# EXPERIMENTAL STUDY OF HYDRAULIC JUMP CHARACTERISTICS IN TRIANGULAR CHANNEL 

# ETUDE EXPERIMENTALE DES CARACTERISTIQUES DU RESSAUT HYDRAULIQUE DANS UN CANAL TRIANGULAIRE 

R. MASMOUDI<br>University of Biskra, Algeria P.O.Box 145 RP 07000 Biskra.Algeria<br>masmoudirachid@hotmail.com


#### Abstract

The hydraulic jump is the turbulent transition from supercritical to subcritical flow. The aim of this work is to contribute to a best understanding of the behavior of hydraulic jump in a symmetrical triangular channel of central angle of $90^{\circ}$. New experiments were conducted to quantify its characteristics for a wide range of Froude numbers ( $2<\mathrm{F} 1<11$ ). The analysis of the experimental results shows the insufficiency of the equations already suggested, deducted from the half triangular channel studied by Hager and Wanoschek (1985). The influence of the upstream sequent depth on the features of the hydraulic jump and especially the surface profile is considered. Some relationships, translating the sizes of the characteristics of the hydraulic jump in asymmetrical triangular channel, are presented in dimensionless terms, in order to show their general practical validity. The effects of the cross-sectional are investigated as well.


This research is applied particularly in the conception of dams' stilling basins.

KEYWORDS: Hydraulic jump; stilling basin; triangular channel; classical jump; Energy dissipation.


#### Abstract

RESUME L'expérimentation du ressaut hydraulique classique dans un canal triangulaire symétrique à angle d'ouverture de $90^{\circ}$ a permis une meilleure compréhension de son comportement. Les analyses des mesures expérimentales ont montré les insuffisances des relations jusqu'alors proposées, déduites du profil semi-triangulaire étudié par Hager et Wanoschek (1985). Des relations traduisant les grandeurs des caractéristiques du ressaut hydraulique dans un canal triangulaire symétrique sont présentées en termes adimensionnels pour que leurs validités pratiques soient générales. L'influence de la hauteur initiale sur les différentes caractéristiques du ressaut, notamment le profil de surface, est mis en évidence. Cette recherche trouve son application dans la conception des bassins amortisseurs des barrages.


MOTS CLES : Ressaut hydraulique ; canal triangulaire ; dissipateur d'énergie ; bassin d'amortissement; ressaut classique.

## 1 INTRODUCTION

Many hydraulic structures such as surface spillways and shaft spillways are designed to remove water and restore it further downstream of a river. In most cases this removal of water results in the conversion of the potential energy stored in a reservoir into high kinetic energy downstream of the inflow stream release structure. The driving forces generated, proportional to the square of the velocity, can seriously threaten the structure due to their highly erosive nature. Therefore it is often necessary to dissipate the
greater part of this kinetic energy.
The flow at the entry of the inflow stream release structure is in supercritical regime, characterised by a Froude number higher than one. The principle of dissipation consists in transforming this supercritical regime into a subcritical river regime, characterised by a relatively low velocities downstream of the energy dissipater. A hydraulic jump is thus formed throughout the length of the stilling basin, often of rectangular shape and sometimes equipped with baffles, sills, etc. positioned width-wise.

The main characteristics of a classical or forced jump are the efficiency, representing the head loss ratio caused by the initial load in its upstream section, the length and upstream and downstream sequent depths.

According to Debabeche et al (2006), Debabeche et al (2011) and Chanson (2012), from the quantitative view point, the characteristics of the jump mainly depend on the geometry of the stilling basin, the inflow Froude number and the relative sill height $\mathrm{s} / \mathrm{h} 1$ for the case of the forced jump, with s and h 1 representing the geometric height of the sill and the upstream depth of the jump.

For the sake of economy and efficiency, the jump length as well as the downstream depth must be as short as possible, and dissipating the maximum amount of energy. Until now, no theoretical development has been carried out to established a relationship making it possible to the evaluation of the roller length Lr and the jump length Lj ; this has only been achieved through experimental tests (Bessaih et al., 2002; Abdul, 2008; Wang et al., 2015). Sanjeev et al. (2010) and Richard et al. (2013) have proposed non-dimensional relations to calculate the relative length $\lambda_{\mathrm{r}}$.

