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# Reliability Analysis of Public Transit Systems Using Stochastic Simulation

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## Abstract

Unreliable public transport systems cause excessive waiting times, late or early arrivals at destinations and missed connections for passengers. Also, unreliability results in economic losses to transit operators through under utilization of vehicles, equipment and work force. The reliability analysis of bus transit, covered in this paper, is based on numerical estimation of headway variations at different bus stops along the route. A number of simulations are conducted to determine the variation of performance of bus operation due to the variability of departure headways. The average waiting time of passengers is used as an indicator of operational performance. Simulation results show that the spread of passenger waiting times widens as the headway variation increases. Impact of size of vehicle on waiting time distributions is also investigated. Irregular headways lead to uneven passenger loads on buses. Such variation in passenger counts result in some buses becoming full and being unable to serve certain stops. Thus, average waiting time increases with smaller bus size. Simulation also reveals that the average waiting time increases for passengers waiting further along the route.

## 1. Introduction

Reliability is an important indicator of level of service in public transport systems. Unreliability causes increase in waiting time, late or early arrivals at destinations and missed connections (Bowman and Turnquist, 1981; Turnquist, 1978; Wilson et al., 1992). User surveys reveal that reliability is implicitly valued by passengers in stated preference surveys (Bates et al., 2001). Reliability is also seen as a governing factor in selection of transport modes by users (Prioni and Hensher, 2000).

Reliability affects the amount of time passengers need to wait at a transit stop for a vehicle. Therefore, passenger waiting time is an important criterion to assess the performance of public transport system. There are number of studies that describe the arrival and waiting process of passengers at bus stops. The most frequently used function to express average waiting time is based on vehicle headway. Under the assumption of random passenger arrivals, buses with adequate capacity and regular arrival of buses at stops, in a traditional model, average waiting time of passengers for buses is expressed as half of bus headway. However, average passenger waiting time can be longer when bus arrival is irregular. For this purpose, Welding (1957), Holroyd and Scraggs (1966) and Osuna and Newell (1972) showed the suitability of theoretical equation to estimate the expected waiting time of passengers as a function of mean headway and headway variation of bus:

$$W = H \times [1+C^2]/2 \dots\dots\dots(1)$$

Where W = expected value of passenger waiting time

H = mean bus headway

C = Coefficient of variation of bus headway

The parameter C in Equation 1 is defined as the ratio of the standard deviation of headway divided by the mean headway. This coefficient of variation of bus headway is frequently used as a measure of service reliability. A study by Bly and Jackson (1974) showed that, generally, the

service reliability weaken along the bus route from deterministic headway ( $C=0$ ) at the commencement of the route to a situation where headway variation becomes that can be represented by an exponential distribution ( $C=1$ ) at the terminus of the route. However, waiting time of passengers vary with their arrival behaviour at transit stops. Some studies (e.g. Seddon and Day (1974), Jolliffe and Hutchinson (1975), Turnquist (1978), and Bowman and Turnquist (1981)) focused on the fact that passenger may time their arrival at transit stop for infrequent bus services with published time tables. Seddon and Day (1974) showed by empirical research that passengers arrive at stops randomly for headway less than 10-12 minutes. Jolliffe and Hutchinson (1975) explained behavioural association between bus and passenger arrival at a bus stop. They showed that number of passengers who time their arrival at transit stops decrease exponentially with frequency and unreliability of bus service. Turnquist (1978) modified Jolliffe and Hutchinson model and identified the effect of service frequency and reliability for both random and non-random arrival of passenger on waiting time. Bowman and Turnquist (1981) analysed sensitivity of passengers waiting time to service frequency and schedule reliability. Ceder and Marguier (1985) developed the probability distribution of passenger waiting time at transit stops for random arrival of passengers considering two types of bus headway distributions of deterministic and exponential distribution. Fan and Machemehl (2002) developed a mathematical model to predict bus passenger waiting times in both random passenger arrivals and real-world situations. They identified 10 minute vehicle headway as the transition from random passenger arrivals to non-random passenger arrivals.

These research works presented a foundation for further research on passenger waiting time at transit stops. However, there is a lack of analysis of transit reliability regarding passenger waiting time in a situation when bus headways are irregular and buses have limited passenger carrying capacity. Limitation of bus capacity leads to passengers denied boarding when the bus is full. The rejected passengers need to wait for next available bus and such rejection adversely affects their waiting time. Moreover, variability of bus dispatch headway amplifies this effect along the route. The available softwares to analyse such kind of problem are too complex that need large amount of data or too general that is difficult to modify. This paper explains an analysis tool currently being developed to estimate the performance of a bus route subjected to variation of bus dispatch headway and bus capacity limitation.

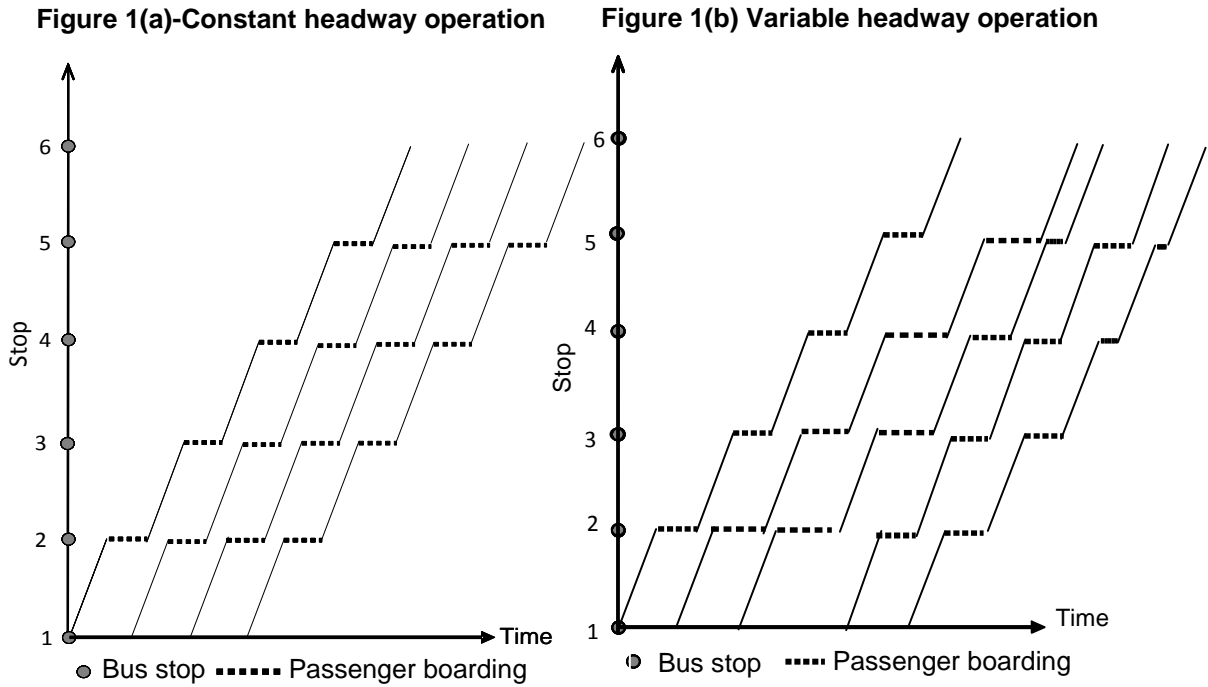
In the next section, bus operation in a hypothetical route is described as a lead into the description of the simulation model. Section 3 describes scenario analysis, model inputs, and model outputs. Analysis and validation are shown in subsequent sections. The final section offers conclusions on the developed simulation model.

