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Pre-Collegiate Factors Influencing Students' Success in their First University Mathematics Course: A Quantitative and Qualitative Study

Michele Iiams

PRE-COLLEGIATE FACTORS INFLUENCING STUDENTS' SUCCESS IN THEIR FIRST UNIVERSITY MATHEMATICS COURSE: A QUANTITATIVE AND QUALITATIVE STUDY

by

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This dissertation, submitted by Michele A. Iiams in partial fulfillment of the requirements for the Degree of Doctor of Philosophy from the University of North Dakota, has been read by the Faculty Advisory Committee under whom the work has been done and is hereby approved.

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This dissertation meets the standards for appearance, conforms to the style and format requirements of the Graduate School of the University of North Dakota, and is hereby approved.

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Dean of the Graduate School

July 8, 2002
Date

PERMISSION

Title	Pre-collegiate Factors Influencing Students' Success in Their First University Mathematics Course: A Quantitative and Qualitative Study
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Signature Michele Iiams

Date 7/3/02

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ABSTRACT

The quantitative investigation conducted considered the effect of pre-collegiate factors such as high school size, high school grade point average, ACT mathematics component score, ACT composite score, highest level of high school mathematics available, highest level of high school mathematics completed with at least a grade of C, and sex on the initial level of mathematics students' enrolled in and the grade earned in their first university mathematics course. The qualitative inquiry obtained the students' points of view on the transition from high school math to university mathematics and the influence of their past experiences on their success or lack of success in their first university mathematics course. Student interviews also provided insight into how the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) were implemented in their high school mathematics courses.

In particular it was concluded that quantitative measures of the individual students' abilities (i.e. ACT scores, high school grade point average, highest level of high school math completed with a C or better, etc) are better predictors of students' initial university mathematics course and subsequent success than general high school descriptors. Qualitatively, it was concluded that the use of technology, students' study habits and strategies for success, and relationships between the student and their peers and the student and their teachers all play an important role in student success in

university mathematics. From the student interviews it was also concluded that, in spite of the recommendations of the NCTM *Standards* , high school and university mathematics classrooms were still predominantly teacher-centered.

CHAPTER I

INTRODUCTION

Motivation for This Study

As an undergraduate at Northeast Missouri State University (NMSU), it seemed to me that the students from small high schools had a much different pre-collegiate experience than the students from large high schools. In talking with classmates and new friends from other high schools, it appeared that the academic and extracurricular options were far greater for students from the larger high schools than the students from the smaller high schools. It seemed that the students from the small high schools were at a disadvantage as they began their university studies. Throughout my first semester at NMSU, I saw many of the students from the small high schools struggle with writing papers, preparing for math class and, in general, working to succeed in college. Not all students from the larger schools thought college was easy, but their transition from high school to college did not seem as difficult personally or academically. Studying to become a secondary math teacher, I questioned whether the different experiences students had in high school might affect their experiences at the university level. At that time, it appeared that the differences were most likely due to the difference in high school size. Research on this topic indicates that the answer is not quite that simple.

As a high school math teacher, first in a small Missouri school district (graduating class size of about 50) and later in a large school district in Colorado (graduating classes of over 350), the pros and cons, from the teacher's viewpoint, of both settings were evident. Currently, as an instructor in the Mathematics Department at the University of North Dakota (UND), and again from the teacher's viewpoint, it appears that many students, particularly from the smaller schools, struggle in their first university mathematics course. Yet, what is frustrating and bewildering to these students and the Mathematics Department faculty is that many of them report being successful, with little or no effort, in their high school math courses.

What are the differences between the pre-collegiate experiences of students from large and small high schools? Specifically, does one size high school prepare students for university mathematics better than another? Are there factors, besides high school size, which play a role in determining students' success in their first university mathematics course? In a search of the Dissertation Abstracts, ERIC, and the Ingenta databases from 1980 forward, research was found regarding the differences in secondary education environments and student success at the university level.

Research has shown that large high schools are able to offer a wider range of courses, particularly in mathematics, than small high schools (DeYoung, 1995; Lee, Smith, & Croninger, 1997; Monk, 1987; Southwest and West Central Educational Planning Task Force, 1980), possibly indicating that students from large schools would enter college with a better academic background and hence be more successful in university level mathematics than students from small high schools. Due to a perennial shortage of mathematics teachers, small schools are typically forced to hire mathematics teachers with less education in the teaching of mathematics than teachers in large schools

(DeYoung, 1995; Fowler & Walberg, 1991). Such teachers may have graduated with a degree in science or English education, for example, but also completed enough math credits in college to be able to teach math courses when needed. In addition, once employed in a rural school district, these teachers may not have district-supported opportunities for continuing education courses in the teaching of mathematics that are often available in large school districts.

In a study conducted by Morgan (1993), data from UND and North Dakota State University students indicated that students from small high schools were less likely to major in mathematics or science than were students from larger high schools. In addition, they were more likely, than students from larger high schools, to transfer out of mathematics or science intensive majors. Students from small North Dakota high schools were also more likely to come from high schools where fewer mathematics and/or science courses were offered and smaller numbers of teachers certified in mathematics and science taught.

There are various quantitative studies in which researchers have attempted to develop regression models that determine which pre-college factors (i.e., ACT scores, high school rank, high school grade point average (GPA), placement test scores, high school size, etc.) predict student success in college, in general, and in mathematics specifically. Wright (1984) reported that he was unable to find an acceptable regression model. That is, of the regression models formed, none of them accounted for a sufficient amount of the variance in the dependent variable. If an acceptable model to predict success in a university-level mathematics course is determined, it typically includes high school rank, high school GPA, and/or that particular university's placement exam score (Edge & Friedberg, 1984; Maxwell, 1988; Nejadsadeghi, 1985) or college success in general (Gallagher, 1986; Willingham et al., 1985).

Seymour and Hewitt (1997) conducted a large-scale qualitative study which considered why students drop out of university science, mathematics, and engineering programs at alarming rates. During interviews and focus groups, students described the shock they felt when they realized how unprepared they were to succeed in their chosen major. The areas of poor high school preparation indicated by the students included lack of study skills, low teacher expectations and grade inflation, lack of available higher-level mathematics and science courses, and lack of school resources for lab materials and technology.

In addition to the search for prediction models for university or mathematics success, and the results of the study conducted by Seymour and Hewitt, in the past decade a shift in the paradigm for the teaching of mathematics at the pre-collegiate level has taken place. In 1989, the National Council of Teachers of Mathematics (NCTM) published the *Curriculum and Evaluation Standards for School Mathematics* (*Standards*). This document outlined a vision for the teaching of mathematics that has been implemented in varying degrees through many different approaches over the past 13 years. In general, the vision highlighted the need to move from teacher-centered classrooms toward more student-centered learning environments, and the need to incorporate technology. The mathematics education programs under which most current university freshmen were instructed have, in some way, been influenced by the

Standards. In spite of changes recommended for the K-12 mathematics curriculum in general, the Mathematics Department at UND does not endorse the use of graphing calculators in any of its mathematics courses through Calculus III, and the faculty predominantly use the lecture format in their teaching. For this reason, what students experience in their secondary mathematics courses and how these experiences affect their success or lack of success in their first university mathematics course is of particular interest.

In general, there are a number of studies that consider the relationship of high school size and various other pre-collegiate factors and achievement at the post-secondary level. There are fewer studies that focus on success in university mathematics and an even smaller number of studies that present the student perspective of high school math and the implications for success in university mathematics. The intention of this study is to add to the results based on quantitative data currently in existence, to add to the limited number of studies based on qualitative data, and to search within a single student sample for connections between the two. This study will also add to the literature that discusses the level to which the NCTM *Standards* are being implemented at the secondary level and the impact of this implementation on students' success in college mathematics. General concerns regarding students' preparation and readiness for college-level mathematics will also be addressed.

Purpose of the Study

The investigation conducted was intended (a) to consider the effect of pre-collegiate factors such as high school size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school mathematics available, highest level of high school mathematics completed with at least a grade of C, and sex on the initial level of mathematics students enroll in and the grade earned in their first university mathematics course; and (b) to obtain students' points of view on the transition from high school math to university mathematics and how their past experiences influenced their success or lack of success in their first university mathematics course.

The results of this study will be made available to the Mathematics Department at UND so as to provide the faculty with additional knowledge about the students they serve and how the services offered might be improved. In addition, as an educator of mathematics teachers, I will share the results of this study with both pre-service and in-service teachers, with the hope that they will consider how their choices as teachers affect their students' later math experiences. On a larger scale, it is expected that this research study will add to the literature already available on school district consolidation and how pre-collegiate experiences affect students' success in university mathematics courses.

Delimitations

The sample consisted of students who had not previously completed a college mathematics course, who had graduated from high school within the previous year (graduated summer 1998 or later), and who were enrolled in an entry-level mathematics course at UND in the fall of 1999. Entry level mathematics courses are the courses students may be placed in according to their UND Placement Test scores, ACT Mathematics scores, College Level Examination Program (CLEP) scores, or Advanced Placement (AP) scores. The population excluded students with transfer credit from any college or university. In an initial survey, the students were asked to provide demographic information and to indicate whether they were willing to participate in the study. The signed consent/survey form allowed the researcher to access students' academic records.

In the spring semester of 2000, 16 students were chosen to be interviewed. These students were selected based on a willingness to be interviewed, which they indicated on the initial consent form (see Appendix A), and the fact that they had completed a Precalculus type course, with a grade of C or better, as their highest level of math in high school and then enrolled in UND's Precalculus course in the fall of 1999. The decision to interview only UND Precalculus students who had taken Precalculus in high school helped to focus the qualitative piece of this study. By limiting the math level under consideration, it was expected that themes and assertions would emerge more fully and clearly. Upon analysis of the interview data, it was concluded that the information about students' high school experiences was incomplete. Thus, this set of student interviews served as a pilot study and the interview process was repeated in the spring of 2001. (NOTE: It was not necessary to repeat the quantitative aspect of the study as the data collected were sufficiently complete.) The 13 students interviewed in the spring of 2001 were chosen in the same manner as the students interviewed in the spring of 2000. They had graduated from high school within the previous academic year (summer 1999 or later), completed a Precalculus course, with a grade of C or better as their highest level of math in high school, and then enrolled in the Precalculus course at UND in the fall of 2000.

Limitations

The limitations of the quantitative aspect of this study include:

1. The sample size, as the data were collected over a short period of time (one semester), at a single university in a rural setting.

2. Course titles and content vary greatly from one high school to another. For example, one high school may offer a course titled Functions, Statistics and Trigonometry to prepare students for that high school's Calculus course, whereas another high school may offer both Functions, Statistics and Trigonometry, and Precalculus, with the latter course being the prerequisite for that high school's Calculus course and the former being the highest level of math available to students who are not interested in or ready to take high school Calculus.

3. Data for socio-economic status were unavailable for this analysis.

4. The UND Mathematics Placement Program examination was under revision thus; Placement Test scores were not used in this analysis.

5. Participants' ACT Mathematics scores, ACT Composite scores, and high school GPAs were available only if they requested the scores be sent to the UND.

6. The high school GPA, as reported by the students, was their GPA at the time they took the ACT Assessment. This is typically in their junior year or early in their senior year of high school.

7. High school graduating class size, highest level of high school math available, and highest level of high school math completed with at least a grade of C were student-reported data.

8. A limitation of the qualitative aspect of this study was the inability to obtain adequate qualitative data from students who graduated from small high schools. Of the students who indicated a willingness to participate in the interview process, only one was from a small high school.

Research Questions

1. Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex, which are the most efficient predictors of the level of mathematics students enroll in their first semester at UND?

2. Is there a significant difference in the average level of math students enroll in their first semester at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

3. Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex which are the most efficient predictors of students' success in their first mathematics course at UND?

4. Is there a significant difference in the average grade earned by students in their first mathematics course at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

5. How are students' high school mathematics experiences connected to their success or lack of success in their UND Precalculus course?

Terminology

American College Test (ACT) – The ACT Assessment is a set of tests taken by high school juniors, and sometimes seniors, to evaluate their abilities in four skill areas. The areas evaluated are English, mathematics, reading, and science reasoning. These tests are designed to assess students' general educational development and their ability to complete college-level work. The test emphasizes reasoning, analysis, problem solving, and the integration of learning from various sources, as well as the application of these proficiencies to the kinds of tasks college students are expected to perform. In addition to providing a composite score and scores for each of the four skill areas, the ACT Assessment also provides two subscores in English, three subscores in mathematics, and two subscores in reading. A student's results from the ACT Assessment are used by universities as one part of the admission criteria and to place freshmen students in appropriate courses (ACT, 2001).

ACT Composite score – The overall score the student earned on the ACT Assessment. The range of possible scores is from 1 to 36 (ACT, 2001).

ACT Mathematics score – The score earned by a student on the mathematics component of the ACT Assessment. The range of possible scores is from 1 to 36. The use of calculators is permitted on the ACT Mathematics Assessment. The student may use any four function, scientific, or graphing calculator that meets ACT's requirements. Calculators with built-in Computer Algebra Systems are prohibited (ACT, 2001).

Entry-level mathematics courses – Entry-level mathematics courses are the courses students are placed into according to their UND Placement Test scores, ACT Mathematics scores, and CLEP or AP scores. Entry-level mathematics courses are the first (or initial) university mathematics course in which a student can enroll. The entry-level mathematics courses were assigned numerical values based on the Placement Program criteria used to place students in each of the courses, how the courses are related to each other in terms of which course is a prerequisite for another, and the math "track" to which each course belongs. The Intermediate Algebra course is a remedial mathematics course and does not count toward graduation. The Precalculus, Discrete Mathematics, Calculus I and Calculus II are designed for students in math-intensive majors, while the College Algebra, Finite Mathematics, Trigonometry, and Applied Calculus courses are designed for students in non-math intensive majors. The numerical values were assigned as follows:

1 = Intermediate Algebra
2 = College Algebra, Finite Mathematics, and Trigonometry
3 = Precalculus, Applied Calculus, and Discrete Mathematics
4 = Calculus I and Calculus II.

High school grade point average (GPA) – A high school grade point average is determined first by assigning a point value for each letter grade given at the end of a course. Typically, an A = 4.00 points, a B = 3.00 points, a C = 2.00 points, a D = 1.00 points, and an F = 0.00 points. The point value of the letter grade is then multiplied by the number of credit hours associated with the course and the product is divided by the total number of credits the taken by the student. The range for this variable is 0.00 to

4.00. The high school GPA for each participant was obtained from information reported by the student on the ACT Course/Grade Information Section of the ACT Assessment.

High school size – This is measured by the number of students in the students' high school graduating class. This information was reported, by the participant, on the survey form.

Highest level of math completed with a grade of C or better – Given the choice of Algebra II, Trigonometry, Precalculus, Calculus, or AP Calculus, the student participants indicated the highest level of mathematics they completed in high school with a grade of C or better. This information was reported, by the participant, on the survey form.

Highest level of high school math available – Given the choice of Algebra II, Trigonometry, Precalculus, Calculus, and AP Calculus, the student participants were asked to indicate which courses were available in their high school. This information was reported, by the participant, on the survey form.

Large high school – This is a high school with a graduating class of more than 320 students.

Medium high school – This is a high school with a graduating class of more than 75 students and fewer than 320 students.

Small high school – This is a high school with a graduating class of fewer than 75 students.

University of North Dakota (UND) – UND is a co-educational, state-supported university located in Grand Forks, North Dakota, that includes undergraduate academic programs in 85 fields through seven academic units: Arts and Science, Aerospace

Sciences, Business and Public Administration, Engineering and Mines, Fine Arts, Education and Human Development, and Nursing. Courses are taught in the semester system, and undergraduate students are expected to satisfactorily complete a minimum of 125 semester hours in order to receive a baccalaureate degree (University of North Dakota, 2001).

University mathematics course grade (UGRADE) – Based on a four-point scale, this is the semester grade earned by students in their first university mathematics course.

In this chapter, the motivation for and the purpose of this study have been discussed. In addition, the delimitations and limitations of the study have been outlined. The purpose and the motivation provide a basis for the chosen research questions. The delimitations and limitations are offered as further clarification of the intentions of this study. In closing, a set of definitions was offered to assist in the understanding of the terminology used throughout this document. In Chapter II, a review of the literature related to the research topic addressed in this dissertation will be presented.

CHAPTER II

REVIEW OF THE LITERATURE

In the previous chapter, the discussion of the purpose of this study contained a brief explanation of and reference to previous research related to the topic of this dissertation. In this chapter, a more detailed review of the literature is provided. The purpose of the literature review is to provide a context in which to consider this research project. In considering the factors related to students' high school settings, particularly size of graduating class, and how these factors may contribute to their success or failure in university mathematics, it is necessary to discuss research from the areas of school size, achievement in high school mathematics, best practices in secondary education, student success at the university level, the teaching and learning of mathematics, teacher preparation, and technology in the mathematics classroom.

Research regarding the best circumstances under which to educate students at the secondary level is diverse. Even with a focus on high school size and academic success, research motives, approaches, and findings vary greatly. The motives include the search for results to inform the ongoing debate about the consolidation of public schools, determining the best conditions under which to educate secondary students, and indicating the differences and similarities of successful versus unsuccessful students at the collegiate level. Regardless of the research motives and the approaches taken, the findings of the studies presented in this chapter provide a broader perspective of the research questions addressed in this study.

School Size

The research on school size has a long history and is often associated with the debate surrounding the consolidation of schools. The most commonly referred to researcher on this subject is James B. Conant. In the book, *The American High School Today*, he offers the results of his national study. Conant concluded that no high school should have a graduating class of fewer than 100 pupils. This conclusion was based on evidence that small schools, in general, were unable to offer students a rich curriculum, especially upper-level courses in foreign language, science, and mathematics (Conant, 1959).

Later research questioned the methods employed by Conant and the validity of his results (Clements, 1970). While others agree that large high schools offer more math classes (DeYoung, 1995; Morgan, 1993), several authors assert that these classes are not necessarily the upper-level math courses Conant found small high schools unable to offer (Lee, Smith & Croninger, 1997; Monk, 1987; Southwest and West Central Educational Planning Task Force, 1980).

In 1964, Barker and Gump published *Big School, Small School.* These authors limited their research to high schools in the eastern part of Kansas and studied students' opportunities to interact with each other and with teachers. In limiting the region from which they drew their sample, they "were able to investigate the relation between size of schools and other institutional characteristics across a number of schools that were otherwise remarkably similar" (Barker & Gump, 1964, p. 44). Their study did not focus on academic success, but instead considered students' participation in and responsibility to the school. Barker and Gump concluded that students attending small schools have many more opportunities to participate in school life. There were very few students who did not participate (Downey, 1978), whereas students in large schools were more polarized, with a group of active participants at one end and a large group of students who did not participate at the other end. Large schools include a substantial group of students with poor academic records and no extracurricular involvement, a group almost unknown in small schools (Boyer, 1983). In the end they wrote,

> What size should a school be? The data of this research and our own educational values tell us that a school should be sufficiently small that all of its students are needed for its enterprises. A school should be small enough so that students are not redundant. (Barker & Gump, 1964, p. 202)

According to a study conducted by Fowler and Walberg (1991), the idea of student participation is an important factor in considering academic success. Their study showed an inverse relationship between school size and student outcomes. As a possible explanation, based on previous research, they state the following:

> Increased school size has negative effects upon student participation, satisfaction, and attendance and adversely affects the school climate and a student's ability to identify with the school and its activities. Students who are dissatisfied, who do not participate in school activities, who are chronically absent, and who do not identify with the school will achieve less, whether on achievement tests or on postschooling outcomes. Small schools differ from large schools in terms of staff interaction . . . small schools may be friendlier institutions, capable of involving staff and students psychologically in their educational purposes. (p. 200)

In studies with a dependent variable of academic achievement, the measurement instrument for achievement is often a standardized test such as the ACT, the Scholastic

Aptitude Test (SAT), state-developed proficiency tests, exams developed for a particular research project, or a combination of these exams.

The results of a study conducted in Arkansas (Smith, 1961) indicated that students from larger school districts performed significantly higher on the ACT exam. The population for this study was a sample of 3,250 college-bound students who participated in the ACT program during the 1959-1960 academic year. Since a sample of this population was taken, the number of participants in each size category only ranges from 72 to 80 students. The smallest schools each enrolled less than 150 students, in grade 7 through 12, while the largest schools each enrolled over 800. Smith concluded that students from medium (400-550 students), large (600-750 students), and very large (more than 800 students) high schools achieved significantly higher ACT Composite scores than students from the very small (less than 150 students) and small (150-350 students) high schools. Students from the large and very large high schools achieved significantly higher mean ACT scores on each of the four subject components of the ACT.

Twenty years later, in a study limited to undergraduate applicants to New Mexico State University, who had graduated from New Mexico high schools (n = 3,446), Edington (1981) found that students from the smallest high schools (total enrollment of less than 100 students) had the highest mean ACT score on the Math, Social Science, and Natural Science components of the ACT. Students from the smallest high schools achieved a first-place ranking, out of six size categories, on all components of the ACT, except English. The highest mean score on the English component of the ACT was achieved by the students from the largest high schools (over 2,000 students enrolled).

The ranks of fifth and sixth place were consistently held by students from the high schools of 100 to 199 students and 200 to 499 students. "The schools of size 100 to 499 were in predominantly rural areas in the state, but they are large enough not to give the individualized instruction that you find in extremely small schools" (Edington, 1981, p. 3). Edington indicates that the results of this study may be misleading due to the small percentage of graduates from the smallest size high schools who applied to New Mexico State University. Only 15 out of an estimated 375 students who graduated from high schools of less than 100 students applied to New Mexico State University. In spite of this shortcoming, he still concludes that "students from smaller high schools, due to individualized instruction, and other characteristics inherent in small schools, did receive a better education which enabled them to score higher on the ACT" (p. 2).