The works of Sanjeev et al. (2010) showed that the length of the jump is as long as $M$ tends to infinity. The increasing values of M lead to a trapezoidal profile geometrically shrunk at the base, which gives rise to a more a triangular profile.

However, jumps placed in channels with triangular crosssections have not attracted as much interest as those in rectangular cross-sections (Abdul, 2008; Chyan-Deng et al., 2009; Sanjeev et al., 2010 ; Richard et al., 2013), although it appears that the triangular profile is better adapted to the role of dissipation.
The first authors to have studied this type of jump were Argyropoulos and Rajaratnam, by analysing the results obtained by Silvester. The former carried out his experiment in a channel with aperture angle of $47.3^{\circ}$, whereas the latter tested a channel with an aperture angle of $60^{\circ}$. The results showed that, overall, the equation of the quantity of movement is generally verified although certain measurement points tend to show that the sequent depth ratios Y are slightly lower than the theoretical values. This variance, estimated at $5 \%$, was also observed in the analysis of the results obtained by (Hager et al., 1985) for a triangular channel with an aperture angle of $90^{\circ}(\mathrm{m}=1)$. A jump controlled by a weir was analysed experimentally by (Achour et al., 2003; Debabeche et al., 2006) and the effect of the weir on the compactness of the stilling basin was observed and quantified.
Mention must be made of the specific approach taken by (Hager et al.,1985) in a jump experiment. A wall inclined at $45^{\circ}(\mathrm{m}=1)$ y was positioned in a channel with a rectangular cross-section. The geometrical profile obtained and on which the tests were performed was semi-triangular, formed by a vertical wall and in inclined wall. The experimental results obtained using this profile, were extrapolated to a symmetrical triangular profile. The
reliability of this extrapolation may appear poor with respect to the influence of the vertical wall on the characteristics and behaviour of the hydraulic jump.

In this context, the present study first presents an experimental examination of the behaviour and the characteristics of hydraulic jump placed in a channel with a symmetrical cross-section with an aperture angle of $90^{\circ}$. The series of measurements performed by varying the initial depth of the jump ( $\mathrm{h}_{1}=$ constant for each series of tests) allowed us to observe the influence of the upstream depth $h_{1}$ on the jump characteristics, especially the profile of the axial surface. Secondly, the experimental results obtained are compared with those obtained by the semitriangular cross-section channel jump, and with the theoretical development.

## 2 MATERIALS AND EXPERIMENTAL PROCEDURE

The tests were performed in an experimental set-up (Figure 1), composed by a stilling basin linked to a filling tank with a capacity of 2000 litters, by the mean of a pipe with 110 mm of diameter equipped with a horizontal pump with a flow rate ranged from 0 to $30 \mathrm{l} / \mathrm{s}$, a horizontal channel 4 meters long, sufficient for the full formation of a jump for the entire range of flow values $\mathrm{Q}(4 \mathrm{l} / \mathrm{s} \leq \mathrm{Q}(1 / \mathrm{s}) \leq 30)$. The flow values were measured using an orifice type velocity meter with an accuracy of $\pm 0.5 \mathrm{l} / \mathrm{s}$. The installation of a convergent nozzle at the outlet of the filling tank generated a high velocity flow. A sill was positioned at the downstream of the channel to return the jump to the upstream preferential position.

For each upstream depth $h_{1}$, the final depth $h_{2}$ was measured, as well as the length $L_{r}$ of the roller, the length $L_{j}$ of the jump, the flow rate Q and the heights of the free surface water stream.