## 2. Description of a bus route operation

In bus transit services, overall travel time of passengers depends on (i) dispatch time from bus stops and (ii) travel time along the route. We consider here a bus route that consists of multiple stops. There are five events that relate to passengers considered in the model. These events are (a) passenger arrives at the bus stops, (b) passenger waits for bus, (c) passenger boards the bus if space is available, (d) bus departs with passengers and (e) bus arrives at the next stop. From the point of view of a bus, this process of serving, boarding and alighting passengers continues stop after stop. The bus movement is presented in the form of trajectory diagram in Figure 1(a). The vertical axis shows the distance traveled by buses along the route stopping at designated bus stops and the horizontal axis indicates the time elapsed during bus travel. Inclined lines between two stops show bus travel from one stop to the next stop. The dotted lines between two inclined lines show the time spent at stops allowing mainly for boarding and alighting of passengers.

Passengers may have early or late arrival at the destination due to variability in dispatch of buses even if we eliminate all variations in bus travel times (Figure 1(b)). Only variability of bus dispatch is considered in the analysis of this paper, although the model developed is able to handle variation of bus travel times as well.

Figure 1– Trajectory of progress of a bus along the route



### 3. Description of Simulation Model

A simulation model has been developed to estimate the impact of variation of dispatch headway and bus size limit on performance of bus operation along a bus route. In this analysis, average waiting time of passengers at bus stops is adopted as the indicator of performance. Variability of dispatch headway at the first stop of the bus route is an input in this model. This variability of dispatch time at the first stop affects arrival and departure times at downstream stops because of the linked nature of travel times and stop times as shown in Figure 1.

#### 3.1. Scenario analysis

For the purpose of analysis, we have set up four operational scenarios as shown in the Table 1. All scenarios have the same timetable headway and same route characteristics. In scenario 1, buses

Table 1 – Simulation scenarios

Scenario	Coefficient of Variation of Bus Dispatch Headway	Bus Capacity	Bus Dispatch Headway
1	1(a)	0.0	6 minutes
	1(b)	0.25	
	1(c)	0.5	
	1(d)	0.75	
	1(e)	1.0	
2	2(a)	0.0	
	2(b)	0.25	
	2(c)	0.5	
	2(d)	0.75	
	2(e)	1.0	
3	3(a)	0.0	
	3(b)	0.25	
	3(c)	0.5	
	3(d)	0.75	
	3(e)	1.0	
4	4(a)	0.0	
	4(b)	0.25	
	4(c)	0.5	
	4(d)	0.75	
	4(e)	1.0	

can accommodate all waiting passengers at bus stops because of infinite vehicle capacity. In the other scenarios, bus capacity is constrained. Importantly, scenarios with limited bus capacity are chosen to analyze the impact of passengers being left behind when buses are full. Six-minute headway service is considered where passengers arrive at stops in a random fashion. Each of four scenarios is again sub-divided into five scenarios depending on the value of coefficient of dispatch headway of buses. These five scenarios have a coefficient of dispatch headway of 0.0, 0.25, 0.5, 0.75 and 1.0 which represent 0, 1.5, 3.0, 4.5 and 6.0 minute standard deviation of headway respectively. For simplicity, the simulated bus route in this paper consists of six stops, although the model can handle more than six stops. The first and last five buses are excluded from calculation of the output to account for “warming-up” of the model. Each simulation scenario is repeated 30 times and each simulation run represents 180 minutes of bus operation.

### 3.2. Simulation model inputs

Following conditions are assumed for model input:

1. **Dispatch headway of bus:** Bus arrival is generated according to a gamma distribution. The model can accept distributions such as normal distribution, exponential, Poisson distribution and gamma distribution. The gamma distribution has been selected for this application because a case study of a bus routes operated by the Chicago Transit Authority (CTA) documented by Lin and Ruan (2009) has shown that dispatch headway followed a gamma distribution.
2. **Arrival of passengers:** Passengers arrive randomly at a stop following a homogeneous Poisson process. Hence, inter-arrival times between passengers were generated using a negative exponential distribution with passenger arrival rates shown in Table 2. If passengers and buses arrived at a stop at fixed headway, the number of passengers expected to arrive between consecutive bus arrivals at different stops and cumulative values of passenger counts is depicted in Figure 2. We presume that passenger demand does not change over the period of interest.
3. **Boarding time:** A portion of onboard passengers alight upon arrival of the bus at a stop. Subsequently, waiting passenger board the bus until available space is filled. In most bus operations, alighting time per passenger is low, hence in this simulation dwell time of buses is assumed to be dominated by the boarding activity. Boarding time is assumed to be a constant three seconds per passenger.
4. **Number of alighting passengers:** The number of alighting passengers depends on the total number onboard bus when it arrives at a stop. If passengers are independent, the number of alighting passengers at each stop is assumed to follow a binomial distribution as suggested by Andersson and Scalia-Tomba (1981). Furthermore, the passenger alighting probability is assumed to be same during the period of interest. The alighting probability of passengers at each stop is given in Table 2. In a deterministic setting, total number of alighting passengers at stops and cumulative are shown in Figure 2.
5. **Travel time:** A constant travel time of 20 minutes between two consecutive stops is arbitrarily selected. This value may be seen as large for bus stop spacing, but as this is a constant, there is no contribution from this stop spacing to analysis presented here. To isolate consequences of departure headway variability, bus travel time between stops is kept constant without any scope for variability.

