



Stability analysis of the extended Kalman filter for Permanent Magnet Synchronous Motor

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Abstract: This paper presents a sensorless direct field oriented control fed interior permanent magnet synchronous motor (IPMSM) by using a known mathematical tool. The Kalman filter is an observer for linear and non-linear systems and is based on the stochastic intromission, in others words, noise. It is a question of studying the state and measurement noise covariance matrices Q and R on the stability of the Extended Kalman Filter. This last is used for the d, q stator current, mechanical speed, rotor position, stator resistance and the load torque estimation. The simulation tests carried out on Matlab Simulink showed that the matrix R improves much more quality of the estimated states while the matrix Q allows the improvement of the estimation process convergence.

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1. Introduction

Current industry trends suggest the Permanent Magnet Synchronous Motor (PMSM) as the first preference for motor control application designers. Its strengths, such as high power density, fast dynamic response and high efficiency in comparison with other motors in its category, coupled with decreased manufacturing costs and improved magnetic properties, make the PMSM a good recommendation for large-scale product implementation (Achour and Mendil 2009; Janiszewski 2000; Samosir and Yatim 2010).

Various control algorithms have been proposed for the elimination of speed and position sensors: estimators using state equations, artificial intelligence, direct control of torque and flux and Model Reference Adaptive System (MRAS). This paper proposes a sensorless direct field oriented strategy based on the EKF (Benchabane and Titaouine 2012a; Chikh and Saad 2011).

Kalman filter is a unique observer which offers best possible filtering of the noise in measurement and of the system if the noise covariances are known. If rotor speed considered as an extended state and is incorporated in the dynamic model of a permanent magnet synchronous motor then the extended Kalman filter can be used to relinearize the nonlinear state model for each new value of estimate. As a result, the extended Kalman filter is measured to be the best solution for the speed estimation of a PMSM (Khan 2009; Alonge 2014).

In this paper, an effective approach is proposed for parameter estimation, (d, q) stator current, speed, rotor position, stator resistance and the load torque, using the EKF technique. The investigations show that the EKF is capable of tracking the actual (d, q) stator current, speed, rotor position, stator resistance and the load torque, provided that the elements of the covariance matrices are properly selected.

Moreover, the performance of the EKF is satisfactory even in the presence of noise or when there are variations in the PMSM parameters. Computer simulation tests of the transient and steady-state performances are presented to highlight the effectiveness of the proposed methods.

2. Model of PMSM analysis

The vector diagram of PMSM d-q axis is shown in figure 1 (Achour and Mendil 2009; Benchabane and Titaouine 2012b).

The voltage and flux equations for a PMSM in the rotor oriented coordinates d-q can be expressed as (Achour and Mendil 2009; Benchabane and Titaouine 2012b):

$$V_d = R_s \cdot I_d + \frac{d}{dt} \lambda_d - p \cdot \lambda_q \quad (1)$$

$$V_q = R_s \cdot I_q + \frac{d}{dt} \lambda_q + p \cdot \lambda_d \quad (2)$$

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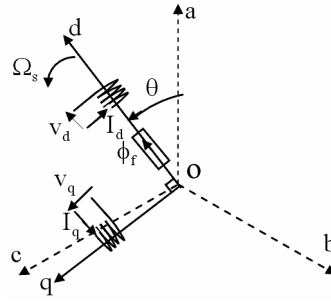


Fig. 1. Model of PMSM in d-q axis.

$$v_d = R_s I_d + \frac{d}{dt} (L_d I_d + \phi_{sf}) \quad (3)$$

$$v_q = R_s I_q + \frac{d}{dt} (L_q I_q) + \omega \phi_f \quad (4)$$

Where I_d and I_q are the d - q axis stator currents, R_s is the stator resistance, ϕ_{sf} is the flux linkage of the rotor magnets linking the stator, L_d and L_q are the d - q axis stator inductances, p is the number of pole pairs and ω is the mechanical speed, ϕ_d and ϕ_q are d - q components of the stator flux linkage.

And the electromagnetic torque equation in the rotor oriented coordinates d - q can be expressed as:

$$T_e = p \cdot [(L_d - L_q) I_d \cdot I_q + \phi_{sf} \cdot I_q] \quad (5)$$

Finally, the motion equation is expressed as:

$$J \cdot \frac{d}{dt} \omega + f \cdot \omega = T_e - T_r \quad (6)$$

Where J moment of inertia, T_r motor load and f damping constant.