The tests were performed with the upstream sequent depths $\mathrm{h}_{1}$ which corresponds to the opening of the C nozzle outlet such as $38 \leq h_{1}(\mathrm{~mm}) \leq 79$ and with a Froude number ranging from 2 to 11.
For each of the values of depth $h_{1}$ measurements were made of the flow rate Q , the sequent depth downstream $\mathrm{h}_{2}$, using a dual scale staff gauge accurate to $\pm 1$ to 1.5 cm , due to the considerable fluctuations of the free surface flow downstream. The horizontal distances, in particular the length of roller $L_{r}$ and the length of jumper $L_{j}$, were measured using a graduated strip fixed along the wall of the channel. To obtain the profile of the free surface flow of the water in the centre of the channel, about ten staff gauges were installed on it to measure the water depths along the whole length of the jump.


## Figure 01: Schematic layout of experimental setup

## 3 RESULTS AND DISCUSSION

### 3.1 Variation of sequent depth ratio with approach Froude number

The experimental verification of the relationship giving the sequent depth ratio Y resulting from the application of the momentum equation was necessary. Firstly, to confirm experimentally the validity of the equation of the quantity of movement momentum equation expressing the ratio Y of the sequent depths $h_{1}$ and $h_{2}$ as a function of the Froude number $\mathrm{F}_{1}$. It should be noted that the theoretical ratio Y for a triangular channel is given by the following relation Hager et al. (1985):
$\mathrm{F}_{1}^{2}=\frac{2 \mathrm{Y}^{2}\left(\mathrm{Y}^{2}+\mathrm{Y}+1\right)}{3(\mathrm{Y}+1)}$
(1)

Secondly, this was done to verify the effect of friction of the channel bed and walls on ratio Y and confirm that this ratio did not depend on the Froude number $\mathrm{F}_{1}$.

Curve $\mathrm{Y}=\mathrm{f}\left(\mathrm{F}_{1}\right)$ of figure 2, resulting from the experiment shows that the influence of the head loss on Y ratio is negligible, or non-existent. The coincidence of the experimental points with the theoretical curve of equation 1 indicates the absence of the effect of friction on the sequent depth ratio throughout the range of $F_{1}$ tested, i.e. 2
$\leq \mathrm{F}_{1} \leq 11$. We can confirm the validity of the theoretical relation (1) for the classical jump in a triangular channel with an
aperture of $90^{\circ}$ and invalidate the consideration of the effect of friction expressed by Hager et al. (1985).


Figure 02: Variation of sequent depth ratio with approached Froude number
(--) Equation 1,
( $\times x \times$ ) Experimental points
for $\mathbf{4 5 m m} \leq h_{1} \leq 79 \mathrm{~mm}$

We propose to express ratio $\mathrm{Y}_{\mathrm{T}}$ of the jump moving in a
basin with a triangular cross-section as a function of ratio YR given by the equation of Belanger :
$: Y_{R}=\frac{\sqrt{1+8 \mathrm{~F}_{1}^{2}}-1}{2}$

This ratio is presented by the following relation:
$\mathrm{Y}_{\mathrm{T}}=0.9 \mathrm{Y}_{\mathrm{R}}^{2 / 3}+0.1$
(2)

Compared to the approximated solution proposed by Hager et al, (1985),
$\mathrm{Y}=\sqrt{2 \mathrm{~F}_{1}}-\frac{1}{2}$ for $\mathrm{M}=0$

And
$Y=\left(\frac{3 \mathrm{~F}_{1}^{2}}{2}-1\right)^{1 / 3}$
$1 / \mathrm{M}=0$
(3)

The relative error is higher than $1 \%$.Relation (2) presents a simpler approximated solution with better precision as its relative error does not exceed $0.3 \%$ (Figure 3). For a Froude number $F_{1}=1$, the value of $Y_{T}$ according to equation (2) is equal to one, which moreover correctly expresses the critical phase of the flow.
However, for $\mathrm{F}_{1}=1$, relation (3) gives a ratio $\mathrm{Y} \neq 1$.