In the continuing effort to determine the optimal size for a high school, Lee and Smith (1997) asked the following questions: "(a) Which size high school is most effective for students' learning?, (b) In which size high school is learning most equitably distributed?, and (c) Are size effects consistent across high schools defined by their social compositions" (p. 205)? To answer these questions, they analyzed data collected from three waves of the National Educational Longitudinal Study of 1988 (NELS:88). The data from this study were designed to measure student achievement growth from the beginning of high school to the end in reading, mathematics, history, and science, using cognitive tests specifically designed for the NELS:88 project. Lee and Smith limited their analysis to the reading and mathematics scores collected in the eighth, tenth, and twelfth grade, for the same students. With the determination of "optimal" being defined in terms of students' learning over the course of high school in reading comprehension and mathematics, Lee and Smith reached several conclusions:

> [1] Students learn more in relatively small high schools; learning is more equitable in small places. [2] High schools can be too small. . . . In general terms and considering both outcomes [equity and effectiveness], our results lead us to recommend an enrollment size of between 600 and 900 students as "ideal" for a high school. [3] Ideal size does not vary by the types of students who attend. . . . Schools whose sizes fall in the moderate size range (600-900 students) produced greater achievement gains for low- and high-SES [socio-economic status] schools and for schools with low and high minority concentrations. (p. 217)

High School Size and Mathematics Achievement

In reaching their "optimal" high school size, Lee and Smith (1997) examine the effects of high school size on students' academic gains in mathematics and reading. For obvious reasons, only the findings related to mathematics achievement gains will be discussed in this paper. When considering only the effects of high school size on achievement gains in mathematics, the results indicate that students who attend high schools that enroll between 600 and 900 students have optimal learning. Gains are less in smaller schools (particularly those with less than 300 students); learning is also considerably less in large schools (with more than 2,100 students).

In determining the effects of school size on the equity of learning, the authors introduce the variable SES (socio-economic status) to the model. The relationship between SES and achievement growth is positive; higher SES students learn more. For low SES schools, the high school size, which shows the greatest mathematics achievement gain, is the 600 to 900 students category. The largest (more than 2,100 students), low SES schools show the smallest achievement gain in mathematics closely followed by low SES schools with less than 300 students. For high SES schools, again schools with 600 to 900 students have the greatest gain in mathematics achievement and the largest and smallest schools post the smallest gains. It should be noted that the gains in mathematics achievement between the low SES schools of different sizes have a greater variance than the gains between the students from high SES schools of different sizes. In general, size effects are larger for students from low SES schools. The variable SES appears in several other studies conducted in a variety of settings, with the results being a positive relationship between SES and academic achievement and size as a less significant (Fowler & Walberg, 1991) or insignificant factor (Bidwell & Kasarda, 1975; Sares, 1992), especially when SES is controlled (Amos & Moody, 1981; McIntire & Marion, 1989).

The participants, in a study conducted by Morgan (1993), graduated from North Dakota high schools and attended either the University of North Dakota or North Dakota State University. Morgan examined the effects of school size on mathematics and science ACT scores. Her findings indicate that students from high schools with a graduating class size of 26 to 55 students scored significantly higher on the Mathematics Assessment of the ACT than students from high schools with graduating classes of less than 25 students and students from high schools with a graduating class of more than 256 students ($p \leq .05$). She also found that the number of different mathematics courses offered by a school has a significant ($p = .007$), positive relationship ($r = .0719$) to students' ACT Mathematics score. These findings may seem contradictory and, in her discussion, Morgan provides the following explanation:

> Students from smaller sized high schools in this sample scored in the top range of the ACT, maintained higher grade point averages while at UND and NDSU, and completed their degree programs in fewer numbers of semesters.
>
> High scores on the ACT mathematics and natural science assessments by students in the sample who eventually majored in these fields probably reflected both an ability and an interest in mathematics and/or science related fields.
>
> Students from largest high schools (more than 255 students in the graduating class) did not necessarily represent the top of their class. A comparison of ACT mean scores of the sample with ACT mean scores of the state implies that many students in high schools with more than 255 in their graduating class may have chosen other post secondary institutions over the University of North Dakota and North Dakota State University. Students in the sample scored considerably higher than the state ACT scores; this indicates that the University of North Dakota and North Dakota State University attract many excellent students from across the state. Students in high schools with 25 or fewer students in the graduating class appeared to represent those students scoring in the highest range on the ACT tests and, most likely, graduating at the top of their classes. On the other hand, it appears that many top students from high schools that graduated more than 400 students in their high school graduating classes did not attend the University of North Dakota or North Dakota State University.
>
> Furthermore, because students from high schools with fewer than 26 in the graduating class majored in mathematics and science significantly less often than did students in group 2 or group 3, students from these small schools – even those students who are intellectually prepared for college – may not have been prepared to major in mathematics and sciences fields as often as other students were, particularly students in graduating classes of 26 to 255. (pp. 119-120)

Best Conditions for Educating Secondary Students

In the book, *High School: A Report on Secondary Education in America*, Boyer (1983) presents a synthesis of the research on the American high school available at that time. He addresses the issue of high school size and draws the following conclusion:

> Research suggests that small schools appear to provide greater opportunity for student participation and greater emotional support than large ones. The authors prefer the arrangement of having both bigness and smallness. Small schools should bring in mobile classrooms, have off-campus sites, etc. Large schools should organize schools within the school. (p. 235)

This statement reflects the idea that the depth of classes that can be offered in large schools and the feeling of being needed that is inherent in the small school are both necessary to create the ideal school atmosphere for educating secondary students.

Success at the University Level

Regardless of the size of a high school, one of its primary responsibilities is to offer an education to students that will allow them to gain admission to a post-secondary educational institution and prepare them for the work involved in earning a post-secondary degree.

The most common route to admission to a post-secondary educational institution, particularly a college or university, is to obtain a high school diploma and an acceptable score on either the ACT or SAT examination. Even though there are differences between colleges and universities, for the remainder of this discussion the terms college and university will be used interchangeably. The admission criteria used by each university are developed by the individual institution and vary greatly from school to school. In addition to the completion of a high school degree and an acceptable ACT or SAT score, some universities also require students to have an acceptable high school GPA and/or class rank. While these criteria have been, and continue to be, the most frequently used when screening applicants for admission to a university, researchers have questioned their validity and are in search of a better set of criteria on which to base university admission.

Given that research has shown that larger schools offer their students more math and science and that the adjustment from a high school to a university setting is more difficult for students from rural or small-town schools (Aylesworth & Bernard, 1976), it is often thought that students from larger high schools should be more prepared for university work than students from smaller schools. These beliefs, in addition to the ease with which information about students' high school size is readily obtained, have led researchers to ask whether high school size should be considered during the university admissions process.

In a study limited to students who had graduated from Kansas high schools and entered Kansas State University in 1974, Downey (1978) concluded that students from different size high schools did differ. This difference was of a nonacademic nature related to student participation in activities outside of the classroom. For the population of his study Downey found

> only minor evidence that students from smaller high schools were doing less well in college than students from larger high schools. The overall GPA for the various groups was not different and only the students from the smallest schools showed a lower persistence rate after three complete semesters. (p. 357)

Just as Morgan (1993) indicated, Downey also considered that his results might be "due to the self-selection out of KSU of the weaker students [from small schools]" (p. 358). Gallagher (1986) completed a similar study in which the participants had graduated from a South Dakota public high school, and entered either the University of South Dakota or South Dakota State University in the fall of 1980. The findings from this study were remarkably similar to Downey's results.

In a similar study conducted at Illinois State University, Cashen (1970) found that students from medium size high schools achieved a significantly higher first semester

GPA than students from all other size categories. He offers two possible explanations for this:

> First, students from medium sized high schools matriculating to a medium sized university may find the press and the needs to be rather congruent. Second, perhaps the various advantages of both small and large schools are reflected in the medium sized school. (p. 259)

In an attempt to look beyond students' ACT or SAT scores, high school GPA, high school rank, and high school size, Willingham et al. (1985) incorporated "personal qualities" into a study they conducted at nine liberal arts colleges located in the eastern United States. Their mission was to find factors, other than the typical numerical data collected on students' college applications, to predict success at the university level. Given the various educational goals set by each of the nine institutions, Willingham et al. developed a definition of success based on three categories:

> 1) Scholarship which represents traditional forms of cognitive achievement;
> 2) Leadership places more emphasis on such affective qualities as maturity, personal effectiveness, and the ability to inspire confidence in others; and
> 3) Accomplishment refers to productive capability, i.e., exemplary instances of independent effort resulting in a meritorious project or developed skill. (p. 42)

They note that these categories overlap and can be mutually dependent, but they can be usefully distinguished. The measure for success in the area of scholarship was determined by the students' final university GPA. Willingham et al. (1985) concluded that high school rank (HSR) and SAT scores were good predictors of students' academic success. However, as they stated,

> A central issue in this study was whether there is supplemental information over and above HSR and SAT that can be helpful in identifying applicants likely to be "most successful" in the institution's eyes. The answer was clearly yes. Four types of information – high school honors, follow-through [in extracurricular

> activities], the personal statement, and the school reference – were useful additions. These measures improved the prediction of "most successful" students by 25 percent. They improved prediction of leadership by 65 percent, accomplishment by 42 percent, but scholarship by only 7 percent. (p. 178)

In regards to the question of the effect of high school size, Willingham et al. found "no differences among small (schools with fewer than 100 seniors), medium (100-500 seniors), and large (more than 500 seniors) public schools as to predicted versus actual success in any areas" (p. 103). Their results, regarding the significance of high school rank and the insignificance of high school size, are supported by Gallagher (1986) and Sanford (1982).

In a qualitative study conducted at Boise State University (BSU), Belcheir, Michener, and Gray (1998) held weekly interviews with 25 first semester freshmen.

> The purpose of the study was to get to know the student experience as a whole, to allow the student to tell about their expectations of college, to learn how they personally changed during their first year of college, what things impressed them, and where stumbling blocks occurred. How college and home experiences related to decisions to continue or withdraw was also of interest. (p. 1)

In addition to the weekly interviews, the 25 students, chosen to represent the Boise State University population, kept journals and attended a group meeting, which gave them the opportunity to discuss their experiences. The findings from this study indicated that local residents were more likely to leave BSU than students who left their families to go to college. "The very act of moving to Boise to enroll requires a level of action and commitment far beyond that of a local student who leaves the house and drives 15 minutes to campus" (p. 14). Living in the dorms was also determined to be a plus, particularly if the student was able to find

> ways to create structure in an often chaotic environment. [If] they set aside specific times during the week to study, attend religious services regularly and made housekeeping a scheduled chore. This structure provided the discipline these students needed to succeed in college. (p. 11)

In addition to these findings, it was important for students to "latch onto the 'culture of learning' ... [The students] found that college was hard work and they needed to love what they did" (p. 14).

From the research presented previously, it appears that overall success at the university level is not significantly related to a student's high school size. However, when considering general success at the university level, often students opt for a minimum number of mathematics courses. In other words, it is possible for students to graduate from a university without taking any college-level mathematics. In the discussion of research that follows, the focus is on the search for factors that are related to student success in university mathematics.

When students enter a university they are typically advised of the level of mathematics for which the university considers them prepared. This placement is commonly determined by students' ACT or SAT scores and/or their scores on the university's placement exam. Whether or not these pieces of information are the best factors to use to place students in an appropriate mathematics course is a question that has been explored in a variety of settings. Again, because of the lack of availability of upper-level mathematics courses in small schools, the variable high school size is frequently considered as a possible predictor for success in a university mathematics course.

In another study conducted at Kansas State University, Nejadsadeghi (1985) considered the following set of possible predictor variables – sex, number of mathematics units in high school, high school GPA, ACT Mathematics score, ACT Composite score, high school rank, and high school size – in an attempt to develop a linear regression model to predict success for freshmen students enrolled in Intermediate Algebra and their first semester overall GPA. He also looked for a model to predict students' College Algebra grade. The model developed for predicting a student's grade in Intermediate Algebra included the variables high school GPA, ACT Mathematics score, and sex ($R = 0.51$, $p < .0001$). The regression model determined to predict a student's first semester GPA included the variables high school GPA and ACT Composite score ($R = 0.48$, $p < .0001$). To develop the linear regression model to predict a student's College Algebra grade he included the Intermediate Algebra grade as an additional independent variable. The final model for College Algebra included only the grade earned in Intermediate Algebra ($R = 0.54$, $p < .0001$).

In a similar study conducted at The Ohio State University (OSU), the independent variables OSU Math Placement Test score, high school rank, high school GPA, units of high school college preparatory mathematics, math senior year in high school, OSU English Placement Test level, ACT Mathematics, ACT English, ACT Social Science, ACT Natural Science, and ACT Composite scores were used to develop a linear regression model to predict students' grades in Math 104 (Basic College Mathematics) or Math 075 (Precollege Mathematics). The researcher hypothesized that there was a linear combination of variables, which would provide a better prediction of success for students with the lowest placement scores than just the Math Placement Test. For Math 104, the combination of Math Placement Test score, quality points in high school math (QUALPTS), and high school rank accounted for 23% of the variance in students' course grades. The variable QUALPTS was defined as follows:

> $Q=4X_A+3X_B+2X_C+X_D$ where X_A is the number of semester grades of A the student earned in high school math; X_B is the number of semester grades of B the student earned in high school math; X_C is the number of semester grades of C the student earned in high school math; and X_D is the number of semester grades of D the student earned in high school math. (pp. 8 – 9)

For Math 075, the combination of ACT Mathematics, QUALPTS, and high school rank explained 25% of the variance in students' course grades. Each of these models provided a significant improvement over using just the Math Placement Test (Maxwell, 1988).

From 1976 to 1980, Edge and Freidberg (1984) used three groups of students at Illinois State University as subjects to determine which factors were significant predictors of success in the first course in Calculus. The first group was used to develop the regression model and the second and third groups were used to perform replications of the initial study. The academic independent variables considered were ACT scores, high school rank, high school GPA, high school Algebra grades, and the score from an Algebra pretest. Biographical independent variables considered were sex, birth order, family size, and high school size. The dependent variable was a function of the student's course grade in the first semester of Calculus. The use of stepwise and all-subsets regression procedures on the three groups revealed in each case that the best combination of predictors consisted of the algebra pretest [placement exam] and high school rank. From this result, the investigators concluded that the combination of algebraic skills, as represented by the score on the algebra pretest, and long-term perseverance and competitiveness, as measured by high school rank, play a significant role in the prediction of achievement in the first semester of Calculus. The regression model (R = .60), determined by data taken from Group I, was applied to Group II with a resulting correlation of .71. The authors also state that 84% of the subjects received a grade at least as high as predicted. In a subsequent regression model obtained by taking a random sample of Group III, the Algebra pretest score and high school GPA were found to be the best predictors of success in Calculus I. The authors claim this is consistent with earlier results since high school GPA and high school rank are both indicators of a student's long term academic achievement.

While ACT and SAT scores are commonly used as criteria for acceptance to a university and placement into a student's first college-level mathematics course, research has indicated that these exams are not necessarily the best predictors of success, particularly in mathematics. Kessel and Linn (1996) reported that, over the years, a collection of SAT Mathematics and ACT Mathematics scores from a voluntary sample revealed that female students scored 0.4 standard deviation units below male students. In the 1980s, this gap was explained by a difference in high school experiences. However, recent data from the Educational Testing Service indicate that although the gap between the average number of math classes taken by male and female students has decreased in recent years, the difference in SAT Mathematics and ACT Mathematics scores has remained the same. Kessel and Linn also note, "Females earn higher grade point averages than males, both in high school and in college. . . . Moreover, there is evidence that women complete their undergraduate degrees more quickly than men. . . . Women also tend to earn higher grades in college mathematics" (p. 10). Thus, scores tend to underpredict the grades of females relative to those of males in mathematics courses and in overall college GPA. After synthesizing several studies, the authors suggest that

> females tend to have study procedures which are more likely to yield a comprehensive and robust understanding of mathematics than those of males. Females report spending more time reflecting on similarities among problems, organizing and linking their ideas, and reviewing material. (p. 11)

Referring to a publication by Gallagher, who studied the solution method used by students with high SAT mathematics scores, females are more likely to use classroom procedures in solving SAT items than males. This practice is to their disadvantage on a test such as the SAT mathematics, where speed is essential. They also indicated that "the performance of males and females on non-high-stakes performance examinations such as general tests of mathematical ability does not show a gender gap" (Kessel & Linn, 1996, p. 12).

In a further discussion about success in mathematics, Kessel and Linn (1996) contend that "using entrance examinations as a measure of mathematical ability is consistent with the view that mathematical activity is not the creation or discovery of new mathematics but clever and speedy manipulation of that which is already known" (p. 13). This, of course, is contrary to the professional mathematicians' view of mathematics, in which emphasis is placed on the solution of difficult problems (Kessel & Linn, 1996).

In the book, *Talking About Leaving: Why Undergraduates Leave the Sciences*, Seymour and Hewitt (1997) report the findings of their landmark qualitative study. The authors interviewed and conducted focus groups with 335 students from seven four-year institutions of different types and locations. They triangulated their initial findings by conducting focus group discussions on six other campuses with 125 students (n = 460).

> All seven institutions took mathematics scores (SAT or ACT) into account for admissions purposes, enrolled freshmen in introductory S.M.E. [science, mathematics, or engineering] courses on the basis of mathematical placement tests, and considered the SAT (or equivalent) mathematics scores targeted for our samples (i.e., 650 or more) as adequate for entry to an S.M.E. major. If S.M.E. freshmen displaying mathematical competence at this level or above are expected to be capable of understanding the material presented in introductory S.M.E. classes, the quality of high school mathematics preparation should not have created difficulties serious enough to contribute to switching decisions for most of the students in our sample. Neither should we have found many students who had switched because of serious conceptual problems in these classes. This proved to be the case: neither inadequate high school preparation (which contributed to 14.8% of switching decisions), nor serious conceptual difficulties (which contributed 12.6%) were major factors in switching decisions. However, the effects of inadequate high school preparation were the most common contributor to early decisions to switch. Approximately 40 percent of both switchers and non-switchers reported some problems related to high school preparation, and around 25 percent of each group reported conceptual difficulties. . . .
>
> Students' accounts of under-preparation were broadly of two types: deficiencies of curriculum content and subject depth, and failure to acquire appropriate study skills, habits and attitudes. However, both facets of under-preparation were, in reality, inter-connected. Some switchers and non-switchers had received no high school teaching in calculus, or described the content and depth of their high school science or mathematics as insufficient for their first college classes. (pp. 78 – 79)

The authors go on to discuss how students' above average scores on the SAT or ACT had given them a false sense of their preparation for college work. Students provided the following insights:

> I just kicked back and got straight As – with zero effort on my part. And I came here thinking I was going to coast through college like that. In the very first week, I had a terrible eye-opener. There were students sitting all around me that *knew* what was going on, and I had absolutely no idea what they were talking about. I was completely lost. I was expected to *be* at that level the very first day.

> I realized I hadn't had any of the stuff at high school that I needed – like physics. I'd never even seen a computer. (Male white science switcher)
>
> I went to such a small high school. Teachers weren't all that great, and there were no advanced math or college prep classes – just the basic core classes up to pre-calculus – which meant I had to start at the beginning in engineering. My roommate, who went to a large, suburban school in a wealthy area, started with the second year of calculus, and circuits too. I had other friends who were way ahead of me – just because of the better education they had had in high school. (Female white engineering non-switcher). (p. 81)

While Seymour and Hewitt (1997) did not attempt to generalize about the quality of education at various types of high schools, they did offer the following observation:

> It is difficult to generalize about the types of high schools which are more likely to under-prepare students for college mathematics and science in terms of curriculum content or depth. However, students from small rural schools, large inner-city schools, reservation schools, and poorly-endowed schools located in working-class areas (white or minority), were those who most often cited insufficient resources and limited access to well-qualified teachers as salient features in their high school science and mathematics education. (p. 81)

Differences in the quality of education between schools within the United States and schools from other countries were also revealed in their interviews with students who had moved during their high school years (Seymour & Hewitt, 1997):

> Going back to the Philippines, I realized that my peers there were doing more science than I ever did. And, you know, they live in the provinces and go to these dinky little schools. It was kind of alarming to admit. (Male Asian-American science non-switcher)
>
> The teachers in my school in Switzerland were really into teaching. They really pushed you hard. . . . remember once I had a class I wasn't doing very well in, and the vice-principal called me in to talk about it. That wouldn't happen here. But I guess they don't have the same standards here. (Male white mathematics non-switcher). (p. 83)

However, most students were unaware of their deficiencies in mathematics and science until they took their first university mathematics or science courses. This was a source of frustration and a psychological barrier that students had to overcome to be successful in their S.M.E. major (Seymour & Hewitt, 1997).

The Teaching and Learning of Mathematics

The controversy over what mathematics is of value and how to teach the chosen topics, from kindergarten through the twelfth grade, has surfaced and submerged repeatedly for more than a century. With the publication of *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), the debate heated up again.

The *Standards* (NCTM, 1989) was in response to *A Nation at Risk* by the National Commission on Excellence in Education, and other publications of the 1980s that called for changes in the way mathematics was being taught. "Inherent in this document [*Standards*] is a consensus that all students need to learn more, and often different, mathematics and that instruction in mathematics must be significantly reviewed" (NCTM, 1989, p. 1). This document defined the following five goals for students of K-12 mathematics:

> 1) that they learn to value mathematics [make connections]; 2) that they become confident in their ability to do mathematics; 3) that they become mathematical problems solvers; 4) that they learn to communicate mathematically; and 5) that they learn to reason mathematically. These goals imply that students should be exposed to numerous and varied interrelated experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess, and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write, and discuss mathematics; and that they should conjecture, test, and build arguments about a conjecture's validity. (NCTM, 1989, p. 5)

The writers of the *Standards* (NCTM, 1989) also define the phrase "mathematically literate," which sums up the intent of these goals. "This term denotes an individual's ability to explore, to conjecture, and to reason logically, as well as to use a variety of mathematical methods effectively to solve problems" (NCTM, 1989, p. 6).

The NCTM's *Standards* for K-12 mathematics education refueled the debate between the groups that align themselves with the "traditional" or the "back to basics movement" and those affiliated with the "mathematics reform movement."

Those who support the back to basics movement favor teacher-centered classrooms and, in general, hold the following view about the teaching of mathematics:

> Mathematics in the current system is a fixed collection of concepts and skills to be taught and mastered in some strict order. The result is that the curriculum focuses on knowledge divided into subjects for study, such as, arithmetic, algebra, and geometry. Furthermore, in each subject, it is assumed that knowledge can be broken down into clearly defined, independent, self-sustaining parts; there is a logical sequence of development in which each part builds on a preceding foundation; and, if knowledge were acquired in this manner, students would be able to use and apply their mathematical knowledge as needed. (Romberg, 1990, p. 471)

Classrooms, which reflect this view of mathematics, frequently employ some form of direct instruction as a method of teaching.

