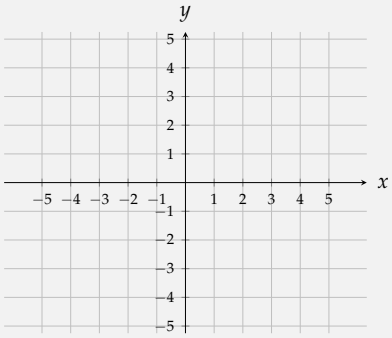


A y-intercept of 4 $\implies$ the line crosses the y-axis at 4, That is, the line passes through the point $(0, 4)$

An x-intercept of -3 $\implies$ the line crosses the x-axis at -3, That is, the line passes through the point $(-3, 0)$

YOU TRY IT:

63. Graph the line whose x-intercept is 5 and whose y-intercept is 1.

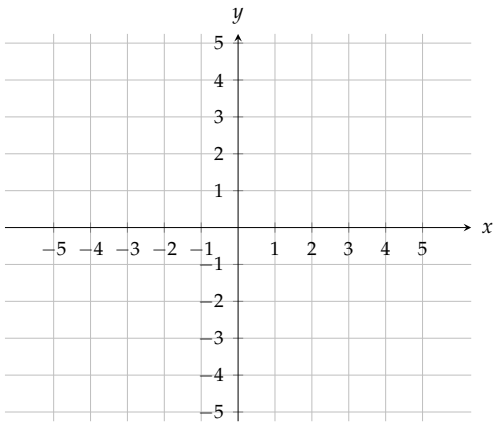


Graphing a line given its equation in standard form

Watch the video *Graphing a Linear Equation by Using a Table of Points* and to complete the following.

Graph the equation _____

x	y



EXAMPLE:
Complete the table and then graph the equation $4x + 3y = 6$

x	y
	2
3	
	-1

Substitute the given value into the equation to find the missing value of the ordered pair.

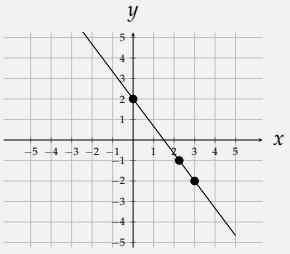
x	y
0	**2**
3	-2
$\frac{9}{4}$	**-1**

$4x + 3(2) = 6 \Rightarrow x = 0$

$4(3) + 3y = 6 \Rightarrow y = -2$

$4x + 3(-1) = 6 \Rightarrow x = \frac{9}{4}$

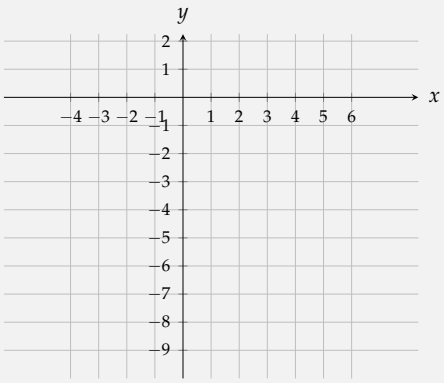
Plot the three ordered pairs on the graph. Draw a line through all of the points.



YOU TRY IT:

64. Complete the table and then graph the equation $3x - 2y = 15$

x	y
1	
4	
	0



Division involving zero

Click on the Why? icon to complete the following:

Understanding division with 0

$20 \div 4 \implies$ "What number multiplied by 4 gives 20? The answer is 5.

Since _____ we get that _____.

Next, $0 \div 4 \implies$ "What number multiplied by 4 gives 0?

The answer is _____. Since _____ we get that _____.

Finally, $4 \div 0 \implies$ "What number multiplied by 0 gives 4?

There is _____, because multiplying any number by 0 gives 0.

Thus _____. We get the same result with any _____ number, not just 4.

So, dividing any nonzero number by 0 is undefined.

YOU TRY IT:
Evaluate each expression.

65. $\frac{13}{0} =$

66. $\frac{0}{13} =$

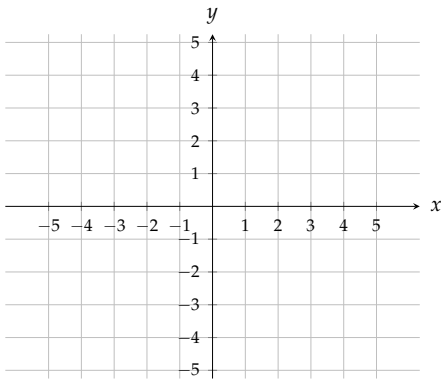
Graphing a vertical or horizontal line

Watch *Video 13: Graphing Horizontal and Vertical Lines* to complete the following.

Graph the equations.

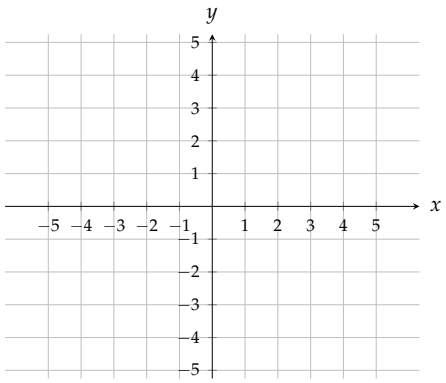
1.

x	y



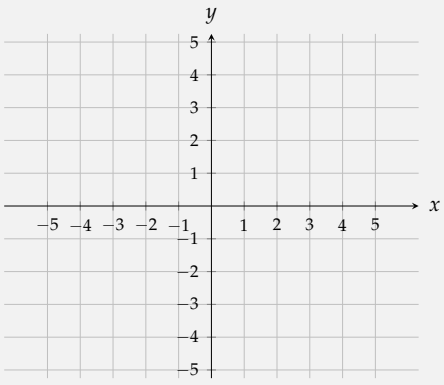
2.

x	y

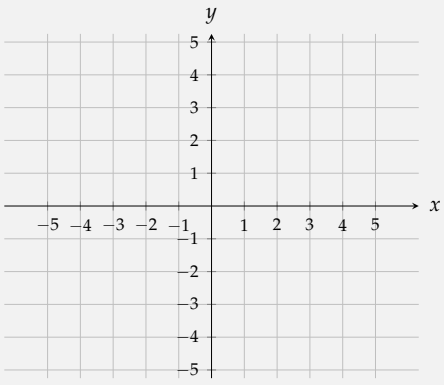


YOU TRY IT:

67. Graph $x = 3$.



68. Graph $y = 2$.

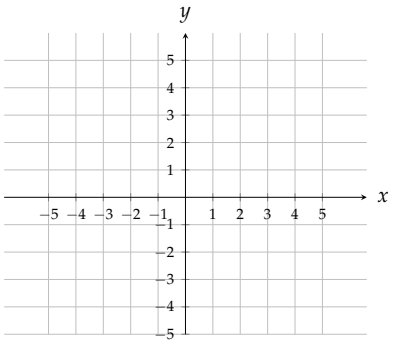


Classifying slopes given graphs of lines

Watch the video *Orientation of a Line and the Sign of the Slope* to complete the following.

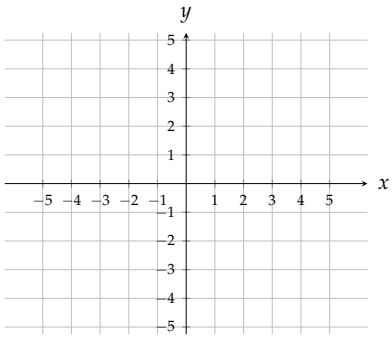
Show the change in y and the change in x on each graph.

positive slope



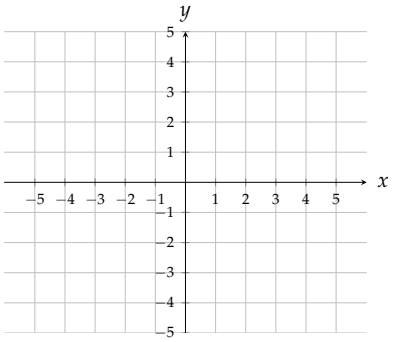
$$m = \frac{\text{change in } y}{\text{change in } x} =$$

negative slope



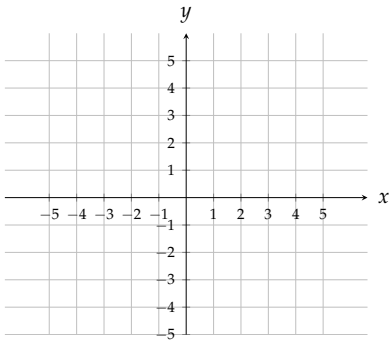
$$m = \frac{\text{change in } y}{\text{change in } x} = 0$$

zero slope



$$m = \frac{\text{change in } y}{\text{change in } x} =$$

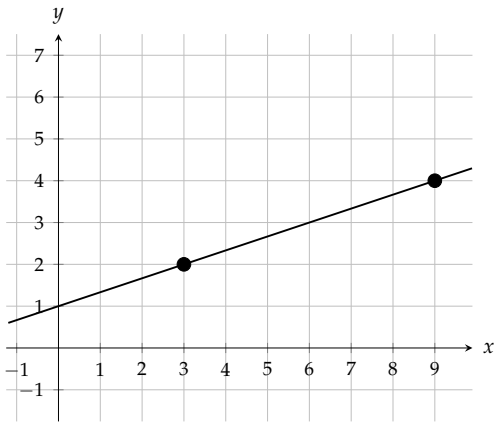
undefined slope



$$m = \frac{\text{change in } y}{\text{change in } x} =$$

Finding slope given two points on the line

Watch *Video 3: Introduction to the Slope Formula* to complete the following.



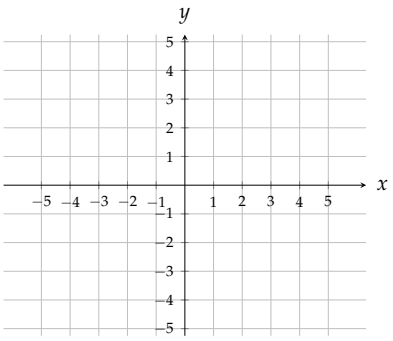
YOU TRY IT:

69. Find the slope of the line containing $(-1, 4)$ and $(3, 7)$.

Finding the slope of horizontal and vertical lines

Watch the video **Video 7:** *Determining the Slope of a Vertical Line* to complete the following.

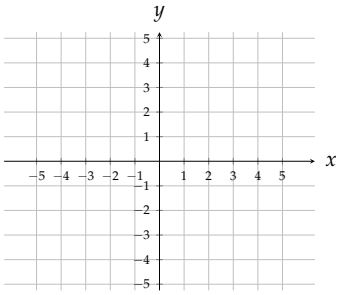
Determine the slope of the line containing the points _____.



$$m = \frac{y_1 - y_2}{x_1 - x_2} =$$

Watch the video **Video 8:** *: Determining the Slope of a Horizontal Line* to complete the following.

Determine the slope of the line containing the points _____.



$$m = \frac{y_1 - y_2}{x_1 - x_2} =$$

EXAMPLE:
Determine the slope of the line containing the points $(-1, -3)$ and $(5, -3)$.

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-3 - (-3)}{-1 - 5} = \frac{0}{-6} = 0$$

The slope is 0.

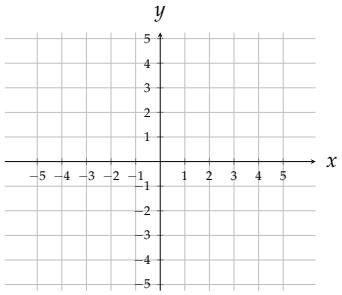
YOU TRY IT:

70. Determine the slope of the line containing the points $(3, -2)$ and $(3, -5)$.

Finding the slope and y-intercept of a line given its equation in the form y = mx + b

Watch *Video 3: Graphing a Line From its Slope and y-Intercept* to complete the following. *NOTE:* This may not be the first video that pops up. Select it from the list of videos.

Graph the equation by using the slope and y-intercept.



YOU TRY IT:

71. Find the slope and y-intercept of the line $y = -2x + 4$.

Graphing a line through a given point with a given slope

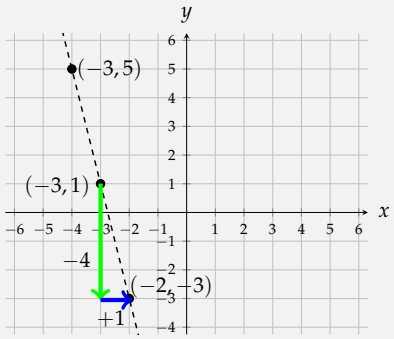
EXAMPLE:
Graph the line with slope -4 passing through the point $(-3, 1)$.