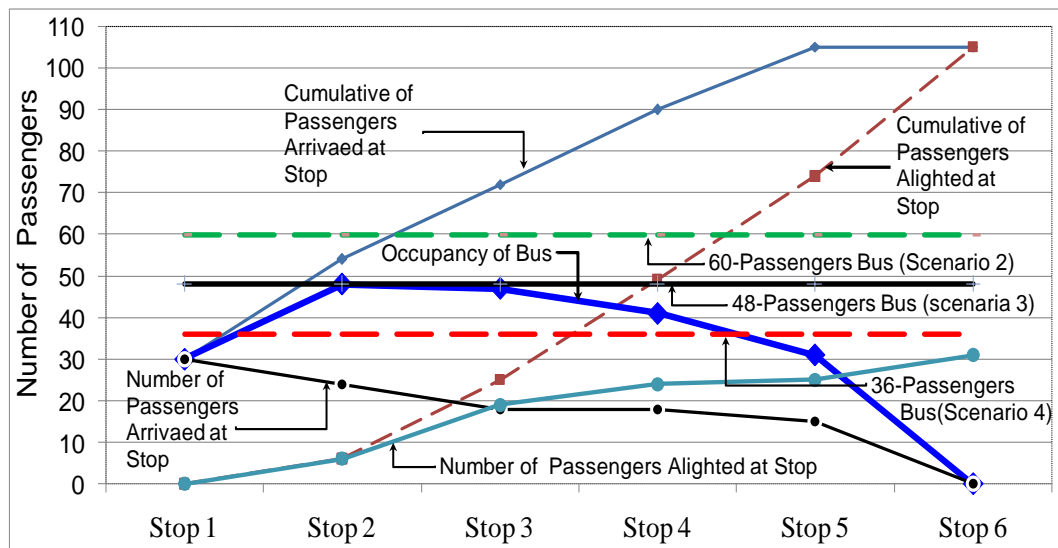
Figure 2 shows a graphical representation of the input characteristics presented in Table 2 in a deterministic setting. This figure shows that under the assumption both passengers and buses arrive at stops at fixed intervals; occupancy of buses leaving stops can be calculated from the

difference between the total number of passengers arrived and alighted. For example, total expected number of passengers arrived at the second stop between two consecutive buses is  $6 \times 4 = 24$  and cumulative arrivals is  $24 + 30 = 54$ . After alighting 20% (Table 2) of passengers carried by a bus from previous stop (i.e. stop1), the number of remaining passenger in the bus is  $30 - (0.2 \times 30) = 24$ . If bus can accommodate all passengers waiting at stop, the occupancy can be calculated as  $24 + 24 = 48$  or  $54 - (0.2 \times 30) = 48$  passengers. In this instance, the 60-passenger bus (scenario 2) and 48-passenger bus (scenario 3) can accommodate all arrived passengers at stop 2, however, the 36-passenger bus size (scenario 4) failed to accommodate all arrived passengers due to capacity constraints. 12 passengers will be denied boarding with 36-passenger buses and have to wait for the next bus. The effect of such bus capacity limits along with bus dispatch headway variation will be explored in next sections of this paper.

**Table 2 – Passenger arrival and alighting probability at stops**

Stops	Passenger arrival rate at stops (passenger/minutes)	Proportion of alighting passengers
1	5.0	0.0
2	4.0	0.2
3	3.0	0.4
4	3.0	0.5
5	2.5	0.6
6	0	1.0

**Figure 2 – Input characteristics in a deterministic setting**



### 3.3. Simulation model outputs

The following output measures are produced from the simulation:

- Passenger waiting time:** Passenger waiting times at stops are calculated from the time elapsed since passenger arrival at a stop until the time of arrival of the bus that the passenger is able to board. In this simulation model, average waiting time is calculated from individual waiting times of all passengers arrived at a particular stop, during the simulation period.
- Number of rejected passengers:** when buses are full, they deny passenger entry. Number of passengers left behind at stops because of such rejection is recorded.

- Coefficient of variation of bus headway at stops:** Bus headway is computed from the time interval between departure times of two successive buses at stops. Coefficient of variation of bus headway is calculated dividing the standard deviation of headway by mean headway.

## 4. Simulation results

This section describes simulation results for scenarios described earlier with the aid of Table 1. Probability distributions of waiting times for coefficient of bus dispatch headway of 0.5 are shown in Figures 3. Peak values and spreads of waiting time distributions are depicted later to show the effect of dispatch headway is subject to different degrees of fluctuation (coefficient of variation of dispatch headway of bus ranging from 0.0 to 1.0). Cumulative distributions of passengers rejected to board the bus due to the unavailability of space for different capacity of buses for coefficient of bus dispatch headway of 0.5 are explained later. Average waiting time of passengers at stops for each scenario is also investigated. Coefficient of variation of bus headway is shown Table 3.

### 4.1. Distribution of passengers waiting time

Figure 3 – Waiting time distribution of passengers

Figure 3(a)

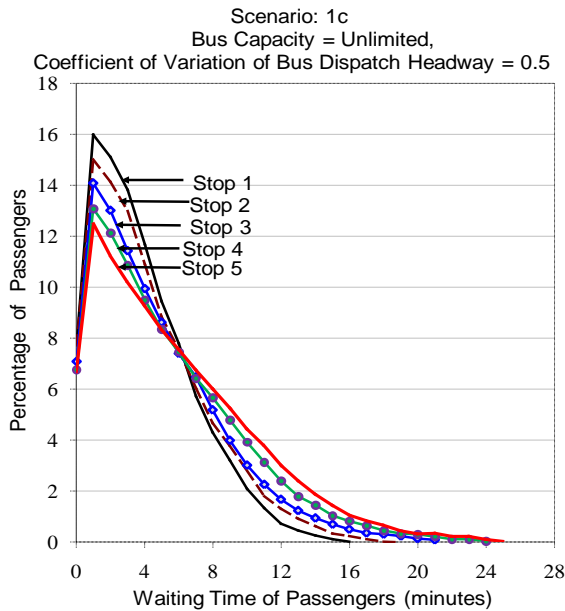


Figure 3(b)