> A fully developed direct-instruction approach involves more [than lecturing]. It is faster paced, includes more student involvement, contains a highly organized set of interactions under the control of the teacher, and focuses more on students' learning than on teacher performance.
>
> The key to direct instruction is not lecture; rather, it is the presence of your control, as the teacher, over the flow of new information. In general, a complete direct instruction lesson includes these characteristics:
>
> - Academic focus,
> - Formal delivery to the whole class,
> - Constant monitoring to check for understanding, and
> - Controlled classroom practice. (Armstrong & Savage, 2002, pp. 238-239)

Academic focus refers to the idea that the lesson concentrates on teaching specific academic content or skills. The teacher keeps the class focused on the topic throughout the lesson so that students can master the objective. Digressions from the content associated with the lesson are avoided as much as possible. In a direct instruction class the teacher provides logical, step-by-step directions. While delivering the necessary information, the teacher frequently poses questions to the students to ensure they understand the material and to keep them actively involved in the lesson. It is expected that students will demonstrate an understanding of each step before the teacher proceeds to the next.

> Direct instruction lessons feature many teacher-to-student questions. A large number of these tend to be recall questions. When your instruction is effective, students are able to respond to a high percentage of them. Questions often focus either on a request for specific answers or a request for an explanation of how a student arrived at an answer. (Armstrong & Savage, 2002, p. 241)

By posing questions to the students during the lesson, the teacher is checking for understanding. In considering the students' responses, the teacher can then decide how to adjust the pace of the lesson and, if needed reteach a topic. The goal here is to ensure mastery of the subject. The end of the lesson provides in-class practice of the day's material.

> Effective direct instruction lessons feature substantial opportunities for students to engage in controlled practice. The key word here is controlled. Before students are allowed to engage in application activities requiring the use of presented information, you need to ensure that they have the necessary understanding to successfully complete the application exercises. (Armstrong & Savage, 2002, p. 241)

As with most philosophies of teaching, how direct instruction has been implemented from classroom to classroom has varied greatly.

The direct instruction model of teaching "is more appropriate for the content that can readily be divided into parts, teaching basic skills, teaching students with an external locus of control, introductory material, and a prescribed body of content" (Armstrong & Savage, 2002, p. 256).

It has been argued that most students do not really know mathematics when they are taught in this manner (Bransford, Brown, & Cocking, 2000; Romberg, 1990; Schoenfeld, 1988). This type of learning has been referred to as "parrot math" (O'Brien, 1999) or "mindless mimicry mathematics" (Battista, 1999). By proponents of reform mathematics, the school mathematics taught under the traditional approach has been described as

> an endless sequence of memorizing and forgetting facts and procedures that make little sense. . . . Though the same topics are taught and retaught year after year, the students do not learn them. Numerous scientific studies have shown that traditional methods of teaching mathematics not only are ineffective but also seriously stunt the growth of students' mathematical reasoning and problem-solving skills. (Battista, 1999, p. 426)

Those who support the mathematics reform movement favor student-centered classrooms in which students actively create mathematical understanding. The NCTM *Standards* (1989) were designed to provide a guideline for the teaching of K-12 mathematics. They were not meant to be prescriptive or to define a specific curriculum or required activities for the teaching of mathematics (Hirsch, 1990). However, as is obvious from previous quotes, the *Standards* called for a change in what we consider important in school mathematics and the ways to facilitate student understanding of mathematics. Classrooms which reflect the ideas presented in the NCTM *Standards* commonly employ a constructivist approach to teaching.

There are many visions of constructivist practices; this review of the literature will give a broad description of the idea of constructivism.

> A constructivist learning environment prioritizes and facilitates the active role of the student. Active learning denotes learning activities in which students are given considerable autonomy and control of the direction of learning activities. Active learning activities include investigational work, problem solving, small-group work, collaborative learning, and experiential learning. . . . Primarily, a constructivist learning environment shifts the focus from teacher dissemination, which promotes the passive role of the student, to student autonomy and reflection, which promotes the active role of the student. . . . This de-emphasis on text books and teacher dissemination of knowledge doesn't imply that teachers shouldn't explain content to students; it does suggest that we should be skeptical of how much understanding learners develop on the basis of our explanations and their recording of that knowledge. (Jacobsen, Eggen, & Kauchak, 2002, p. 5)

The basis for constructivist practices is credited to the educational philosopher, John Dewey. In 1897 Dewey wrote,

> With the child, instruction must take the standpoint not of the accomplished results, but of the crude beginnings. We must discover what there is lying within the child's present sphere of experience (or within the scope of experiences which he can go get)... It is not the question of how to teach the child geography [or mathematics], but first of all the question of what geography [mathematics] is for the child (Dewey, 1897).

Different interpretations of this quote give rise to various beliefs within constructivism. In considering the question, "what [mathematics] is for the child," the following interpretation could be appropriate: Given the child's current interests, mathematical understanding, and ability, what experiences do we need to provide for him or her so that he or she can grow and develop knowledge of the necessary mathematical topics, with these topics being defined by the school or district curriculum committee? A second interpretation is to ask: Given the child's interests and abilities, what math topics are appropriate for him or her and what experiences will help him or her develop these concepts? In other words, what is it that the child is curious about and demonstrates a desire to learn?

In addition to the idea of student-centered experiences, Dewey noted the necessity of reflective thinking if actual learning is to occur. According to Dewey, "thinking ... is an affair of the *way* in which the vast multitude of objects that are observed and suggested are employed, the way they run together and are *made* to run together, the way they are handled" (Archambault, 1964, p. 230). A child's participation in an activity does not guarantee learning. The activity needs to be followed up by time for students to reflect on their experiences, particularly in the context of their previous knowledge and experiences. Finally, Dewey also suggested that learning is not accomplished by a child in isolation, but is facilitated through interaction with others (Archambault, 1964).

John Dewey brought together the basic ideas, which form the basis for constructivist theory, but he is not the sole supporter of such thoughts. Specifically, Bruner (1964) advanced the ideas of starting with what the student knows and can experience and how the student advances to a more abstract understanding. He discusses three modes of representation that are a part of learning with understanding. In the enactive mode, students are physically active, manipulating materials to illustrate their solution to the current problem or new concept. In the second mode, the iconic representation, students have the ability to call upon their own visual images of the manipulatives to problem solve. Finally, students develop the ability to represent the visual images with appropriate symbols. Bruner (1966) writes,

> If it is true that the usual course of intellectual development moves from enactive through iconic to symbolic representation of the world... it is likely that an optimum sequence will progress in the same direction. Obviously, this is a conservative doctrine. For when the learner has a well-developed symbolic system, it may be possible to by-pass the first two stages. But one does so with the risk that the learner may not possess the imagery to fall back on when his symbolic transformations fail to achieve a goal in problem solving. (p. 49).

In other words, "Students who truly make sense of this situation are not manipulating symbols, oblivious to what they represent. Instead, they are purposefully and meaningfully reasoning about quantities. They are not blindly following the rules invented by others. Instead, they are making personal sense of the ideas" (Battista, 1999, p. 428).

In support of constructivism, Bruner (1966) writes,

> A body of knowledge enshrined in a university faculty and embodied in a series of authoritative volumes, is the result of much prior intellectual activity. To instruct someone in these disciplines is not a matter of getting him to commit results to mind. Rather, it is to teach him to participate in the process that makes possible the establishment of knowledge. We teach a subject not to produce little living libraries on that subject, but rather to get a student to think mathematically for himself, to consider matters as an historian does, to take part in the process of knowledge-getting. Knowledge is a process, not a product. (p. 72)

The key components of a constructivist classroom include a student-centered, active learning community where the activities build on students' previous knowledge and experiences and are followed by a period of reflection. The constructivist model of teaching is more appropriate for content in which distinct parts are difficult to define, teaching higher-level thinking, teaching students with an internal locus of control, affective outcomes, and learning demanding creative thinking (Armstrong & Savage, 2002).

Opponents of this approach to teaching mathematics consider it "fuzzy math." It appears that the teaching of mathematics has been reduced to playing with toys and games without any substance. In *Principles and Standards for Mathematics Education*, the NCTM (2000) acknowledged that implementation of the *Standards* varied greatly from classroom to classroom (Burrill, 1997) and, in many cases, reflection and mathematical connections were not included in the lessons (Battista, 1999; Olson, 1999).

A third group believes that implementation of one approach to teaching and learning, with total exclusion of the other, has been detrimental to our mathematics education programs and that false dichotomies have been formed. In support of this perspective, mathematician Hung-Hsi Wu (1999) states,

> "Facts vs. higher order thinking" is another example of a false choice that we often encounter these days, as if thinking of any sort – high or low – could exist outside of content knowledge. In mathematics education, this debate takes the form of "basic skills or conceptual understanding." This bogus dichotomy would seem to arise from a common misconception of mathematics held by a segment of the public and the education community: that the demand for precision and fluency in the execution of basic skills in school mathematics runs counter to the acquisition of conceptual understanding. The truth is that in mathematics, skills and understanding are completely intertwined. In most cases, the precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding. There is not "conceptual understanding" and "problem-solving skill" on the one hand and "basic skills" on the other. Nor can one acquire the former without the latter. . . .
>
> The desire to achieve understanding in a technical subject such as mathematics while minimizing the component of skills is a most human one. . . . In the context of school mathematics, however, such a desire cannot be indulged without doing great harm to students' education. There are many reasons. Sometimes a simple skill is absolutely indispensable for the understanding of more sophisticated processes. For example . . . the fact that the arithmetic of

> ordinary fractions (adding, multiplying, reducing to lowest terms, etc.) develops the necessary pattern for understanding rational algebraic expressions. At other times, it is the *fluency* in executing a basic skill that is essential for further progress in the course of one's mathematics education. The automaticity in putting a skill to use frees up mental energy to focus on the more rigorous demands of a complicated problem. . . . Finally, when a skill is bypassed in favor of a conceptual approach, the resulting conceptual understanding often is too superficial. (pp. 14 – 15)

The belief that students need to learn basic skills and have a conceptual understanding of mathematics has been endorsed by mathematicians and educators for more than a century. Mathematician Felix Klein (1932) wrote,

> The problem of teaching children the properties of integers and how to reckon with them, and of leading them on to complete mastery, is very difficult and requires the labor of several years, from the first school year until the child is ten or eleven years old. The manner of instruction as it is carried on in this field in Germany can perhaps best be designated by the words *intuitive* and *genetic*, i.e., the entire structure is gradually erected on the basis of familiar, concrete things, in marked contrast to the customary *logical* and *systematic* method at the university.
>
> One must know what 5 X 7 or 3 X 8 is in one's sleep, so to speak. Consequently the pupil must learn the multiplication table by heart to this degree of thoroughness, to be sure only after it has been made clear to him visually with concrete things.
>
> Calculation with numbers of more than one digit [is taught], based on the known simple rules whose general validity is evident, or should be evident [based on previous work done with concrete and pictorial models], to the pupil. (pp. 6 – 7)

Klein (1932) advocated teaching the concepts of mathematics through connections between the different subject areas that constitute the field of mathematics. He noted that in the early 1900s, mathematics in Germany was frequently taught based on what he refers to as Plan A.

> *Plan A is based upon a more particularistic conception of science which divides the total field into a series of mutually separated parts and attempts to develop each part for itself, with a minimum of resources and with all possible avoidance*

> *of borrowing from neighboring fields.* Its ideal is to *crystallize out each of the partial fields into a logically closed system.* (p. 78)

In contrast an advocate of Plan B,

> *lays the chief stress upon the organic combination of the partial fields, and upon the stimulation which these exert one upon another. He prefers, therefore, the methods which open for him an understanding of several fields under a uniform point of view.* His ideal is the *comprehension of the sum total of mathematical science as a great connected whole.* (p. 78)

After a discussion of the mathematical discoveries of the 17th and early 18th centuries, he concludes,

> As a summary, we might say that, *in the history of mathematics during the last centuries, both of our chief methods of investigation were of importance; that each of them, and sometimes the two in succession, have resulted in important advances of the science.* It is certain that mathematics will be able to advance uniformally in all directions, only if *neither of the two methods of investigation is neglected.* May each mathematician work in the direction which appeals to him most strongly.
>
> Instruction in the secondary schools, however, as I have already indicated, has long been *under the one-sided control of the Plan A.* Any movement toward reform of mathematical teaching must, therefore, press for *more emphasis upon direction B.* (Klein, 1932, p. 85)

Later in the mid 1900s in his work, "The Place of Meaning in the Teaching of Arithmetic" Brownell (1947) discussed the difference between the "*meaning of* a thing" and the "*meaning of* a thing *for* something else." He remarked that delineating the difference between these two ideas was not "hair-splitting" but of great value. These two phrases, which sound similar, but have very different meanings, were frequently represented by the same words, "meaningful" and "meaningfully," in the literature of his time. The first phrase, "*meaning of* a thing," refers to the meaning of the subject of a mathematical topic within the context of mathematics. The phrase, "*meaning of* a thing *for* something else," refers to the meaning or application of a mathematical topic in the context of daily living, for example. While the NCTM documents favor developing both aspects of "meaningful" mathematics, in this article, Brownell focused on the *meaning of* mathematics. In his concluding remarks, he provided a list of 10 benefits of meaningful mathematics education.

From the standpoint of the pupil, meaningful arithmetic –

1. Gives assurance of retention.

2. Equips him with the means to rehabilitate quickly skills that are temporarily weak.

3. Increases likelihood that arithmetical ideas and skills will be used.

4. Contributes to ease of learning by providing a sound foundation and transferable understandings.

5. Reduces the amount of repetitive practice necessary to complete learning.

6. Safeguards him from answers that are mathematically absurd.

7. Encourages learning by problem-solving in place of unintelligent memorization and practice.

8. Provides him with a versatility of attack which enables him to substitute equally effective procedures for procedures normally used but not available at the time.

9. Makes him relatively independent so that he faces new quantitative situations with confidence.

10. Presents the subject in a way which makes it worthy of respect. (Brownell, 1947, pp. 263 - 264)

Through his work, Brownell promoted helping students develop a conceptual understanding of mathematics as a way to memorize and maintain the basic skills of mathematics. While it is clear that the *Standards* called for a shift in the content of school mathematics and the way in which it was taught, the intention was not to eliminate the methods and the skills promoted by the back to basics movement, but to encourage the use of a diverse set of teaching strategies to develop mathematical skill and conceptual understanding (NCTM, 1989).

Further support for integrating basic skills and conceptual understanding in the teaching of mathematics is provided by Battista (1999).

> Students who truly make sense of this situation are not manipulating symbols, oblivious to what they represent. Instead, they are purposefully and meaningfully reasoning about quantities. They are not blindly following the rules invented by others. Instead, they are making personal sense of the ideas.
>
> Obviously, not all problems can be easily solved using such intuitively appealing strategies. Students must also develop understanding of and facility with symbolic manipulations and even an appreciation for the workings of axiomatic systems that describe how to deal formally with mathematical symbols. Thus, it is not enough to involve students only in sense making, reasoning, and the creation of new mathematical knowledge. Sound curricula must include clear long-range goals for ensuring that students become fluent in employing the abstract concepts and mathematical perspectives that our culture has found most useful. Students should be able to apply, readily and correctly, important mathematical strategies and lines of reasoning in numerous situations. They should possess knowledge that supports mathematical reasoning. For instance, students should know the "basic number facts" because such knowledge is essential for mental computation, estimation, performance on computational procedures, and problem solving. (p.428)

Tanner and Tanner (1995) add:

> Everyone agrees that what is learned should transfer to new situations, both in and out of school. But when interested citizens and educators talk about what should be included in the school curriculum there is strong disagreement about the probability of transfer. It has long been argued that the study of the traditional academic disciplines and classical languages is the best kind of education and will provide for mental discipline and transfer of learning. When the evidence fails to reveal these claimed benefits, instead of seeking ways of integrating these studies and relating them to the life of the learner and to social reality, there is the

> tendency to look to extreme learner-centered approaches and to negate systematically organized knowledge. These viewpoints are not merely extremes, they are dangerous dualisms. (p. 65)

Finally, after his experience with the progressive schools, Dewey (1938) wrote:

> ...those who are looking ahead to a new social order, should think in terms of Education itself rather than in terms of some 'ism about education, even such an 'ism as "progressivism." For in spite of itself any movement that thinks and acts in terms of an 'ism becomes so involved in reaction against other 'isms that it is unwittingly controlled by them. For it then forms its principles by reaction against them instead of by comprehensive, constructive survey of actual needs, problems, and possibilities. (p. 6)

For more than a century mathematicians and educators have been calling for the integration of the best ideas from both the back to basics movement and the reform movement. In order to provide this type of education for our students, teachers must have a strong mathematical knowledge and the ability to facilitate student learning though various teaching methods.

Teacher Preparation

As has been noted in the discussion of both the direct instruction and constructivist ideals of teaching, the way these theories are implemented in the classroom is inconsistent. Clearly, how these theories appear in the classroom is determined by the teachers. In 2000, the National Commission on Mathematics and Science Teaching for the 21st Century released what has become known as the *Glenn Commission Report*.

> Two core premises of this report are simply stated and they undergird every change we recommend: (1) *Now, more than ever, America's students must improve their performance in mathematics and science.* That is the burden of the case presented thus far. (2) The second premise points in the direction of a solution: *The most direct route to improving mathematics and science achievement for all students is better mathematics and science teaching.* In other words, better teaching is the lever for change. (p. 18)

The Commission provides the following definition of "high quality" teaching in mathematics and science:

- A core premise of high-quality teaching is that the ability to teach, contrary to myth, is not "something you are born with"; it can be learned and refined over time. Specific teaching skills – for example, the ability to distinguish between what is most important for students to learn and what is hardest for them to understand – can only be acquired through training, mentoring, collaboration with peers, and practice.

- High-quality teaching requires that teachers have a deep knowledge of subject matter. For this there is no substitute.

- In high-quality teaching, the process of *inquiry*, not merely "giving instruction," is the very heart of what teachers do. Inquiry not only tests what students know, it presses students to put what they know to the test. It uses "hands on" approaches to learning, in which students participate in activities, exercises, and real-life situations to both learn and apply lesson content. It teaches students not only what to learn but how to learn.

- High-quality teaching not only encourages students to learn, it *insists* they learn.

- High-quality teaching fosters healthy skepticism. It encourages students to submit their work to questioning by others, to pull things apart and put them back together, and to reflect on how conclusions were reached.

- High-quality teaching allows for, recognizes, and builds on difference in the learning styles and abilities of students. It has the deepest respect for students as persons; it corrects without squelching; it builds on strengths rather than trying to stamp out weaknesses.

- High-quality teaching is grounded in a careful and thorough alignment of curriculum, assessment, and high standards for student learning.

- To keep its edge, high-quality teaching must be continually reshaped by the institutional structures that support it, i.e., by professional development, continuing education, the effective use of technology, and recognition and rewards.

- Finally, the effectiveness of high-quality teaching can be evaluated by the performance and achievement of the students who receive it. (National Commission on Mathematics and Science Teaching for the 21st Century, 2000, p. 22)

In the same time period that the *Glenn Commission Report* was developed, *The Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences [CBMS], 2001) was produced. In this document, the CBMS reiterated the changing expectations for mathematical knowledge and for school mathematics instruction. Based on these new expectations and realizations about teacher preparation and the way mathematics continues to be taught, they developed 11 recommendations for improving the quality of mathematics teaching:

1. Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.

2. Although the quality of mathematical preparation is more important than the quantity, the following amount of mathematics course work for prospective teachers is recommended.
   a) Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.
   b) Prospective middle grades teachers of mathematics should be required to take at least 21 semester-hours of mathematics, that includes at least 12 semester-hours on fundamental ideas of school mathematics appropriate for middle grades teachers.
   c) Prospective high school teachers of mathematics should be required to complete the equivalent of an undergraduate major in mathematics that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics.

3. Courses on fundamental ideas of school mathematics should focus on a thorough development of basic mathematical ideas. All courses designed for prospective teachers should develop careful reasoning and mathematical "common sense" in analyzing conceptual relationships and in solving problems.

4. Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching.

5. Teacher education must be recognized as an important part of mathematics departments' mission at institutions that educate teachers. More mathematicians should consider becoming deeply involved in K-12 mathematics education.

6. The mathematical education of teachers should be seen as a partnership between mathematics faculty and mathematics education faculty.

7. There needs to be greater cooperation between two-year and four-year colleges in the mathematical education of teachers.

8. There needs to be more collaboration between [university] mathematics faculty and school mathematics teachers.

9. Efforts to improve standards for school mathematics instruction, as well as for teacher preparation accreditation and teacher certification, will be strengthened by the full-fledged participation of the academic mathematics community.

10. Teachers need the opportunity to develop their understanding of mathematics and its teaching throughout their careers, through both self-directed and collegial study, and through formal coursework.

11. Mathematics in the middle grades (grades 5-8) should be taught by mathematics specialists. (pp. 7 – 11)

These recommendations encompass numerous aspects of teacher education. The need for teachers to have a deep conceptual understanding of mathematics (Ball, 1988), the opportunity to learn through the teaching methods they are expected to apply in their own classrooms (Ball, 1990), and the opportunity for continued education as they refine their skills (Manouchehri, 1998) are all components of a teacher education program with the ability to produce "high quality" teachers (Feiman-Nemser, 1983; NCTM 1991; Committee on Mathematics and Science Teacher Preparation, 2000). As Will Rogers, the 1920s "cowboy philosopher," once remarked, "You can't teach what [or with a method] you don't know any more than you can come back from where you ain't been."

Technology in the Mathematics Classroom

In addition to reevaluating what mathematics in the K-12 curriculum should be considered important and how those mathematical topics should be taught, the *Standards* (NCTM, 1989) also supported the use of technology in the classroom.