Figure 03: Theoretical and experimental variation of $Y$

### 3.2 Characteristic jump lengths

### 3.2.1 Relative length of Roller

The lengths of both the roller and jump are difficult to measure due to the instability of the jump and the quite
considerable imprecision with which the corresponding sections were localised experimentally. We have deliberately represented the variation of the relative length of the roller $L_{r} / h_{1}$ as a function of Y-1 instead of $F_{1}$ as is usually done. The main reason is that this representation permits eliminating errors made on the flow rate measurements and thus on the calculation of the Froude numbers F1. Figure 4 shows the experimental data resulting from seven series of measurements performed under upstream depth h1 and varying between 45 and 79 mm . Thus, we observed a cloud of points distributed between two enveloping curves defined as:
$6.0(\mathrm{Y}-1)^{1.3} \leq \frac{\mathrm{L}_{\mathrm{r}}}{\mathrm{h}_{1}} \leq 8.25(\mathrm{Y}-1)^{1.21}$
(4)
$45 \mathrm{~mm} \leq \mathrm{h}_{1} \leq 79 \mathrm{~mm}$


Figure 04: Variation of relative length of roller as a function of $Y$ 1. $(+++$ ) Experimental data,
$45 \mathrm{~mm} \leq \mathrm{h}_{1} \leq 79 \mathrm{~mm}$

The error usually accepted in the measurement of characteristic jump lengths is $\pm 20 \mathrm{~cm}$; the uniform distribution of the data is sufficiently significant. For the values $\mathrm{Y}-1>1$, i.e. $\mathrm{Y}>2$ (a value greatly exceeded in practice), we can assume that the relative lengths $L_{r} / h_{1}$ vary around a mean curve with a practically linear trend. This line is located between the envelope curves defined by relation (4) and has the following equation:
$\frac{\mathrm{L}_{\mathrm{r}}}{\mathrm{h}_{1}}=11(\mathrm{Y}-1)-4$
$\mathrm{Y}>2$
(5)

### 3.2.2 $\quad$ Relative length of Jump

This length, which is difficult to measure, especially for quite high upstream Froude numbers ( $\mathrm{F}_{1}>5$ ), is shown in Figure 5 on the basis of experimental values of lengths relating to jump $L_{j} / h_{2}$ as a function of the Froude number $F_{1}$. This graph clearly shows that the mean experimental curve for the jump is located far above the curve plotted
according to the following equation Hager et al. (1985):

$0.4<m \leq 1$
(6)


Figure 05: Variation of relative length of jump as a function of $F_{1}$. (--) Equation 6, (x x x) Experimental data for

$$
45 \mathrm{~mm} \leq \mathrm{h}_{1} \leq 79 \mathrm{~mm}
$$

This considerable variance between the two curves, first expresses the effect of the vertical wall of the semitriangular channel on the length of the jump and, second, the non-validity of equation (6) for the case of the symmetrical triangular jump. The lengths for jump $\mathrm{L}_{\mathrm{j}} / \mathrm{h}_{2}$ vary around a mean curve whose values can reach 9 for Froude numbers between 6 and 8 .

### 3.3 Generalised surface profile

Variables $\mathrm{X}=x / L_{j} \quad$ and $\quad \mathrm{Y}=\frac{h(x)-h_{1}}{h_{2}-h_{1}}$
Plot the surface profile of the jump in non-dimensional form.

The relation governing the generalised non dimensional profile $\mathrm{y}(\mathrm{x})$ along the axis of a non-symmetric triangular channel for several Froude numbers $\mathrm{F}_{1}$ obtained by Hager et al. (1985) is written as follows:

## $y=(1+\infty X) \tanh (\beta X)$

(7)

For $0 \leq X \leq 0.2$ the surface profile has an approximately linear slope $\beta=\mathrm{dy} / \mathrm{dx}=2.5$

In figure 6 it is presented the curve resulting from relation (7) for the case of a semi-triangular jump with $h_{1}$ values varying between 70 and 76 mm and the measurement points corresponding to the h1values between 45 and 79 mm . It was able to observe that for $76 \mathrm{~mm} \leq \mathrm{h}_{1} \leq 79 \mathrm{~mm}$ the experimental points of the symmetrical triangular jump were located around the curve of equation (7). However, for the values of $h_{1}<76 \mathrm{~mm}$, the distance between the
curves obtained increased progressively while $h_{1}$ decreased. This indicated that relation (7) can only be applied to values of $h_{1} \geq 76 \mathrm{~mm}$ and shows the influence of $\mathrm{h}_{1}$ on the shape of the generalised surface profile of the triangular jump $(\mathrm{m}=1)$.