3. Kalman filter

Kalman filter takes care of the effects of the disturbance noise of a control system and the errors in the parameters of the system are considered as noise. The Kalman filter can be expressed as a state model (Barut et al. 2003; Guohan and Qin 2012; Yahia and Zouzou 2009):

$$\dot{x}(t) = Ax + Bu + U(t)w(t) \text{ (System equation)} \quad (7)$$

$$y = Cx + v(t) \text{ (Measurement equation)} \quad (8)$$

where

$U(t)$ = weight matrix of noise

$v(t)$ = noise matrix of output model (measurement noise)

$w(t)$ = noise matrix of state model (system noise)

$U(t)$, $v(t)$, and $w(t)$ are assumed to be stationary, white, and Gaussian noise, and their expectation values are zero. The covariance matrices Q and R of this noise are defined as

$$Q = \text{covariance}(w) = E\{ww'\} \quad (9)$$

$$R = \text{covariance}(v) = E\{vv'\} \quad (10)$$

where $E\{.\}$ denotes the expected value.

The basic configuration of the Kalman filter is shown in Figure 2.

The state equations of the Kalman filter can be made as follows

$$\hat{x} = (A - KC) \hat{x} + Bu + Ky \quad (11)$$

The Kalman filter matrix is based on the covariance of the noise and denoted by K . The measure of quality of the observation is expressed as follows:

$$L_x = \sum E\{[x(k) - \hat{x}(k)]^T [x(k) - \hat{x}(k)]\} = \min \quad (12)$$

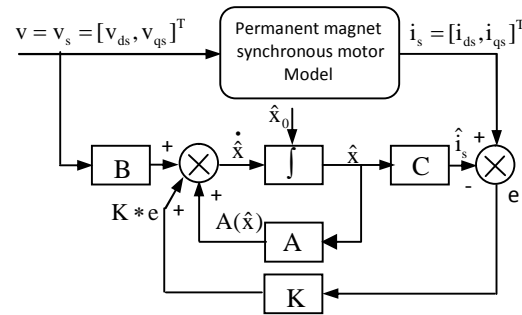


Fig. 2. The basic configuration of the Kalman filter observer.

The value of K should be such that as to minimize L_x . The result of K is a recursive algorithm for the discrete time case. The discrete form of Kalman filter may be written by the following equations, in which all symbols denote matrices or vectors (Benchabane and Titaouine 2011; Deng 2005).

(i) System state estimation

$$x(k + 1) = x(k) + K(k)(y(k) - \hat{y}(k)) \tag{13}$$

(ii) Renew of the error covariance matrix

$$P(k + 1) = P(k) - K(k)h^T(k + 1)P(k) \tag{14}$$

(iii) Calculation of Kalman filter gain matrix

$$K(k + 1) = P^*(k + 1)h^T(k + 1)[h(k + 1)P^*(k + 1)h^T(k + 1) + R]^{-1} \tag{15}$$

(iv) Prediction of state matrix

$$f(k + 1) = \frac{\partial}{\partial x} (A_d x + B_d v)|_{x=\hat{x}(k+1)} \tag{16}$$

(v) Estimation of error covariance matrix

$$P^*(k + 1) = f(k + 1)\hat{P}(k)f^T(k + 1) + Q \tag{17}$$

Discretization of (9) and (10) yields

$$x(k + 1) = A_d(k)x(k) + B_d(k)u(k) \tag{18}$$

$$y(k) = C_d(k)x(k) \tag{19}$$

where $K(k)$ is the feedback matrix of the Kalman filter. $K(k)$ gain matrix calculates how the state vector of the Kalman filter is updated when the output of the model is compared with the actual output of the system. The Kalman filter algorithm can also be used for nonlinear systems (e.g. permanent magnet synchronous motor). However, the optimal performance may not be obtained and it is impractical to verify the convergence of the model. To realize the recursive algorithm of the extended Kalman filter, a state model of the permanent magnet synchronous motor is required. After knowing the matrices A_d , B_d , and C_d , the matrices $x(k)$ (state prediction) and $y(k)$ (output prediction) can be calculated.

4. Sensorless control

The scheme of sensorless control of PMSM is very similar to sensor control (Barut and Bogosyan 2005; Bolognani and Tubiana 2003; Dhaouadi and Mohan 1991). Proposed scheme is presented on figure 3. There is one significant different that the system have no any mechanical sensors. It can be possibility to replaced physical sensors by mathematical algorithm.