> Some aspects of doing mathematics have changed in the last decade. The computer's ability to process large sets of information has made quantification and the logical analysis of information possible in such areas as business, economics, linguistics, biology, medicine, and sociology.
>
> Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. . . . The new technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them. Because technology is changing mathematics and its uses, we believe that –
>
> - appropriate calculators should be available to all students at all times;
> - a computer should be available in every classroom for demonstration purposes;
> - every student should have access to a computer for individual and group work;
> - students should learn to use the computer as a tool for processing information and performing calculations to investigate and solve problems.
>
> We recognize, however, that access to this technology is not a guarantee that any student will become mathematically literate. Calculators and computers for users of mathematics, like word processors for writers, are tools that simplify, but do not accomplish, the work at hand. (pp. 7-8)

In the *Standards* (NCTM, 1989) asserted that students still need to know how to perform pencil-and-paper computations; but, most importantly, students must be able to make appropriate choices about when and how to use technology.

> Similarly, the availability of calculators does not eliminate the need for students to learn algorithms. Some proficiency with paper-and-pencil computational algorithms is important, but such knowledge should grow out of the problem

> situations that have given rise to the need for such algorithms. Furthermore, when one needs to calculate to find an answer to a problem, one should be aware of the choices of methods. . . . When an approximate answer is adequate, one should estimate. If a precise answer is needed, an appropriate procedure must be chosen. Many problems should be solved by mental computation. . . . Some calculations, if not too complex, should be solved by following standard paper-and-pencil algorithms. For more complex calculations, the calculator should be used (column addition, long division). And finally, if many iterative calculations are required, a computer program should be written or used to find the answers (finding the sum of squares).
>
> Contrary to the fears of many, the availability of calculators and computers has expanded students' capability of performing calculations. There is no evidence to suggest that the availability of calculators makes students dependent on them for simple calculations. Students should be able to decide when they need to calculate and whether they require an exact or approximate answer. Students should have a balanced approach to calculation, be able to choose appropriate procedures, find answers, and judge the validity of those answers. (p. 9)

The *Standards* was not the first NCTM document to support the use of calculators in the teaching and learning of mathematics (NCTM, 1974). In 1984, Hembree and Dessart (1986) began a meta-analysis of 79 studies conducted on the use of calculators in the classroom, published from 1969 to 1983. To synthesize the findings of these reports, the researchers defined a "common metric" called "effect size." An effect size was calculated for each of the dependent variables of attitude and achievement, with a positive effect size indicating a study supporting the use of calculators. They reached the following conclusions:

> 1. In Grades K-12 (except Grade 4), students who use calculators in concert with traditional instruction maintain their paper-and-pencil skills without apparent harm. Indeed, a use of calculators can improve the average student's basic skills with paper and pencil, both in basic operations and in problem solving.
>
> 2. Sustained calculator use by average students in Grade 4 appears counterproductive with regard to basic skills.

3. The use of calculators in testing produces much higher achievement scores than paper-and-pencil efforts, both in basic operations and in problem solving. . . . The overall better performance in problem solving appears to be a result of improved computation and process selection.

4. Students using calculators possess a better attitude toward mathematics and an especially better self-concept in mathematics than noncalculator students.

5. Studies with special curricula indicate that materials and methods can be developed for enhancing student achievement through instruction oriented toward the calculator. However, such special instruction has been relatively unexamined by research.

   It no longer seems a question of *whether* calculators should be used along with basic skills instruction but *how*. (pp. 96 – 97)

In general, Hembree and Dessart (1986) concluded that calculators should be used in all K-12 mathematics classes, with calculator use in Grade 4 approached with caution and students in Grade 5 and above should be allowed to use calculators in all problem-solving activities and testing situations. Their recommendations for further research included investigating the following: effective procedures for calculator use in learning basic facts, computational skills, and problem-solving strategies; the effects of curriculum emphasis and sequence changes; effective education programs to help teachers incorporate calculators in their classrooms appropriately; and the effects of calculator use in grade 4 so that the negative effects can be understood and avoided.

Since the research of Hembree and Dessart and the publication of the NCTM *Standards* in 1989, the power of hand-held computers or calculators and their use by teachers and students has increased dramatically. Continued research has shown that calculators need to be used wisely in all grades.

In an extensive review of the research literature about the uses and effects of the graphing calculator in the classroom, Penglase and Arnold (1996) summarized the findings of the research results produced from 1986 to 1995. In their review, the authors noted that, in general, the literature focused on the effects of the use of graphing calculators within specific areas of mathematics and drawing conclusions about the effectiveness of such use. They also noted that defining appropriate methods of measuring student achievement was a difficult task and often poorly done in the research reviewed. This was particularly true when studies compared a calculator group to a non-calculator group. In these studies, it was not always possible to determine if the test questions were biased toward one group over the other. Another challenge in determining the effects of graphing calculators on student achievement is the variety of newly implemented instructional programs under which the effects of calculator use are evaluated. For several of the studies reviewed, the authors concluded that the effects of the calculators could not be distinguished from the possible effects of the instructional program. With these limitations in mind, a discussion of several of the studies included in the review follows.

In the Precalculus arena of mathematics, Penglase and Arnold (1996) reported on studies conducted by Dunham and Tolias. In the study conducted by Dunham, the effect of the calculator was evaluated while a field test of a new technology-based Precalculus course was being conducted. Fifty-five schools taught the course and 22 schools served as the control group. Student achievement was measured by student performance on a "calculus readiness" test.

> [Dunham] reported that the 55 schools which taught this [technology-based] course attained significantly higher means on a "calculus readiness" test than did the 22 control schools. Students from the experimental schools who did not achieve calculus placement on the pre-test, achieved post-test calculus placement-level scores at nearly twice the rate of those in traditional classes. (p. 64)

The study conducted by Tolias "found that the effect of [graphing] calculator use varied in the development of different types of knowledge" (Penglase & Arnold, 1996, p. 63). She divided Precalculus knowledge into three subcategories: procedural knowledge, relational knowledge, and transfer of knowledge. Her study consisted of two experimental and two control classes.

> The experimental classes were taught using a Precalculus curriculum with an emphasis on functions and their graphs. Their program was designed specifically with the intention of facilitating the integration of algebraic and graphic procedures. Control subjects were taught using a more traditional precalculus outline. (p. 63)

A post-test designed to measure procedural and relational knowledge and transfer of knowledge was administered. "No significant differences were found between the experimental and control groups concerning procedural knowledge, [however,] significant differences which favoured the experimental group in relational knowledge and transfer of knowledge" (p. 63) were found. In addition, "subjects in the experimental group who chose algebraic procedures to solve items which tested relational knowledge were found to perform better on these items than students in the control group" (p. 63). For both of these studies, Penglase and Arnold concluded that the effects of the calculators could not be distinguished from that of the instructional program.

Another study reviewed by Penglase and Arnold (1996) discussed student attitudes toward the graphing calculator. In a study by Boers and Jones (Penglase &

Arnold, 1996)) a focus group of university students formed a list of benefits and concerns they had regarding the graphing calculator:

> The five benefits that the students felt were most important were: the ease of sketching and obtaining information from graphs; being able to check quickly the correctness of derivatives, integrals, and solutions; being able to understand and interpret graphs and derivatives more easily; the ease of calculation and checking procedures regarding difficult formulae; and the increase in confidence and enthusiasm associated with the use of the tool. . . . The concern which was thought to be most important by the students was the possibility of becoming dependent upon the tool, since they had noticed in themselves a tendency to rely solely upon its use (p. 78)

Penglase and Arnold (1996) concluded that the research available for their review did not provide conclusive results concerning the use and the effects of graphing calculators. Further research is still needed, particularly in regards to the "de-skilling" concern raised by the students in the study by Boers and Jones, the effect of graphing calculators on the nature of tests, and the effect of graphing calculators on the current mathematics curricula.

In a report published by the Educational Testing Service (Wenglinsky, 1998), data from the 1996 National Assessment of Educational Progress (NAEP) in mathematics were analyzed at the fourth and eighth grade level. From the NAEP data, Wenglinsky was able to obtain information about the frequency of students' computer use at school and home, the professional development available to mathematics teachers in computer use and how mathematics teachers and students use computers in the context of mathematics instruction. The results of this report indicated that "the use of computers to teach higher-order thinking skills was positively related to academic achievement in mathematics and the social environment of the school . . . [and that] the use of computers to teach lower-order thinking skills was negatively related" (p. 3).

On a smaller scale, Zhang and Patzer (2001) also addressed the issue of technology and basic skills. In their study, 104 sixth and seventh grade students assigned to participate in a remedial math lab were divided into two groups. Both groups used computer-assisted instruction (CAI) for drill and practice of addition, subtraction, multiplication, and division facts. The treatment group used additional software to study surface area of a cylinder, basic math facts, and the subtraction and division of three- or or four- digit numbers, while the control group used paper-and-pencil, textbook, and workbook exercises. They found no significant difference in the post-test scores between the groups. They did note that anecdotal evidence provided by the students' regular classroom teachers indicated that students in the experimental group experienced a positive change in their attitude toward math. However, the authors caution teachers with the following statement:

> The one important lesson we learned as teacher educator and classroom teacher is that simply putting a student in front of a computer does not equate with integrating technology in a math curriculum. Desirable learning outcomes from technology-based learning cannot be realized in the absence of meticulous planning, identification of appropriate resources and materials and carefully organized presentations dedicated to learning the students through the entire problem-solving process. (p. 93)

Clearly, there is no universally accepted view of the best use of calculators in mathematics classrooms. Goldenberg (2000) states,

> The right questions about technology are not the broad ones about which hardware or software to use, but about how each works with in a certain curriculum, right down to its effect on how individual problems are posed to the

> student. . . . It is the problems posed, not the technology with which they are attacked, that make all the difference. (p. 1)

Technology changes the types of question students can be expected to attack. It can provide students with a visual representation of a mathematical concept, thus allowing them to "manipulate" mathematics with their hands and eyes – similar to the way concrete objects are used in teaching elementary school mathematics. Goldenberg (2000) goes on to develop six principles to help teachers decide "whether, when, and how to use computers or calculators and how to maximize the gains and minimize the dangers of their use" (p. 2). His six principles are as follows:

> *The Genre Principle: Good decision making requires us to be aware of the different roles for technology, to think clearly about our own classroom goals, right down to the particular needs of particular students, and to choose technologies expressly to further those goals, rather than merely adding technology to the classroom in ways that may be attractive but tangential or even detrimental to the goals we set. . . .*
>
> *The Purpose Principle: Allow calculator use when computational labor can get in the way of the purpose of the lesson. When learning how to perform the computation is the purpose of the lesson, calculators may be a bad idea. . . .*
>
> *The Answer vs. Analysis Principle: . . . even when the process of calculation is not the point of the lesson, stepping through that process and seeing the intermediate details explains the results that it produces. At such times, a technology that obscures the details and skips directly to the answer is no help. . . .*
>
> *The Who Does The Thinking Principle: . . . we might ask - specific to a lesson or even to a particular problem - whether the role of technology is to replace a capacity that the student might, otherwise need to develop or to develop the student's capacity to think, independent of the technology. Some of each may be warranted, but good use of technology depends on making such decisions consciously. . . .*
>
> *The Change Content Carefully Principle: Decisions about what is or is not obsolete content must be made thoughtfully, attending not just to what technology*

> *can do, but to a careful, analysis of what students need to be able to do - especially how they need to be able to reason. . . .*
>
> ***The Fluent Tool Use Principle:*** *"Touching" several computer or calculator tools but not really mastering them may do more harm than good: it costs time and teaches little. Learning a few good tools well enough to use them knowledgeably, intelligently, mathematically, confidently, and appropriately in solving otherwise difficult problems makes a genuine contribution to a student's mathematical education. . . .* (pp. 2 – 4, 6 – 7)

Usiskin (1998) makes the following prediction regarding the use of calculators and the teaching of mathematics:

> When the paper-and-pencil algorithms used today were introduced – most about four or five hundred years ago – they constituted the "advanced technology" of their time. . . . These algorithms were not immediately accepted by all because some people feared that a loss of mental power would result from algorithms that enabled one to do mathematics with less mental arithmetic or other mental work. Even though paper-and-pencil algorithms finally triumphed and became used for almost all mathematics, still one was expected to memorize certain facts and formula. The old ways did no die out completely, because it was not efficient to have to recall or reconstruct or even write mathematics every time it was needed. And so we can expect that although calculator and computer algorithms will overtake virtually all paper-and-pencil algorithms, a few of these old algorithms will remain. Those that remain will be in our curriculum not because they are curiosities and not because they train the mind but because they have the qualities of good algorithms. (p. 19)

The NCTM *Standards* (NCTM, 1989) and the *Principles and Standards* (NCTM, 2000) clearly call for using technology "appropriately" in the teaching and learning of mathematics. From the research presented previously, it appears that the mathematics education community is still striving to determine precisely what constitutes an appropriate use of technology and how to translate that into classroom practice.

Summary

The literature discussed in this chapter has set the context in which to consider this study. It has been shown that, in general, high school size does not seem to influence student performance on college entrance examinations or subsequent success in college. However, for high schools striving to provide an educational experience that includes depth in the math courses offered and opportunities for students to develop a sense of responsibility to school, a medium size high school appears to be most appropriate. The linear regression models developed in the various studies typically included students' high school GPA, high school rank, ACT Mathematics or Composite scores, and/or mathematics placement test scores. These models lead to the conclusion that perseverance and good algebra skills are important for university mathematics success. For each of the regression models found there was still a significant amount of variance in the dependent variable for which there was no explanation. The quantitative data do not provide an indication of why the success of certain students cannot be predicted by the regression model. The few qualitative studies presented offered some insight into unquantifiable factors that appear to influence students' success in college and in college mathematics. The factors put forward in the research included: the general transition students experience when they live away from home for the first time; graduating from a high school with an inadequate curriculum; and failure to acquire appropriate study skills, habits and attitudes in high school.

After the publication of the *Standards* (NCTM, 1989), the 1990s were expected to be a time of transition for teachers and students of K-12 mathematics. However, a review of the research related to mathematics education in the past 12 years illustrates that the debate surrounding the ideals of the *Standards* is ongoing and implementation of the *Standards* varies greatly from classroom to classroom. The teaching strategies encouraged by the *Standards* also affect the professional education of teachers and the use of technology in the mathematics classroom. "High quality" teaching, as defined by the research, requires that teachers have the opportunity to learn in the same kind of environment as they are expected to create for their students. It is essential that pre-service teacher preparation and in-service teacher professional development programs adjust to meet this need.

The literature presented in this chapter has provided a diverse sample of the previous research questions studied, methodologies applied, and conclusions drawn. In this study the quantitative data examined and the method of examination used is a variation of previous research on pre-collegiate factors as indicators of student placement and success in university mathematics. This study also provides insight into the unquantifiable factors which influence student success in university mathematics; an area in which there is insufficient literature. In the next chapter, the research questions and methodologies that guided this research will be discussed.

CHAPTER III

METHODOLOGY

In this chapter, a description of the setting and the participants of the study are provided. The quantitative and qualitative research methodologies used to study the research questions posed in Chapter I are discussed.

Given my interest in learning more about students' high school math experiences and how they are related to their experiences in mathematics at the university level, it was necessary to employ both quantitative and qualitative research methods. Used together, quantitative and qualitative methodologies can provide a more informed understanding of the problem under consideration than can be obtained from using one method or the other. The use of quantitative methods allows the researcher to study a larger number of subjects with the focus being the analysis of data that can be measured numerically. Qualitative methods allow the researcher to obtain a thorough description of the situation from the participants' perspective. "Seeing" the situation from the participants' point of view gives valuable insight into the phenomenon being studied and can also lead to an understanding of why certain statistical relationships may exist.

Background

The participants in this study were students at the University of North Dakota, a liberal arts college in the upper Midwest. The University is a co-educational, state-supported institution, enrolling approximately 10,000 undergraduate students and 1,900 graduate students. Within the student population, approximately 52% are male and 48% are female; 88% are White/Non-Hispanic American and 12% are American Indian, Black, Asian, Hispanic American, or non-resident aliens. The average age of the undergraduate population is 22.5 years. Geographically, 55% of the student population is from North Dakota, 42% from other states in the United States with the highest percentage coming from Minnesota (25%), and 3% from other countries. The University of North Dakota offers 85 undergraduate degree programs, through seven academic units: the colleges of Arts and Sciences, Business and Public Administration, Education and Human Development, and Nursing; and the schools of Medicine and Health Sciences, Engineering and Mines, and Aerospace Sciences. Approximately 50 undergraduate degree programs require the equivalent of at least one university-level mathematics course. Courses are taught on the semester system, and undergraduate students are expected to satisfactorily complete a minimum of 125 semester hours in order to receive a baccalaureate degree.

In the fall semester, the Mathematics Department offers approximately 40 sections of entry-level mathematics courses, which include: Intermediate Algebra, which does not count toward graduation, College Algebra, Finite Mathematics, Trigonometry, Precalculus, Applied Calculus, Calculus I, and Discrete Mathematics. Intermediate Algebra, Finite Mathematics, and Applied Calculus are taught in large lecture sections with 100 to 130 students per section. The remaining introductory courses are taught in smaller sections of 30 to 40 students each. Typically, College Algebra is taught by graduate teaching assistants seeking a master's degree in mathematics. The Finite

Mathematics, Trigonometry, and Applied Calculus courses are taught by lecturers. Lecturers have earned at least a master's degree in mathematics. The Precalculus and Calculus I sections are taught by either lecturers or tenure-track faculty, and Discrete Mathematics is traditionally taught by tenure-track faculty.

Data Collection

Access to the student participants was gained during the summer of 1999 by discussing the research topic with the chairman of the Mathematics Department at the University of North Dakota and by obtaining approval from the Institutional Review Board (IRB). As a senior lecturer in the Mathematics Department, the Department's approval was readily obtained, as was permission from the 19 faculty and graduate teaching assistants to collect survey and consent forms from the students during class time. The collection of the survey and consent forms was completed the third and fourth weeks of the semester. This was just after the last day to add a course and just before the first round of exams in most classes. It was hoped that students would still be attending class regularly during this time period and would not have a test score, either positive or negative, to influence their willingness to participate in the quantitative or qualitative aspect of this research project.

A total of 1,426 surveys were collected. Of the 1,426 students who completed a survey, 532 consented to participate in the study, indicated that they had graduated from high school in 1999, and were enrolled in their first university-level mathematics course. Two of these students did not sufficiently complete the survey form and had to be dropped from the study. Thus, 530 students formed the sample for this study.

On the survey the students were asked to indicate whether or not their current math class was their first university-level math course, their sex, the date of their high school graduation, the name and location of their high school, the size of their graduating class, the highest level of math offered in their high school (five choices), and the highest level of math they completed in high school with a grade of C or better (seven choices). See Appendix A for a copy of the consent and survey forms.

Information about students' high school GPA, ACT Composite, ACT Mathematics, and grade in their first university level math course was obtained through the Registrar's Office. The students' high school GPA was available only if the students reported it when they took the ACT and only if they requested their ACT scores be sent to UND.

The Sample

The 530 students involved in this study were graduates of high schools in 25 different states and Canada. Over one half of the students came from North Dakota high schools (n = 298) and one third of the students from Minnesota high schools (n = 148). With regard to sex, 44.2% of the students were female and 55.8% were male. The number of students in the entry-level mathematics courses was as follows: 63 students were enrolled in Intermediate Mathematics, 148 students in College Algebra, 73 students in Finite Mathematics, 14 students in Trigonometry, 92 students in Precalculus, 64 students in Applied Calculus, and 76 students in Calculus I or above.

The Quantitative Aspect of the Study

The following questions guided the quantitative aspect of this study:

1. Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex, which are the most efficient predictors of the level of mathematics students enroll in their first semester at UND?

2. Is there a significant difference in the average level of math students enroll in their first semester at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

3. Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex, which are the most efficient predictors of students' success in their first mathematics course at UND?

4. Is there a significant difference in the average grade earned by students in their first mathematics course at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

Statistical Analysis

The questions posed regarding the predictive ability of the pre-collegiate factors measured and the dependent variables, level of mathematics course enrolled in the first semester at UND, and grade earned in that course are best answered by attempting to define a significant regression model. When the purpose of determining a regression model is to develop a set of independent variables to predict the value of a dependent variable, a stepwise linear regression procedure is considered appropriate. In this procedure, the

> independent variables are treated as predictors, and the researcher seeks to determine the best subset of them to predict the criterion at a high level of accuracy. The goal is to maximize R^2 (or some other statistical criterion that may be maximized or minimized) while minimizing the number of predictors. [In the] stepwise regression. . . . the order of entry of the independent variables into the regression equation is determined by a selection algorithm. . . . The most commonly employed stepwise procedure capitalizes on the advantages of both adding and subtracting predictor variables. The procedure automatically adds a variable or subtracts a variable depending on which step would most significantly enhance the model (i.e. increase R^2). Short of the all subsets regression, this approach will yield the model that best fits the sample data. (Newton & Rudestam, 1999, pp. 253 – 254)

To answer the research questions regarding differences in the average math level students enroll in their first semester at UND and the average grade students earn in their first university mathematics course, while considering the factors of high school graduating class size and the availability of a particular course, the use of the two-factor analysis of variance (ANOVA) procedure is required.

> One advantage of a two-factor design is that it allows the researcher to control some of the variability within the study. . . . Often the variability within the categories of a factor is not all unexplained and uncontrolled variance. (Gravetter & Wallnau, 1996, p. 481)

It is possible for the introduction of a second factor to explain or predict the variance, thus reducing the error variability and increasing the chances of obtaining a significant difference in the effect of the first factor. The type I two-factor analysis tests the main effect while controlling for a second effect under the assumption that no significant interaction is occurring between the two factors.

> When a two-factor ANOVA produces a significant interaction, you should be very cautious about accepting the main effects (whether significant or not significant) at face value. In particular, a significant interaction can distort, conceal or exaggerate the main effects. (Gravetter & Wallnau, 1996, p. 477)

If the interaction is significant further statistical analysis may be warranted.

When performing the two-way ANOVAs for this study, in some cases, it was necessary to collapse the three levels of high school graduating class size into two categories. This was done to reduce the likelihood of skewed results that might be caused by cells with a small number of cases. Consistently, the levels medium and large were collapsed to form the category called "big." This choice was made because the characteristics of medium and large high schools seemed more compatible than those of small and medium high schools. Given the availability of mathematics courses and the different size high schools, large and medium high schools were more likely to offer every course under consideration than small schools.