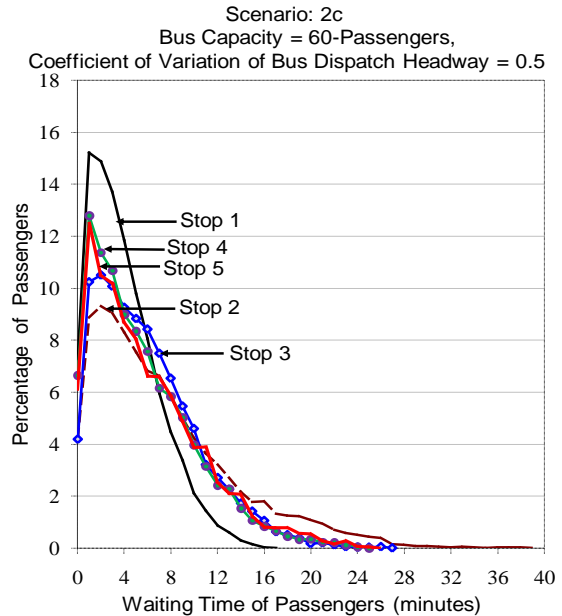


Figure 3(c)

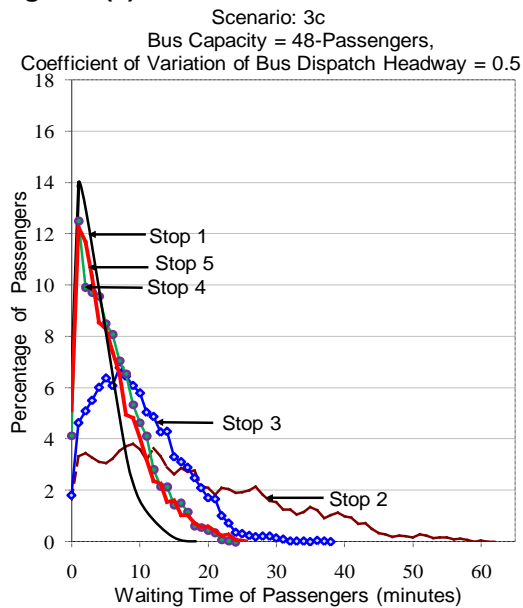
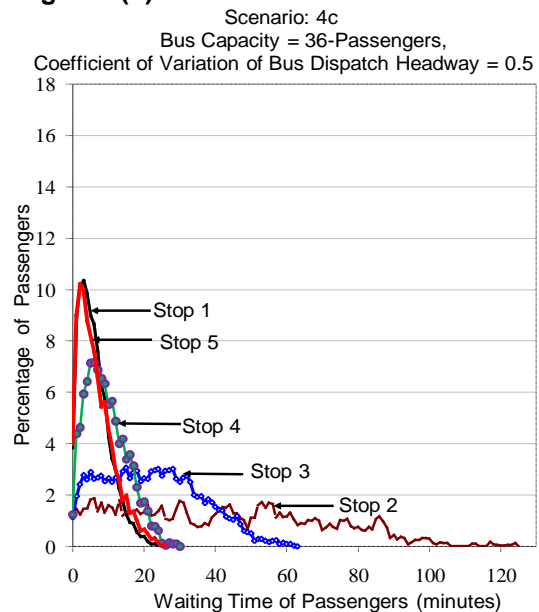


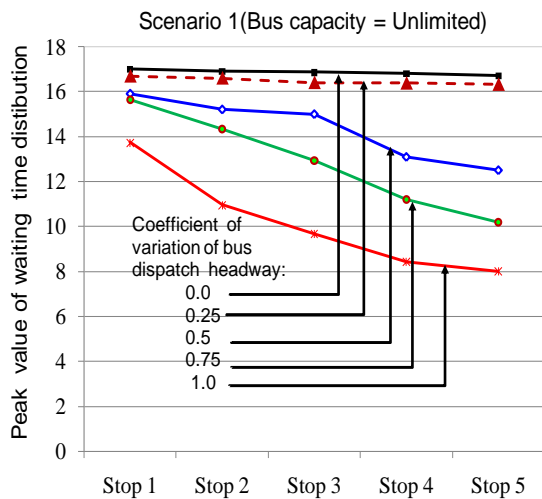
Figure 3(d)



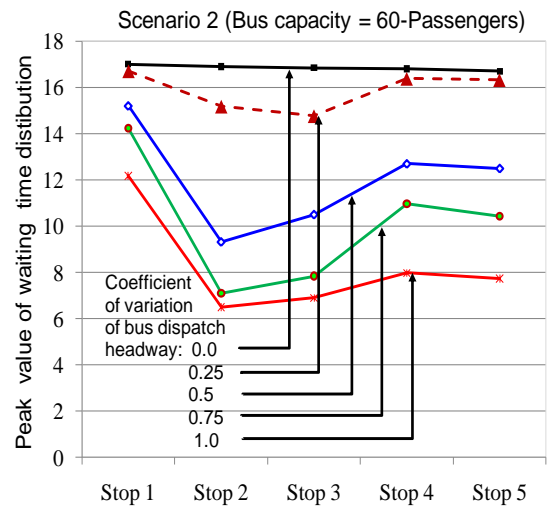
Simulation results are shown in Figures 3(a) to 3(d) display waiting times under different bus capacity limits. Bus capacity is reduced in a stepwise manner from Scenario 2 to 4. In this analysis, bus capacity includes standing passenger spaces. Figures 3(a) to 3(d) reveal that as bus dispatch headway variation increases, the tail of the waiting time distribution extends increasing skewness toward the right of the diagrams. It can also be seen that the peak value of the distribution lowers in values from the first stop to the last stop and the tail of the distribution becomes longer. As an example, for Scenario 1(c) in Figure 3(a), where dispatch variation of bus headway is 0.5 and bus capacity is unlimited, the peak value of distribution is 16, 15, 14, 13 and 12 percent for the first, second, third, fourth and fifth stops respectively. At the same time, the tail of distributions end up at 16, 19, 21, 24 and 25 minutes for first, second, third, fourth and fifth stop respectively. Since bus capacity is be unconstrained, the effect of passenger arrival rate and alighting ratio of passengers will not be visible. When bus arrival at a stop is irregular, passengers may miss their bus and need to wait for next bus and hence experience an increase in their waiting time. Thus, the tails of waiting time distributions become longer for passengers downstream of the bus route.

