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Re-describing Surface Roughness

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Objectives

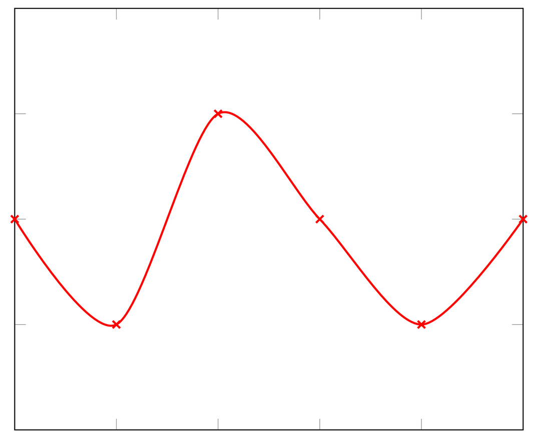
- Provide more intuitive mathematical description of roughness.
- Preserve as much information as possible.
- Relate findings to traditional variance calculations.

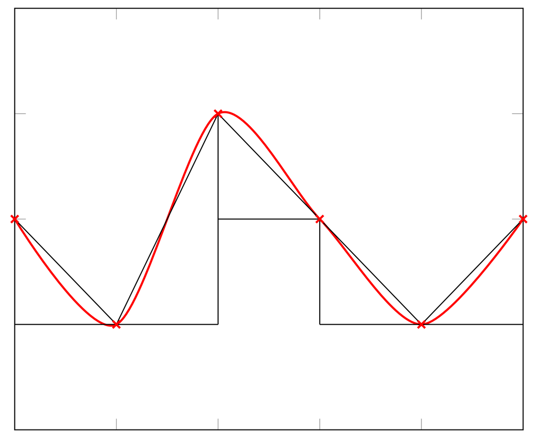
Overview of Method

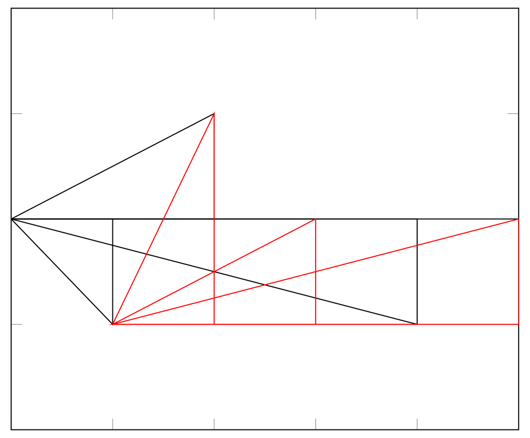
Maintain information of ratio between arc-length to domain

It is useful to know how the arc-length relates to the linear distance measurement. However, for energy-loss calculations it is also useful to know about the 'steepness' of each variation along the y-axis. This can be easily described by the angle of incidence to the horizontal that an arc-section tends to follow.

The method being described uses right-triangle approximations to calculate the angle between any point, and any *other* following point. This allows a sample size of $\frac{n(n-1)}{2}$ when analyzing the data.