Throughout the gathering of the quantitative data and the statistical analysis process, the Statistical Package for the Social Sciences (SPSS) software was employed to organize the data and perform statistical tests.

The Qualitative Aspect of the Study

The question, how are students' high school mathematics experiences connected to their success or lack of success in their UND Precalculus course?, guided the qualitative research conducted for this study. The goal was to obtain a description of the various high school mathematics experiences students come from and to determine how these experiences impact their level of success in the Precalculus course at the University of North Dakota.

Data Collection

Students who completed the consent form in the fall of 1999, in addition to giving consent for the release of numerical data, were asked if they would be willing to participate in an interview process to be conducted in the spring of 2000. During the spring 2000 semester, the researcher, and an outside interviewer, interviewed 13 Precalculus students who had consented to be part of the interview process. The participants in this study were chosen because Precalculus was their first university-level mathematics course, a Precalculus type course was the highest level of mathematics they had completed in high school, with a grade of C or better, and they had graduated from high school within the previous year.

The decision to interview only Precalculus students was based on several factors. First, it was thought that limiting the math level of the students being interviewed would focus the qualitative piece of this study and thus allow themes and assertions to emerge more fully and clearly. Second, learning about how students' high school math experiences have or have not been influenced by the NCTM *Standards* and how this relates to their math experiences at UND was of particular interest. Choosing students who had demonstrated an interest in math during high school and showed a continued interest at the university level seemed an appropriate choice. Finally, since Precalculus is the prerequisite course for the Calculus sequence and thus subsequent mathematics, student success in Precalculus has long-term implications for the Mathematics Department.

It was necessary and beneficial to include an outside interviewer in this process. Four of the students interviewed had been students in my Precalculus section in the fall of 1999 or were my students in Calculus I during the spring of 2000. These students were assured that I would not listen to their interviews until after grades for the spring 2000 semester had been submitted. The tapes from these interviews were stored in the locked file cabinet of a third party. A benefit of involving an outside interviewer is the opportunity to gain another perspective on the interview process and the data being gathered. (See Appendix C for the list of questions used to guide the interviews).

After completing the interviews in the spring of 2000 and proceeding with the analysis of the data, it was decided that the information was incomplete and would be used as a pilot study. In particular, given the variety of settings in which students completed their high school years, pieces of information about various students' high school experiences seemed to be missing. Thus, the interview process was refined and preparations were made to repeat the process in the spring of 2001.

The same process, as was implemented in the fall of 1999, was used to obtain consent and survey information from students in Precalculus in the fall of 2000. Revisions made to the survey form were due to information gained through the pilot study (see Appendix B). Again, students chosen for the interview process indicated that a Precalculus course was the highest level of high school math they had completed, with a grade of C or better, and they were enrolled in Precalculus as their first university mathematics course. A total of 151 survey/consent forms were administered and collected during the third week of the semester. From the collected survey/consent forms, 16 students met all of the requirements. That is, they indicated a willingness to be interviewed, that Precalculus was their first university-level mathematics course, and that the highest level of math they completed in high school, with a grade of C or better, was a Precalculus course. Of those 16 students, I was able to contact 13. One of the students was not available at the phone number he or she had provided and a current phone number could not be determined. The other two students did not return any of the several phone messages I left on their answering machines and/or with roommates. Four out of the 13 students chosen to be interviewed had been my students in Precalculus during the fall 2000 semester and/or were currently students in my Calculus I course during the spring 2001 semester. The person who interviewed my students for the pilot study also conducted these interviews. I assured the students that I would not listen to the tapes of their interviews until after the spring 2001 semester grades were submitted. The tapes from these interviews were stored with a third party in a locked file cabinet until after the end of the semester.

A change that occurred as a result of the pilot study experience was for me to interview each student twice instead of once. The first interview focused on their high school experiences, particularly their experiences in mathematics. Before the second interview, I listened to the tape of the first interview to determine appropriate follow-up questions to clarify what they told me about their high school experiences and to prepare for the second interview regarding their Precalculus experience at UND.

Qualitative Study Sample

Of the 13 students interviewed only 11 interviews were fully analyzed. Initial analysis of the interviews indicated that two students had completed a high school math course beyond Precalculus with a grade of C or better, consequently, it was necessary to exclude them from the study. Six of the 11 students included in the study were female. Three women graduated from North Dakota high schools, two from Minnesota high schools and one from another state. The female students all attended public high schools with graduating classes ranging from 200 to 340 students. Of the five male students, all graduated from public high schools: four from high schools in North Dakota, and one from a Minnesota high school. The graduating class sizes for the male students ranged from 35 to 675 students. These students earned the following final grade for their UND Precalculus course: one grade of A, one grade of B, three grades of C, two grades of D, one grade of F, and three withdrew. (See Appendix D for more information about each of the students who contributed to this paper.)

Qualitative Data Analysis

Before starting the data analysis, the text of the interviews was read and re-read. In addition to reading the text, in order to gain a sense of the tone underlying the written representation of each interview, the interview recordings were also reviewed. Throughout the data analysis process, I employed the Ethnograph software package. This software package allows for multiple codes for the same section of text, the designation of parent codes that can group together several codes, and the search function is an efficient feature.

I began my analysis with the process Strauss and Corbin (1998) refer to as "open coding." They write,

> Broadly speaking, during open coding, data are broken down into discrete parts, closely examined, and compared for similarities and differences. Events, happenings, objects, and actions/interactions that are found to be conceptually similar in nature or related in meaning are grouped under more abstract concepts termed "categories" [or concepts]. (p. 102)

Through this process, I developed what I will refer to as codes, which are synonymous with Strauss and Corbin's idea of "categories" or "concepts."

The experiences the students shared with me were about similar events occurring in two different time periods and in two different settings: high school and their first semester at UND. Thus, there are many similar codes for each time period/setting. For example, for the code that represents the strategies employed by a student to overcome difficulties experienced in high school math, there is a corresponding code for strategies students used to overcome difficulties experienced in UND Precalculus. From the 11 interviews, 47 codes emerged from the data. Those directly related to the students' high school experiences included high school (HS) favorite math course/teacher, HS least favorite math course/teacher, what students found helpful in their learning of math in HS, what students believe hindered their learning of math in HS, strategies students employed when struggling with HS math, experiences in HS Precalculus (PC), homework in HS PC, actions/attitudes of their HS PC teacher, students' actions in HS PC, grading in HS PC, the use of technology in any of the students' HS math classes, the students' views on the benefits/drawbacks of using technology in HS math classes, the students' view of their math ability in HS, the students' beliefs about grades, and the students' attitudes about math and studying for math.

During the interviews, students also discussed their experiences with high school math in general. This required codes similar to those for PC and were referred to as HS other math codes. Codes reflecting the students' corresponding experiences in the UND Precalculus course were also developed. For text related to only the students' experiences at UND, it was also necessary to develop several codes that did not correspond to their high school experiences. They included the transition from high school math to UND math, which contains the comparisons directly stated by the students about their high school experiences and their UND experiences; the students' expectation of the UND PC course; and how the student was placed in the UND PC course.

The next phase of the analysis was to group codes into categories. This "enables the analyst to reduce the number of units with which he or she is working. In addition, categories have analytical power because they have the potential to explain and predict" (Strauss & Corbin, 1998, p. 113). After considering the codes that had emerged and reading the interviews through again, the following categories emerged: Graphing Calculators, Transition, and Strategies for Success.

The last stage of analysis was the formation of my assertions. During this stage of analysis, I read through my data with respect to the four categories mentioned above. Within each category, I looked for recurring themes mentioned by the students and created statements that I felt offered a synthesis of the interview data. Seidman (1998) describes this "stage of interpretation" and its relationship to the interview process:

> The last stage of interpretation, then, consistent with the interview process itself, asks researchers what meaning they have made of their work. In the course of interviewing, researchers asked the participants what their experience meant to them. Now they have the opportunity to respond to the same question. In doing so they might review how they came to their research, what the research experience was like, and finally, what it means to them. How do they understand it, make sense of it, and see connections in it? Some of what researchers learn may lead them to propose connections among events, structures, roles, and social forces operating in people's lives. (p. 111)

Throughout the data analysis process, I was aware of the biases that my position in the Mathematics Department and my previous experiences could inflict on the data. I have made every effort to use my past knowledge to help in this process while also making every effort to avoid seeing only those pieces of information that support my biases. Strauss and Corbin (1998) write the following:

> As we come across an event of interest in our data, we ask, "What is this?" Later, as we move along in our analysis, it is our knowledge and experience (professional, gender, cultural, etc.) that enables us to recognize incidents as being conceptually similar or dissimilar and to give them conceptual names. It is by using what we bring to the data in a systematic and aware way that we become sensitive to meaning without <u>forcing</u> our explanations on data. (p. 47)

In order to maintain the integrity of this research project, throughout the analysis of the data, every effort was made to discern any discrepancies that might exist between the assertions that developed and the interview data.

In this chapter, an explanation of how this study was carried out has been provided. The details of the data collection process, the chosen sample, and the data analysis procedures were described for the quantitative and qualitative facets of this research project. In Chapter IV, the findings of the research project conducted according to the methods described will be presented.

CHAPTER IV

RESULTS

The purpose of this study was to examine factors influencing students' success in their first university mathematics course. The results of the data analysis are presented in this chapter. The following research questions guided the data analysis process:

Research Questions

1. Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex, which are the most efficient predictors of the level of mathematics students enroll in their first semester at UND?

2. Is there a significant difference in the average level of math students enroll in their first semester at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

3. Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex which are the most efficient predictors of students' success in their first mathematics course at UND?

4. Is there a significant difference in the average grade earned by students in their first mathematics course at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

5. How are students' high school mathematics experiences connected to their success or lack of success in their UND Precalculus course?

The data analysis is organized and presented in two major sections. The first section contains a presentation of the descriptive data and the results related to each of the four quantitative research questions. In the second section the qualitative results for the fifth research question are presented.

Quantitative Results

Descriptive Data

This part of the study focused on undergraduate students enrolled in their first university mathematics course in the fall of 1999, at the University of North Dakota, who graduated from high school between May of 1998 and August of 1999. The number of students included in this part of the study was 530.

Table 1 presents variables that describe characteristics of the students in the study. Section A describes the students by sex; section B describes the size of students' high school graduating classes; section C describes the highest level of high school math completed with a grade of C or better by the students; section D describes the first UND mathematics course enrolled in by the students; and section E describes the students' high school GPA, the grade earned in their first UND mathematics course, and their ACT Math and Composite scores.

More than half of the participants (55.8%) were male students. By design, each of the graduating class size categories was assigned approximately one third of the participants. One and one half percent of the students completed a maximum of two years of high school mathematics. An additional 25% completed an Algebra II course, which is typically the third year of high school math. A little less than three fourths of the students completed high school math courses beyond Algebra II.

Almost 12% of the students were enrolled in the Intermediate Algebra course, which is considered a remedial math course and does not count toward graduation. Approximately 42% of the students were enrolled in either College Algebra or Finite Math, both of which satisfy the university general education mathematics requirement and serve as the prerequisite course for Applied Calculus. Slightly more than 12% of the students were enrolled in Applied Calculus as their first university level mathematics course. Close to 32% of the students were enrolled in the courses for math intensive majors, with 17.4% enrolled in Precalculus and 14.4% in Calculus I or above.

The mean high school GPA reported by the students was a 3.50, with a range of 1.58 to 4.00. The mean grade earned by the students in their first university mathematics course was 2.36, with a range of 0.00 to 4.00. The mean ACT Math score was 24.01 and the mean ACT Composite score was 23.61; the range for these scores was 15 to 35 and 15 to 34, respectively.

Table 1. Characteristics of UND Students Enrolled in Their First University Mathematics Course in the Fall of 1999

Characteristic	Number	Percent
A. Sex		
Male	296	55.8
Female	234	44.2
B. HS Senior Class Size		
Small: 1 – 74 students	179	33.8
Medium: 75 – 319 students	178	33.6
Large: 320 + students	173	32.6
C. Highest HS Math Completed With a C or Better		
Algebra I	2	0.4
Geometry	6	1.1
Algebra II	133	25.1
Trigonometry	99	18.7
Precalculus	195	36.8
Calculus (non – AP)	55	10.4
A P Calculus	40	7.5
D. Course Enrolled in at UND		
Intermediate Algebra	63	11.9
College Algebra	148	27.9
Finite Math	73	13.8
Trigonometry	14	2.6
Precalculus	92	17.4
Applied Calculus	64	12.1
Calculus I	65	12.3
Calculus II	8	1.5
Discrete Math	3	0.6
E. Additional Descriptive Data	Range	Mean
HS GPA	1.58 to 4.00	3.50
Grade in UND Math Course	0.00 to 4.00	2.36
ACT Math Score	15 to 35	24.01
ACT Composite Score	15 to 34	23.61

Research Question 1

Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex, which are the most efficient predictors of the level of mathematics students enroll in their first semester at UND?

The entry-level of mathematics course students enrolled in was defined numerically as follows:

1 = Intermediate Algebra
2 = College Algebra, Finite Mathematics, and Trigonometry
3 = Precalculus, Applied Calculus, and Discrete Mathematics
4 = Calculus I and Calculus II.

The results of the stepwise regression, presented in Table 2, indicate that ACT Mathematics score and the highest level of high school mathematics completed by the student are significant predictors of the level math students enrolled in for their first UND mathematics course. These two variables accounted for 54.9% of the variance in the level of mathematics taken by students ($F = 235.287$, $p < .001$).

Table 2. Regression Model for the Level of Mathematics Students Enrolled in as Their First University Mathematics Course

Variable	B	t	p
ACTMATH	0.156	18.254	<.001
HSMC	0.094	2.997	.003

Research Question 2

Is there a significant difference in the average level of math students enroll in their first semester at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

All students reported that their high schools offered Algebra II. Thus, it was not possible to determine whether there was a difference in students' entry-level university mathematics course between the three high school size categories when accounting for the availability of Algebra II.

The frequencies, means, and standard deviations for the three levels of high school graduating class size and the availability of Trigonometry are summarized in Table 3.

Table 3. Frequency, Mean, Standard Deviation of the Mathematics Level Students Enrolled in as Their First University Mathematics Course Given the Factors High School Graduating Class Size and the Availability of a Trigonometry Course

	Trigonometry Available			No Trigonometry		
HSGCS	n	M	SD	n	M	SD
Small	132	2.33	0.82	47	2.36	0.85
Medium	170	2.52	0.92	8	2.00	1.07
Large	173	2.54	0.85	0	N/A	N/A
Total	475	2.47	0.87	55	2.31	0.88

Since all of the large high schools were reported as offering a Trigonometry course, it was not appropriate to perform a two-way ANOVA using three levels of high school graduating class size. In order to facilitate further analysis, the medium and large levels of high school graduating class size were collapsed into the category called "big." The results are reported in Table 4.

Table 4. Two-way ANOVA Results for the Level of the First University Mathematics Course Enrolled in by Students When Considering Two Levels of High School Graduating Class Size and the Availability of a Trigonometry Course

Source	Type I SS	df	MS	F	p
SIZE	3.984	1	3.984	5.275	.022
TRIG	0.198	1	0.198	0.263	.609
SIZE*TRIG	2.048	1	2.048	2.711	.100
ERROR	397.272	526	0.755		
TOTAL	3602.000	530			

There was a significant difference in the mean level of university mathematics students enroll in for their first mathematics course between students from small and big high school graduating classes when accounting for the availability of Trigonometry ($p = .022$). Students from big high schools enrolled in a higher level of university mathematics ($M = 2.52$) than students from smaller high schools ($M = 2.34$).

Four students from medium and three students from large high school graduating classes reported that a Precalculus course was not available at their high school (see Table 5).

Table 5. Frequency, Mean, Standard Deviation of the Mathematics Level Students Enrolled in as Their First University Mathematics Course Given the Factors High School Graduating Class Size and the Availability of a Precalculus Course

	Precalculus Available			No Precalculus		
HSGCS	n	M	SD	n	M	SD
Small	119	2.30	0.83	60	2.40	0.83
Medium	174	2.49	0.93	4	2.75	1.26
Large	170	2.55	0.84	3	2.00	1.00
Total	463	2.46	0.88	67	2.40	0.85

In order to minimize the possibility of skewed results due to small cell sizes, collapsing the medium and large high school graduating class sizes into the category "big" was justified. To evaluate whether or not there was a significant difference in the university math level students enrolled in when considering two levels of high school graduating class size when the availability of Precalculus was taken into account a type I two-way ANOVA was performed. The results in Table 6 show a significant difference in the mean level of university mathematics students enrolled in for their first mathematics course between students from small and big high school graduating classes when accounting for the availability of Precalculus ($p = .022$). Students from high schools with big graduating classes enrolled in a higher level of UND mathematics (M = 2.52) than students from smaller high schools (M = 2.34).

Table 6. Two-way ANOVA Results for the Level of the First University Mathematics Course Enrolled in by Students When Considering Two Levels of High School Graduating Class Size and the Availability of a Precalculus Course

Source	Type I SS	df	MS	F	p
SIZE	3.984	1	3.984	5.251	.022
PRECALC	0.227	1	0.227	0.299	.585
SIZE* PRECALC	0.210	1	0.210	0.276	.599
ERROR	399.081	526	0.759		
TOTAL	3602.000	530			

As shown In Table 7, when considering three categories of high school graduating class size and the availability of a Calculus course, only two students from large schools reported that their high school did not offer Calculus.

Again, the small and big high school size categories were used and a two-way ANOVA for the dependent variable level of mathematics taken by students for their first university math course, when considering two levels of high school graduating class size and accounting for the availability of Calculus, was performed. The results in Table 8 show a significant difference in the mean level of university mathematics students enrolled in for their first mathematics course between students from small and big high

Table 7. Frequency, Mean, Standard Deviation of the Mathematics Level Students Enrolled in as Their First University Mathematics Course Given the Factors High School Graduating Class Size and the Availability of a Calculus Course

	Calculus Available			No Calculus		
HSGCS	n	M	SD	n	M	SD
Small	62	2.48	0.82	117	2.26	0.82
Medium	121	2.42	0.93	57	2.67	0.93
Large	171	2.53	0.84	2	3.50	0.71
Total	354	2.48	0.87	176	2.40	0.88

Table 8. Two-way ANOVA Results for the Level of the First University Mathematics Course Enrolled in by Students When Considering Two Levels of High School Graduating Class Size and the Availability of a Calculus Course

Source	Type I SS	df	MS	F	p
SIZE	3.984	1	3.984	5.302	.022
CALC	0.016	1	0.016	0.021	.885
SIZE* CALC	4.288	1	4.288	5.707	.017
ERROR	395.214	526	0.751		
TOTAL	3602.000	530			

school graduating classes when accounting for the availability of Calculus ($p = .022$). Similar to the results found when accounting for the availability of Trigonometry or the availability of Precalculus, students from big high schools enrolled in a higher level of UND mathematics than students from smaller high schools.

It is also interesting to note that in this case the interaction is significant ($p < .02$) as well. Figure 1 provides an illustration of how the interaction affects the dependent variable. The average math level for students from small schools without Calculus ($M = 2.25$) was lower than the average math level for students from big schools without Calculus ($M = 2.70$), while the average math level for students at small and big high schools with Calculus appeared to be equal ($M = 2.48$).

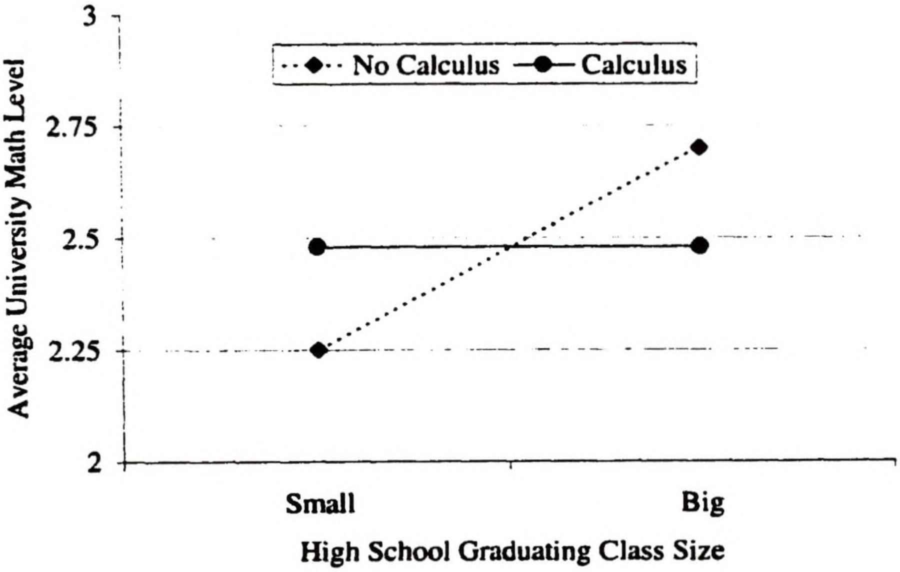

Figure 1. Average Level of the First University Mathematics Course Students Enrolled in When Considering Small and Big High School Size and the Availability of Calculus.

For the dependent variable university math level taken by students when considering three levels of high school graduating class size and the availability of AP Calculus, only four small schools were reported as offering an AP Calculus course (see Table 9). No valid conclusions could be drawn regarding the effects of these two factors on the dependent variable due to the small cell size. Collapsing the three high school graduating class sizes into small and big was not appropriate in this situation.

Table 9. Frequency, Mean, Standard Deviation of the Mathematics Level Students Enrolled in as Their First University Mathematics Course Given the Factors High School Graduating Class Size and the Availability of AP Calculus

	AP Calculus Available			No AP Calculus		
HSGCS	n	M	SD	n	M	SD
Small	4	2.50	0.58	175	2.33	0.83
Medium	89	2.44	0.90	89	2.56	0.96
Large	158	2.54	0.80	15	2.47	1.25
Total	251	2.51	0.84	279	2.41	0.90

In summary, the results from the two-way ANOVAs performed indicated that the average level of the first university mathematics course taken by students was significantly different between the students from small and big high school graduating classes when taking into account the availability of either a high school Trigonometry or Precalculus or Calculus class. In addition, the interaction between the factors small and big high school graduating class size and the availability of Calculus was significant when evaluating the average level of students' first university mathematics course.