Sensorless control diagram consists of PI current controllers subordinated to speed controller and adequate frame coordinate transformations. There are no differences in this part. The main state of presented system is an estimator, realized by Kalman Filter theory operation. The PI speed controller feeds current i_d^* in q axis in order to keep Field Oriented Control (Sturtzer and Smigiel 2000). The demanded current is computing by using the difference between requested speed (ω_r^*) and speed ($\hat{\omega}_r$) estimated by Kalman filter. Motor operating does not require the field weakening, as assumed. Therefore desired current i_d^* in d axis is maintained to zero. These signals are inputs of PI current controllers, which provide desired voltages in dq reference frame. Basing on estimating shaft position $\hat{\theta}$, voltages are converted into the stationary two axis frame ($\alpha\beta$) and send to control Pulse Width Modulation inverter (Janiszewski 2000).

$$f[x(k), u(k)] = \begin{bmatrix} (1 - a_1 R_s(k)) I_d(k) + a_2 \omega_r(k) I_q(k) + a_3 u_d(k) \\ a_4 \omega_r(k) I_d(k) + (1 - a_5 R_s(k)) I_q(k) - a_6 \omega_r(k) + a_7 u_q(k) \\ a_8 I_d(k) I_q(k) + a_9 I_q(k) + (1 - a_{10}) \omega_r(k) + a_{11} T_r(k) \\ a_{12} \omega_r(k) + \omega_r(k) \\ T_r(k) \\ R_s(k) \end{bmatrix} \quad (22)$$

$f(x(k), u(k))$ Nonlinear function vector of the states, $x(k)$ extended state vector, A_d system matrix, $u(k)$ is the control input vector, B_d input matrix, $h(x(k), v(k))$ function vector of the outputs, C_d Measurement matrix, $w(k)$ and $v(k)$ process and measurement noise respectively. Where:

$$\begin{aligned} a_1 &= \frac{R_s}{L_d} T_s; \quad a_2 = \frac{L_q}{L_d} T_s; \quad a_3 = \frac{1}{L_d} T_s \\ a_4 &= \frac{L_d}{L_q} T_s; \quad a_5 = \frac{R_s}{L_q} T_s; \quad a_6 = \frac{sf}{L_q} T_s \\ a_7 &= \frac{1}{L_q} T_s; \quad a_8 = P \frac{L_d - L_q}{J} T_s; \quad a_9 = P \frac{sf}{J} T_s \\ a_{10} &= \frac{B}{J} T_s; \quad a_{11} = \frac{1}{J} T_s; \quad a_{12} = P T_s \end{aligned}$$

$$A_d = \begin{bmatrix} (1 - a_1 R_s(k)) & a_2 \omega_r(k) & 0 & 0 & 0 & a_1 \\ a_4 \omega_r(k) & (1 - a_5 R_s(k)) & -a_6 & 0 & 0 & a_5 \\ a_8 I_q(k) & a_9 & 1 - a_{10} & 0 & a_{11} & 0 \\ 0 & 0 & a_{12} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (23)$$

$$B_d = \begin{bmatrix} a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_7 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (24)$$

$$C_d = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (25)$$

The essential matrices and vectors for the recursive algorithm of the extended Kalman filter can be calculated with the discrete system model. With the help of Matlab/Simulink program, speed estimation algorithm of the extended Kalman filter can be simulated, as shown in figure 5. The execution of the S-function block is based on an M-file written as MATLAB code.

The critical step in the EKF is the search for the best covariance matrices Q and R have to be set-up based on the stochastic properties of the corresponding noise. The noise covariance R accounts for the measurement noise introduced by the current sensors and quantization errors of the A/D converters (Benchabane and Titaouine 2012a; He and Jiang 2007; Yahia and Zouzou 2006). Increasing R indicates stronger disturbance of the current. The noise is weighted less by the filter, causing also a slower transient performance of system.

The noise covariance Q reflects the system model inaccuracy, the errors of the parameters and the noise introduced by the voltage estimation (Benchabane et al. 2011; Guohan and Qin 2012). Q has to be increased at stronger noise driving the system, entailing a more heavily weighting of the measured current and a faster transient performance.