We can write -4 as $\dfrac{-4}{1}$ to get

$$\text{slope} = \frac{\text{rise}}{\text{rise}} = \frac{-4}{1}$$

Starting at the given point $(-3, 1)$ we "rise" -4 (down) and then "run" $+1$ (right) to find a second point on the line $(-2, -3)$.

We use $(-3, 1)$ and $(-2, -3)$ to graph the line.



We can find another point on the line by thinking about -4 as $\dfrac{4}{-1}$. Applying the "rise" of 4 (up) and "run" of -1 (left) we find the point $(-3, 5)$.

YOU TRY IT:

72. Graph the line with slope $\dfrac{1}{3}$ passing through the point $(0, 1)$.

Graphing a line given its equation in slope-intercept form: Fractional slope

There are two ways to graph a line when given the slope-intercept form. Complete ONE of the following options:

Option 1:

Use your notes from the previous 2 topics in this notebook to complete the **YOU TRY IT** below.

- Finding the slope and y-intercept of a line given its equation in the form y = mx + b
- Graphing a line through a given point with a given slope

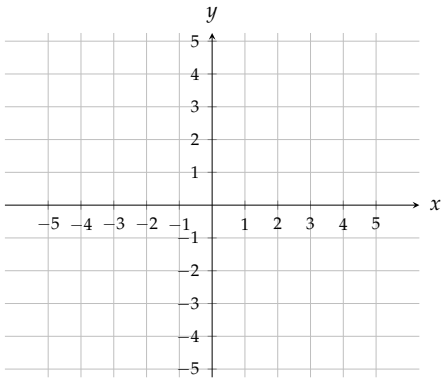
OR

Option 2:

Watch the video *Graphing a linear equation by Using a Table of Points* to complete the following.

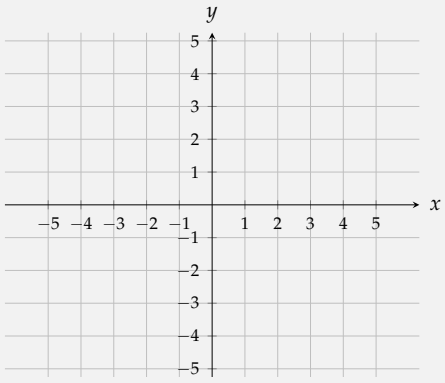
Graph the equation

x	y



YOU TRY IT:

73. Graph the equation $y = \frac{1}{3}x - 4$ by first completing the table.



Module 5

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel, but still count toward your module completion. To prepare for your upcoming exam:

- ☐ Complete this module.
- ☐ At least two days before your focus group, take your ALEKS exam in the MALL.
- ☐ If you score less than 80% you are strongly encouraged to retake the ALEKS exam.
  - ☐ Ask for a ticket to retake from a tutor.
  - ☐ Work in the MALL for one hour.
  - ☐ Have a tutor sign that you have finished your review.
  - ☐ Retake the ALEKS portion of your exam.
- ☐ Take your written exam the day of your focus group. No retakes will be allowed on written exams.

The score on your Scheduled Knowledge Check is the number of topics that you have mastered (including prerequisite topics) out of the number of topics that you should have mastered by this point.

	Score
ALEKS Exam	
ALEKS Exam Retake	
Written Exam	

*Your recorded ALEKS exam score is the higher of your ALEKS Exam score and ALEKS Exam Retake score.

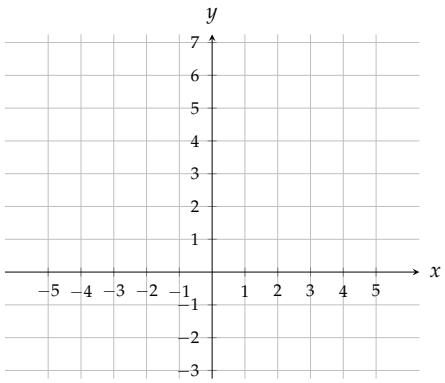
Module 6

Graphing a line by first finding its slope and y-intercept

Watch the video *Exercise: Graphing a Line by Using a Slope and $y-$Intercept* to complete the following.

Write the equation in slope-intercept form (if possible). then graph each line, using the slope and y-intercept.

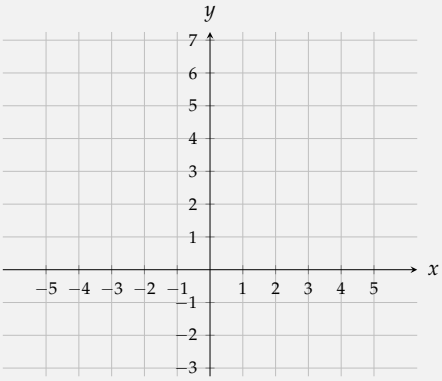
Graph the equation



$m =$ $y-$intercept $=$

YOU TRY IT:

74. Graph the line $6x + 2y = 4$.



$m =$

y-intercept $=$

Writing an equation in point-slope form given the slope and a point

Watch *Video 6: Introduction to the Point-Slope Formula* to complete the following.

DEFINITION **Point-Slope Formula**

Point-Slope Formula: _____

m is the _____, _____ is a known point on the line.

Use the Point-Slope Formula to write an equation of a line passing through the point _____ and having a slope of _____. Write the answer in slope-intercept form and in standard form.

Derive the Point-Slope Formula: $m = \frac{y_1 - y_2}{x_1 - x_2}$ (x_1, y_1) (x, y)

EXAMPLE:

Use the point-slope formula to write an equation of a line passing through the point $(3,8)$ and having a slope of $\frac{1}{3}$. Write the answer in slope-intercept form and standard form.

Start with $y - y_1 = m(x - x_1)$, and substitute $m = \frac{1}{3}, x_1 = 3$ and $y_1 = 8$.

$y - 8 = \frac{1}{3}(x - 3)$

$y - 8 = \frac{1}{3}x - 1$

$y = \frac{1}{3}x + 7$ Solve for y

To change the answer to standard form, subtract $\frac{1}{3}x$ on both sides to obtain: $y - \frac{1}{3}x = 7$. Now multiply both sides of they equation by 3

to get: _____.

YOU TRY IT:

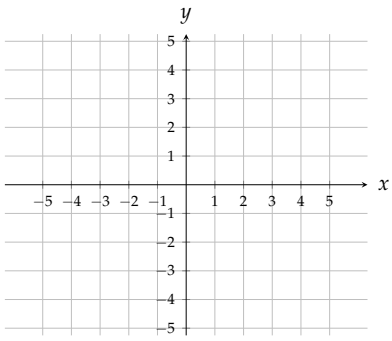
75. Use the point-slope formula to write an equation of a line passing through the point $(-2,-5)$ and having a slope of 3. Write the answer in slope-intercept form and standard form.

Writing an equation in slope-intercept form given the slope and a point

Watch *Video 5: Using Slope-Intercept Form to Determine an Equation of a Line Given a Point on the Line and the Slope*

Use the slope-intercept form to write an equation of a line passing through the point _____ and having a slope of _____. Write the answer in slope-intercept form and in standard form.

$y = mx + b$



EXAMPLE:

Use the slope-intercept form to write an equation of a line passing through the point $(3,8)$ and having a slope of $\frac{1}{3}$. Write the answer in slope-intercept form and standard form.

Start with $y = mx + b$ and substitute $m = \frac{1}{3}$.

$y = \frac{1}{3}x + b$

$8 = \frac{1}{3}(3) + b$ Plug in $(3,8)$ for x and y

$8 = 1 + b$

$7 = b$ Solve for b

$y = \frac{1}{3}x + 7$ Substitute m and b

To change the answer to standard form, subtract $\frac{1}{3}x$ to both sides to obtain: $y - \frac{1}{3}x = 7$. Now multiply both sides of they equation by 3

to get: _____.

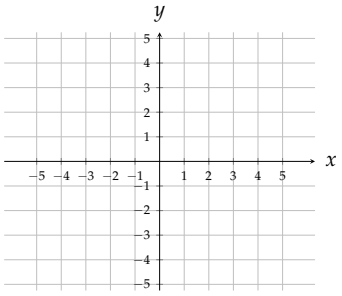
YOU TRY IT:

76. Use the slope-intercept form to write and equation of a line passing through the point $(-2,-5)$ and having a slope of 3. Write the answer in slope-intercept form and standard form.

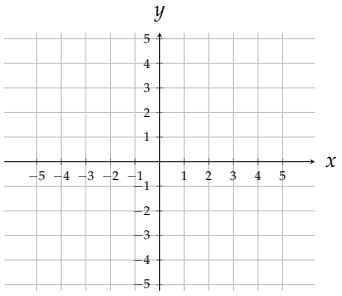
Writing the equations of vertical and horizontal lines through a given point

Watch *Video 11: Determining an Equation of a Horizontal or Vertical Line* to complete the following.

1. Determine an equation of the line passing through the point _____ and parallel to the line defined by _____.



2. Determine an equation of the line passing through the point _____ and perpendicular to the _____.



YOU TRY IT:

77. Write the equations for the horizontal and vertical lines passing through the point $(3,7)$.

Writing the equation of the line through two given points

Watch *Video 7: Writing an Equation of a Line Given Two Points on the Line* to complete the following.

1. Write an equation of the line passing through the points _____ and _____. Write the answer in slope-intercept form. **(Use the point-slope form.)**

   Write an equation of the line passing through the points _____ and _____. Write the answer in slope-intercept form. **(Use the slope-intercept form.)**

YOU TRY IT:

78. Write an equation of the line passing through the points $(-2, -1)$ and $(3, -4)$.

Finding slopes of lines parallel and perpendicular to a line given in slope-intercept form

To find the slopes, we will us the following properties.

Parallel Slope Property:

Two non-vertical lines are parallel if and only if _____.

Perpendicular Slope Property:

Two non-vertical lines are perpendicular if and only if _____.

YOU TRY IT:

79. Consider the line $y = \frac{2}{3}x + 5$. Find the slope a of a line perpendicular and the slope of a line parallel to this line.

Finding slopes of lines parallel and perpendicular to a line given in the form $Ax + By = C$

We will use the **Parallel Slope Property** and the **Perpendicular Slope Property** that are defined above.

First we must convert the form $Ax + By = C$ into the **slope-intercept form** _____.

EXAMPLE:

Consider the line $5x + 3y = 12$.

a. Find the slope of a line parallel to this line.

First we must find the slope-intercept form, so we solve for y.

$$\begin{aligned} 5x + 3y &= 12 \\ 3y &= -5x + 12 \\ y &= -\frac{5}{3} + 4 \end{aligned}$$

The slope of this line is $-\frac{5}{3}$ so the slope of a parallel line is $-\frac{5}{3}$.

b. Find the slope of a line perpendicular to this line.

The slope of the line is $-\frac{5}{3}$ so the slope of a perpendicular line is $\frac{3}{5}$ because

$$-\frac{5}{3} \cdot \frac{3}{5} = -1.$$

YOU TRY IT:

80. Consider the line $-4x + 7y = 21$. Find the slope of a line parallel to this line and perpendicular to this line.

Identifying parallel and perpendicular lines from equations

Watch *Video 4: Determining if Lines are Parallel, Perpendicular, or Neither* to complete the following. *NOTE:* This video may not pop up. Select it from the list in the video box.

In the following examples, information is given about two different lines. Use the slopes to determine if the lines are parallel, perpendicular, or neither.

1. The slopes of the lines are _____ and _____, respectively.

2. The slopes of the lines are _____ and _____, respectively.

3. The slopes of the lines are _____ and _____, respectively.

EXAMPLE:
The equations of two lines are given below. Determine whether they are parallel, perpendicular, or neither.

Line 1: $-6x + 2y = 6$
Line 2: $-y = 3x - 2$

First, find the slope of a each line by writing the equations in **slope-intercept form** (solve for y).

Line 1: $-6x + 2y = 6$
$2y = 6x + 6$
$y = 3x + 2$
$\implies m = 3$

Line 2: $-y = 3x - 2$
$y = -3x + 2$

$\implies m = -3$
$3 \neq -3 \implies$ not parallel.
$3 \cdot (-3) \neq -1 \implies$ not perpendicular.

$\implies$ the lines are neither.