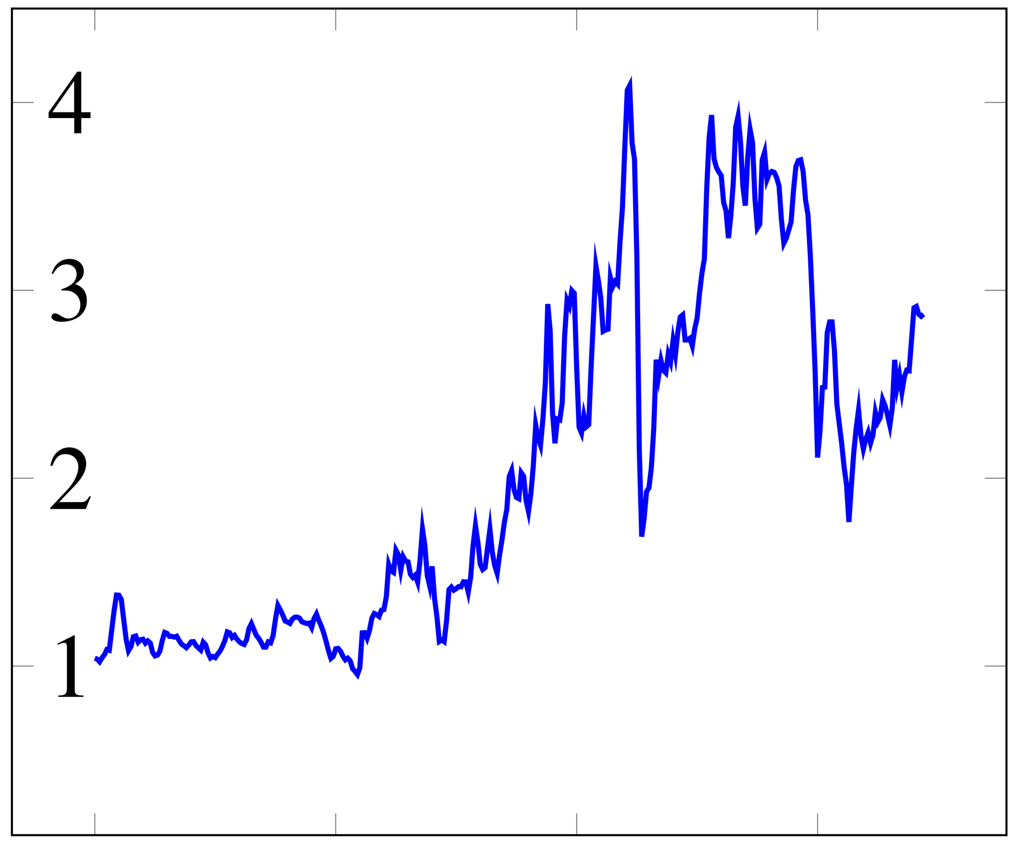




$$\bar{\theta} = \frac{2}{n(n-1)} \sum_{i=0}^{n-1} \left(\sum_{k=1}^{n} \tan^{-1} \left| \frac{y_i - y_k}{x_i - x_k} \right| \right)$$

Monthly Gas Prices

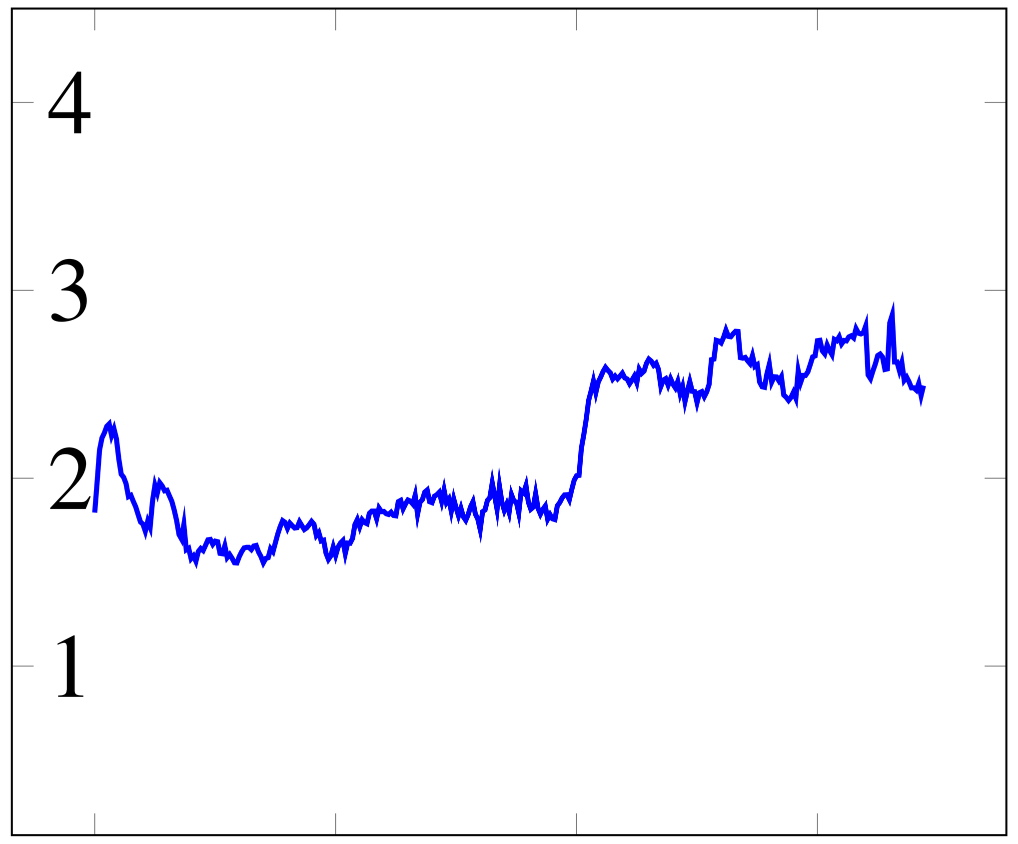
Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2008	3.05	3.03	3.26	3.44	3.76	4.07	4.09	3.79	3.70	3.17	2.15	1.69
2009	1.79	1.93	1.95	2.06	2.27	2.63	2.54	2.63	2.57	2.56	2.66	2.62
2010	2.73	2.66	2.78	2.86	2.87	2.74	2.74	2.75	2.70	2.80	2.85	2.99
2011	3.09	3.17	3.55	3.82	3.93	3.70	3.65	3.63	3.61	3.47	3.42	3.28
2012	3.40	3.57	3.87	3.93	3.79	3.55	3.45	3.71	3.86	3.79	3.49	3.33
2013	3.35	3.69	3.74	3.59	3.62	3.63	3.63	3.60	3.56	3.38	3.25	3.28
2014	3.32	3.36	3.53	3.66	3.69	3.70	3.63	3.48	3.40	3.18	2.89	2.56
2015	2.11	2.25	2.48	2.49	2.78	2.83	2.83	2.68	2.39	2.29	2.19	2.06
2016	1.97	1.77	1.96	2.13	2.26	2.36	2.23	2.16	2.21	2.24	2.19	2.23
2017	2.35	2.30	2.32	2.42	2.39	2.34	2.28	2.37	2.63	2.48	2.55	2.46
2018	2.54	2.58	2.57	2.74	2.91	2.91	2.87	2.86	2.87	2.89		