An initial matrix P_0 represents the matrix of the covariance in knowledge of the initial condition. Varying P_0 affects neither the transient performance nor the steady state condition of the system. In this study, the value of these elements is tuned “manually”, by running several simulations. This is maybe one of the major drawbacks of the Kalman filter.

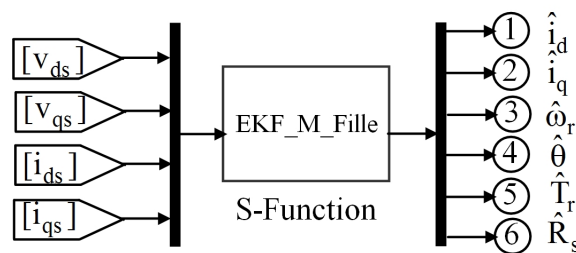


Fig. 5. Simulink based extended Kalman filter speed estimator.

6. Simulation results

Figure 6 and figure 7 show the responses of the speed, position, stator resistance and load torque between the actual and estimated states for step reference of 100 rad/s with 75% of the rated load (5 Nm) at $t=0$ s. Its value is increased from the value of 5 Nm to 5.5 Nm at 0.5 s, and then, it is 3 Nm at 1 s. The stator resistance is changed abruptly during steady-state operation of the drive. Its value is increased from the nominal value of 0.6 Ω to 1 Ω at 0.8 s, and then, decreased to 0.79 Ω at 1.3 s. Figure 6 and figure 7 illustrate that Simulation results between the actual and estimated of speed, position, stator resistance and load torque with various R covariance matrix of measurement noise and with various Q covariance matrix of system noise for Cases a, b, and d respectively. The matrix Q gives the statistical description of the drive model. The increasing Q would indicate the presence of either heavy system noise or increased parameter uncertainty. An increment of the elements of Q will likewise increase the EKF gain, resulting in a faster filter dynamic. On the other hand, matrix R is related to measurement noise. The increasing the values of the elements of R will mean that the measurements are affected by noise and thus they are of little confidence.

7. Measurement noise

In many applications, the measurement noise caused by either hardware or environment affects the system significantly. The estimated states or sensed variables are the inputs into the nonlinear control. These states or variables are often multiplied with other states or constant coefficients and amplified which degrades the outputs of the system. In this case, the filtering characteristic of the observer becomes more critical.

Since the Kalman filter has the measurement noise uncertainty modeling property, one can eliminate measurement noise up to a certain limit. The estimation accuracy of EKF is tested in this paper under noisy current measurement. In Figure 8, the injected noise to the d and q stator currents in the range of [0- 0.5 A] is shown. The noise is zero mean, white Gaussian. The aim of the current injection is to observe the low pass filter characteristics of EKF. Figure 9 (overleaf) shows the responses of the speed, position, the currents (d, q), stator resistance and load torque between the actual and estimated states for step reference of 100 rad/s with 75% of the rated load at $t=0.4$ s. Its value is reversed from the value of 5 Nm to -5 Nm at 0.8 s, and then, at $t=1.2$ s the speed is reversed from +100 rad/s to -100 rad/s and at $t=2.2$ s, the reference speed becomes 10 rad/s. The stator resistance is changed abruptly during steady-state operation of the drive. Its value is increased from the nominal value of 0.6 Ω to 1 Ω at 0.8 s, and then, decreased to 0.79 Ω at 1.3 s. As shown in Figure 9, the estimated speed, currents, and flux are not affected too much from the injected noise. The state estimation accuracy may be increased by increasing the measurement noise covariance under noisy conditions thus the system model will have more importance.

8. Conclusion

The intent of this paper was to show the utility of the EKF as a fundamental method for solving range of problems in sensorless control of PMSM. There is presented the design and implementation of high-performance motor drive with speed, position, stator current (d, q), stator resistance and the load torque estimation. The direct field-oriented control based speed controlled PMSM requires no shaft sensors for rotor position and speed due to rotor asymmetry.

The simulation tests carried out on Matlab Simulink showed that the matrix R improves much more quality of the estimated states while the matrix Q allows the improvement of the estimation process convergence. Moreover, a simple estimation algorithm based on EKF in direct vector control with PMSM has been presented. Good results are obtained through the simulations performed under load variation, reference variation and low speed. The effectiveness and the validity of the proposed EKF estimator are verified.