Research Question 3

Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex which are the most efficient predictors of students' success in their first mathematics course at UND?

The results of the stepwise linear regression presented in Table 10 show that high school GPA, the highest level of mathematics completed by the student with a C, and the availability of high school Calculus are significant predictors of the grade earned in students' first university-level mathematics course. These three variables account for 18.8% of the variance in the grade earned ($F = 26.176$, $p < .001$).

Table 10. Regression Model for the Grade Earned in Students' First University Level Mathematics Course

Variable	B	t	p
HSGPA	0.971	6.103	< .001
HSMC	2.820	3.824	< .001
CALC	- 0.379	-2.502	.013

Research Question 4

Is there a significant difference in the average grade earned by students in their first mathematics course at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

As in the analysis of the level of the students' first mathematics course, when considering the students' university math grade among the three levels of high school graduating class size and the availability of the different high school math courses, difficulties with cell size were encountered.

As was previously noted, all students reported that their high school offered Algebra II. Thus, it was not possible to determine whether there was a difference in the grade earned in students' first university mathematics course when considering three levels of high school graduating class size and the availability of Algebra II.

The frequencies, means, and standard deviations of the grade earned by students in their entry-level university mathematics course, for the three levels of high school graduating class size and the availability of Trigonometry are summarized in Table 11. The students' entry-level university mathematics grade is based on a four-point scale.

Since all the large high schools were reported as offering a Trigonometry course, it was not appropriate to perform a two-way ANOVA using three levels of high school graduating class size. To facilitate further analysis, the small and big classifications for size of high school graduating class were used. The results are presented in Table 12.

Table 11. Frequency, Mean, Standard Deviation of the Grade Students Earned in Their First University Mathematics Course Given the Factors High School Graduating Class Size and the Availability of a Trigonometry Course

	Trigonometry Available			No Trigonometry		
HSGCS	n	M	SD	n	M	SD
Small	114	2.47	1.38	41	2.41	1.53
Medium	150	2.31	1.42	5	2.40	1.52
Large	156	2.29	1.40	0	N/A	N/A
Total	420	2.35	1.40	46	2.41	1.51

Table 12. Two-way ANOVA Results for the Grade Students Earned in Their First University Mathematics Course When Considering Two Levels of High School Graduating Class Size and the Availability of a Trigonometry Course

Source	Type I SS	df	MS	F	p
SIZE	2.409	1	2.409	1.201	.274
TRIG	0.049	1	0.049	0.024	.876
SIZE* TRIG	0.102	1	0.102	0.051	.822
ERROR	926.308	462	2.005		
TOTAL	3516.000	466			

There was no significant difference in the mean grade students earned in their first university mathematics course between students from small and big high school graduating classes when accounting for the availability of Trigonometry.

Two students from medium and three students from large high school graduating classes reported that a Precalculus course was not available at their high school (see Table 13). To minimize the possibility of skewed results due to small cell size, the small and big categories for high school size were used in the statistical analysis.

Table 13. Frequency, Mean, Standard Deviation of the Grade Students Earned in Their First University Mathematics Course Given the Factors High School Graduating Class Size and the Availability of a Precalculus Course

	Precalculus Available			No Precalculus		
HSGCS	n	M	SD	n	M	SD
Small	102	2.52	1.41	53	2.34	1.44
Medium	152	2.32	1.42	3	2.33	2.08
Large	154	2.29	1.40	2	3.00	1.41
Total	408	2.36	1.41	58	2.36	1.45

The two-way ANOVA results are reported in Table 14. There was no significant difference in the mean grade students earned in their first university mathematics course between students from small and big high school graduating classes when accounting for the availability of Precalculus.

Table 14. Two-way ANOVA Results for the Grade Students Earned in Their First University Mathematics Course When Considering Two Levels of High School Graduating Class Size and the Availability of a Precalculus Course

Source	Type I SS	df	MS	F	p
SIZE	2.409	1	2.409	1.203	.273
PRECALC	0.580	1	0.580	0.290	.591
SIZE* PRECALC	0.991	1	0.991	0.495	.482
ERROR	924.887	462	2.002		
TOTAL	3516.000	466			

When considering three categories of high school graduating class size and the availability of a Calculus course, only two students from large high schools did not have the opportunity to take Calculus (see Table 15).

For two levels of high school graduating class size when accounting for the availability of Calculus, the two-way ANOVA showed that there was no significant difference in the mean grade students earned in their first university mathematics course between students from small and big high school graduating classes (see Table 16).

When considering three levels of high school graduating class size and the availability of AP Calculus, only four students from small schools reported having the opportunity to take AP Calculus (see Table 17). Thus, no valid conclusions could be drawn regarding the effects of these two factors on the grade students earned in their first

Table 15. Frequency, Mean, Standard Deviation of the Grade Students Earned in Their First University Level Mathematics Course Given the Factors High School Graduating Class Size and the Availability of a Calculus Course

	Calculus Available			No Calculus		
HSGCS	n	M	SD	n	M	SD
Small	55	2.27	1.45	100	2.56	1.40
Medium	104	2.18	1.48	51	2.59	1.26
Large	154	2.30	1.41	2	2.00	1.41
Total	313	2.26	1.43	153	2.56	1.35

Table 16. Two-way ANOVA Results for the Grade Students Earned in Their First University Mathematics Course When Considering Two Levels of High School Graduating Class Size and the Availability of a Calculus Course

Source	Type I SS	df	MS	F	p
SIZE	2.409	1	2.409	1.211	.272
CALC	7.252	1	7.252	3.645	.057
SIZE* CALC	0.014	1	0.014	0.007	.933
ERROR	919.192	462	1.990		
TOTAL	3516.000	466			

university mathematics course. In addition, collapsing the medium and large categories of high school graduating class size into "big" did not eliminate the small cell. Thus, the two-way ANOVA was not performed.

Table 17. Frequency, Mean, Standard Deviation of the Grade Students Earned in Their First University Mathematics Course Given the Factors High School Graduating Class Size and the Availability of an AP Calculus Course

	AP Calculus Available			No AP Calculus		
HSGCS	n	M	SD	n	M	SD
Small	4	2.25	1.71	151	2.46	1.42
Medium	77	2.16	1.44	78	2.47	1.39
Large	146	2.28	1.40	10	2.50	1.43
Total	227	2.24	1.42	239	2.47	1.40

In summary, the results of the two-way ANOVAs performed for the dependent variable, the grade students earned in their first university mathematics class, indicate that the main effect of high school graduating class size was not significant when taking into account the various high school math courses.

The results reported in this section of Chapter IV focused on pre-collegiate factors previously considered relevant to students' success in their first university mathematics course. The linear regression models that were found included measures related to students' academic choices and performance and not factors that described their high school. However, the results of the two-way ANOVAs indicated that when high school size was considered, while taking into account the availability of particular high school math courses, there was a significant difference in the average entry-level university mathematics course students enroll in, but not in the grade students earned in their first university mathematics course.

The quantitative data cannot provide information about the unquantifiable factors that influence students' success in their first mathematics course. In addition, the quantitative data do not provide an indication of why the success of certain students cannot be predicted by the regression models. In the next section, the results from the qualitative piece of this study will provide a description of the factors the students considered important as they transitioned from high school mathematics to the UND Precalculus course.

Qualitative Results

Research question 5

How are students' high school mathematics experiences connected to their success or lack of success in their UND Precalculus course?

The four assertions discussed in this section are themes that evolved from the categories Graphing Calculators, Transition, and Strategies for Success. Quotations have been selected from the student interviews to clarify the assertions. The selected quotes are representative of the experiences of the majority of the students; however, what one student said is not precisely what others stated. The quotes which are used are similar in context and meaning to other student responses and descriptions. Any disconfirming quotes are also noted. Considered together, the quotes provide a rich description of students' experiences in high school Precalculus and the transition to university Precalculus. It should be noted that the quotes have been edited to eliminate idiomatic phrases which can be distracting for the reader. However, the content and the intent of the students' statements have been preserved. To further inform the discussion that follows, the reader is referred to Appendix D for a brief description of each of the students who contributed to this study. A pseudonym has been substituted for each participant's real name.

Assertions

Assertion 1. In high school, students became dependent on graphing calculators even when their teachers stressed using the calculator as a tool and not as a replacement for learning.

All 11 of the students interviewed reported using a graphing calculator in their high school math courses. In high school, students were regularly allowed to use a graphing calculator for homework, class work, and exams. As they began their UND Precalculus course, they were surprised to learn that graphing calculators would not be allowed on the exams; nor would they be instructed on how to use them during the course. As one student said, "I could use graphing calculators in pretty much all of those classes [HS math], which, coming here now, is a slap in the face (chuckle), cuz no one lets us use it, and honestly it hurts a lot, I think" (Katherine). The use of calculators outside the classroom was left to the students' discretion.

All of the students agreed that using the graphing calculator made math easier. How the calculator made math easier varied from student to student, but included the following: computations, even those that could have been done mentally, were easier to do on the calculator; graphs were created quickly, without much effort or thought; and there were fewer formulas to memorize. In short, the graphing calculator was a time saver. Ten of the students acknowledged a certain amount of dependence on the graphing calculator throughout high school. They turned to the calculator even when they could have answered the question without it and began to rely on it to help them do problems when they were not quite sure how to do the work using paper and pencil.

> Well, I would just use it for adding stuff and stuff that I could usually do in my head or would be able to do in my head, I just used it, and, when our teacher was like, okay you guys are gonna need to know how to graph this, it really didn't matter, cuz on our tests we could use our graphing calculator anyways, so, I didn't try to learn how to do it. (Denise)

> [The graphing calculator made it] easy to do the problems. It's a lot easier just to punch it in, have the graphing calculator do all the work and then copy down the graph, or whatever, so it was easy to do the problems but maybe not so . . . easy to learn. You weren't really understanding what you were getting, you were just kind of punching it in because you knew how to punch it in. (Rachel)

> Which we also had the graphing calculator [for the tests], you can type in your equations and store them, you never have to memorize them. . . . Well you could graph things, and if you had to find your asymptotes or certain points on something, you could kind of zero in on 'em and people would actually like test numbers until it came out right. . . . If they didn't know how to do it. I've done that a few times. (chuckle.) I wish I wouldn't have relied on them so much. . . . [You could] fib a little bit. (Katherine)

In some of the high schools, students were expected to buy a particular graphing calculator. "It was a requirement that we had to buy the TI-86 and we did all our stuff on that, we did experiments and all that stuff" (Jacob). Often the larger high schools were able to provide calculators for the students. One student recalled,

> At [my] high school they had graphing calculators they allowed everyone to use during class, every once in a while they'd let us take them home, if we needed to or whatever, just sign them out, ya know, name and whatever so they knew who took them and, yeah, they were, they were more than helpful. (Travis)

Another said,

> We always got to use calculators. . . . I mean as far as I can remember, I don't remember not being able to use a calculator. Um, if you didn't have a graphing calculator, one was provided for you. (Kayla)

Typically, the graphing calculator was integrated into the daily lessons. The most frequent use of the calculator reported was that the teacher would introduce the new topic on the chalkboard or overhead projector without the calculator, and then they would show the students how to use the calculator to solve particular problems related to the topic. If the teacher did not include a presentation of how to use the calculator, the students would ask the teacher to do so. If the teacher did not have sufficient class time to show students how to use the calculator, the students would figure out, on their own, how to use the calculator to help them solve the problems.

> He'd kind of show you how, how to do most of the work by hand, and then he'd show us at the end how to do it by calculator and usually he'd run out of time and then he'd say that he'd start doing it the next day but he never would. So we kind of had to just kind of figure it out by ourselves a little bit, some of the stuff. (Jacob)

> I think, it [the teacher's lesson] was just focused on the math. Like I mean they did not disallow calculators so, people in the class would ask, okay, how do you do it on the calculator? And they'd explain it. And that seemed to be the first thing everyone wanted to know. There were a lot of people that, I mean they understand the math, but they wanted to know how to use the calculator. (Katherine)

The extent to which teachers proclaimed and enforced the idea that the graphing calculator should be used as a tool and not as a replacement for learning math varied greatly. Some teachers indicated that the graphing calculator (or technology) would always be available to students and should be freely used. Other teachers restricted calculator use to the exploration of mathematical concepts or the checking of work done with paper and pencil and did not allow students to use them on exams.

> We had graphing calculators, usually TI-83 and he'd throw [the calculator projection device] on the overhead and show everybody how to do them and stuff and you'd graph this and flip that over and do that stuff. . . . it was pretty much you get your calculator for everything; there wasn't much that we didn't get our calculators for. There might have been one where we were doing circles with radians, we didn't use calculators so much then because . . . it was easier not to use calculators for those. I don't think he let us use calculators for those cuz it was just counting around the circle . . . it depended on what kind of problem we were doing, sometimes he'd say, okay, everybody whip out your calculators, I'm gonna show ya how to do something for the homework today and the thing was he showed us pretty much everything on calculators . . . sometimes he'd show us both ways and then he was like just pull out your calculators, I'll show you how to do it fast, but sometimes he'd show us the other way too, but he'd only show us that other way once maybe and then he'd always use the calculator or every other time cause it was faster and, and he'd let us use them on the test. (Janelle)

Sam remembered,

> First she'd use the overhead to show us how it was done then she'd let us see how it was done using the calculator, she'd plug it into the little deal and start typing in numbers on the calculator and stuff. . . . She never said that [we should know how to do the work without the calculator] she'd say that it'd be good to know how to do it without the calculator but she never expected us to do it without the calculator.

Even for students in classes where calculators were used frequently it was expected that the students would show their work on tests.

> Most of the time [we were expected to show our work on the tests]. And sometimes it's like, oh, this is the button I pushed on the calculator. So, like inverse cosine of this and then you'd put what it equals but for most things we had, we had to show work to at least get some points so even if we did get it wrong it would be, he'd give you points for showing your work for how far you

got when it was right. . . . He still wanted us to at least show him how we got it, or attempted to get the answer. (Janelle)

In addition to expecting students to show their work, some teachers cleared the calculator memory to keep students from storing formulas and still expected students to memorize certain basic identities.

> Yeah, we had to show the written work [on our exams] and we were able to use the calculator ... we could use the calculator to help us out. . . . [We couldn't have formulas in our calculator], she'd come around and she'd hit some buttons that would erase everything off your memory. . . . [We used the calculators for] Trigonometry a little bit like for finding out the cosines and stuff. . . . Oh, yeah, we had to memorize that [trigonometric values for common angles] . . . but like those harder ones like the cosine of like other numbers, we could use our calculator to figure them out. (Sam)

The following quote describes the use of graphing calculators in the high school Precalculus course of the only student who did not indicate that he was dependent on the graphing calculator.

> These days now, kids can do everything on a calculator and never understand where it's coming from, but she really, really emphasized doing it on your paper, she said, you go ahead and use two pieces of paper before you use that graphing calculator or whatever but, but, then it was stressed that if you have any doubts about it, go back and check it on the graphing calculator, and then you can go through and do it again by hand and then you can understand where your problem was, it helped more or less understand where your problems were and how did you, how'd you get to this point or whatever. . . . In the precalc class [on the tests], we were able to use just the Texas Instruments solar powered scientific ones but not graphing calculators. (Travis)

The extent of their dependence was not necessarily apparent to the students until they started their UND Precalculus course. When asked if they felt like they were learning the mathematics while using the graphing calculator in high school, most students' responses were similar to these two:

> Yeah, I did, there were a couple of things that I needed to tie up, like little absolute value rules or whatever, but otherwise I, pretty much, I knew my math behind it and just the calculator, it didn't help me get the entire grade. It wasn't just like a cheat method, it was a learning aid more or less. (Lee)

Another student remembers,

> We used the calculator very well (chuckle). [I can use the calculator] a little better than I can do my math, I suppose, um. I kind of have the idea of the basic concepts, it seems. If you put a problem and ask me to do it without a calculator, I, I can do it. It takes me a while and there'd be a lot of scratches here and there, but, I could probably get it. (Katherine)

However, at some point in the interviews, most students indicated that they felt like they were starting over with at least some of the mathematics.

> Basically I had to relearn everything [at UND]. I, I'm not sure if we went over that stuff in high school or not, but we could use the calculator and it was way easier to just graph it on the calculator, so that was a lot different (laugh). (Jacob).

Paul presented an idea that merits further investigation. "I learned a lot [in high school Precalculus]. I learned concepts, but I didn't learn, basically how to do it without a calculator is what I didn't learn."

<u>Assertion 2.</u> The transition from high school math to university mathematics was affected by differences in the expectations of their high school Precalculus course and their University of North Dakota Precalculus course.

In addition to the use of technology in high school but not at UND, the students interviewed discussed several other differences, which presented challenges for them in making the transition from high school Precalculus to the UND Precalculus course. These differences included: (a) in high school, students used formula and/or identity sheets and other test aids on exams; (b) in high school, there is typically time for students to work together in class on the next homework assignment, and this was not the case in their UND Precalculus course; (c) the material in the UND Precalculus course was covered at a faster pace and more in-depth than in high school; and 4) the students did not need to study for math in high school and they did not know how to study for math at UND.

The students in the UND Precalculus course were not allowed to use any external aids on their exams. They were expected to memorize the necessary formulas and identities. In high school, as well as using the graphing calculator from which they could retrieve stored formulas and trigonometric values, students were often allowed other aids, such as the unit circle or an identity sheet on exams.

> Like I said in high school we were able to write down a whole bunch of formulas and stuff on note cards so we wouldn't have to remember 'em and you can just basically breeze through a test and look something up when you need it but when you take a test [at UND] you have to have everything in your head and you have to have everything memorized you have to have the concepts memorized and you have to know how to do everything without the help of a note card or stuff like that. (Paul)

> I remember [in high school Precalculus], here's a sheet with all the identities on it, you can use it on the test. (chuckle.) Thanks. You know, here [at UND] it's like, okay you got 27 identities to memorize, hope you know 'em. And you're like, ahhhh, you have to be joking you know. (Kayla)

Overall, the students agreed that the Precalculus material was covered in greater depth and at a faster pace at UND than in high school.

> I'd been out of math for a while. But, I could tell there was a difference in the college, the way they taught pre-calc and the way I learned pre-calc in high school. . . . It's just more, I guess more faster here. You didn't spend as much time on certain chapters and that kind of thing so. . . . Um, glad I decided to go with pre-calc first. Just cuz, a lot of the stuff [the instructor is] teaching, I did learn in pre-calc in high school but there's concepts that we hadn't covered, and, I

> knew I was probably gonna need 'em for calc. . . . I'm studying, I'm definitely studying harder, for this math. . . . Spending a lot more time on it, outside of class. In high school I could usually get everything done in class, so there were very few problems to do outside of class. And now I spend, ah, a lot more time working the problems, trying to understand it. (Rachel)

> Um, again it's tougher it's more rigorous. It's more demanding to learn the subject matter. Bit by bit and through the guts of it than just kind of punching it in on your calculator and seeing how everything turns out there. (Lee)

As a result of the increase in pace and depth of the material Paul stated, "I definitely have to work a lot harder and even when I work a lot har,der I don't achieve the results I did in high school."

Over half of the students claimed that they did little or no studying for math in high school and did not know how to study for Precalculus here at UND. One student reported not doing anything to prepare for tests. She kept up with her daily homework assignments and that was all she needed to do to get As.

> I don't know, in high school, I couldn't study for math. Just cuz, for me it's hard to sit down and stare at a book and numbers and letters, and I don't know, it's just, for me math is one of my, one of my subjects, and I'm just kind of like, you either know it or you don't. . . . I mean I kind of have, that's my philosophy on things, . . . sit down and try and, stare at it, and I mean you know, unless you're gonna work it out, you can't really, you have to sit down and put out some paper and work problems, but I just was too lazy to do that I think. And I did fine, I passed at the top of my class. And, um, it was really easy for me. (Kayla)

Five students, while in high school, "studied" by looking over their notes and homework assignments and working a few problems only if something was unclear. Frequently, this was done the night before or morning of the test. This effort was sufficient to earn them As and Bs in high school Precalculus.

> Um, I'd look over like different formulas really quick and then try to memorize how to do some stuff and that would be pretty much it. . . . Well, [I might work]

> a couple problems, if I looked over something that was old and I didn't remember how to do it I'd like go to the, the problems in the book and see if I could work them out. . . . If you went to class it would be easy to know stuff and, that would be pretty much what was on the test was what they talked about in class. (Sam)

Another student chose particular problems based on what was reviewed in class the day before the test:

> Usually the review day helped a lot. Cuz the review was usually what was on the test. And so, I mean if I got the review, or if I didn't, then I would just work harder on those problems and try to get it. (Denise)

For Denise, working harder on problems was mostly looking at them more. "My studying in high school was just look over a section and then that was good enough. Or, I would try, um, doing a few example problems, in the section. But only if I was having trouble."

As a result of an intervention by his teacher, Travis' study skills underwent a transformation during high school Precalculus. His original study skills included looking over the sections and practicing the easy problems, for which he earned Ds and Fs. He described his revised study skills as follow:

> I found what helped me the best is I would go through, and, two nights before a test, a night before a test, I would go through and I would just go to the sections, I wouldn't really realize what problems I did and I would just start, and go through a certain problem and I would do that problem, and I'd just do it, and if I got it wrong, I'd do, I'd do ones that I knew the answer to in the back of the book. I'd force myself not to look in the back of the book till I got 'em, and, ah, so I'd do those and I'd try and understand why I got 'em and I would sit there and I would spend a lot of time on one problem and, it kind of made it the long way of studying but I think it helped me the best because then I would try to understand why I got this answer from this question that sort of stuff so I'd try and spend, I'd go through and do problems from each, cause each section then divided up into different little sections you say you have the simple ones and the harder ones and then you got more elaborate ones, and I'd try and do maybe one or two from each one and try and get everything in that sort of way, but as homework it was just

> kind of, doing the problems that were assigned to you, whereas studying you could do what you want on your own, and you could spend more time on what you were having problems with yourself.

This was similar to how the remaining students studied for math in high school. These students earned As in high school Precalculus.

The students also discussed the fact that in their UND Precalculus course they did not have time to work on assignments during class time. This was a change from high school math courses for 10 of the students. The one student who did not have time in class to work on his assignments was in a class taught on interactive television. Thus, class time was used strictly to answer students' questions about the homework that had been assigned the day before and to present the new material.