**Figure 4 – Peak value of waiting time distribution of passengers (percentage)**

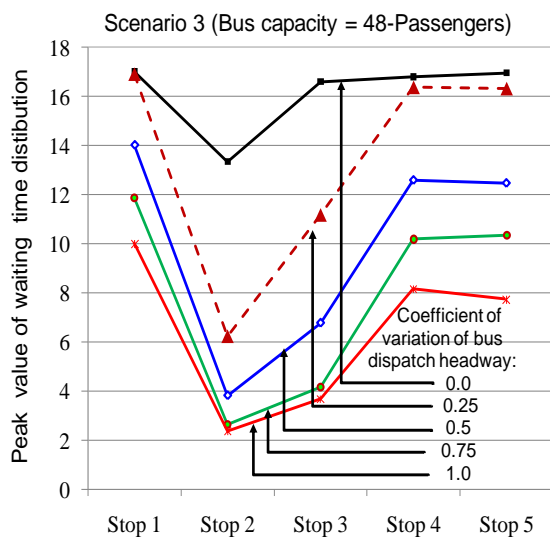
**Figure 4(a)**



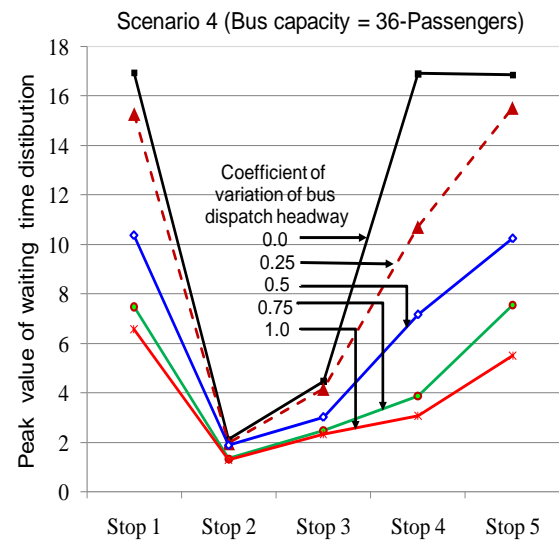
**Figure 4(b)**



**Figure 4(c)**



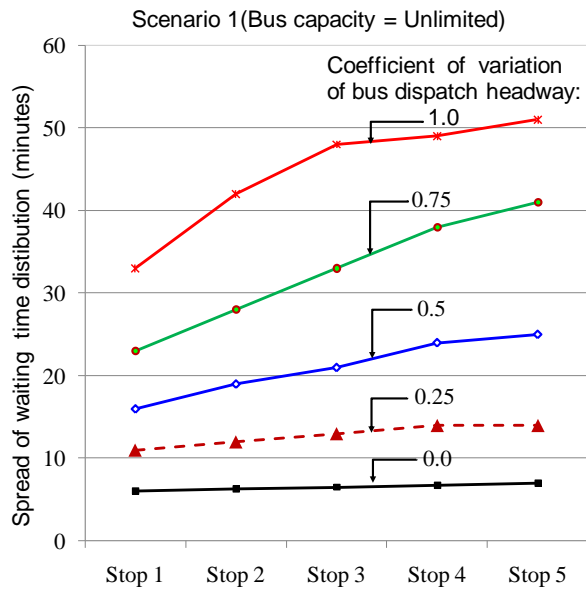
**Figure 4(d)**



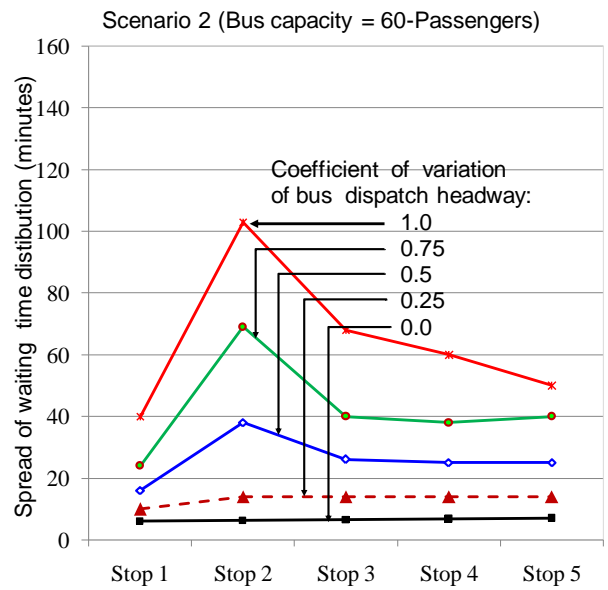


**Figure 5 – Spread of waiting time distribution of passengers**

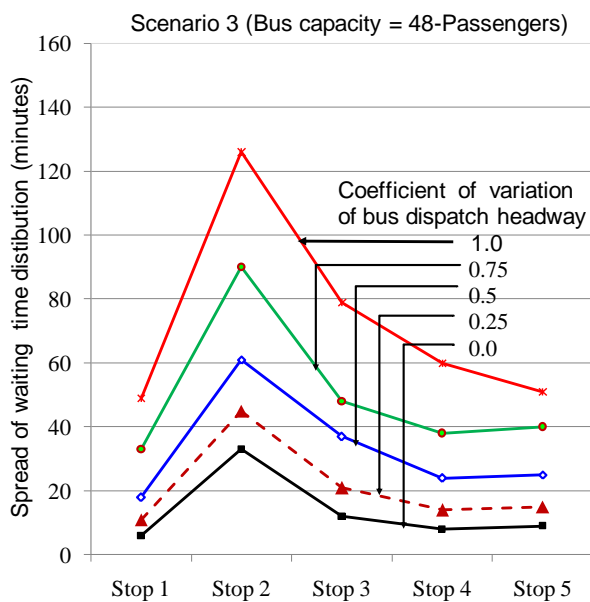
**Figure 5(a)**



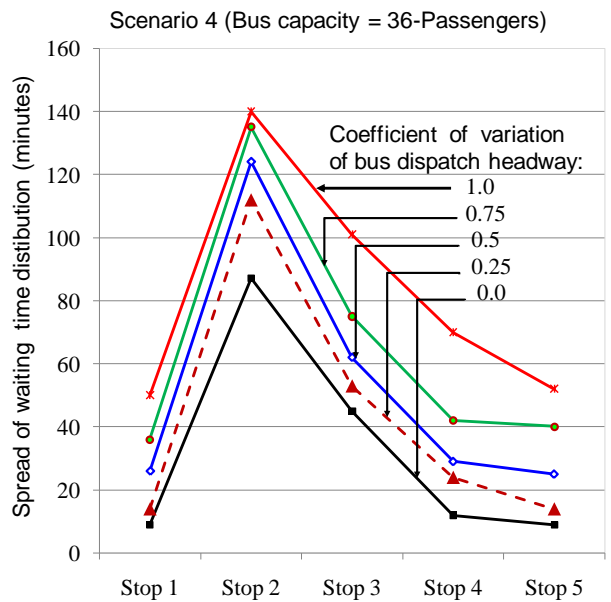
**Figure 5(b)**



**Figure 5(c)**



**Figure 5(d)**



Figures 4 and 5 show the peak value and spread of waiting time distribution respectively for different capacities of bus ranged from unlimited passenger capacity bus to 36-passenger capacity bus. Each figure shows the peak value and spread of waiting time distribution corresponding to the coefficient of variation of dispatch headway of 0.0, 0.25, 0.5, 0.75 and 1.0. In scenario 1, as the variation of bus dispatch headway is increased from 0.0 to 1.0 i.e. bus service is deteriorated, the peak of waiting time distributions falls to lower values (shown in Figure 4(a)) and tails of distributions become longer (shown in Figure 5(a)) to increase the average waiting time of passengers (this will be shown later in Figure 7(a)). This means larger headway variations causing a higher percentage of passengers facing longer waiting times.

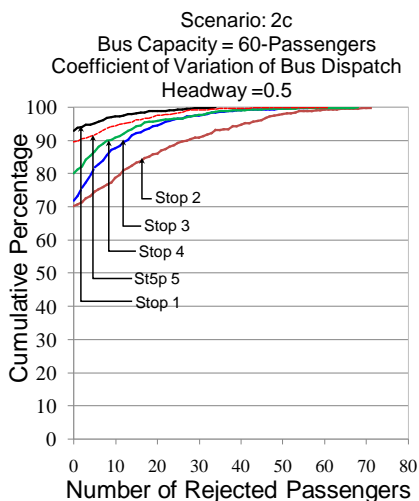
Figure 4(b) and 5(b) show the peak value and spread of waiting time distribution when bus capacity is limited to 60-passengers (Scenario 2). It can be seen from these Figures that the peak values of waiting time distribution and the tails of the distributions move further to the right in time scale as variation of dispatch headway increases. The observation is similar to scenario 1 as observed in previous simulations.

In scenario 2, where bus capacity is limited to 60-passengers, the spread of waiting time distribution extends and the probability of a long wait for passengers becomes larger compared to scenario 1 that has unlimited bus capacity. It can be noticed from Figure 5(b) that the spread of waiting time distribution at second stop is longest in Scenario 2(b) to 2(e). With limited capacity, buses could be nearly full at stop 2 that has become maximum load point as shown by the bus occupancy profile for a deterministic operation shown in Figure 1. This is an outcome of the specific arrival passenger rates and alighting ratios selected for these simulations. When bus size limit is imposed, passengers are denied entry to buses with a relatively high probability at this