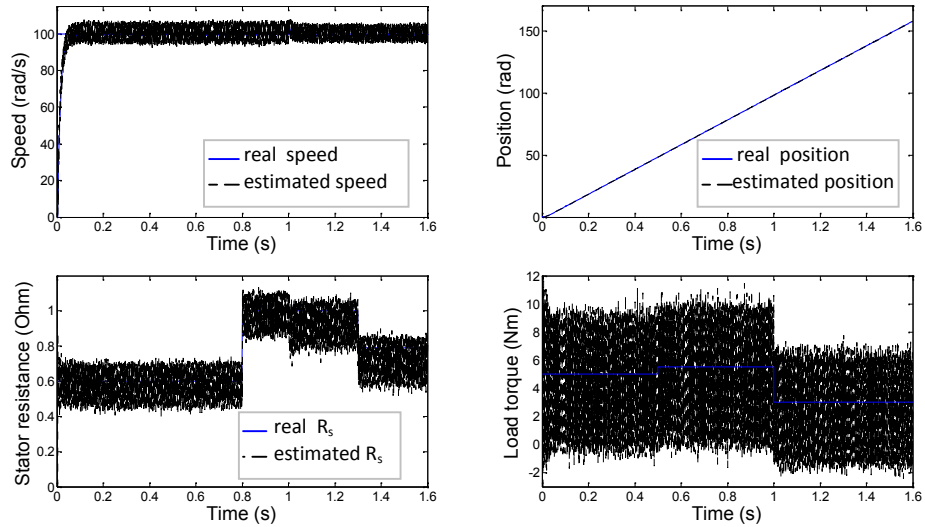
Appendix A

A.1. Parameters of the system used in simulation (Parameters of the PMSM)

- Rated power 1500 W, Rated speed 3000 r/min;
- Stator winding resistance 1.5 Ω ;
- Stator winding direct, quadrature inductance 0.0424 H, 0.0795 H;
- Rotor flux 0.314 Wb;
- Viscous friction 0.00008/(N•m•s•rad⁻¹);
- Inertia 0.003/(kg•m⁻²);
- Pairs pole number 4;
- Nominal current, voltage line 15 A, 220 V.

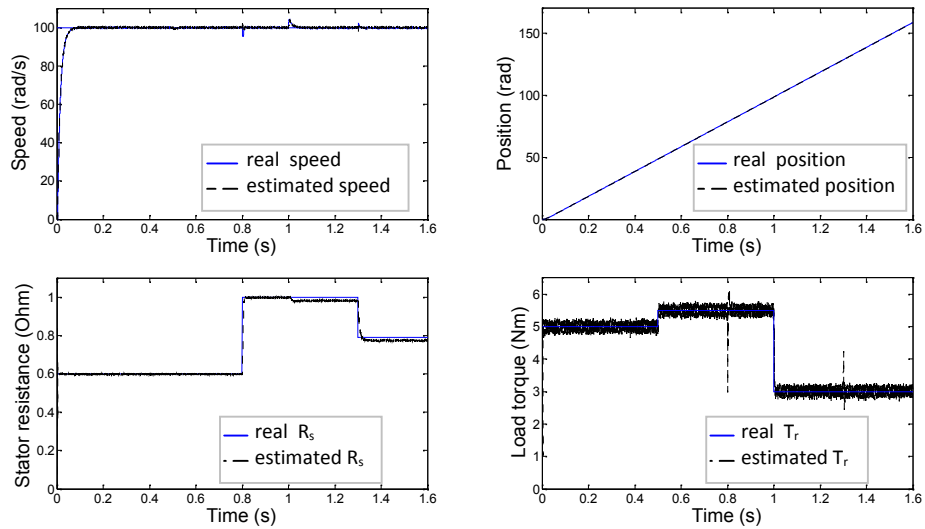
A.2. Parameters of the pulse-width modulation (PWM) converter

- Supply's voltage and frequency: 220 V(rms), 50 Hz;
- Line's inductor and resistance 0.002 H, 0.08 Ω ;
- Output capacitors 0.0025 F and PWM carrier frequency 1 kHz.