YOU TRY IT:

81. Determine whether the lines are parallel, perpendicular, or neither.

$-4x + 7y = 21$
$7x = 4y + 20$

Writing equations of lines parallel and perpendicular to a given line through a point

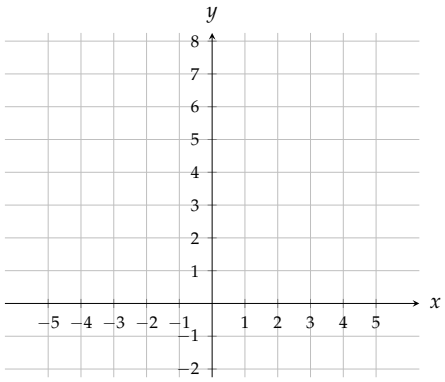
Watch the video **Video 8:** *Writing an Equation of a Line Passing through a Given Point and Parallel to a Given Line* to complete the following.

Write an equation of the line passing through

_____ and parallel to the line

_____.

Pause the video and try graphing the given line and the parallel line yourself.



Play the video and check your answers.

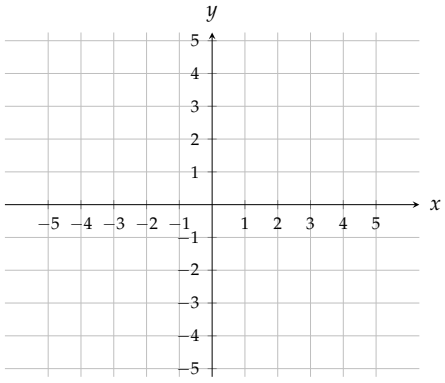
Watch the video **Video 9:** *Determining an Equation of a Line Passing Through a Given Point and Perpendicular to a Given Line* to complete the following.

Write an equation of the line passing through

_____ and perpendicular to the line

_____.

Pause the video and try graphing the given line and the perpendicular line yourself.



Play the video and check your answers.

YOU TRY IT: Consider the line $4x + 3y = -6$. Find the equation of a line that is:

82. perpendicular to $4x + 3y = -6$ and contains $(4, -2)$.

83. parallel to $4x + 3y = -6$ and contains $(4, -2)$.

Identifying functions from relations

Watch the video *Definition of a Function* to complete the following.

DEFINITION

Given a relation in x and y, we say that y is a **function** of x if for each element x in the _____, there is _____ corresponding y value in the _____.

For each relation, determine if the relation defines y as a function of x.

a. {_____}

b. {_____}

YOU TRY IT:
For each relation, determine whether or not it is a function.

84. $\{(2,3), (-5,1), (0,3), (5,-4)\}$.

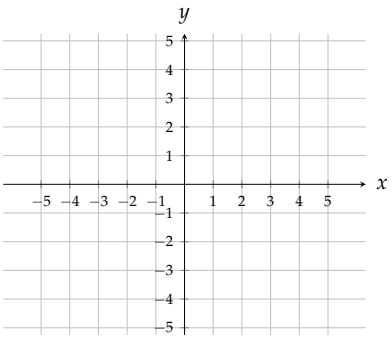
85. $\{(1,-2), (-7,3), (1,5), (0,8)\}$.

Vertical line test

Watch the video *Vertical Line Test* to complete the following.

Determine if the relation is a function.

a.

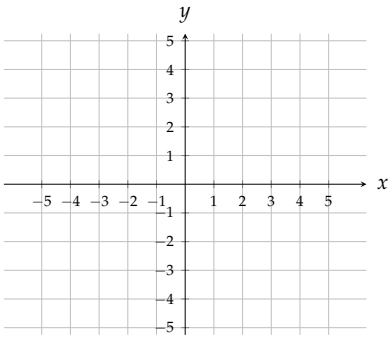


Yes or No?

PROCEDURE **Vertical Line Test**

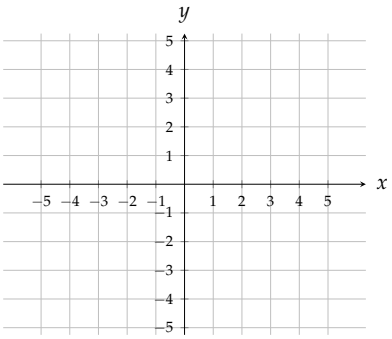
Given a relation in x and y, the graph defines y as a function of x if no vertical line intersects the graph in _____.

b.



Yes or No?

c.



Yes or No?

Domain and range from ordered pairs

Watch *Video 1: Introduction to Relations, Domain, and Range* to complete the following.

DEFINITION

A **relation in** x **and** y is a _____ of the form (x, y).

- The set of _____ is called the _____ of the relation.
- The set of _____ is called the _____ of the relation.

1. The table shows a relation between the number of minutes played and the number of points scored for a college basketball player.

Minutes played x	Points scored $f(x)$
	18
24	
30	
	26
30	22

   (a) Write the relation given in the table as a set of ordered pairs.

   (b) Determine the domain and range of the relation.

   Domain:

   Range:

YOU TRY IT:

86. Find the domain and range of the relation $S = \{(2, 3), (-5, 1), (0, 3), (5, -4)\}$.

Table for a linear function

Watch the video *Function Notation* to complete the following.

Given the function defined by _____, determine the function values.

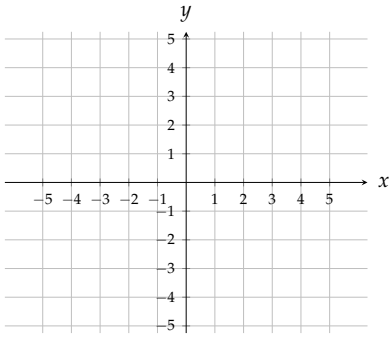
a. $f(-2) =$

b. $f(-1) =$

c.

d.

e.



YOU TRY IT:

87. Complete the function table for

$$f(x) = 2x + 1.$$

x	$f(x)$
-2	
-1	
0	
1	
2	

Additional Notes:

Module 7

Evaluating functions: Linear and quadratic or cubic

Watch the video *Exercise: Evaluating a function* to complete the following.

Consider the functions defined by $f(x) =$ _____, $g(x) =$ _____, _____,

and _____. Find the following.

YOU TRY IT:

88. Given $f(x) = -2x^2 + 3x - 4$, find $f(-3)$.

Finding outputs of a one-step function that models a real-world situation: Function notation

EXAMPLE:
A one-day admission ticket to Valleyfair amusement park is \$37. The cost, C (in dollars), of admission for a group of n people is given by the function

$$C(n) = 37n.$$

What is the cost of admission for a group of 5 people?

We need to find the cost of admission $C(n)$, where the number of people, n is 5.

We let $n = 5$ and write $C(5) = 37 \cdot 5 = \$185$.

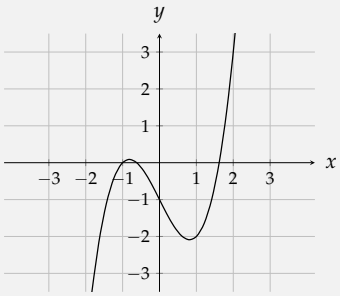
YOU TRY IT:
Lettie and 6 of her friends are sharing the cost of pizza Rhombus Guys Pizza. If p is the total cost of pizza, the amount to be paid by each person, $A(p)$ (in dollars) is given by the function

$$A(p) = \frac{p}{7}.$$

If the total cost of the pizza is \$80 how much does each person owe?

Finding inputs and outputs of a function from its graph

EXAMPLE:



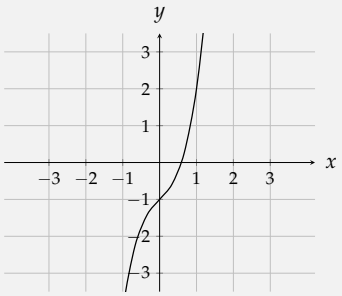
a. Use the graph to find $f(2)$.

   We see the point $(2, 3)$ on the graph, so $f(2) = 3$.

b. Use the graph to find one value of x for which $f(x) = -1$.

   From the graph we see that $f(0) = -1$ so $x = 0$. There are two other values of x where $f(x) = -1$.

YOU TRY IT:

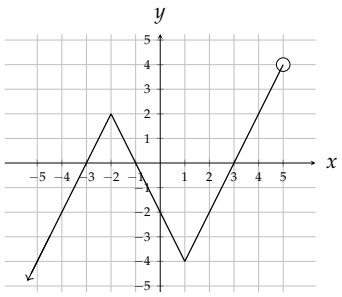


89. Use the graph to find $g(1)$.

Finding an output of a function from its graph

Watch *Video 6: Estimating Function Values from a Graph* to complete the following.

The graph of $y = f(x)$ is shown here.



1. .

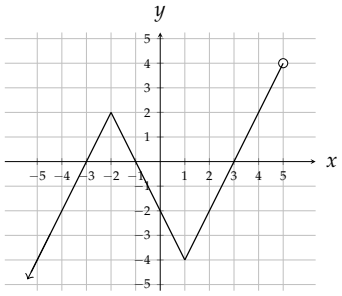
Pause the video and try these yourself.

2. 3. Find

Play the video and check your answers.

Play the video to complete the following.

The graph of $y = f(x)$ is shown here.



4. For what values of x is _____?

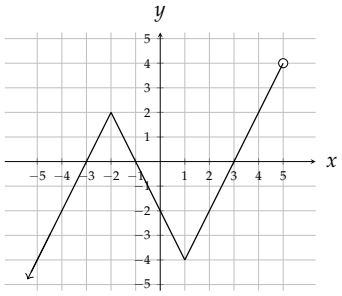
Pause the video and try this yourself.

5. For what values of x is _____?

Play the video and check your answers.

Play the video to complete the following.

The graph of $y = f(x)$ is shown here.



6. Write the domain of f.

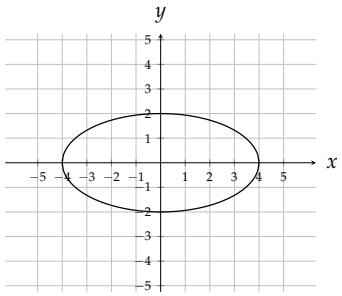
7. Write the range of f.

Domain and range from the graph of a continuous function

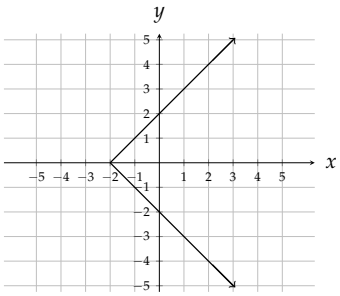
Watch the video *Determining Domain and Range of a Relation Containing an Infinite Number of Points* to complete the following.

Determine the domain and range of each relation.

a.

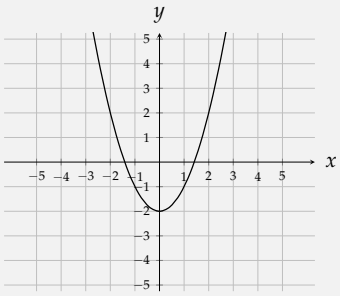


b.



YOU TRY IT:

90. Find the domain and range.



Graphing an integer function and finding its range for a given domain

Take notes from the Explanation Page.

Identifying solutions to a system of linear equations

Watch *Video 1: Determining if an Ordered Pair is a Solution to a System of Linear Equations* to complete the following.

Determine if the given ordered pair is a solution to the system.

a. $(3,1)$

Pause the video and try this yourself.

b. $(-2,6)$

Play the video and check your answer.

EXAMPLE:
Determine if $(3,7)$ is a solution to the system

$$\begin{aligned} 3x - y &= 2 \\ y &= x + 2 \end{aligned}$$

Equation 1:

$3(3) - 7 \stackrel{?}{=} 2$

$9 - 7 = 2$ *TRUE*

Equation 2:

$7 \stackrel{?}{=} 3 + 2$

$7 = 5$ *FALSE*

Because the ordered pair $(3,7)$ does not satisfy *both* equations, the ordered pair is NOT a solution to the system.