Figure 06: Evolution of generalised non dimensional profile

## $y(x)$ along the axis of channel

(--) Equation 7, ( $\times \times x$ ) Experimental data for

$$
45 \mathrm{~mm} \leq \mathrm{h}_{1} \leq 79 \mathrm{~mm}
$$

### 3.4 Jump efficiency

Theoretically, for $\mathrm{F}_{1}>3$ the efficiency of the triangular jump is higher than that of the rectangular one; the variance can reach $10 \%$ above $\mathrm{F}_{1}=5$. The efficiency is then evaluated approximately by relation (8):

$$
\begin{equation*}
\eta_{R}=\left(1-\frac{\sqrt{2}}{F_{1}}\right)^{2} \tag{8}
\end{equation*}
$$

$\eta_{\mathrm{T}}=\left[1-\left(\frac{12}{\mathrm{~F}_{1}^{4}}\right)^{1 / 3}\right]_{1 / M=0}^{2}$

## 4 CONCLUSIONS

The main characteristics of the classical jump moving in a symmetrical triangular channel with an aperture of $90^{\circ}$ were evaluated experimentally. The installation of a convergent nozzle under pressure generated a high velocity flow and made it possible to avoid measuring the upstream depthh ${ }_{1}$. The analysis of the experimental values of sequent depths $h_{1}$ and h2showed that the effect of friction on ratio $Y=h_{2} / h_{1}$ was negligible and that the equation of the quantity of movement was verified. An approximate relation of ratio Y of the symmetrical triangular jump was proposed. In addition, the measures of the relative lengths of roller $\mathrm{Lr} / \mathrm{h} 1$ for $45 \mathrm{~mm} \leq \mathrm{h}_{1} \leq 79 \mathrm{~mm}$ led to the establishment of a mean curve with a practically linear situated between two envelope curves. The relative length of the jump $L_{j} / h_{2}$ could reach 9 for $6 \mathrm{~mm} \leq \mathrm{h}_{1} \leq 8 \mathrm{~mm}$, whereas for the same range of $F_{1}$, the value of $L_{j} / h_{2}$ was in
the region of 5 in the semi-triangular channel. The influence of the initial depth h1on the shape of the surface profile of the jump was observed. The application of the equation proposed up to then remains limited to the values $h_{1} \geq 76 \mathrm{~mm}$. The experimental points of the surface profile progressively tended away from the curve already established as $h_{1}$ decreased. The maximum efficiency of the rectangular jump was about $71 \%$, whereas it could reach $73 \%$ for the triangular jump, i.e. $2 \%$ more for the same Froude number $F_{1} \cong 9$. The study showed that the symmetrical triangular jump was characterised by a very different behaviour compared to the jump in the semitriangular channel. This difference stemmed from the symmetry of the cross-section of the jump which permitted performing measures along the centre of the channel, which was not possible in the semi-triangular section; however, it also stemmed from the difference in the quantitative and qualitative characteristics of the jump.