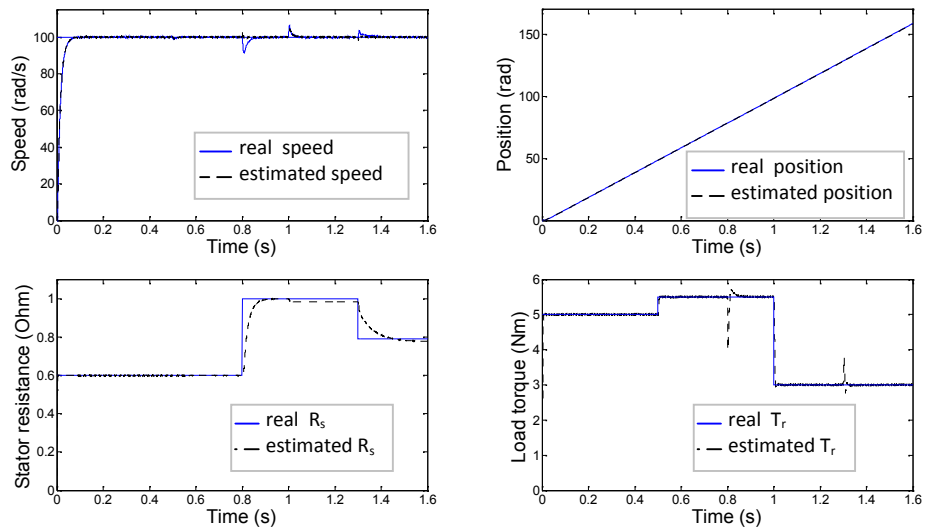
```
Q=[1e-3 0 0 0 0 0;0 1e-3 0 0 0 0;0 0 1e3 0 0 0;0 0 0 1e3 0 0;0 0 0 0 1e3 0;0 0 0 0 0 1e3 0;0 0 0 0 0 0 0.5];
P=1e7*eye(6); R=1e2*eye(2).
```

(a)



```
Q=[1e-3 0 0 0 0 0;0 1e-3 0 0 0 0;0 0 1e3 0 0 0;0 0 0 1e3 0 0;0 0 0 0 1e3 0;0 0 0 0 0 1e3 0;0 0 0 0 0 0 0.5];
P=1e7*eye(6); R=1e5*eye(2).
```

(b)



```
Q=[1e-3 0 0 0 0 0;0 1e-3 0 0 0 0;0 0 1e3 0 0 0;0 0 0 1e3 0 0;0 0 0 0 1e3 0;0 0 0 0 0 1e3 0;0 0 0 0 0 0 0.5];
P=1e7*eye(6); R=1e7*eye(2).
```

Fig. 6. Simulation results with various R covariance matrix of measurement noise.