YOU TRY IT:

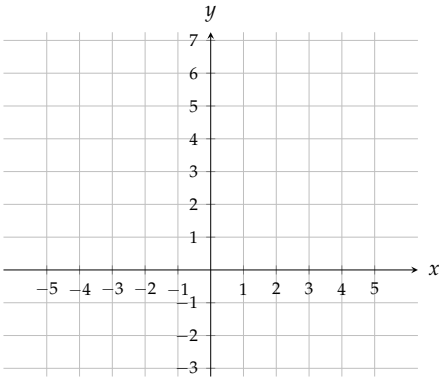
91. Determine if $(2,4)$ is a solution to the system

$$\begin{aligned} 3x - y &= 2 \\ y &= x + 2 \end{aligned}$$

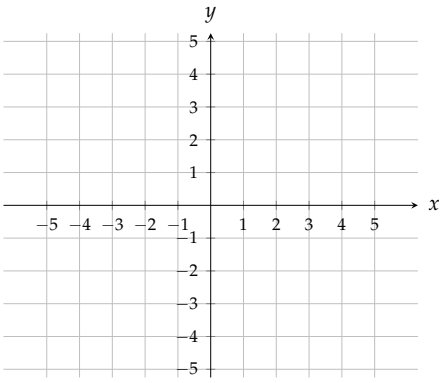
Classifying systems of linear equations from graphs

Watch the video *Solutions to a System of Linear Equations in Two Variables: A Summary* to complete the following.

Example 1

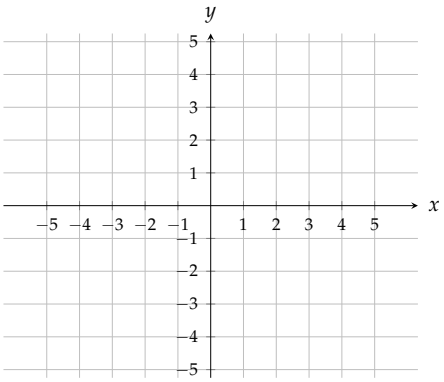


Example 2



- A system is _____ if it has no solution.
- A system is _____ if it has one or more solutions.

Example 3



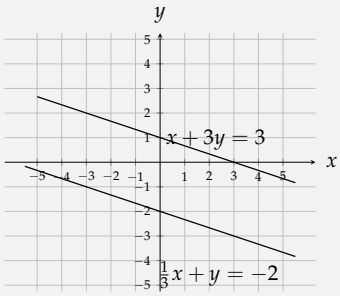
- If the equations in a system of equations represent the same line, then the equations are _____.
- If the equations represent different lines, then the equations are _____.

EXAMPLE:
Graph the system of equations and determine the solution set.

$$x + 3y = 3$$
$$\frac{1}{3}x + y = -2$$

Equation 1: Slope-intercept form:
$y = -\frac{1}{3}x + 1$

Equation 2: Slope-intercept form:
$y = -\frac{1}{3}x - 2$

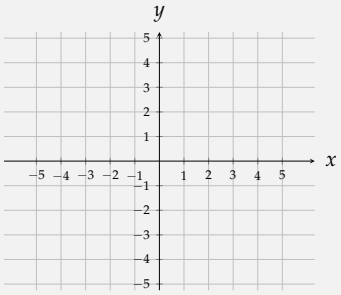


- The line each have a slope of _____, but different $y-$intercepts.
- The lines are parallel and have not point of intersection. Thus, the number of solutions is _____.
- The solution set is _____.

YOU TRY IT:

92. Graph the system of equations and determine the solution set.

$$\frac{1}{2}x + \frac{1}{3}y = \frac{1}{3}$$
$$2y - 1 = -3x$$



a. The lines are (*circle one)*
parallel coinciding.

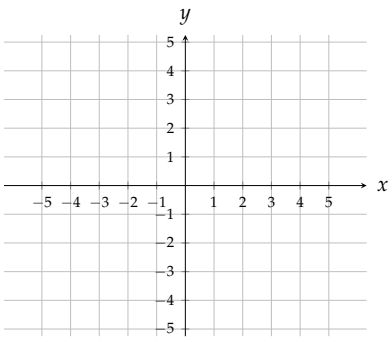
b. How many solutions does the system have? _____

c. The solution set is _____.

Graphically solving a system of linear equations

Watch *Video 2: Solving a System of Linear Equations by Using the Graphing Method* to complete the following.

Graph the system of equations and determine the solution set.



To solve a system of linear equations by graphing, graph the individual equations in the system. The solution set to the system consists of all points of intersection of the graphs.

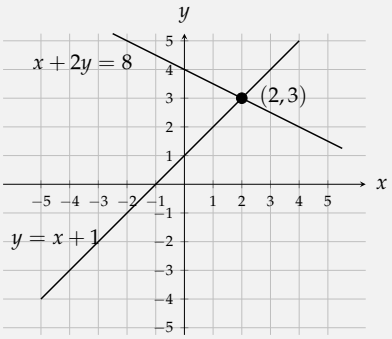
EXAMPLE:

Graph the system of equations and determine the solution set.

$$x + 2y = 8$$
$$y = x + 1$$

Graph each equation and determine the point(s) of intersection.

Equation 1: Slope-intercept form: $y = -\frac{1}{2}x + 4$
Equation 2: Slope-intercept form: $y = x + 1$

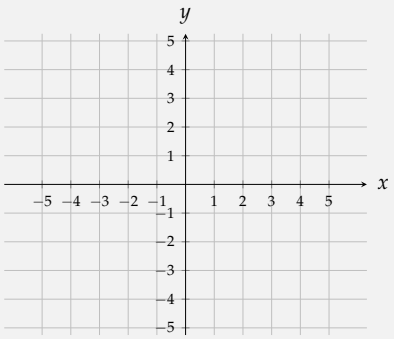


The solution set is $(2, 3)$.

YOU TRY IT:

93. Graph the system of equations and determine the solution set.

$$x - 3y = 9$$
$$2x - y = -2$$



The solution set is _____.

Solving a system of linear equations using substitution

Watch the video *Solving a System of Equations by Using the Substitution Method* to complete the following.

Solve the system by using the substitution method.

Step 1: Isolate one of the variables from one of the equations.

Step 2: Substitute the quantity found in step 1 into the other equation.

Step 3: Solve the resulting equation.

Step 4: Substitute the value of the variable found in step 3 into one of the other equations. Then solve for the remaining variable.

Step 5: Check the solution in both original equations.

EXAMPLE:
Solve the system by using the substitution method.

$$\begin{aligned} -10x + 2y &= 0 \\ -3x + y &= 2 \end{aligned}$$

Step 1: The $y-$ variable in the second equation is the easiest variable to isolate because its coefficient is 1.

$$\begin{aligned} -10x + 2y &= 0 \\ -3x + y &= 2 \Rightarrow y = 3x + 2 \end{aligned}$$

Step 2: Substitute the quantity $3x + 2$ for y in the other equation.

$$-10x + 2(3x + 2) = 0$$

Step 3: Solve for x.

$$\begin{aligned} -10x + 2(3x + 2) &= 0 \quad \text{Simplify} \\ -4x &= -4 \quad \text{Divide by } -2 \\ x &= 1 \end{aligned}$$

Step 4: Substitute the known value for x into one of the original equations to solve for y.

$$\begin{aligned} -3x + y &= 2 \\ -3(1) + y &= 2 \\ y &= 5 \end{aligned}$$

Step 5: Check the ordered pair $(1,5)$ in both original equations.

-10(1)+2(5)=0	-3(1)+(5)=2
-10+10=0	-3+5=2
0=0	-2=2

The ordered pair $(1,5)$ checks in both equations.

Therefore, the solution to the system is $\{(1,5)\}$

YOU TRY IT:

94. Solve the system by using the substitution method.

$$\begin{aligned} 4x + 5y &= -3 \\ x - 2y &= -4 \end{aligned}$$

Solving a system of linear equations using elimination with addition

EXAMPLE:
Solve the system of equations using elimination.

$$\begin{aligned} 2x &= 3y - 2 \\ 3x - 12 &= 2y \end{aligned}$$

Step 1: Write both equations in standard form.

$$\begin{aligned} 2x = 3y - 2 &\Rightarrow 2x - 3y = -2 \\ 3x - 12 = 2y &\Rightarrow 3x - 2y = 12 \end{aligned}$$

Step 2: There are no fractions or decimals to clear.

Step 3: Multiply one or both equations by a nonzero constant to create opposite coefficients for one of the variables.

To get opposite coefficients on x multiply the first equation by -3 and the second equation by 2.

$$\begin{aligned} -3(2x - 3y) &= -2(-3) \\ 2(3x - 2y) &= 12(2) \end{aligned}$$

Simplify the equations from step 3.

$$\begin{aligned} -6x + 9y &= 6 \\ 6x - 4y &= 24 \end{aligned}$$

Steps 4 & 5: Add the simplified equations and solve for the remaining variable.

Add the two equations together and solve for y.

$$\begin{aligned} -6x + 9y &= 6 \\ 6x - 4y &= 24 \\ \hline 5y &= 30 \\ y &= 6 \end{aligned}$$

Step 6: Substitute the known quantity into one of the original equations.

Use one of the equations to solve for x.

$$\begin{aligned} 2x - 3(6) &= -2 \\ 2x - 18 &= -2 \\ 2x &= 16 \\ x &= 8 \end{aligned}$$

The solution is the ordered pair $(8, 6)$.

Step 5: Check the ordered pair $(8, 6)$ in both original equations.

2(8) =3(6)-2	3(8) -12=2(6)
-16+16=0	12=12

The ordered pair $(8, 6)$ checks in both equations.

Therefore, the solution to the system is $\{(1, 5)\}$

YOU TRY IT:

95. Solve the system of equations using elimination.

$$\begin{aligned} -2x + 5y &= 14 \\ 7x + 6y &= -2 \end{aligned}$$

Solving a system of linear equations using elimination with multiplication and addition

Watch *Video 1: Solving a System of Equations by Using the Addition Method* to complete the following.

Solve the system by using the elimination method (also called addition method).

Step 1: Write both equations in standard form, $Ax + By = C$.

Step 2: Clear fractions or decimals (optional).

Step 3: Multiply one or both equations by a nonzero constant to create opposite coefficients for one of the variables.

Step 4: Add the equations from step 3.

Step 5: Solve for the remaining variable.

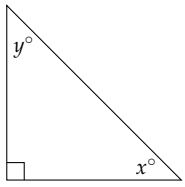
Step 6: Substitute the known quantity into one of the original equations.

Step 7: Check the ordered pair in both original equations.

Solving a word problem involving a sum and another basic relationship using a system of linear equations

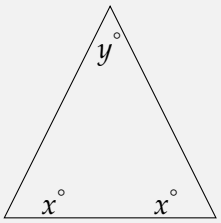
Watch the video *Applications of Systems of Linear Equations Involving Geometry* to complete the following.

In a right triangle, one acute angle measures _____. If the sum of the measures of the two acute angles must equal 90°, find the measures of the acute angles.



YOU TRY IT:

96. An isosceles triangle has two angles of the same measure. If the angle represented by y measures 3° less than the angle x, find the measures of all angles of the triangle.



Solving a word problem using a system of linear equations of the form Ax + By = C

Watch *Video 1: An Application of Systems of Linear Equations (Popcorn and Drink Sales* to complete the following.

At a movie theater, Maria bought ___ large popcorns and ___ drinks for _____. Annie

bought ___ large popcorns and ___ drinks for _____.

_____

YOU TRY IT:

97. Tim and Traci bought school supplies. Tim spent $10.65 on 4 notepads and 5 markers. Traci spent $7.50 on 3 notepads and 3 markers. What is the cost of 1 notepad and what is the cost of 1 marker?

Solving a 2×2 system of linear equations that is inconsistent or consistent dependent

Watch the video *Solving a System of Dependent Equations* and work along.

Solve the system by using the addition method.

EXAMPLE:
Solve each system of equations.

a)

$$3x - y = 9$$
$$-6x + 2y = 7$$

We use substitution here. Solve for y in the first equation.

$$y = 3x - 9$$

Substitute this expression for y into the other equation.

$$-6x + 2(3x - 9) = 7$$
$$-6x + 6x - 18 = 7$$
$$-18 = 7$$

The last equation is always false, so the system is inconsistent and has no solution.

b)

$$\frac{1}{2}x - \frac{2}{3}y = -2$$
$$-3x + 4y = 12$$

Again we use substitution here. Solve the second equation for y.

$$4y = 3x + 12$$
$$y = \frac{3}{4}x + 3$$

Substitute into the other equation.

$$\frac{1}{2}x - \frac{2}{3}\left(\frac{3}{4}x + 3\right) = -2$$
$$\frac{1}{2}x - \frac{1}{2}x - 2 = -2$$
$$-2 = -2$$

The last equation is an identity, so the system is dependent. The solution is $\{(x, \frac{3}{4}x + 3) | x \text{ is any real number}\}$

YOU TRY IT:
Solve each system of equations.