> It was nice to have time at the end to work on the assignment, um, a lot of times we were just working through the first couple of problems of a new section, you know, there might be a question or something, and you could refer back to your notes or refer back to the book, and, you could usually answer the question, but, sometimes it's nice to have someone, you know, just right there to explain it to you step by step. (Danielle)

In general, the students agreed that time to work on an assignment in class was beneficial. It gave them a chance to work with their peers as well as get immediate feedback from the instructor. When they had time to work in class, they left with a better sense of what they understood from the lecture and where they were having difficulties.

<u>Assertion 3</u>. The strategies students used to overcome tough times in high school math did not transfer to overcoming struggles in university mathematics.

During the interview, students were asked about the times they struggled with understanding a mathematical topic and what they did to overcome their difficulties.

This question was posed to the students twice, first in the context of their high school experiences and again when discussing their experiences at UND. The student responses for the two settings, high school and UND, differed greatly.

At the high school level, 10 students cited their teachers as a major resource in times of trouble. They felt that their high school teachers were interested in their success and they were readily available in a convenient location. These things made it easier for students to call upon their teachers for help both in and out of class.

> The teacher I had would answer any questions you had, you could go to him outside of class and he'd talk to you and help you with stuff. . . . You could usually find him in his classroom, cuz he pretty much spent the whole day there, I think teaching different math classes. (Rachel)

> Actually, my Precalculus teacher was really helpful, [when] I started to have problems she noticed right away at the beginning of the semester and she actually got my parents involved. She took time after school and to meet with me and she tutored me, um, two times, twice a week or something like that for an hour . . . she really, she took the initiative to help me get involved in the stuff. (Travis)

According to the students, their high school teachers were available during lunch and before and after school. The students knew where to find their teachers and, when their particular math teacher was not available, they were often able to ask another math teacher for help.

> There was a math office and all the teachers had their offices within there. And you could go and get help from any teacher there, whether your [teacher] was there or not. But, they always said, come in, we have people in the math office if you need help or tutoring or anything . . . they were available. (Kayla)

As discussed earlier, in high school there was frequently time in class for the students to work together and to ask the teacher questions. This time in class gave students and teachers the opportunity to form a relationship that made it more comfortable for the student to approach the teacher outside of class.

In addition to their teachers, asking their peers for help was another commonly used strategy for overcoming difficulties. Most of the time these peer relationships had developed long before the students were in math class together.

> In high school we always scheduled our classes, you know, so we have classes with friends, so I probably would have called one of my friends first. If I hadn't been able to get a hold of one of them, or they didn't know what they were doing either, um, I would have gone to the school the next day, early, and gone in and talked to her. If she wasn't available, we'd go to anyone in the math department, any of the teachers, we kind of just knew everybody. It was a pretty small school. (Danielle)

In contrast, when the students started their UND Precalculus course in the fall they were new to the university setting and faculty and did not have pre-existing peer relationships to call upon. The students did not find their UND teachers to be as available or approachable as their high school teachers. Several of the students indicated that their UND teachers did not appear to be as interested in them and their success as their high school teachers.

> I never went in to talk to [the instructor] because I didn't really like [the instructor], I didn't really, just didn't like [the instructor's] personality. It's a "you failed, too bad" kind of personality, so I didn't, I didn't go in and ask for help and I literally did not have time to go to the math learning center for help because I had class all day and then practice all night and then all afternoon and then I'd go home and do homework all night. (Paul)

Several of the students thought their UND teacher was approachable but never went in to get help, even when they were struggling.

> [The instructor] did, I never went in, but, I believe that you could always, a lot of people went in and asked questions that they were having, problems that they

> were having, he was, I think [the instructor] was fairly, pretty willing to help people. (Katherine)

At the university when students were struggling, they were more likely to refer to the textbook or their notes than go to see the instructor. Even though they were not inclined to go to office hours, several of the students would ask questions in class.

> In class, sometimes [the instructor would] try to relate things to other things to help, a problem, to something where you might use it, so that might help, oh, maybe I should remember this or figure out how to do this and then when you asked a question [the instructor would] try to explain it and I'm like well [I don't really understand]. [The instructor would] say, well, let's try this and then [the instructor would] try it again, a whole, try it till you got how to do it. (Janelle)

Even when the instructor encouraged students to come in during office hours, the students were reluctant:

> Yeah, [the instructor was] like, if you don't understand sometime come to my office after class or come during my office hours and I'll help you. . . . [I went in to see the instructor] a few times. I didn't do it as often as I should have and it was more towards, sometimes it was more towards test times that I'd go in for help, [the instructor would] be like, well, we've been doing those for a long time, and I'm like well, I'm here now, I'm gonna try. (Janelle)

Students also found it difficult to make connections with other students. All of the students interviewed were in their first semester at UND. The students usually did not know anyone in the class at the beginning of the semester. If they were able to connect with others to study with, they would meet sporadically at best and organizing the meeting time and place was a challenge:

> Yeah, there were a few students in that class that had other classes with me that we'd work together sometimes, but I never really got a math study group going. I heard that's really helpful to do. . . but, no one in my, my dorm was in my class which made it kind of hard to arrange meeting times and places but we did once or twice before tests and it helped a little bit. (Paul)

The differences between the high school and university settings make it more difficult for students and university faculty to form the same type of connections the students are accustomed to establishing with their high school teachers. They see their high school teacher more frequently in and out of class, throughout the day and the school year. In addition, in their first semester of college, students do not have the pre-existing peer relationships that they depended on in high school for academic support.

Travis and Kayla provide two interesting perspectives on what students need to make this transition. Travis placed greater responsibility for the transition on the high schools.

> I think that [high school teachers], someway need to instill that, these [high school] kids are on their own in the next year and they need to learn to take initiative to, seek out help on their own instead of having help come to them.

While Kayla would like the university faculty to help smooth the transition.

> The thing is, I think there needs to be more encouragement to come in to get help or, have some sort of, um, sessions be set up, like once a week for just your class. Be like there's gonna be a study session, I'll be there, I mean kind of being office hours type of thing, but be, and have it be a group thing, have us all come in and just kind of, you know, not all of us, but just some people that feel that they need extra help, come in and, you know, get help one on one or, you know, I think or set up individual appointments, office hours sound so broad. I feel bad. It's like they're working on something and I'm gonna interrupt. . . . set up appointments or something. Make it sound like they're waiting for you, it's like your appointment, [the instructor] has you written down, and knows you're coming in. Something like that, cuz then you feel like they want you there. And they're expecting you to come in. You yourself, not just a broad range of students that you have in your classes, you know. Something like that, make it more personable, you know.

<u>Assertion 4.</u> More than a decade after the release of the NCTM *Standards*, the instructional paradigm at the high school and college levels has remained teacher-centered.

In spite of the NCTM *Standards*, published in 1989, the description of a typical day in the students' Precalculus class included the same set of events that occurred daily in high school math classes in the decades before the *Standards*. The events of a typical day in high school Precalculus were essentially the same for all of the students interviewed. First, they would go over the homework that was assigned for that day; next, the teacher would present the new material, usually through a form of direct instruction; and often there would be time at the end of class to work on the assignment due for the next class meeting.

> Well, they were 45 minute classes I think and, maybe 10 minutes was goofing around. And then, he'd go through the [homework] assignment, then go through the notes for the next assignment, and show us how to do stuff like that. And then he asked us if we have any questions. And then he'd give us the rest of the time to work on our assignment. So, usually we were done with our assignments before we even got out of the class. (Denise)

Danielle was the only student to include group work in her description of a typical day in Precalculus.

> We had like a warm up question every day that we went through. We'd come into class and the overhead would be on and there'd be a problem or two on there and we'd work through those and it would usually be about what we had gone through the day before. And, then, we'd work through that and then we'd get in groups and go over those problems and then if anybody had any questions, they'd write the problem numbers on the board and then she would go over those, and, generally, it sounds like a lot to do, but we'd go through it pretty quick, like in about 15 minutes. We had block scheduling in our high school, so we had hour and a half classes, so it wasn't that big of a problem. . . . And then, um, then after she'd answer any questions, we'd go through the lecture and get our assignment.

> We'd generally have like 15 or 20 minutes to work on the assignment before our class was over. Then the assignment was due the next day.

When asked specifically if there were "out of the ordinary" class days, students frequently mentioned review days as being different. A few of the students indicated that occasionally they worked in groups on projects or experiments in class. In general, the students whose high school Precalculus class included projects indicated that the projects were relevant to the mathematics they had been studying and provided a welcome change of pace.

> I think we did three projects. We did two from the book; there was a project section after every three sections, or it might have been every, at the end of every chapter. And he would group us into groups of three or four and then we'd all have to do this project. And he'd just pick it out of the book, or whatever. And we did that twice and then we did another group project but it was an internet project. He had assigned each of us a famous mathematician and then we just had to do like a page book report on it. . . . I think he just wanted to do something different like. It was a nice break for us. I mean in high school it's five days a week for a whole year. (Kayla)

Since the projects came directly out of the text, Kayla indicated that the connection between the mathematics they had learned in class and the projects was clear. Jacob reported doing several projects.

> We did an experiment [that had] to do with dropping a basketball and counting, or figuring out the velocity that it fell, and we did one where we stuck a thermometer into something and calculated the rate of change as it cooled off as it came out and different things like that, and one with pitch and tuning forks, we did that too. Trying to relate them to waves. . . . I could see where [the math] came from in most of the stuff. Some of it really helped, I think we used derivatives a couple of times. . . . I finally got to use them for something else other than busy work (laugh).

However, the students reported that, outside of completing a worksheet or answering questions from the text while they worked on the project, they were not held accountable for the knowledge gained from it at any other time.

The most consistent difference, discussed by the participants, in how mathematics was taught at the high school level before the *Standards* and how it is taught now is the use of the graphing calculator. In Assertion 1, the use of the graphing calculator and how students perceived the graphing calculator was discussed. Most of the students considered it a crutch more often than a tool for learning. Two students described a class activity in which the intention was to use the calculator as a tool for learning. Lee said,

> Just to graph the equations that you didn't know how to graph, he would let us graph them and study them through our calculator, and study how they go, it was just a learning aid, teaching aid in a way.

The second student discussed the same basic activity.

> We used them to look at what a parabola looked like and what a hyperbola and an ellipse and stuff looks like, and what the equations would be, and how the different degree of the variable or the exponent would make it look different. (Paul)

However, more often than not, the calculator seemed to replace the need to perform algorithms or to graph functions using paper and pencil, instead of increasing students' understanding of these mathematical concepts.

When asked to describe a typical day in their UND Precalculus course, students described a sequence of events similar to what they had experienced in high school. Rachel described a typical day in her UND Precalculus class:

> Oh, typically, we would come in, [the instructor] would ask if we had any questions on previous homework assignments, covering anything we didn't get, or

> understand. Spend about 10 or 15 minutes just going through that. And then move into the lesson for the day. You know, covering whatever we were covering and then assign you a homework assignment and then it's time to go.

A student from a different section gave a similar description.

> Well we'd come in and, [the instructor] asked if there were any questions, we usually had an assignment, we never turned anything in, but, it was like a list of what we should do from each section, as we went over it. [The instructor would] go over quick questions and then start on the next section and do, [the instructor would] do quite a bit of examples from the, um, and work maybe harder ones than, than we'd have in homework I think. They were a little harder, you know, from the board. (Katherine)

Students reported the various forms of group work were either regularly or randomly used in their university Precalculus course. In one section of Precalculus, group quizzes were given once a week.

> On Fridays we had group quizzes. The questions were hard, I mean they were a lot harder than they were on the test, cuz [the instructor] figured they, you know, four people working together, somebody's bound to be able to, figure it out and help everyone else. (Katherine)

In the other sections group work occurred occasionally. "Every so often he'd have us group up together and work with groups on an in-class assignment thing." (Sam).

For these students, the predominant instructional technique for presenting new topics (the teacher talking and students taking notes) did not change from the high school setting to the university. What was consistently different was the emphasis placed on performing algorithms and producing graphs <u>without</u> the assistance of the graphing calculator. After finishing a semester of Precalculus at UND, seven of the students agreed that they learned mathematics better without the calculator than when they were allowed to use it. Denise said the following:

> But, [the UND instructor] said that the calculator graph isn't necessarily what the graph really looks like anyway. So, I guess I was depending upon something that isn't even true. . . . Well I miss it still because I was so dependent upon it, but, I guess I better learn how to do the real graph rather than the one that I think it looks like.

When asked what she would tell her high school Precalculus teacher after her first semester at UND, Katherine replied:

> I would suggest that he teach a little more and not rely on the calculator. I've warned my sister. Cuz she's going through the same math track that I took and I told her, don't use your calculator a lot. It's there, but you don't need to use it a lot and it'll get you in trouble later.

After having been through a high school Precalculus class where the graphing calculator was always available and then the UND Precalculus where the calculator was not used Katherine concludes:

> Somewhere in the middle would have been nice. I think it's better here because we have to think a little more without the calculator. You actually, you have to know how to do it. Not just know how to do it, but be able to do it. But then, then again if you have numbers that are a little tedious I don't think that it would hurt anything, being allowed to use, you know, a calculator to add or multiply. I wouldn't be in pre-calculus if I didn't know how to add. It's not that I don't know how and I can't do it, it's just a little tedious. But I think probably here because you're forced to think a little bit more, there's not something to do your thinking for you.

The technology "added on" to the "traditional" instructional format in high school appears to have hurt more than it helped the students' understanding of mathematics. Eight of the students agreed that their high school Precalculus course had not prepared them for their first university mathematics course, with seven of those students indicating that the use of the graphing calculator was a major contributor to their lack of knowledge.

In summary, the assertions that evolved from the interview data indicated that the use of technology, students' study habits and strategies for success, and relationship between the students and peers and the students and their teachers all play an important role in students' success in university mathematics.

In Chapter IV, the results of the analysis of the quantitative and qualitative data have been presented. The next chapter will offer a discussion of the findings in light of previous research, as well as additional conclusions and recommendations.

CHAPTER V

FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

The purpose of this study was (a) to consider the effect of pre-collegiate factors such as high school graduating class size, high school grade point average, ACT Mathematics score, ACT Composite score, highest level of high school mathematics available, highest level of high school mathematics completed with at least a grade of C, and sex on the entry-level of mathematics course students enroll in and the grade earned in their first university mathematics course; and (b) to obtain students' points of view on the transition from high school math to university mathematics and how their past experiences influenced their success or lack of success in their first university mathematics course.

In this chapter, the findings of the study are addressed in the same order in which the research questions were presented. Each research question is restated and is followed by a discussion of the major findings of this study and their relationship to previous findings in the literature. These relationships, however, must be interpreted with caution because the terminology and methodological procedures of other studies were not always similar in nature. Thus, even though various studies may have achieved similar findings, the perceived similarities may be misleading, as numerous other factors may influence their interpretation.

Research Question 1

Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex, which are the most efficient predictors of the level of mathematics students enroll in their first semester at UND?

For this sample, students' ACT Mathematics scores and the highest level of high school mathematics they completed with a grade of C or better were the most efficient predictors of the entry-level of mathematics course students will enroll in at UND. These two factors accounted for 54.9% of the variance in the dependent variable. Students' ACT Mathematics scores alone accounted for 53.9% of the variance in the students' entry level of mathematics. This is not surprising, since UND uses ACT Mathematics scores as part of their mathematics placement process.

The fact that high school graduating class size did not enter into the regression model is not surprising. The level of UND mathematics students enroll in is based completely on their high school academic performance as represented by the students' ACT score, high school GPA, and placement exam score, which measures high school level math skills. Three of the studies discussed in the literature review provided conflicting results regarding high school size and high school academic performance. Smith (1961) found a direct relationship between the two variables, while Edington (1981) and Fowler and Walberg (1991) reported the existence of an inverse relationship. In addition, the 'optimal" high school size determined by Lee and Smith (1997) was a medium size high school of 600 to 900 students. Overall, the research available concluded that SES more than size was an indicator of high school academic performance (Amos & Moody, 1981; Bidwell & Kasarda, 1975; Fowler & Walberg, 1991; Lee & Smith, 1997; McIntire & Marion, 1989).

Research Question 2

Is there a significant difference in the average level of math students enroll in their first semester at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

When performing the two-way ANOVAs necessary to answer this question, it became apparent that the categories of medium and large needed to be collapsed into the category called "big." The results of this study indicated that there was a significant difference in the average level of the first university mathematics course taken by students between the categories small and big high school graduating class size when taking into account the availability of either a high school Trigonometry or Precalculus or Calculus class. For the participants in this study, the average mathematics level enrolled in by students from small schools was consistently lower than the level enrolled in by students from big high schools. The entry-level mathematics course students enroll in is based on several pieces of information: their ACT Mathematics scores, their UND Placement Test scores, and their intended majors. If students do not plan to study a math intensive major, they are less likely to enroll in the higher-level mathematics courses, even if they have the ACT Mathematics and/or Placement Test scores to place into those courses. In her study, Morgan (1993) concluded that students from small high schools are less likely to choose math intensive majors. Thus, it is reasonable to expect the UND math level for students from small high school graduating classes to be lower than the UND math level for students from big high school graduating classes. Another plausible explanation, which is not supported by the more recent research reviewed, is that students from small high school graduating classes scored lower on the ACT Mathematics section than students from big schools and were required to take a lower entry level mathematics course. In fact, through additional analysis of the data collected for this study, it was found that the mean ACT Mathematics score for students from small high schools was significantly lower than the mean ACT Mathematics score for students from big high schools ($p < .02$).

The interaction between small and big graduating class sizes and the availability of Calculus provide an interesting twist when evaluating students' average entry-level university mathematics course. That students from small high schools with no Calculus, on average, enroll in lower levels of UND mathematics than students from big high schools where Calculus is not available was expected. The finding that students from small high schools with Calculus enroll in essentially the same mean level of UND mathematics as students from big high schools with Calculus was not expected (see Figure 1). Another way to say this is, when Calculus is not offered, size makes a difference in the level of the first UND math course; but when Calculus is offered the effect of size disappears. This difference may be linked to the mathematical background of teachers at different size high schools. Small schools often struggle to attract teachers qualified to teach mathematics (DeYoung, 1995; Fowler & Walberg, 1991). As a result, in general, the students are less likely to be encouraged to pursue university majors that require higher levels of mathematics. However, when a teacher in a small high school offers a Calculus course, it is an indication that he or she has a strong background in math and, thus, is able to provide students with the background and encouragement to continue their studies in math intensive disciplines.

Research Question 3:

Considering the variables high school graduating class size, high school GPA, ACT Mathematics score, ACT Composite score, highest level of high school math available, highest level of high school math completed with at least a C, and sex which are the most efficient predictors of students' success in their first mathematics course at UND?

The variables high school GPA, the highest level of high school mathematics completed with a grade of C or better, and the availability of Calculus were the most efficient predictors of the average student grade earned in the entry-level university mathematics courses. The regression model is significant ($p < .05$); however, the three factors do not account for a sufficient amount of the variance in the dependent variable to consider the model useful.

Of the studies reviewed in Chapter II, the linear regression models produced to predict success in a university-level mathematics course typically included high school rank, high school GPA, and/or that particular university's placement exam score (Edge & Friedberg, 1984; Maxwell, 1988; Nejadsadeghi, 1985). Edge and Friedberg (1984) stated that high school GPA and high school rank indicate a student's level of long-term, academic perseverance. In light of the fact that students are required to take only two years of high school math to satisfy graduation requirements, it could be asserted that the level of mathematics a student completes in high school is an indicator of perseverance with respect to mathematics. The inclusion of the availability of Calculus in the model may be related to the mathematical background of the high school teachers and, thus, the quality of mathematics education they provide for their students.

The fact that high school size is not a significant predictor in this model aligns with previous research (Downey, 1978; Edge & Friedberg, 1984; Gallagher, 1986; Maxwell, 1988; Nejadsadeghi, 1985). Downey (1978) and Gallagher (1986) stated that student "self-selection" may have eliminated the influence of high school graduating class size on student success. "Self-selection" refers to the idea that students from small high schools who choose to attend a "large" state university are the top students in their class, while the top students from large high schools are more likely to choose a more competitive and possibly larger university.

Research Question 4

Is there a significant difference in the average grade earned by students in their first mathematics course at UND among the three levels of high school graduating class size, taking into account whether or not a particular math class is offered at their high school?

The results of the two-way ANOVAs performed for the dependent variable, grade students earned in their first university mathematics class, the main effect of high school graduating class size was not significant when taking into account the various high school math courses.

The general consensus of the research reported earlier was that high school size, in and of itself, is not significantly related to student success at the university level in general (Downey, 1978; Gallagher, 1986; Willingham et al., 1985) or in mathematics specifically (Edge & Friedberg, 1984; Maxwell, 1988; Nejadsadeghi, 1985). In spite of introducing the availability of particular high school classes, in an attempt to account for some of the variance in the average university grade within the size categories, the results of this study agree with the current literature. Based on the previous discussion of the relationship between high school size and the initial level of students' first university mathematics course, it is realistic to conclude that the appropriate placement of students reduced the influence of high school size on the students' university math course grade to a statistically insignificant level ($p > .05$).

In general, the results of the quantitative aspect of this study have confirmed the findings of previous research related to this topic. The discussion of the findings of the first four research questions and their relationship to the literature reviewed presents conclusions and rationale based on the quantitative data. The next section of this chapter discusses the assertions which emerged from the interview data.

Research Question 5

How are students' high school mathematics experiences connected to their success or lack of success in their UND Precalculus course?

Assertion 1

In high school, students became dependent on graphing calculators even when their teachers stressed using the calculator as a tool and not as a replacement for learning.

A hotly debated topic in mathematics education is the use of technology in the teaching and learning of mathematics. From the interviews conducted for this study, it appears that the use of graphing calculators, in the high schools these students graduated from, is replacing instead of supplementing student knowledge and understanding of mathematical algorithms and graphing techniques.

The students in this study admitted to relying on the calculators more than they should have, even when the task they needed to perform was easily done without the calculator. They said that, even though the teacher exposed them to the mathematics beyond the button pushing on the calculator, and expected them to show paper-and-pencil work, their homework and tests were a reflection of technological knowledge just as much or even more than mathematical knowledge.