Standard Deviation: $\sigma = .899$
Average Angle: $\bar{\theta} = .748$

Monthly Orange Juice Prices

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2008	2.54	2.53	2.54	2.56	2.53	2.53	2.50	2.52	2.54	2.51	2.58	2.56
2009	2.57	2.61	2.63	2.62	2.60	2.61	2.58	2.49	2.52	2.53	2.50	2.53
2010	2.50	2.48	2.52	2.45	2.49	2.41	2.46	2.51	2.46	2.46	2.40	2.46
2011	2.46	2.43	2.46	2.50	2.63	2.63	2.73	2.73	2.72	2.75	2.79	2.76
2012	2.75	2.77	2.78	2.78	2.64	2.64	2.64	2.63	2.61	2.65	2.60	2.60
2013	2.51	2.49	2.48	2.56	2.61	2.51	2.54	2.54	2.52	2.54	2.44	2.43
2014	2.41	2.43	2.46	2.43	2.57	2.51	2.55	2.55	2.57	2.61	2.65	2.65
2015	2.73	2.73	2.68	2.66	2.71	2.68	2.66	2.74	2.73	2.75	2.72	2.73
2016	2.73	2.75	2.76	2.75	2.80	2.77	2.77	2.77	2.81	2.55	2.53	2.57
2017	2.61	2.65	2.66	2.65	2.58	2.58	2.83	2.87	2.62	2.62	2.57	2.62
2018	2.52	2.54	2.51	2.48	2.48	2.47	2.50	2.44	2.49	2.48		



Standard Deviation: $\sigma = .408$
Average Angle: $\bar{\theta} = .312$

Comments on Comparison

This data was selected to show an obvious difference in roughness to allow comparison between σ and $\bar{\theta}$. It should be emphasized that $\bar{\theta}$ represents an angle in degrees, as such, the similarity in resulting numbers from these particular data sets should not be misconstrued.

Does Multiplying Each Data Point by a Constant Affect Roughness?

Using σ implies that the data would be a factor of "C" more rough.

$$C\sigma = \sqrt{\left(\frac{\sum (CX - \frac{C}{n} \sum x_i)^2}{n-1} \right)} \quad (1)$$

This is not the case with $\bar{\theta}$.

$$C\bar{\theta} \neq \tan^{-1} \left| \frac{Cy_i - Cy_k}{x_i - x_k} \right| \quad (2)$$

Conclusion

$\bar{\theta}$ interacts with data in a similar fashion to standard deviation but is not affected in the same way by changes in data. Additionally, the amount of metadata provided during the calculation of $\bar{\theta}$ is significant, providing a very large data-set for traditional statistical analysis of the angles involved.

The findings of this project show that there likely exists a significantly different approach to analyzing and describing data than traditional statistical analysis. Additionally, the $\bar{\theta}$ method can be applied to any set of data allowing it to be experimented with in many applications.

Summary

- $\bar{\theta}$ behaves similarly to σ.
  - $\bar{\theta}$ is not affected by scaling in the same manner as σ.
- Calculating $\bar{\theta}$ generates a large volume of metadata.
- This project opens the door to a geometric approach to analyzing data.

References

- Bureau of Labor Statistics: Orange Juice Prices
- Bureau of Labor Statistics: Gas Prices

Special Thanks

- Dr. Bruce Dearden
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And Most Importantly:

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