98.

$$y - 3x = 5$$
$$3(x + 1) = y - 2$$

99.

$$6 - 3x = 2y$$
$$\frac{1}{2}x + \frac{1}{3}y = 2$$

Additional Notes:

Module 8

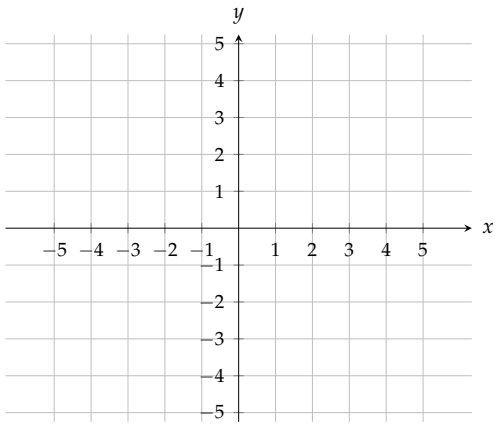
Graphing a linear inequality in the plane: Slope-intercept form

Watch *Video 2: Solving a Linear Inequality in Two Variables* to complete the following.

PROCEDURE Graphing a Linear Inequality in Two Variables

1. Solve for y if possible.
2. Graph the related equation. Draw a dasked line if the inequality is strict, $<$ or $>$. Otherwise, draw a solid line.
3. Shade above or below the line using these guidelines.
   - Shade above the line for $y > ax + b$ or $y \geq ax + b$.
   - Shade below the line for $y < ax + b$ or $y \leq ax + b$.
   - **OR** Use test points to determine which side of the line to shade.

Graph the solution set.



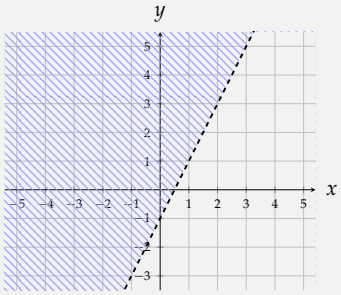
Identifying solutions to a linear inequality in two variables

EXAMPLE:
Graph the inequality $y > 2x - 1$ in the coordinate plane.

- First sketch the line $y = 2x - 1$ using a dashed line.
- Choose a test point on one side of the line. We use $(0, 0)$.

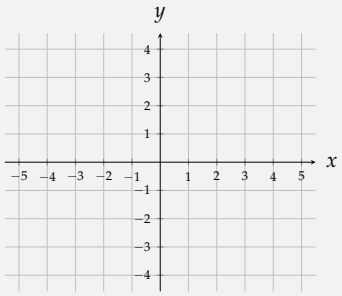
$$0 \overset{?}{>} 2(0) - 1$$
$$0 > -1$$

We shade this region.



YOU TRY IT.

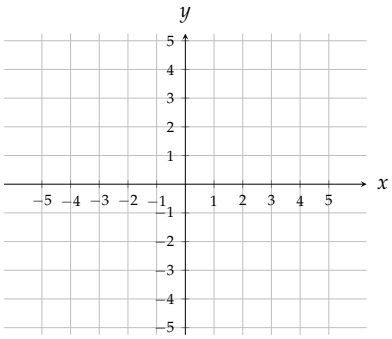
100. Graph the inequality $y \leq -x + 2$ in the coordinate plane.



Graphing a system of two linear inequalities: Basic

Watch *Video 5: Graphing the Intersection of Two Linear Inequalities* to complete the following.

Graph the solution set to the compound inequality.



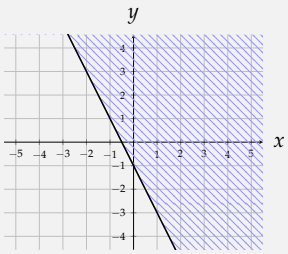
Graphing a system of two linear inequalities: Advanced

EXAMPLE:
Graph the solution to

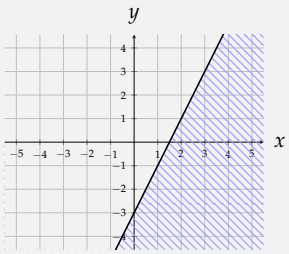
$$2x + y \geq -1$$
$$y - 2x \leq -3$$

- First sketch the graphs of $2x + y = -1$ and $y - 2x = -3$ and shade the regions that satisfy each individual inequality.

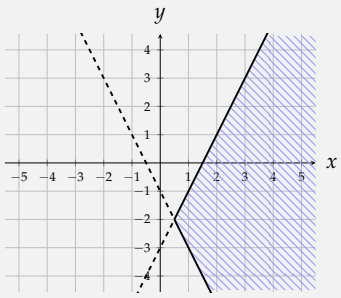
$2x + y = -1$



$y - 2x = -3$



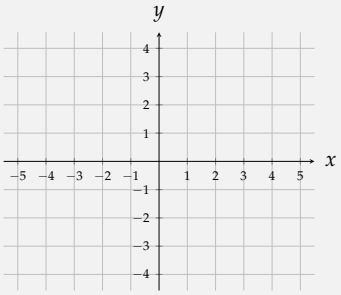
- We want the overlap of the two regions. It is also useful to find the intersection point of the two lines.
  - The intersection point is $(\frac{1}{2}, -2)$.



YOU TRY IT.

101. Graph the solution to

$$3x - 2y \leq 6$$
$$2y - 3x < -8$$



Introduction to the product rule of exponents

Simplify $w^6 \cdot w^6$

Method 1:
Using the definition of **exponent**, we can rewrite this product until we have a single power of w.

$w^6 \cdot w^6 = \overbrace{\qquad\qquad}^{w^6} \cdot \overbrace{w \cdot w \cdot w \cdot w \cdot w \cdot w} = _____$

Method 2:
The method above suggests a rule called the **product rule of exponents**. It says that for any integers

_____ and _____ we have the following.

_____

So, when multiplying powers with the _____, we _____.

Using the rule with the current problem, we get the following.

$w^6 \cdot w^6 = w^{6+6} = w^{12}$

EXAMPLE: Simplify $m^2 \cdot m \cdot m^4$
Method 1:

$m^2 \cdot m \cdot m^4 = \overbrace{m \cdot m}^{m^2} \cdot \overbrace{m}^{m_1} \cdot \overbrace{m \cdot m \cdot m \cdot m}^{m^4} = m^7$

Method 2: $m^2 \cdot m \cdot m^4 = m^{2+1+4}$

YOU TRY IT:

102. Simplify $p^5 \cdot p^3 \cdot p$.

Product rule with positive exponents: Univariate

Watch *Video 1: Multiplying Monomials* to complete the following.

Multiply the monomials.

1. 2. 3.

Product rule with positive exponents: Multivariate

YOU TRY IT:

103. Simplify $4x^3 \cdot (-2x^5)$.

104. Simplify $3y^7 \cdot 4x^3y^4 \cdot x^5$.

Introduction to the quotient rule of exponents

Simplify $\dfrac{x^7}{x^2}$

The exponents tell us how many x's to multiply.

$$\frac{x^7}{x^2} = \frac{\overbrace{\qquad\qquad}^{\text{seven } x\text{'s}}}{\underbrace{x \cdot x}}$$

Dividing out (cancelling) gives us the following.

$$\begin{aligned} \frac{x^7}{x^2} &= \frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}} \\ &= \frac{x \cdot x \cdot x \cdot x \cdot x}{1} \\ &= x^5 \end{aligned}$$

Click on the More about the answer link to complete the following.

More about this answer

We get the exponent in the answer by _____ with the exponents 7 and 2.

$$\frac{x^7}{x^2} = _____ = y^5$$

We call this idea the **quotient rule of exponents**: $\dfrac{a^m}{a^n} = a^{m-n}$ where m and n are integers and a is any number.

So, when dividing powers with the same base, we _____ the exponents.

Quotient of expressions involving exponents

EXAMPLES:

- Simplify $\frac{m^3}{m^5}$.

$$\frac{m^3}{m^5} = \frac{\overbrace{m \cdot m \cdot m}^{\text{three } m\text{'s}}}{\underbrace{m \cdot m \cdot m \cdot m \cdot m}_{\text{five } m\text{'s}}}$$
$$= \frac{\cancel{m} \cdot \cancel{m} \cdot \cancel{m}}{\cancel{m} \cdot \cancel{m} \cdot \cancel{m} \cdot m \cdot m}$$
$$= \frac{1}{m \cdot m}$$
$$= \frac{1}{m^2}$$

Or using the quotient rule of exponents:

$$\frac{m^3}{m^5} = \frac{1}{m^{5-3}} = \frac{1}{m^2}$$

- Simplify $\frac{x^8y^6}{x^3y^4}$.

$$\frac{x^8y^6}{x^3y^4} = \frac{x^8}{x^3} \cdot \frac{y^6}{y^4}$$
$$= x^{8-3}y^{6-4}$$
$$= x^5y^2$$

YOU TRY IT:

105. Simplify $\frac{y^9}{y^3}$.

106. Simplify $\frac{y^3}{y^9}$.

107. Simplify $\frac{a^5b^6}{a^7b^4}$.

Introduction to the power of a power rule of exponents

Simplify $(x^4)^2$

Method 1:

By definition, the _____ tells us _____ x^4 appears in the product.

$(x^4)^2 =$ $\underbrace{\qquad\qquad}_{\textbf{two } x^4\text{'s}}$ Each exponent of 4 in this product tells us how many x's to multiply.

$(x^4)^2 =$ $\underbrace{\qquad\qquad}_{\textbf{four } x\text{'s}} \cdot \underbrace{x \cdot x \cdot x \cdot x}_{\qquad} = x^8$ Note that we get 8 by _____.

Method 2:

The method above suggests a rule called the **power of a power rule of exponents**. It says that for any integers _____, and _____ we have the following.

_____ = _____

Using the rule with the current problem, we get the following: $(x^4)^2 = x^{4 \cdot 2} = x^8$

YOU TRY IT:

108. Simplify $(x^5)^6$.

Introduction to the power of a product rule of exponents

EXAMPLE: Simplify $(2y^4)^3$
Method 1:

$$(2y^4)^3 = \underbrace{2y^4 \cdot 2y^4 \cdot 2y^4}_{\textbf{three } 2y^4\text{'s}}$$
$$= 2 \cdot 2 \cdot 2 \cdot y^4 \cdot y^4 \cdot y^4$$
$$= 8y^{4+4+4} \qquad \text{product rule of exponents}$$
$$= 8y^{12}$$

Method 2: An extension of the **power to a power rule** is the **power of a product rule**. From Method 1 we can see that

$$(2y^4)^3 = 2y^4 \cdot 2y^4 \cdot 2y^4$$
$$= 2 \cdot 2 \cdot 2 \cdot y^4 \cdot y^4 \cdot y^4$$
$$= 2^3(y^4)^3$$
$$= 8y^{4 \cdot 3} \qquad \text{power of a power rule}$$
$$= 8y^{12}$$

Power of a product rule of exponents: for any integer n and any numbers a and b, we have the following.

$$(ab)^n = a^n b^n$$

YOU TRY IT:

109. Use the power of a product rule to simplify $(3p^5)^2$.

Power rules with positive exponents: Multivariate products

Watch *Video 2: Summary of Properties of Exponents* to complete the following.

Name	Property/Definition	Example
Product	$b^m b^n = b^{m+n}$	
Quotient	$\frac{b^m}{b^n} = b^{m-n}$	
Power of a power	$(b^m)^n = b^{mn}$	
Power of a product	$(ab)^m = a^m b^m$	
Power of a quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	
Negative exponent	**Definition:** $b^0 = 1$ for $b \neq 0$	
Zero exponent	**Definition:** $b^{-n} = \left(\frac{1}{b}\right)^n = \frac{1}{b^n}$ for $b \neq 0$	

Power and product rules with positive exponents

EXAMPLE:
Simplify $(-5x^2y)^3$.

Using the **power of a product** rule we get

$$\begin{aligned}(-5x^2y)^3 &= (-5)^3(x^2)^3(y^1)^3 \\ &= -125x^6y^3\end{aligned}$$

YOU TRY IT:

110. Simplify $(-3xy^4)^2$.

Power and quotient rules with positive exponents

EXAMPLE:

Simplify $\left(\frac{4p^3}{r^2}\right)^3$.