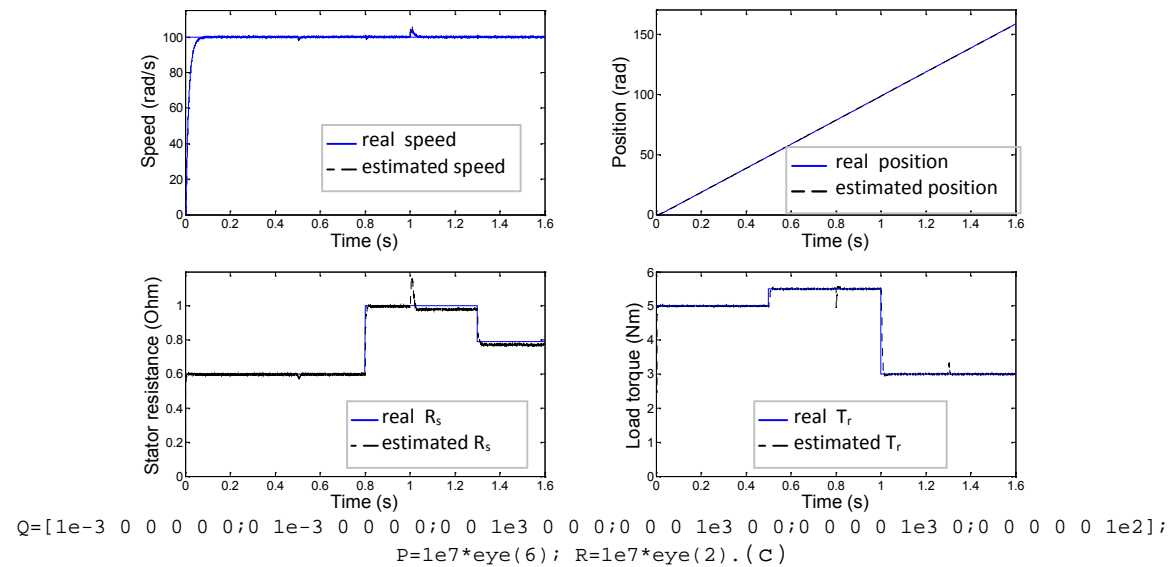
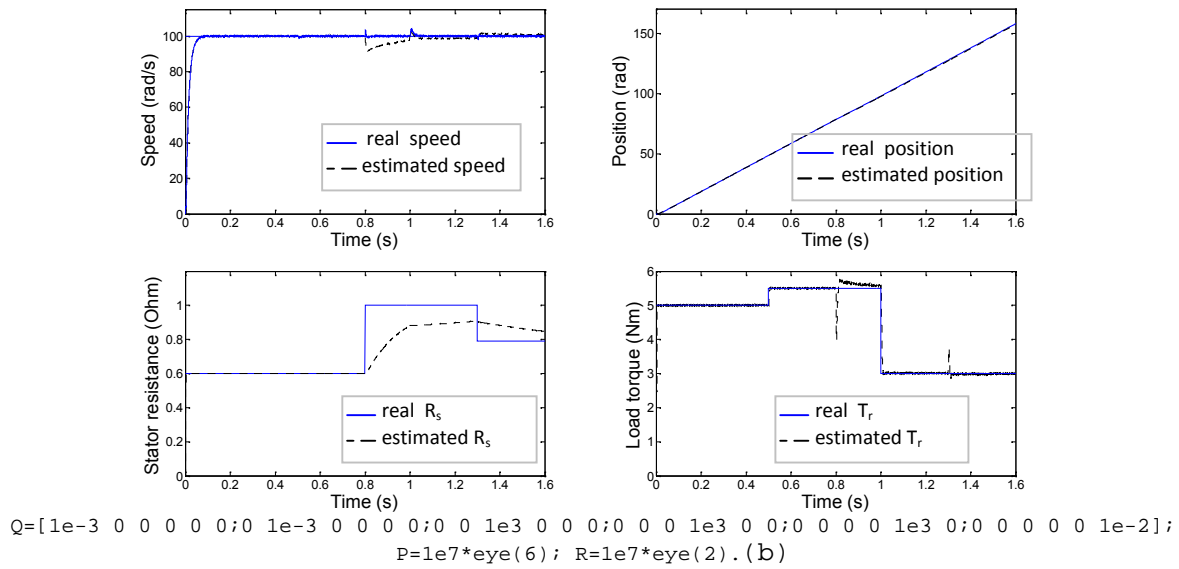
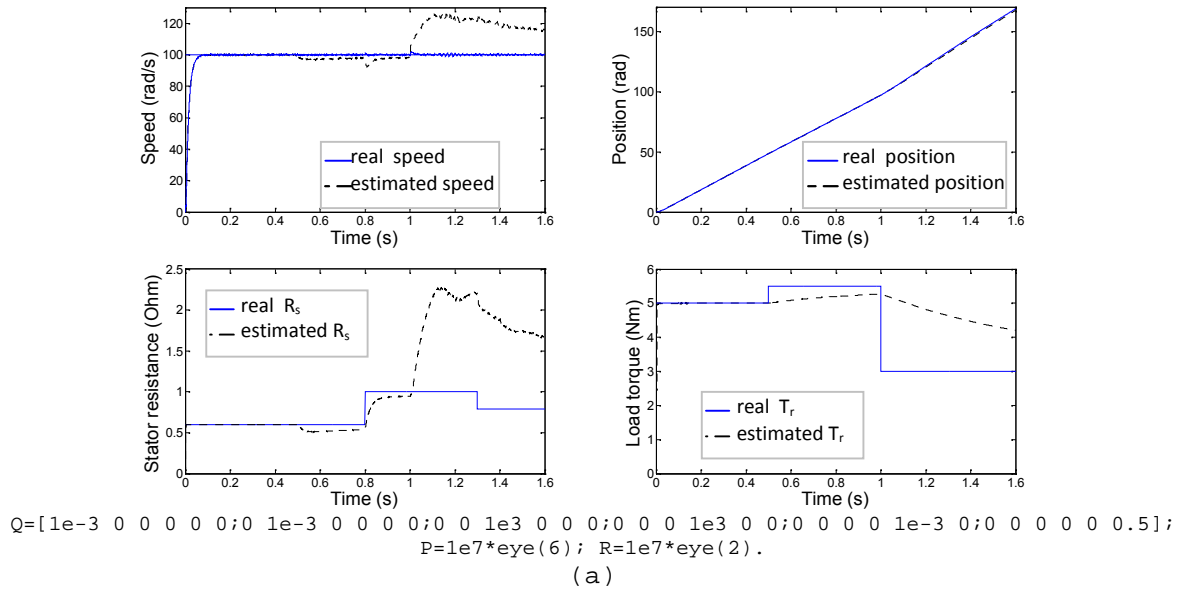