stop (probability is 0.3 as shown in Figure 6(a)). These passengers who were unable to board the bus have to wait for next bus and experience high waiting times. For example, in Scenario 2(c) where the dispatch variation of headway is 0.5 and bus size is 60-passengers, the peak value of the distribution dropped to 9.0 percent from 15.0 percent at stop 2 (Figure 3(b)) compared to Scenario 1(c) (Figure 3(a)). This can also be seen in Figure 4(b) for coefficient of variation of bus dispatch headway of 0.5 for stop 2. The maximum waiting time at stop 2 of Scenario 2(c) (shown in Figure 3(b) and 5 (b)) is 39 minutes which is approximately 6.5 times bus headway (6 minutes).

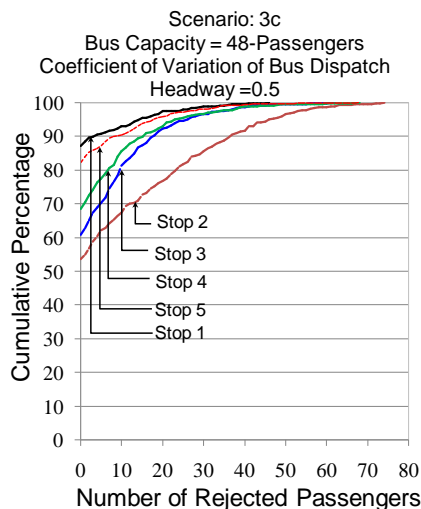
Scenario 3 assumes 48-passenger capacity buses, 20 percent lower than capacity in Scenario 2. The results are shown in Figures 4(c) and 5(c). Compared to scenario 2, in scenario 3 the probability of passenger rejection is increased (rejection probability is 0.45 at stop 2 as shown in Figure 6(b)). This leads to the waiting time distributions to be more spread. This spread of waiting time distribution become longer as variability of dispatch headway is increased. The peak value of distribution for stop 2 and stop 3 dropped sharply and the tails also become rapidly longer. At stop 2 of Scenario 3(c), probability of passenger rejection increased to 0.45. Hence, the maximum waiting time becomes 61 minutes (Figure 3(c) and Figure 5(c)) which is approximately 10 times the bus headway. Compared to Scenario 2(c) (Figure 3(b)), it can be seen that the 20 percent reduction in bus capacity increases the probability of buses operating at capacity by 1.5 times, increasing the maximum waiting time by approximately 1.5 times. Similar trends are found in Scenario 4 where bus capacity is assumed to be 36 passengers. In other words, bus passenger carrying capacity is further reduced by about 25 percent in Scenario 4 from Scenario 3. In the case of Scenario 4 as shown in Figures 4(d) and 5(d), the effect of bus capacity along with dispatch headway variability of buses further intensifies the long passenger waiting time at stops 2, 3 and 4. At stop 2 of Scenario 4(c), for example, probability of passenger rejection increased to 0.68 (Figure 6(c)). Hence, the maximum waiting time becomes 124 minutes (Figure 3(d) and 5(d)) which is approximately 21 times bus headway. Compared to Scenario 3(c), it can be seen that the reduction of bus capacity by 40 percent increases the probability of being denied boarding 2.25 times and the maximum waiting time is increased approximately by 3.0 times.