Using the **power of a quotient** rule we get

$$\left(\frac{4p^3}{r^2}\right)^3 = \frac{(4p^3)^3}{(r^2)^3} = \frac{4^3(p^3)^3}{(r^2)^3} = \frac{64p^9}{r^6}$$

YOU TRY IT:

111. Simplify $\left(\frac{-2}{c^5d}\right)^4$.

Power rules with positive exponents: Multivariate quotients

EXAMPLE:
Simplify $(ab^2c^3)^4(-3ac)^2$.

$$\begin{aligned}(ab^2c^3)^4(-3ac)^2 &= a^4(b^2)^4(c^3)^4(-3)^2a^2c^2\\ &= (-3)^2a^4a^2(b^2)^4(c^3)^4c^2\\ &= 9a^6b^8c^{12}c^2\\ &= 9a^6b^8c^{14}\end{aligned}$$

YOU TRY IT:

112. Simplify $(-3x^3yz^2)^3(yz^2)^4$.

Additional Notes:

Module 9

Simplifying a ratio of multivariate monomials: Advanced

Watch *Video 4: Simplifying a Rational Expression with Monomials* to complete the following.

Simplify the rational expressions.

1.

2.

PROPERTY Fundamental Principle of Rational Expressions

Let p, q, and r represent polynomials such that $q \neq 0$ and $r \neq 0$. then

$$\frac{pr}{qr} = \frac{p}{q} \cdot \frac{r}{r} = _____$$

YOU TRY IT:

113. Simplify $\dfrac{7x^9}{21x^2}$.

Evaluating expressions with exponents of zero

Watch the video *Definition of b to the Zero Power* to complete the following.

DEFINITION **Definition of b^0**

Let b be a nonzero real number. Then $b^0 = 1$

Simplify.

1. **2.** **3.**

4. **5.**

Show the example used to explain why $b^0 = 1$ for any nonzero number b.

YOU TRY IT:
Simplify.

114. -3^0 **115.** $(-5)^0$

Evaluating an expression with a negative exponent: Positive fraction base

Watch the video *Exercise: Simplifying Expressions with Negative Exponents* to complete the following.

Simplify and write the answer with positive exponents only.

Simplify and write the answer with positive exponents.

EXAMPLE:

$$\left(\frac{-2}{3}\right)^{-4} = \left(\frac{3}{-2}\right)^{4}$$
$$= \left(\frac{3}{-2}\right)\left(\frac{3}{-2}\right)\left(\frac{3}{-2}\right)\left(\frac{3}{-2}\right)$$
$$= \frac{81}{16}$$

YOU TRY IT:

116. $\left(\frac{5}{-2}\right)^{-3} =$

Evaluating an expression with a negative exponent: Negative integer base

Watch *Video 4: Definition of b to a Negative Exponent* to complete the following.

DEFINITION **Definition of** b^{-n}

Let b be a _____ and n be an integer. Then $b^{-n} =$ _____ $=$ _____

Simplify. Write the answers with positive exponents.

1. 2. 3.

EXAMPLE:
Write all answers with positive exponents. Simplify $8x^{-2}$

$$8x^{-2} = \frac{8}{1} \cdot \frac{x^{-2}}{1}$$
$$= \frac{8}{1} \cdot \frac{1}{x^2}$$
$$= \frac{8}{x^2}$$

YOU TRY IT:
Simplify. Write all answers with positive exponents.

117. $-7(-2)^{-3} =$

Rewriting an algebraic expression without a negative exponent

EXAMPLE:
Write all answers with positive exponents.
Simplify $\dfrac{1}{8x^{-2}}$

$$\begin{aligned}\frac{1}{8x^{-2}} &= \frac{1}{8}\cdot\frac{1}{x^{-2}}\\ &= \frac{1}{8}\cdot\frac{x^2}{1}\\ &= \frac{x^2}{8}\end{aligned}$$

YOU TRY IT:
Simplify. Write all answers with positive exponents.

118. $\dfrac{2}{-x^{-8}} =$

Introduction to the product rule with negative exponents

We'll be using the following rules for exponents.

Product rule:
For any number a and any integers m and n, we have the following.

$$a^m \cdot a^n = _____$$

Negative exponent rule:
For any number nonzero number a and any integer m, we have the following.

$$a^{-m} = _____$$

Product rule with negative exponents

YOU TRY IT:
Simplify.

119. $x^{-3} \cdot x^{-5}$

120. $5c^2d^{-4} \cdot 2c^3 \cdot 6c^{-2}d^4$

Quotient rule with negative exponents: Problem type 1

We will use the following rules for exponents.

Quotient rule:
For any number a and any integers m and n, we have the following.

$$\frac{a^m}{a^n} = _____$$

Negative exponent rule:
For any number nonzero number a and any integer m, we have the following.

$$a^{-m} = _____$$

Quotient rule with negative exponents: Problem type 2

YOU TRY IT:
Simplify.

121. $\dfrac{x^{-3}}{x^{-5}}$

122. $\dfrac{10x^4y^{-5}}{20x^{-1}y^{-2}}$

Power of a power rule with negative exponents

We'll be using the following rules for exponents.

Power of a power rule:
For any number a and any integers m and n, we have the following.

$$(a^m)^n = _____$$

Negative exponent rule:
For any number nonzero number a and any integer m, we have the following.

$$a^{-m} = _____$$

Power rules with negative exponents

Take notes from the Explanation Page

Power and quotient rules with negative exponents: Problem type 1

Refer to the table at the end of Module 8 for a reminder of the Properties of Exponents.

YOU TRY IT:
Simplify.

123. $(x^{-3})^5$

124. $(y^{-6})^{-7}$

Power and quotient rules with negative exponents: Problem type 2

Watch the video *Simplifying an Exponential Expression* to complete the following.

Simplify. Write the answer with positive exponents.

YOU TRY IT:
Simplify.

125. $\dfrac{(2a^7b^{-4})^3}{(4a^3b^{-2})^2}$

Midpoint of a line segment in the plane

Watch the video *Exercise: Finding the Midpoint Given Two Points* to complete the following.

Find the midpoint of the line segment between the two given points.

YOU TRY IT:

126. Find the midpoint between $(1, 10)$ and $(-3, 4)$.

Distance between two points in the plane: Exact answers

Watch the video *Exercise: Finding the Distance Between Two Points* to complete the following.

Use the distance formula to find the distance between the two points _____ and _____.

YOU TRY IT:

127. Find the distance between $(1, 10)$ and $(-2, 4)$.

Additional Notes:

Module 10

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel, but still count toward your module completion. To prepare for your upcoming exam:

- ☐ Complete this module.
- ☐ At least two days before your focus group, take your ALEKS exam in the MALL.
- ☐ If you score less than 80% you are strongly encouraged to retake the ALEKS exam.
  - ☐ Ask for a ticket to retake from a tutor.
  - ☐ Work in the MALL for one hour.
  - ☐ Have a tutor sign that you have finished your review.
  - ☐ Retake the ALEKS portion of your exam.
- ☐ Take your written exam the day of your focus group. No retakes will be allowed on written exams.

The score on your Scheduled Knowledge Check is the number of topics that you have mastered (including prerequisite topics) out of the number of topics that you should have mastered by this point.

	Score
ALEKS Exam	
ALEKS Exam Retake	
Written Exam	

*Your recorded ALEKS exam score is the higher of your ALEKS Exam score and ALEKS Exam Retake score.

Module 11

Degree and leading coefficient of a univariate polynomial

Watch the video *Exercise: Polynomials: Degree, Descending Order, Leading Term, and Coefficients* to complete the following.

Given the polynomial _____

a. List the terms of the polynomial.

b. Write the polynomial in descending order.

c. State the degree of the polynomial and the leading coefficient.

Pause the video and try this yourself.

Given the polynomial _____

a. Identify the degree of each term.

b. Identify the degree of the polynomial.

Play the video and check your answer.

Simplifying a sum or difference of two univariate polynomials

Watch *Video 3: Adding Polynomials Horizontally and Vertically* to complete the following.

Add the polynomials.

YOU TRY IT:
Add the polynomial.

128. $(-3x^5 + 2x^3 + 5) + (7x^5 - 8x^3 + 9)$

Multiplying a univariate polynomial by a monomial with a positive coefficient

Watch *Video 2: Multiplying a Monomial by a Polynomial* to complete the following.

Multiply the polynomials.

1.

2.

Multiplying a univariate polynomial by a monomial with a negative coefficient

YOU TRY IT:
Multiply the polynomials.

129. $-5x^3(2x^2 - 7x + 6)$

Multiplying a multivariate polynomial by a monomial

YOU TRY IT: Multiply the polynomials.

130. $-4x^3y^7z(2xy^2z^4 - \frac{1}{2}x^5y)$

Multiplying binomials with leading coefficients of 1

Watch *Video 3: Multiplying Binomials* to complete the following.

Multiply the polynomials.

YOU TRY IT:
Multiply the polynomials.

131. $(x-3)(x+5)$

Multiplying binomials with leading coefficients greater than 1

Watch *Exercise: Multiplying Binomials* to complete the following.

Multiply the polynomials by using the distributive property.

Multiplying binomials with negative coefficients

YOU TRY IT: Multiply the polynomials.

132. $(2x-3)(-3x+5)$

Multiplying binomials in two variables

Watch *Video 4: Multiplying Binomials* to complete the following.

Multiply the polynomials.

YOU TRY IT: Multiply the polynomials.

133. $(3a+4b)(7a-2b)$

Multiplication involving binomials and trinomials in one variable

Watch *Video 5: Multiplying a Binomial by a Trinomial* to complete the following.

Multiply the polynomials.

YOU TRY IT: Multiply the polynomials.

134. $(x-3)(3x^2+4x-5)$

Additional Notes:

Additional Notes:

Module 12

Multiplying conjugate binomials: Univariate

Multiplying conjugate binomials: Multivariate

Watch *Video 6: Formulas for Multiplying Conjugates and Squaring Binomials* to complete the following.

Multiply the polynomials. Show the work.

YOU TRY IT:
Multiply the polynomials.

135. $(3x+4)(3x-4)$

136. $(3x-2)^2$

Squaring a binomial: Univariate

Watch the video *Squaring Binomials* to complete the following.

FORMULA Special Case Products

- $(a+b)^2 =$ _____
- $(a-b)^2 =$ _____

The square of binomials reults in a _____.

Multiply.

1.

2.

Squaring a binomial: Multivariate

YOU TRY IT:
Rewrite without parentheses and simplify.

137. $(3a+4b)^2$

Dividing a polynomial by a monomial: Univariate

Watch the video *Dividing a Polynomial by a Monomial* to complete the following.

Divide.

1.

2.

YOU TRY IT: Divide.

138. $\dfrac{3x^4 - 6x^3 + 9x}{3x^2}$

Polynomial Long Division Type 1

Watch the video *Dividing Polynomials Using Long Division* to complete the following.

Check: (divisor)(quotient) + remainder = dividend

Polynomial Long Division Type 2

Watch the video *Video 3: Long Division of Polynomials* to complete the following. *NOTE:* This may not be the first video that pops up. Select it from the list of videos in the video box.

Check: (divisor)(quotient) + remainder = dividend

EXAMPLE:
Use polynomial long division to evaluate:
$(x^4 + 3x^3 + x - 5) \div (x^2 - 3)$

$$
\begin{array}{r}
x^2 + 3x + 3 \\
x^2 - 3 \overline{)\ x^4 + 3x^3 + 0x^2 + x - 5} \\
\underline{-x^4 \qquad + 3x^2} \\
3x^3 + 3x^2 + x \\
\underline{-3x^3 \qquad + 9x} \\
3x^2 + 10x - 5 \\
\underline{-3x^2 \qquad + 9} \\
10x + 4
\end{array}
$$

So the quotient is $x^2 + 3x + 3$ and the remainder is $10x + 4$.

YOU TRY IT:
Use polynomial long division to evaluate:

139. $(6x^3 + 5x^2 - 7x - 1) \div (3x + 1)$

Synthetic Division

Watch *Video 5: Using Synthetic Division to Divide Polynomials* to complete the following.

EXAMPLE:
Use synthetic division to evaluate:
$(x^4 - 14x^2 + 5x - 9) \div (x + 4)$

$$
\begin{array}{r|rrrrr}
-4 & 1 & 0 & -14 & 5 & -9 \\
 & & -4 & 16 & -8 & 12 \\
\hline
 & 1 & -4 & 2 & -3 & 3
\end{array}
$$

So $(x^4 - 14x^2 + 5x - 9) \div (x + 4)$

$$= x^3 - 4x^2 + 2x - 3 + \frac{3}{x+4}$$

YOU TRY IT:
Use synthetic division to evaluate:

140. $(2x^4 - x^3 - 3x - 1) \div (x - 2)$

Greatest common factor of three univariate monomials

Watch *Video 1: Identifying the Greatest Common Factor* to complete the following.