This use of calculators is not consistent with the expectations outlined in the *Standards* (NCTM, 1989). Incorporating technology into the learning of mathematics was intended to extend students' problem-solving abilities, the types of questions they could address, and to enhance their conceptual understanding of mathematics (Burrill, 1992; Goldenberg, 2000). This does not seem to be the case for most of the students interviewed for this study. For these students, it appears that calculators were readily available, as is recommended by the NCTM, but the students did not have or did not exercise the ability to make appropriate choices regarding the use of the calculators.

Instead, the calculator provided a fast and easy alternative to using mental computation and paper-and-pencil algorithms. The interview data collected for this study support the concern about "de-skilling" reported by Boers and Jones (as cited in Penglase and Arnold, 1996). In their study, the students reported that calculators made it possible for them to complete their assignments quickly, provided storage for formulas, allowed them to check their answers to work done using paper and pencil, and provided a sense of confidence when taking exams. Clearly, making the technology available to the students did not guarantee the learning of mathematics (Goldenberg, 2000; Wenglinsky, 1998; Zhang & Patzer, 2001).

Assertion 2

The transition from high school math to university mathematics was affected by differences in the expectations of their high school Precalculus course and their University of North Dakota Precalculus course.

The commonly stated differences between high school Precalculus and UND Precalculus included the use of technology and other aids on exams, the depth of the material covered, the need to study for math, and time provided to work on assignments in class.

In high school, several of the students in this study were allowed to program formulas into their graphing calculators and to use other aids such as the unit circle or a note card on exams. In their UND Precalculus course, the students were not allowed to use this type of "help" on the exams. The students agreed that this made the UND Precalculus exams more difficult than their high school Precalculus exams. In addition to this change, while many of the topics they had covered in their high school Precalculus class were also a part of the UND Precalculus course, at UND the topics were covered in greater depth. The majority of the students struggled to meet these new expectations.

Another factor that contributed to student difficulties in the UND Precalculus course was their attitude, skills, and habits related to studying for mathematics. Several students stated that they did little to study for math in high school. If they did study, it was usually the night before or the morning of the test day. When the students realized they needed to study for Precalculus tests at UND, they did not have an established method or a scheduled time in place. The study conducted by Seymour and Hewitt (1997) reached the same conclusion regarding students' study skills and habits and considered this to be a contributing factor in students' decision to switch out of math intensive majors. This difficulty was also alluded to in the research by Belcheir et al. (1998) when they concluded that students were most likely to be successful if they were able to establish a routine which included time for studying and they "latched onto the 'culture of learning' . . . they needed to love what they did" (p. 14).

Interestingly, the participants in this study seemed to have transitioned from living at home to living on their own (with the exception of Kayla who lived at home) without experiencing the major challenges often associated with the freedoms of college life (Belcheir et al., 1998).

The students consistently discussed the benefits of having time in class to work on mathematics with their peers and to ask the teacher questions. In high school, they found this time to be helpful in learning the mathematics, and they left the class period with confidence in their ability to complete the assignment. In addition to providing time for students to ask questions before leaving class, it appears that this time helps to establish a class atmosphere which promotes peer and student-teacher interaction outside of the classroom. At UND, students did not have time to work on assignments in class.

Assertion 3

The strategies students used to overcome tough times in high school math did not transfer to overcoming struggles in university mathematics.

The two predominant strategies for overcoming difficulties in high school math were to seek help from the teacher and/or one's peers. In high school, they did so with little or no hesitation. Outside of the need for help a particular student required, the factors that seemed to influence the amount of interaction the students reported having with their teachers were the students' perceptions of the accessibility of the teacher and of the level of personal interest the teacher took in their success.

Of the six students who earned a grade of D or F in or withdrew from the UND Precalculus course, five had frequently sought the help of their teacher when struggling in high school Precalculus. In contrast, these five students reported visiting their UND instructor at most twice and possibly not at all throughout the semester. They found it more difficult to go to their UND instructor than they had their high school teacher. The students felt that their individual success was of no interest to the UND instructor. Whether or not students had expected this to happen before coming to college or had just resigned themselves to this being a reality of college after their first semester at UND was unclear.

The extra effort it took for students to meet with their instructors was also cited as a reason students did not go to the instructor for help. In contrast to the students' high school teachers, the UND instructor was not in the building, just down the hall from the student from 8 am to 4 pm. Students had to be aware of their instructors' office hours and of their own schedule, which varied from day to day. Even when the UND instructor encouraged students to come to office hours or make an appointment to come in for help, the students felt uncomfortable doing so. A reasonable connection can be made between the absence of time to work in class on an assignment or project and students' hesitation to seek help from their UND instructor. The time students spent working during their high school Precalculus class gave the students and teachers the opportunity to develop a relationship that made it more comfortable for students to seek the teachers' help outside of class. Time spent working in class provides a time where the teacher can ask individual students if they are struggling and offer the needed assistance on a personal level. Essentially, in high school, the teacher took the first step toward connecting with the student and sending a message of concern. This was not a part of the students' UND Precalculus experience.

Peer relationships were also used in high school to overcome difficulties in mathematics. In high school, students had long-standing peer relationships that were in existence before they were having trouble in mathematics class. To call upon these relationships when they needed help was natural. The first relationships to be developed in college are those with roommates and other nearby dormitory residents, and/or members of extracurricular organizations which they affiliated with at UND. For the students in this study, there was no overlap between the relationships they were developing outside of classes and their fellow Precalculus students. Thus, the students in Precalculus did not need to or necessarily attempt to connect with each other until they realized they were struggling and in need of help. The physical distance between students (i.e., different dorms and conflicting schedules) made establishing "Precalculus friends" an even greater challenge. When students attempted to connect with each other, the establishment of a meeting time and place was often difficult.

Assertion 4

More than a decade after the release of the NCTM *Standards*, the instructional paradigm at the high school and college levels has remained teacher-centered.

More than a decade after the publication of the *Standards* (NCTM, 1989), the students descriptions of high school Precalculus classes were characteristic of practices that would have typically occurred before the *Standards*. The students in this study reported that their high school Precalculus classes were still predominantly teacher-centered, with discrete pieces of mathematics presented daily and an assignment of drill-and-skill type problems to be turned in the next day. The basic structure of the classrooms described by these students does not appear to have been impacted by the *Standards* or other research cited in Chapter II. References cited in the literature review concurred that the changes in K-12 mathematics that were expected as a result of the NCTM *Standards* have been slow to occur and, in some cases, changes made in the name of the *Standards* have not actually reflected their intent (Battista, 1999; Burrill, 1997; NCTM, 2000; Olson, 1999).

The fact that college mathematics instructors predominantly employ methods of direct instruction is not surprising. The *Standards* (NCTM, 1989) were directed at reforming K-12, not post-secondary, mathematics. Due to the diversity of university mathematics programs, there has not been a consistent reform movement at the post-secondary level.

One recommendation made by the NCTM, which appears to have gained widespread acceptance in high school mathematics classrooms, is the use of graphing calculators. Based on the information provided by the students interviewed for this study, it seems that, although graphing calculators were used regularly in their high school math classes, the way they were used did not reflect the intentions of the *Standards*. What seems to be lacking in the way calculators were used is the presentation of problems that "require" calculators, as well as the time for students to reflect on how the calculator informs their understanding of mathematics. Instead, the students indicated that the graphing calculator was "added on" to the traditional curriculum and, as was discussed under Assertion 1, served more as a crutch than a learning tool.

Conclusions

The results of this study show that there is a relationship between high school graduating class size and the entry-level university mathematics course students enroll in, but no relationship exists between size of high school graduating class and students' success in their first university mathematics course. All but one of the factors that entered into the prediction models for each of the dependent variables were related to students' long-term academic perseverance. The availability of high school Calculus, which entered into the model for the grade earned in the students' first university mathematics course, is likely related to the mathematics ability and confidence of the high school teacher.

The results of the qualitative part of this study, while not generalizable, revealed insights into what students experience as they transition from high school to university-level mathematics and provided a snapshot of how the Standards are being implemented. Clearly, the use of technology, students' study habits and strategies for success, and relationships between the student and their peers and the student and their teachers all play an important role in student success in university mathematics.

Recommendations

This section gives recommendations for facilitating students' transition from high school math to university mathematics with the hope of increasing student success in their first university mathematics course. Recommendations for further research regarding the transition from high school math to university mathematics are also presented. The following recommendations are based on the findings of this study.

First and foremost, communication between high school mathematics teachers and university mathematics faculty needs to improve. The students interviewed for this study came from various high schools; nonetheless, improving communication between local high school teachers and UND faculty would be beneficial. It is not expected that these interactions would necessarily cause the establishment of a common curriculum or approach to teaching; however, these discussions should address how to end the "dangerous dualisms" (Tanner & Tanner, 1995, p. 65) that have developed in mathematics education. The literature reviewed on the teaching and learning of mathematics and this study demonstrate the need for all mathematics teachers to adopt practices that will integrate the learning of algorithmic skills, the development of conceptual understanding and the thoughtful use of technology

Second, to assist students as they adjust to the new demands of university life the College of Education and Human Development offers a one-credit Introduction to University Life course. Currently this course is optional; UND might consider making it mandatory for all freshmen or consider integrating aspects of this course into other entry-level university courses. Clearly, there are students who need assistance with initiating interactions with their university instructors.

Third, in the past individual Mathematics Department faculty have attempted to help students establish study groups and have incorporated regular in-class group work in their courses. Further exploration of the strategies these faculty members found effective in helping students develop study groups and how to encourage this practice among students would be worthwhile. In addition, students need time in class to establish student-peer and student-faculty relationships at the university level. The existence of or lack of these relationships appeared to influence the strategies for math success employed by the students while in high school, but were not used at UND.

Finally, to increase student success in mathematics courses, the Mathematics Department opened the Math Learning Center (MLC) in the fall of 2000. Throughout the academic year the MLC is open six days a week with a combination of day and evening hours. The MLC was designed as an informal meeting place for students to get help from undergraduate tutors and to meet with their instructors. The tutors are available to assist students with homework assignments and studying for exams. Initially, several faculty members held some of their office hours in the MLC. This practice has decreased over time. Increasing faculty involvement with the MLC might prove to be advantageous to the students. Interestingly, only two of the eleven students interviewed mentioned the MLC. While the students were not specifically asked about the MLC, they were asked about particular strategies they used to overcome difficulties in their UND Precalculus course. Currently, the Mathematics Department is conducting an evaluation of the MLC. It is expected that a more informed picture of student knowledge about and use of the MLC will result from this process.

The findings of this dissertation indicate the need for further research. The quantitative data from this study provided a reasonable model for the placement of students in their first university mathematics course. However, the fact that the UND Placement Test was under revision and, thus, not included in this study suggests a need for additional research. From the data collected for this study, it was not possible to determine a satisfactory set of pre-collegiate factors to predict student success in entry-level university mathematics. Previous research has shown similar results with the exception of studies that included SES. This suggests that future studies should follow-up on the effects of SES on student success.

The interview data from this study has provided students' perspectives on both their high school math experiences and their first university mathematics course. A study to examine the instructors' perspectives, teacher practices, and teacher intentions – at both education levels – is necessary to complete the picture.

Studies regarding the use of calculators at the secondary level and the implementation of the NCTM Standards (1989) are also warranted. A similar qualitative study conducted at a comparable university where calculators are used in the teaching of Precalculus would certainly complement this study. Coincidentally, this year the North Dakota Education Standards and Practices Board and the North Dakota Curriculum Initiative conducted a web-based survey of teachers regarding the implementation of the NCTM Standards in schools across the state. It is intended that the results of this study will be used to assess the type of professional development needed for teachers to be able to effectively implement the NCTM Standards. In addition to evaluating the needs of in-service teacher professional development, pre-service teacher education also needs to be reviewed.

The research presented in the literature review clearly demonstrates the need for pre-service teachers to experience the same kind of learning environment they are expected to create for their students. A study to determine to what extent this occurs for pre-service teachers graduating from UND and other nearby universities should be conducted and used to inform the curriculum designed and instructional approaches planned for K-12 pre-service mathematics teachers.

APPENDICIES

APPENDIX A
ORIGINAL CONSENT AND SURVEY FORMS

Consent to Participate

Hi! My name is Michele Iiams. I am currently a doctoral student in the Department of Teaching and Learning collecting information for my dissertation. I am interested in becoming a more informed and hopefully better math teacher and educator of future math teachers. Through this study I hope to gain insight into how students experience the curriculum that has been designed for them while considering their academic background.

The purpose of this study is to examine a student's first experience in a university mathematics course and the role played by a student's high school math experiences. Your instructor will not know whether or not you have chosen to participate in this study. Any information obtained will be held in the strictest of confidence and will not affect your grade in this course.

This study could be of benefit to the Mathematics Department by determining demographic and academic characteristics of students commonly enrolled in entry-level mathematics courses. The Math Department could consider this information when placing students in their first mathematics course and when determining appropriate instructional strategies. In looking at the differences between students from large schools versus smaller schools, if any, it may support or refute the common claims of school district consolidation. The information gained in this study may also benefit high school math teachers and math teacher educators.

Your agreement to participate in this study will authorize the release of your academic records including the following information: high school grade point average, ACT scores, Advanced Placement Test scores, UND Placement Testing Program scores, and grade you earn in this course. Grades will not be available to the researcher until after the Fall 1999 semester ends. If at any time throughout the course of this study you are uncomfortable and wish to end your participation in this study, you may do so without penalty.

Are you willing to participate in this study? Yes No

During the Spring semester of 2000 I would like to conduct approximately 40 follow-up interviews to learn more about the math background and experiences of the participants in this study. The interviews will take approximately 30 minutes. A reliable outside interviewer or myself will conduct the interviews in the Education Building. The interviews will be audio taped and later transcribed. Only I will have access to the tapes and transcribed interviews. Your identity will not be matched to the interview tapes or the transcription.

Would you be willing to participate in an interview? Yes No

If so, please provide a phone number where you can be reached: ________________

Name: ________________________________ NAID (or SSN): ______________
(Please Print)

________________________________ ______________
Participant Signature Date

Please feel free to contact me at anytime regarding this study: Michele Iiams, 777-2432.

Survey

Sex: M F

Is this your first university-level mathematics course? _______

Date of high school graduation (month/year): _______________

Approximate size of your graduating class: _____________

Name of high school graduated from: ____

City and state of high school graduated from: ____

Please circle any of the following math courses offered in your high school:

Algebra II Trigonometry Precalculus Calculus AP Calculus

What is the highest level math course you completed with a grade of C or better?

Algebra I Geometry Algebra II Trigonometry Precalculus

Calculus AP Calculus

Do you need to take any university math courses beyond the one in which you are currently enrolled?

YES NO

If you answered NO to the previous question, do you plan to take any math courses beyond the one in which you are currently enrolled?

YES NO

APPENDIX B
REVISED CONSENT AND SURVEY FORMS

Consent to Participate

Hi! My name is Michele Iiams. I am currently a doctoral student in the Department of Teaching and Learning collecting information for my dissertation. I am interested in becoming a more informed and hopefully better math teacher and educator of future math teachers. Through this study, I hope to gain insight into how students experience the curriculum that has been designed for them while considering their academic background.

The purpose of this study is to examine a student's first experience in a university mathematics course and the role played by a student's high school math experiences. Your instructor will not know whether or not you have chosen to participate in this study. Any information obtained will be held in the strictest of confidence and will not affect your grade in this course.

This study could be of benefit to the Mathematics Department by determining demographic and academic characteristics of students commonly enrolled in entry-level mathematics courses. The Math Department could consider this information when placing students in their first mathematics course and when determining appropriate instructional strategies. In looking at the differences between students from large schools versus smaller schools, if any, it may support or refute the common claims of school district consolidation. The information gained in this study may also benefit high school math teachers and math teacher educators.

Your agreement to participate in this study will authorize the release of your academic records including the following information: high school grade point average, ACT scores, Advanced Placement Test scores, UND Placement Testing Program scores, and grade you earn in this course. Grades will not be available to the researcher until after the Fall 2000 semester ends. If at any time throughout the course of this study you are uncomfortable and wish to end your participation in this study, you may do so without penalty.

Are you willing to participate in this study? Yes No

During the Spring semester of 2001, I would like to conduct interviews with approximately 20 students to learn more about the math background and experiences of the participants in this study. I would like to interview each person twice, which should take a total of approximately 60 minutes. A reliable outside interviewer or myself will conduct the interviews in the Education Building. The interviews will be audio taped and later transcribed. Only I will have access to the tapes and transcribed interviews. Your identity will not be matched to the interview tapes or the transcription.

Would you be willing to participate in an interview? Yes No

If so, please provide a phone number where you can be reached: ________________

Name: ________________________________ NAID (or SSN): ________________
(Please Print)

________________________________ ________________
Participant Signature Date

Please feel free to contact me at anytime regarding this study: Michele Iiams, 777-2427.

Survey

Sex: M F

Is this your first university-level mathematics course? ______

Date of high school graduation (month/year): ______________

Approximate size of high school graduating class: ____________

Name of high school graduated from: ______________________________________

City and state of high school graduated from: ______________________________________

Please circle any of the following math courses offered in your high school:

Algebra II   Trigonometry   Precalculus   Calculus   AP Calculus

What is the highest level math course you completed with a grade of C or better?

Algebra II   Trigonometry   Precalculus   Calculus   AP Calculus

What is the last semester you were enrolled in a math course in high school?

Sophomore/Fall semester ______   Sophomore/Spring semester ______

Junior/Fall semester ______   Junior/Spring semester ______

Senior/Fall semester ______   Senior/Spring semester ______

APPENDIX C
INTERVIEW QUESTIONS

Interview Guide

High School Precalculus Experiences

1. What high school did you graduate from?
2. How many math teachers were at your high school?
3. Did you take classes from more than one math teacher?
4. What math classes did you take in high school?
5. Would you have wanted to take more math if it had been offered?
6. Did your high school offer math courses through the Interactive Video Network (IVN)? Which classes?
7. Did you take any of these classes? What was that experience like?
8. Did you have a favorite math class? Was there a math class you disliked or hated?
9. Describe a typical day in your (favorite/least favorite) high school math class.
10. Describe a typical day in your high school precalculus course.
11. How were grades determined?
12. Did math "come easily" to you in high school or did you have to study a lot?
13. When you were struggling in math what did you do to overcome those difficulties?
14. What was good about your high school math classes? Or what factors contributed to your success?
15. What do you feel could have been improved? Or what factors made the class difficult?
16. Was technology part of your high school math curriculum? In what way?

University of North Dakota Precalculus Experiences

1. What made you decided to come to UND?
2. What is your major?
3. How was your math placement determined?
4. Describe a typical day in your UND Precalculus class.
5. How were grades determined in your UND Precalculus course?
6. Is this different from how grades were determined in your high school Precalculus course?
7. What factors contributed to your success?
8. What factors made the class difficult?
9. Did the math "come easily" to you in your UND Precalculus course or did you have to study a lot?
10. When you were struggling in math what did you do to overcome those difficulties?
11. Was technology part of your UND Precalculus course? In what way?
12. What would you go back and say to your high school teacher now that you have been through a semester of math at UND?
13. How could the faculty in the Mathematics Department help students be more successful in our mathematics courses?

APPENDIX D
STUDENT PROFILES

Interview Participants

Kayla is a female who graduated from a public high school in North Dakota. There were 340 students in her graduating class. She took Precalculus her senior year of high school. Her declared major was Atmospheric Science; however, she was considering changing her major to Geology. She earned a D in the UND Precalculus course.

Denise is a female who graduated from a public high school in Minnesota. There were 299 students in her graduating class. She took Precalculus her senior year of high school. At the time of the interview, she intended to major in Mechanical Engineering with a possible second major in Business Finance. She earned a grade of C in the UND Precalculus course.

Lee is a male who graduated from a public high school in Minnesota. There were 675 students in his graduating class. He took Precalculus his junior year and attempted Calculus his senior year of high school. He completed the Calculus course with less than a C. He intends to major in Meteorology. He withdrew from the UND Precalculus course.

Katherine is a female who graduated from a public high school in North Dakota. There were 200 students in her graduating class. She took Precalculus her junior year of high school. She attempted Calculus her senior year but did not complete the class. She had considered majoring in Meteorology but was also considering Biology as a major. It should be noted that Katherine was earning a B in the UND Precalculus course until a family emergency caused her to miss several classes before the third test. She earned a grade of C in the UND Precalculus course.

Travis is a male who graduated from a public high school in North Dakota. There were 496 students in his graduating class. He took Precalculus his junior year and a statistics and probability class his senior year of high school. He had declared Biology as his major but indicated that he was not sure he would stay with that major. It should be noted that Travis took a year off between high school and his first semester at UND. He withdrew from the UND Precalculus course.

Janelle is a female who graduated from a public high school in Minnesota. There were 300 students in her graduating class. She took Precalculus her senior year of high school. Her initial major at UND was Meteorology, but, after difficulties in Physics and Precalculus, she has decided to change to a related major that requires fewer math courses. She withdrew from the UND Precalculus course.

Jacob is a male who graduated from a public high school in North Dakota. There were 35 students in his graduating class. He took Precalculus his senior year of high school. He is an Electrical Engineering major. It should be noted that Jacob was earning a B in the UND Precalculus course until he missed several classes before the third test due to a family emergency. He earned a grade of C in the UND Precalculus course.

Sam is a male who graduated from a public high school in North Dakota. There were 200 students in his graduating class. He took Precalculus his senior year of high school. He intends to major in Electrical Engineering. He earned a grade of F in the UND Precalculus course.

Paul is a male who graduated from a public high school in North Dakota. There were 120 students in his graduating class. He took Precalculus his senior year of high school. He intends to major in Geological Engineering. He earned a grade of D in the UND Precalculus course.

Rachel is a female who graduated from a North Dakota public high school. There were 244 students in her graduating class. She took Precalculus her senior year of high school. Geology is her declared major. She earned a grade of B in the UND Precalculus course.

Danielle is a female who graduated from a public high school in Texas. There were 300 students in her graduating class. She took Precalculus her junior year of high school. She did not take math her senior year. Her initial major at UND was Computer Science. She has decided to change her major and believes it will be "number oriented." She earned a grade of A in the UND Precalculus course.

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