**Figure 6– Cumulative distribution of passengers not able to board the bus**

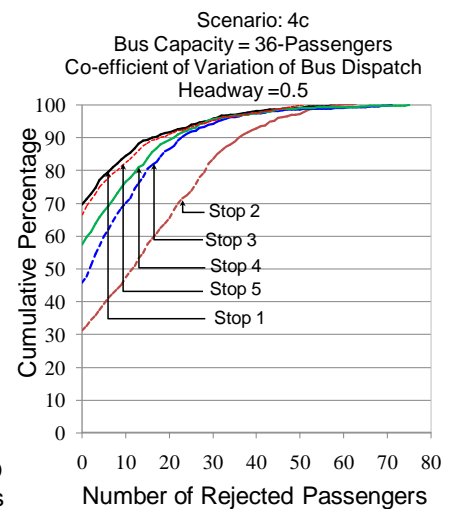
**Figure 6(a)**



**Figure 6(b)**

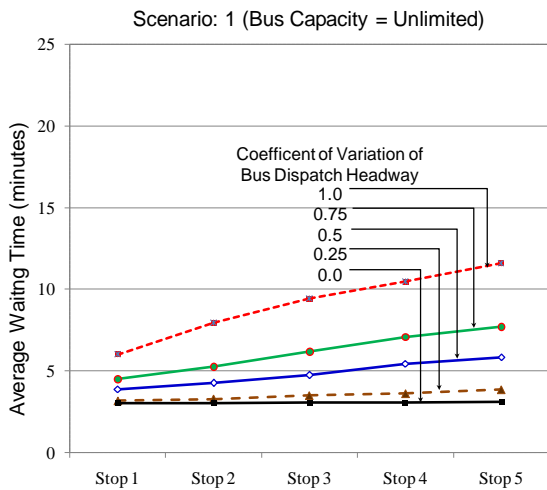


**Figure 6(c)**

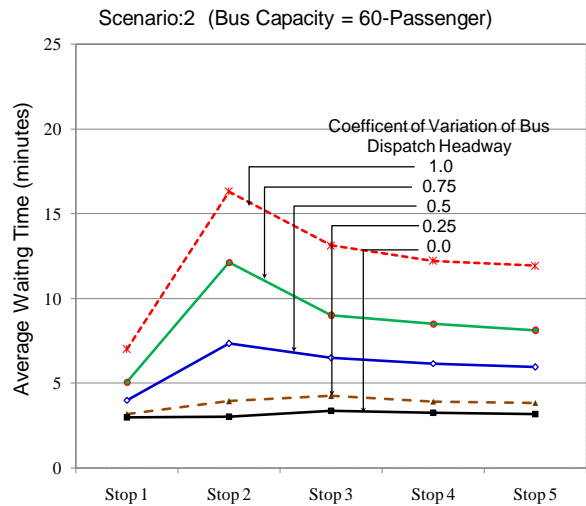


**Figure 7– Average waiting time of passengers**

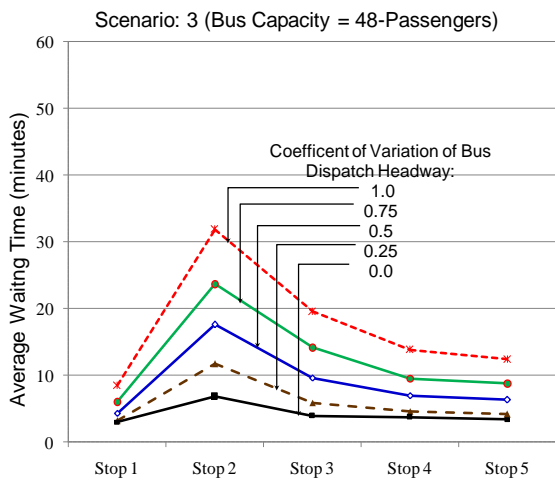
**Figure 7(a)**



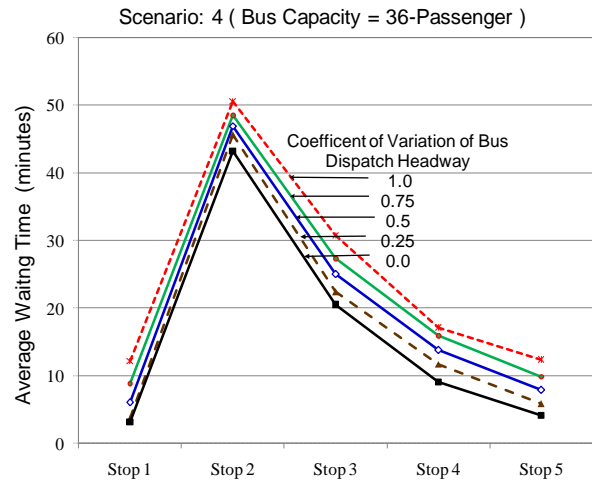
**Figure 7(b)**



**Figure 7(c)**



**Figure 7(d)**



## 4.2. Average waiting time of passengers

Average waiting time of passengers for different levels of variation of bus dispatch headway and different bus capacities are shown in Figures 7(a) to 7(d). Figure 7(a) depicts the effect of bus dispatch headway variation in a bus system where bus capacity is unlimited. The average waiting time of passengers is half average bus headway along the route when there is no variation in bus dispatch headway (i.e. coefficient of variation of bus dispatch headway is 0.0). However, average waiting time increases for downstream passengers as headway variation increases. Since buses have infinite capacity in this scenario, the limiting effect of bus capacity is not observed. Due to irregularity of bus arrival at stops, passenger waiting time increases. This irregularity then affects the arrival of a bus at the next stop which increases the variation of passenger waiting time along the bus route. Thus, average waiting time increases towards downstream stops as headway variation increases.