Identify the greatest common factor.

1.

Pause the video and try this yourself.

2.

Play the video and check your answer.

Greatest common factor of two multivariate monomials

YOU TRY IT:
Find the GCF.

141. $20x^5$, $60x^3$, and $4x^2$

142. $14a^3b^5$ and $49ab^7$

Factoring out a monomial from a polynomial: Univariate

Watch *Video 2: Factor Out the Greatest Common Factor* to complete the following.

Factor out the greatest common factor (GCF).

Factoring a linear binomial

YOU TRY IT:
Factor out the GCF.

143. $14x^4 - 7x^3 + 21x$

Factoring out a monomial from a polynomial: Multivariate

Watch *Video 3: Factoring Out the Greatest Common Factor* to complete the following.

Factor out the GCF.

YOU TRY IT:
Factor out the GCF.

144. $12x^2y^3 - 30x^3y^2 - 3xy$

Additional Notes:

Module 13

Factoring out a binomial from a polynomial: GCF factoring, basic

Watch the video *Factoring Out a Binomial Factor* to complete the following.

Factor out the greatest common factor.

YOU TRY IT: Factor out the GCF.

145. $y^3(y+2) - y(y+2) - 9(y+2)$

Factoring a univariate polynomial by grouping: Problem type 1

YOU TRY IT:
Factor by grouping.

146. $y^3 + 3y^2 - 3y - 9$

Factoring a univariate polynomial by grouping: Problem type 2

Watch *Video 7: Factoring by Grouping* to complete the following.

Factor by grouping.

Factoring a multivariate polynomial by grouping: Problem type 1

Watch the video *Factoring by Grouping* to complete the following.

Factor by grouping. $2ax + 3a + 8bx + 12b$

YOU TRY IT: Factor by grouping.

147. $6xy - 21x + 4y - 14$

Factoring a quadratic with leading coefficient 1

Watch *Video 9: Factoring Trinomials with a Leading Coefficient of 1* to complete the following.

Factor completely.

YOU TRY IT: Factor completely.

148. $x^2 - 12x + 27$

Factoring a quadratic in two variables with leading coefficient 1

Watch *Video 2: Factoring a Trinomial Using the ac-Method* to complete the following. *NOTE:* This video may not pop up. Select it from the list of videos.

Factor completely.

YOU TRY IT: Factor completely.

149. $a^2 + 9ab - 10b^2$

Factoring out a constant before factoring a quadratic

Watch the video *Exercise: Factoring a Trinomial with a Leading Coefficient of 1 and a GCF* to complete the following.

Factor the trinomial completely by using any method. Remember to look for a common factor first.

EXAMPLE:
Factor completely. $4x^2 + 8x - 4$

We factor out 4, the GCF of the trinomial to get

$$4x^2 + 8x - 4 = 4(x^2 + 2x - 1).$$

When we apply the trial and error or ac-method we find that there are no factors of -1 that add to $+2$.

The trinomial cannot be factored any further and we say that $x^2 + 2x - 1$ is **prime**.

$4(x^2 + 2x - 1)$ is the final answer.

YOU TRY IT:
Factor completely.

150. $6a^2 + 21a - 12$

151. $12x^2 + 6x + 18$

Additional Notes:

Module 14

Factoring a quadratic with leading coefficient greater than 1: Problem type 2

Watch *Video 4: Factoring a Trinomial by the Trial-and-Error Method (Leading Coefficient Not Equal to 1)* to complete the following.

Factor completely.

Factoring a quadratic with leading coefficient greater than 1: Problem type 3

YOU TRY IT: Factor completely.

152. $2x^2 - 7x - 15$

Factoring a quadratic with a negative leading coefficient

Watch *Video 7: Factoring a Trinomial with a Negative Leading Coefficient* to complete the following.

Factor completely.

YOU TRY IT: Factor completely.

153. $-x^2 + 2x + 3$

Factoring a quadratic by the ac-method

Watch the videos to complete the following.

Video 1: Factoring a Trinomial Using the ac-Method (1)

Factor completely.

Exercise: Factoring a Trinomial Using the ac-Method

Factor completely.

Exercise: Summary of Factoring a Trinomial Using the ac-Method

Factor completely.

Factoring a perfect square trinomial with leading coefficient 1

YOU TRY IT: Factor completely.

154. $x^2 - 10x + 25$

Factoring a perfect square trinomial with leading coefficient greater than 1

Watch *Video 10: Recognizing and Factoring Perfect Square Trinomials* to complete the following.

Factor completely.

Factoring a difference of squares in one variable: Basic

Watch *Video 1: Introduction to Factoring a Difference of Two Squares* to complete the following.

Factor completely if possible.

1. 2.

YOU TRY IT: Factor completely, if possible

155. $x^2 - 49$

Factoring a difference of squares in one variable: Advanced

Watch *Video 2: Factoring a Difference of Squares* to complete the following. *NOTE:* This video may not pop up. Select it from the list of videos in the video box.

Factor completely.

1.

2.

3.

Perfect squares	Perfect Squares
$1^2 =$	$(x^1)^2 =$
$2^2 =$	$(x^2)^2 =$
$3^2 =$	$(x^3)^2 =$
$4^2 =$	$(x^4)^2 =$
$5^2 =$	
$6^2 =$	
$7^2 =$	
$8^2 =$	
$9^2 =$	
$10^2 =$	
$11^2 =$	
$12^2 =$	
$13^2 =$	

Factoring a polynomial involving a GCF and a difference of squares: Univariate

YOU TRY IT:

156. Factor completely, if possible. $4x^3 - 36x$

Factoring a product of a quadratic trinomial and a monomial

Watch *Video 6: Factoring a Trinomial Using the ac-Method and by Removing the GCF* to complete the following.

Factor completely.

YOU TRY IT:

157. Factor completely. $-20x^3 + 34x^2y - 6xy^2$

Module 15

Module 15 contains all of the topics from Modules 1-14. This is to help you review for your upcoming final exam. If you have already mastered these topics, you will not see them in your carousel.

Additional Notes:

Solutions

Module 1

1. 17°C

2. $-\frac{1}{3}$

3. $-\frac{21}{4}$

4. 125

5. $-\frac{1}{125}$

6. -11

7. 2

8. $-\frac{5}{28}$

9. Associative Property of Addition

10. Distributive Property

11. Commutative Property of Multiplication

12. $-12x + 15$

13. $21x + 14$

14. 13

15. $y = \frac{31}{35}$

16. $y = -\frac{15}{4}$

17. $2x + 17y$

18. $2x + 3y$

Module 2

19. $y = -2$

20. $x = 21$

21. -9

22. 60

23. 126

24. $x = \frac{27}{4}$

25. $y = -2$

26. $y = 13$

27. $x = -\frac{19}{3}$

28. $y = -7$

29. $w = \frac{1}{2}$

30. $x = A + y - 12$

31. $77^{\circ}F$

32. $t - 2$

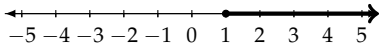
33. $7 + \frac{d}{6} = 9$

Module 3

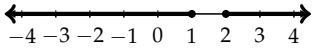
34. $x < 5$

35. $x \geq \frac{7}{8}$

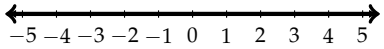
36.



37.



38. All numbers.



39. $x > 0$ and $x \leq 4$

40. $\{a, c, 2, 4\}$

41. $\{a, b, c, d, 1, 2, 4, 6\}$

42. $(-2, 4]$

43. $(-\infty, \infty)$

44. $x < -63$

45. $x \leq \frac{32}{3}$

46. $(-\infty, 2]$

47. $(10, \infty)$

48. $\{x | x \leq -6\}$ or $(-\infty, 6]$

49. $\{x | x \leq \frac{16}{3}\}$ or $(-\infty, \frac{16}{3}]$

50. All real numbers

51. $(-\infty, -3] \cup (-2, \infty)$

52. $(-\infty, -3] \cup (-2, \infty)$

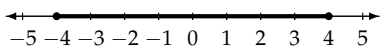
53. $\{3, -3\}$

54. {7,-7}

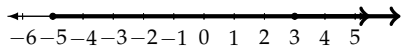
55. $\{8, -5\}$

Module 4

56.



57.



58. $|x| < 1$

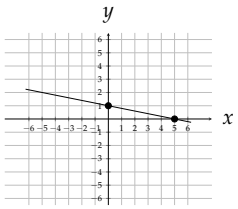
59. no

60. no

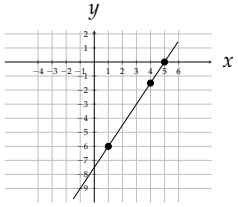
61. yes

62. $(-3, 0), (2, 0), (0, -2)$

63.



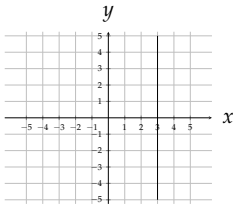
64.



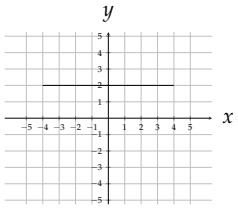
65. undefined

66. 0 (zero)

67.



68.

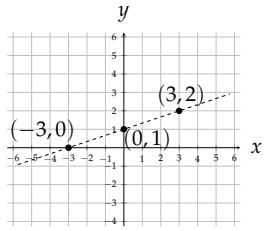


69. $m = \frac{3}{4}$

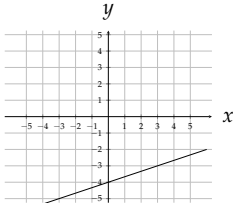
70. m is undefined

71. slope: -2
y-intercept: $(0, 4)$

72.

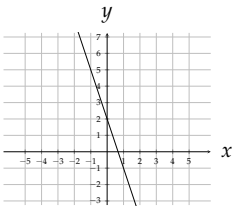


73.



Module 6

74.



$m = -3$
y-intercept:$(0, 1)$

75. Slope-intercept: $y = 3x + 1$
Standard Form: $-3x + y = 1$

76. Slope-intercept: $y = 3x + 1$
Standard Form: $-3x + y = 1$

77. vertical line: $x = -2$ horizontal line: $y = 7$

78. $y = -\frac{3}{5}x - \frac{11}{5}$

79. Slope of parallel line: $\frac{2}{3}$
Slope of perpendicular line: $-\frac{3}{2}$

80. Parallel: $\frac{4}{7}$
Perpendicular: $-\frac{7}{4}$

81. The lines are perpendicular.

82. $y = \frac{3}{4}x - 5$

83. $y = -\frac{4}{3}x + \frac{10}{3}$

84. Function

85. Not a Function

86. domain: $\{2, -5, 0, 5\}$
range: $\{3, 1, -4\}$

87.

x	$f(x)$
-2	-3
-1	-1
0	1
1	3
2	5

Module 7

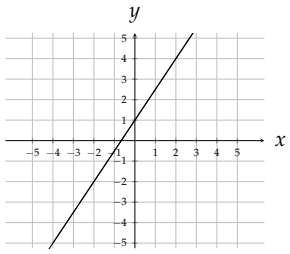
88. -31

89. 2

90. domain: $(-\infty, \infty)$
range: $[-2, \infty)$

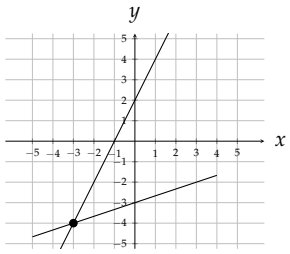
91. $(2, 4)$ is a solution.

92.



a. Coinciding
b. None
c. Empty set

93.



The solution set is $\{(-3, -4)\}$

94. $\{(-2, 1)\}$

95. $(-2, 2)$

96. $x = 61^{\circ}, y = 58^{\circ}$

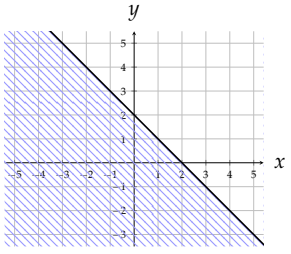
97. notepad: \$1.85
marker: \$0.65

98. $\{(x, 3x+5) | x \text{ is any real number}\}$

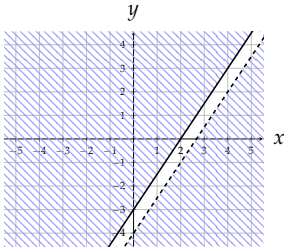
99. no solution

Module 8

100.