## REFERENCES

[1] Achour, B., Debabeche, M., 2003. Ressaut hydraulique contrôlé par seuil dans un canal triangulaire. Journal of Hydraulic Research 3,319325.
[2] Abdul, M., 2008. Experiment on hydraulic jump in sudden expansion in a sloping rectangular channel. Bangladesh, Journal of Civil Engineering (IEB) 36, 65-77.
[3] Bessaih, N., Azua A., 2002. Effect of baffle block with sloping front face on the length of the jump. Journal of Civil Engineering, The Institution of Engineers Bangladesh, Vol. CE 30, No - 2 .
[4] Chanson, H., 2012. Momentum Considerations in Hydraulic Jumps and Bores. Journal Irrigation Drainnage Engineering 138(4), 382-385.
[5] Chyan-Deng, J., Chia-Jung, C., 2009. Hydraulic jump in an inclined rectangular chute contraction. Journal of Hydraulic Engineering, ASCE, Vol. 135, No 11, 949-958.
[6] Debabeche, M., Ghomri, A., Kateb, S. and Achour, B., 2011. Theoretical Study of the Hydraulic Jump in a Triangular Channel with Rough Walls. Dam Engineering, International Water Power an Dam Construction, Vol. XXII, Issue 1.
[7] Debabeche, M., Kateb, S., Ghomri, A., 2006. Etude expérimentale du ressaut hydraulique dans un canal triangulaire à parois rugueuses. Journal du Laboratoire de recherche en Hydraulique Souterraine et de Surface, LARHYSS, Université de Biskra, Algérie 05,187-196.
[8] Hager, W.H., Wanoschek R., 1985. Ressaut hydraulique en canal triangulaire.Journal of Hydraulic Research 5, 549-564.
[9] Rajaratnam, N., 2002. Hydraulic jumps on corrugated beds, J. Hydraul. Engng., ASCE 128, 656-336.
[10] Richard, G. L., Gavrilyuk, S. L., 2013. The classical hydraulic jump in a model of shear shallow water flows. Journal Fluid Mechanics 725, 492-521.
[11] Sanjeev Kumar, G., Mehta, R., C., 2010. Experimental study of hydraulic jump characteristics in sloping prismatic channels. Proceedings of ICTACEM, International Conference on Theoretical, Applied, Computational and Experimental Mechanics, IIT Kharagpur, India.
[12] Wang, H., Chanson, H., 2015. Experimental Study of Turbulent Fluctuations in Hydraulic Jumps. Journal of Hydraulic Eng. 10.1061(ASCE) HY.19437900.

## Nomenclature

| $\lambda_{j}(-)$ | Relative jump length, $\lambda_{\mathrm{j}}=\mathrm{L}_{\mathrm{j}} / \mathrm{h}_{2}$ |
| :---: | :---: |
| $\lambda_{r}(-)$ | Relative roller length, $\lambda \mathrm{r}=\mathrm{L}_{\mathrm{r}} / \mathrm{h}_{1}$ |
| $\mathrm{F}_{1}(-)$ | Inflow Froude number |
| $\mathrm{H}(\mathrm{m})$ | Total load |
| h (m) | Flow depth |
| $\mathrm{h}_{1}(\mathrm{~m})$ | upstream Sequent depth |
| $\mathrm{h}_{2}(\mathrm{~m})$ | downstream Sequent depth |
| $\mathrm{L}_{\mathrm{r}}(\mathrm{m})$ | Length of roller |
| $L_{\mathrm{j}}(\mathrm{m})$ | Length of jump |
| m (-) C with regard | ent of the angle of inclination of the wall he horizontal |
| M (-) R | upstream depth of jump, |
| $\mathrm{M}=\mathrm{mhl} / \mathrm{b}$ |  |
| Q (m3s-1) | flow discharge |
| X (-) | Non-dimensional longitudinal coordinate, $\mathrm{X}=\mathrm{x} / \mathrm{Lj}$ |
| $\mathrm{X}(\mathrm{m})$ | Longitudinal coordinate |
| Y (-) | Sequent depth ratio |
| YR (-) | Sequent depth ratio in rectangular channel |
| YT (-) | Sequent depth ratio in triangular channel |
| $y(-)$ | Non-dimensional vertical coordinate, $y=\frac{h(x)-h_{1}}{h_{2}-h_{1}}$ |
| $\alpha(-)$ | Correction coefficient, for $\mathrm{X}=1$ and $y=1, \alpha=0.014$ |
| $\eta(-)$ | Efficiency of jump |
| $\eta \mathrm{R}(-)$ | Efficiency of jump in rectangular channel |
| $\eta \mathrm{T}(-)$ | Efficiency of jump in triangular channel |