It was described earlier that the spread of waiting time distribution widens as variation of dispatch headway of buses increases and the probability of passenger rejection increases as the size of bus decreases. Consequently, average waiting time of passengers increases differently for different stops. Figures 7(b) to 7(d) show the effect of bus passenger capacity on average passenger waiting times along with variation of dispatch headway of bus. In Figure 7(b), average waiting time remains the same along the route when the coefficient of bus dispatch headway is 0.0 for 60-passenger capacity buses. Average waiting time increases at all stops when the coefficient of bus dispatch headway is increased. This increasing trend in average waiting time is much higher at the second stop, which is the maximum load point. For example, when the

coefficient of bus dispatch headway is 0.5, the average waiting time of passengers at stop 2 increased to 7.5 minutes which is 2.5 times of the average waiting time when there is no variation of dispatch headway and no capacity limits (i.e. 3 minutes). In Scenario 3(c), where bus capacity is reduced 20 percent from Scenario 2 and coefficient of variation bus dispatch headway is 0.5, average waiting time increased approximately 5.5 times (shown in Figure 7(c)) compared to Scenario 1(c) (Figure 7(a)) and approximately 2.0 times compared to Scenario 2(a) (Figure 7(b)). When bus capacity is reduced further 20 percent from Scenario 3 or 40 percent from Scenario 2, average waiting time increases sharply to 45 minutes (shown in Figure 7(d)) which is 15 times compared to Scenario 1(c) (Figure 7(b)) and 6 times compared to Scenario 2(c) (Figure 7(b)). Similar observation can be made from presented figures for other stops for scenario 1 to 4.

Thus, the results of scenarios 2, 3 and 4 reveal that dispatch headway variability yields longer waiting times for a higher percentage of bus passengers. Furthermore, bus passenger capacity limits lengthens the waiting time of passengers. These simulation results highlight the need to produce improvements by reducing dispatch uncertainty and increasing bus capacity to achieve better waiting time outcomes.

### 4.3. Coefficient of variation of bus headway

The simulation results show that the variation of bus headway increases the further along the route. This is because once a bus service becomes irregular at one stop it creates knock on effects downstream. On the other hand, headway variation declines if bus capacity is reduced. However, this reduction of headway variation is quite feeble and shows low sensitivity to reduction of bus capacity. For example, Table 3 shows that the variation of headway increased to 0.870 at stop 5 when dispatch variation of headway is 0.5 at stop 1 with unlimited capacity of bus. Similarly the variation of headway increases to 0.869, 0.867 and 0.866 at stop 5 for 60-passenger, 48-passenger and 36-passenger bus respectively. However, these variations of headway decrease as bus capacity decreases. Besides, this decreasing value of headway variation shows low sensitivity with respect to reduction of bus capacity. This result agrees with the analysis put forward by Marguier (1985).

**Table 3 – Variation of headway along the route when coefficient of bus dispatch headway is 0.5**

Bus capacity	Stop 1	Stop 2	Stop 3	Stop 4	Stop 5
Unlimited	0.5	0.506	0.640	0.766	0.870
60-passenger	0.5	0.504	0.639	0.765	0.869
48-passenger	0.5	0.503	0.637	0.763	0.867
36-passenger	0.5	0.501	0.635	0.762	0.866

## 5. Model validation

The result obtained from simulation is compared against analytical result using statistical t-test at a 0.10 level of significance. The following formula is used for statistical t-test:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \dots \dots \dots (2)$$

Where,

$\bar{x}$  = mean waiting time of passengers obtained from simulations

$\mu$  = mean waiting time of passengers obtained from equation 1 (it should be noted here that equation 1 is applicable for unlimited capacity of bus)

$s$  = standard deviation of waiting times of passengers obtained from simulations

$n$  = number of simulation runs

The test result of all five stops corresponding to coefficient of variation of bus dispatch headway 0.0 to 1.0 is shown in Table 4. Table 4 shows t-values obtained from the test are within the range of +1.70 to -1.70 which is standard t-value at 90 percent confidence interval for a given degree of freedom.

**Table 4 – Model validation**

Coefficient of variation of bus dispatch headway		0	0.25	0.5	0.75	1
Stop 1	Simulation result	3.009	3.177	3.881	4.487	6.005
	Standard deviation	1.778	2.046	2.909	3.815	5.672
	Analytical result	3.000	3.190	3.766	4.527	6.000
	t-value	0.028	-0.034	0.216	-0.058	0.005
Stop 2	Simulation result	3.010	3.275	4.251	5.260	7.935
	Standard deviation	1.789	2.192	3.334	4.579	7.262
	Analytical result	3.000	3.309	4.228	5.349	7.802
	t-value	0.030	-0.085	0.037	-0.107	0.100
Stop 3	Simulation result	3.065	3.504	4.735	6.181	9.417
	Standard deviation	1.823	2.410	3.822	5.471	8.398
	Analytical result	3.000	3.478	4.759	6.303	9.450
	t-value	0.195	0.060	-0.035	-0.123	-0.021
Stop 4	Simulation result	3.046	3.616	5.435	7.087	10.464
	Standard deviation	1.840	2.568	4.362	6.118	9.030
	Analytical result	3.000	3.682	5.268	7.250	10.832
	t-value	0.136	-0.143	0.210	-0.146	-0.223
Stop 5	Simulation result	3.086	3.859	5.837	7.707	11.584
	Standard deviation	1.892	2.775	4.635	6.684	9.648
	Analytical result	3.000	3.857	5.668	8.042	11.785
	t-value	0.248	0.004	0.200	-0.274	-0.114

## 6. Conclusions

The reliability analysis presented in this paper is based on a simulation model of bus operations. The purpose of the simulation model is to demonstrate the application of a user orientated measure of bus operation performance. The analysis has investigated the effect of bus dispatch headway variability and effect of bus capacity on service reliability of a high frequency bus route. Four scenarios are presented. These scenarios consider four different bus capacities: 36-passengers, 48-passengers, 60-passengers and unlimited capacity buses. For each scenario, simulations were undertaken using different bus dispatch headways, ranging from 0.0 to 1.0. The simulations account for random generation of passengers and buses at stops and keep track of start times of activities of individual passengers and vehicles. The simulation results show that for the selected route parameters, the distribution of passenger waiting times widen as the departure headway variation increases. The spread of waiting time as well as average waiting time of passengers increases as bus capacity is reduced. And passenger rejection probability increases as bus capacity decreases. Bus capacity limits compel passengers to wait for the next bus increasing average passenger waiting times. Headway variance shows a tendency to increase along the route as a result of variation of headway at dispatch point and low sensitivity with the changes of bus capacity.

The proposed model is able to vary departure headway of buses and bus travel time according to assigned probability distributions. The transit operator is able to estimate the performance of a particular transit route by evaluating the user waiting time distribution in a realistic manner.

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