101. No solution



102. p^9

103. $-8x^8$

104. $12x^8y^{11}$

105. y^6

106. $\frac{1}{y^6}$

107. $\frac{b^2}{a^2}$

108. x^{30}

109. $9p^{10}$

110. $9x^2y^8$

111. $\frac{16}{c^{20}d^4}$

112. $-27x^9y^5z^{14}$

Module 9

113. $\frac{x^7}{3}$

114. -1

115. 1

116. $-\frac{8}{125}$

117. $\frac{7}{8}$

118. $-\frac{2}{x}^8$

119. $\frac{1}{x^8}$

120. $60c^3$

121. x^2

122. $\frac{x^5}{2y^3}$

123. $\frac{1}{x^{15}}$

124. y^{42}

125. $\frac{a^{15}}{2b^8}$

126. $(-1, 7)$

127. $3\sqrt{5}$

Module 11

128. $4x^5 - 6x^3 + 14$

129. $-10x^5 + 35x^4 - 30x^3$

130. $-8x^4y^9z^5 + 2x^8y^8z$

131. $x^2 + 2x - 15$

132. $-6x^2 + 19x - 15$

133. $21a^2 + 22ab - 8b^2$

134. $3x^3 - 5x^2 - 17x + 15$

Module 12

135. $9x^2 - 16$

136. $9x^2 - 12x + 4$

137. $9a^2 + 24ab + 16b^2$

138. $x^2 - 2x + \frac{3}{x}$

139. quotient: $2x^2 + x - 2$
remainder: 1

140. $2x^3 + 3x^2 + 6x + 9 + \frac{17}{x-2}$

141. $4x^2$

142. $7ab^5$

143. $7x(2x^3 - x^2 + 3)$

144. $3xy(4xy^2 - 10x^2y - 1)$

Module 13

145. $(y + 2)(y^3 - y - 9)$

146. $(y + 3)(y^2 - 3)$

147. $(2y - 7)(3x + 2)$

148. $(x - 9)(x - 3)$

149. $(a - b)(a + 10b)$

150. $3(a + 4)(2a - 1)$

151. $6(2x^2 + x + 3$

Module 14

152. $(2x + 3)(x - 5)$

153. $-(x - 3)(x + 1)$ or $(3 - x)(x + 1)$

154. $(x - 5)^2$

155. $(x - 7)(x + 7)$

156. $4x(x - 3)(x + 3)$

157. $-2x(5x - y)(2x - 3y)$

ARITHMETIC PROPERTIES

Associative:	addition: $a+(b+c)=(a+b)+c$ multiplication: $a(bc)=(ab)c$	**Identity:**	addition: $0+a=a$ multiplication: $1\cdot a=a$
Commutative:	addition: $a+b=b+a$ multiplication: $ab=ba$	**Inverse:**	addition: $a+(-a)=0$ multiplication: $a\cdot\frac{1}{a}=1,\ a\neq 0$
Distributive:	$a(b+c)=ab+ac$		

FRACTIONS

Adding:	$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$	**Multiplying:**	$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$
Subtracting:	$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$	**Dividing:**	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$

FACTORING

Difference of Two Squares	**Sum and Difference of Two Cubes**
$a^2-b^2=(a-b)(a+b)$	$a^3+b^3=(a+b)(a^2-ab+b^2)$
$a^2+b^2=$Does not factor	$a^3-b^3=(a-b)(a^2+ab+b^2)$
Perfect Square Trinomials	
$a^2-2ab+b^2=(a-b)^2$	
$a^2+2ab+b^2=(a+b)^2$	

DISTANCE AND MIDPOINT FORMULAS

Distance between (x_1,y_1) and (x_2,y_2)	Midpoint between (x_1,y_1) and (x_2,y_2)
$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	$m=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

ABSOLUTE VALUE

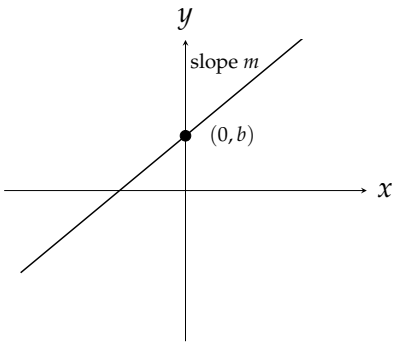
Statement	Equivalent Statement	Statement	Equivalent Statement
$\lvert x\rvert=a$	$x=a$ or $x=-a$	$\lvert x\rvert\leq a$	$-a\leq x\leq a$
$\lvert x\rvert=\lvert y\rvert$	$x=y$ or $x=-y$	$\lvert x\rvert\geq a$	$x\leq -a$ or $x\geq a$

CIRCLE

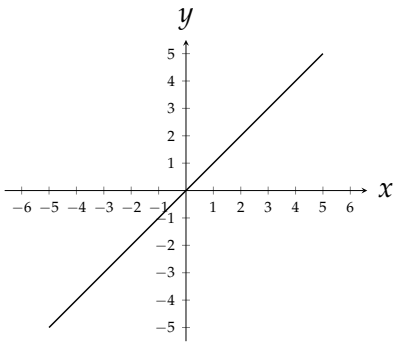
Standard Form of a Circle with center (h,k) and radius r: $(x-h)^2+(y-k)^2=r^2$

COMMON GRAPHS

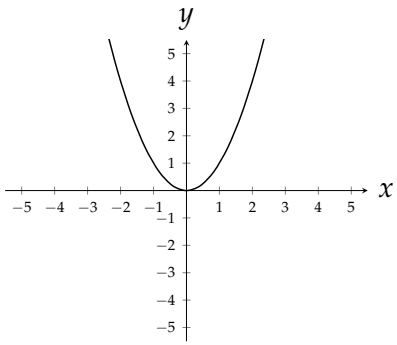
$f(x) = mx + b$



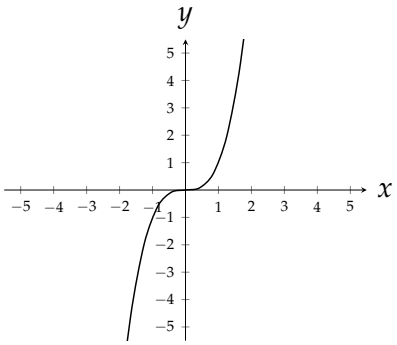
$f(x) = x$



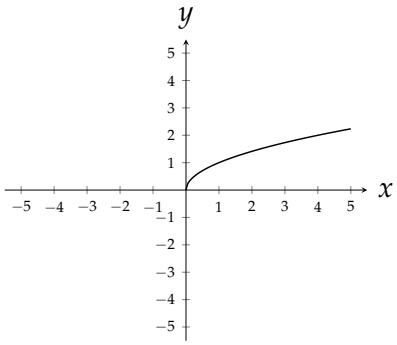
$f(x) = x^2$



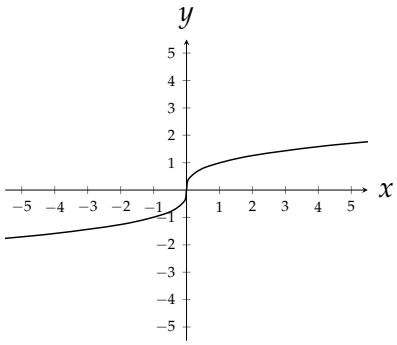
$f(x) = x^3$



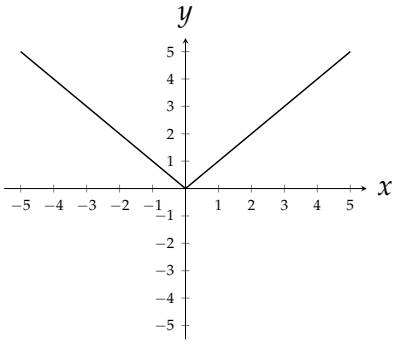
$f(x) = \sqrt{x}$



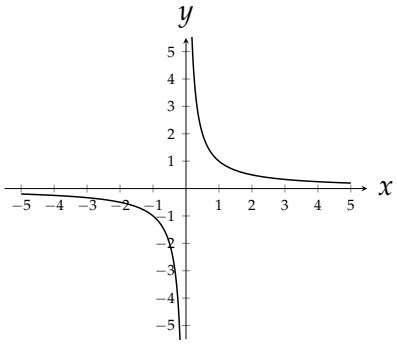
$f(x) = \sqrt[3]{x}$



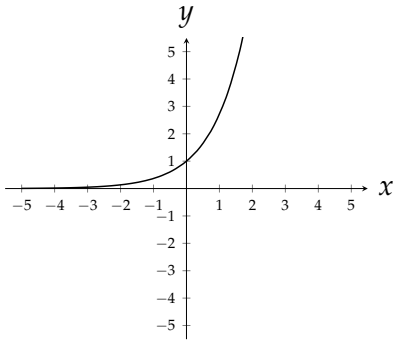
$f(x) = |x|$



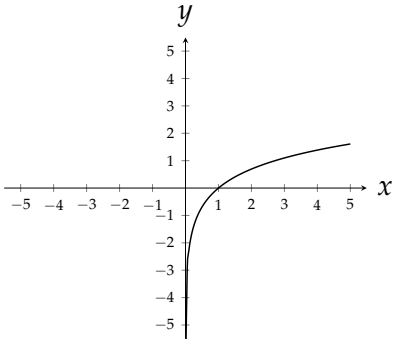
$f(x) = \dfrac{1}{x}$



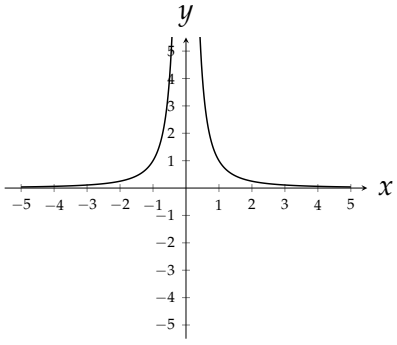
$f(x) = e^x$



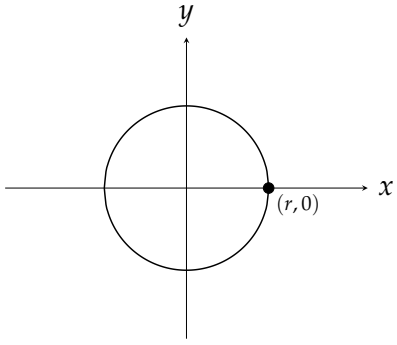
$f(x) = \ln x$

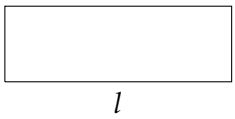
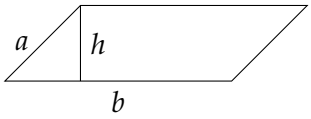
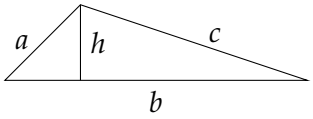
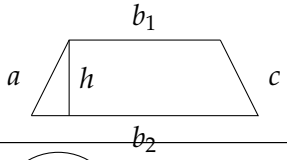
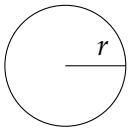


$f(x) = \dfrac{1}{x^2}$



$x^2 + y^2 = r^2$



GEOMETRY			
Rectangle		Perimeter $= 2l + 2w$	Area $= lw$
Parallelogram		Perimeter $= 2a + 2b$	Area $= bh$
Triangle		Perimeter $= a + b + c$	Area $= \frac{1}{2}bh$
Trapezoid		P $= a + b_1 + b_2 + c$	Area $= \left(\frac{b_1 + b_2}{2}\right)h$
Circle		Circumference$= 2\pi r$	Area $= \pi r^2$

PROPERTIES OF EXPONENTS			
$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a^n)^m = a^{nm}$	$(ab)^m = a^m b^m$
$a^0 = 1, a \neq 0$	$a^{-n} = \frac{1}{a^n}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	

DEFINITION OF LOGARITHM	
$log_a x = y \iff a^y = x$	$\ln x = y \iff e^y = x$

LAWS OF LOGARITHMS	
$\log_a m + \log_a n = \log_a mn$	$\ln m + \ln n = \ln mn$
$\log_a m - \log_a n = \log_a \frac{m}{n}$	$\ln m - \ln n = \ln \frac{m}{n}$
$\log_a m^n = n \log_a m$	$\ln m^n = n \ln m$