Fig. 7. Simulation results with various Q covariance matrix of system noise

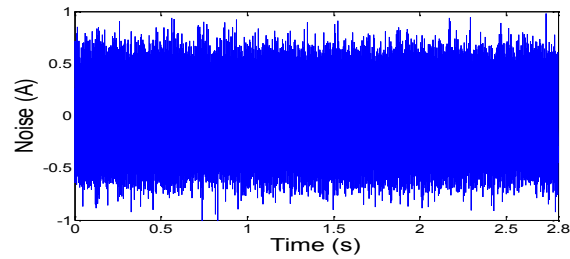


Fig. 8. Injected noise to the currents of PMSM.

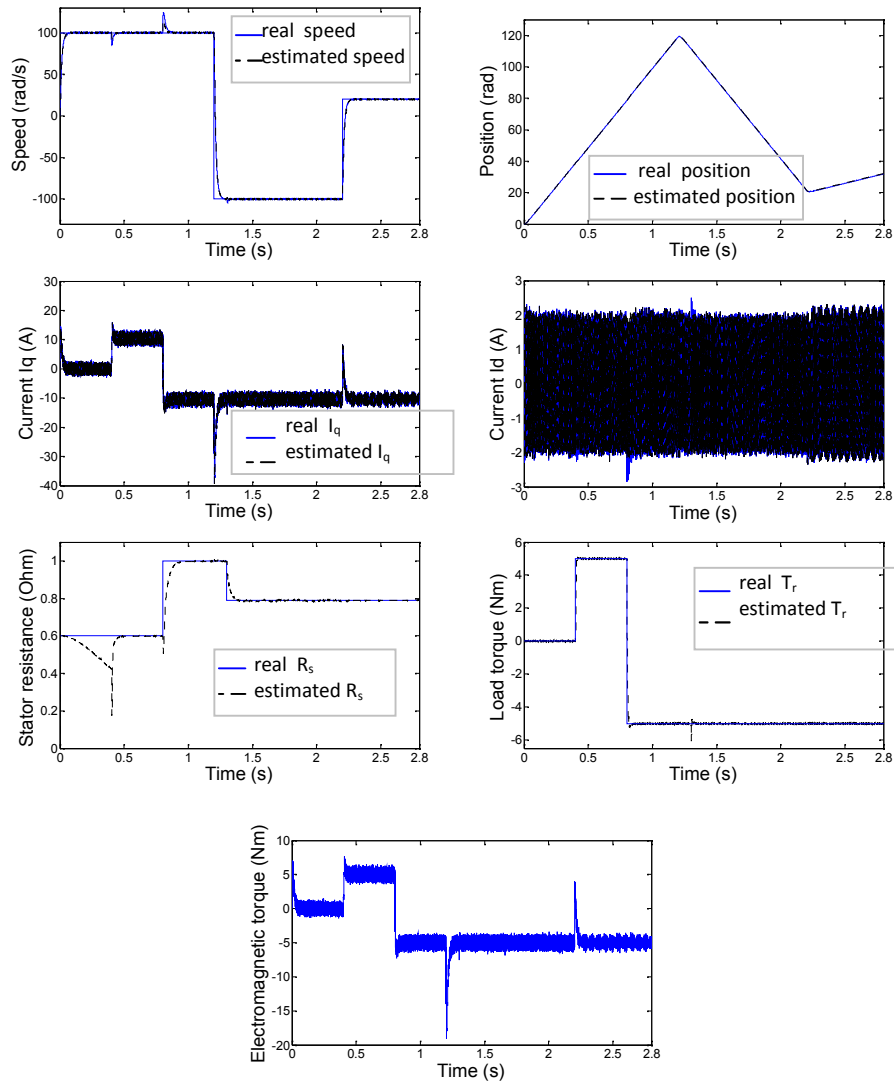


Fig. 9. PMSM estimated and actual speed, position, currents (d, q), stator resistance and the load torque with measured noisy.

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