

# 2DPSK Signal Detection Based on Cascaded Stochastic Resonance

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## Abstract

In the case of poor channel environment, the detection and reception of digital signal often appear errors. In view of this situation, by reducing the error rate of coherent reception of 2D PSK signals, we propose a new method based on the detection efficiency and improved cascaded stochastic resonance theory. A cascaded bistable stochastic resonance model was established by using stochastic resonance theory. The nonlinear receiver was used to receive 2DPSK signal under small signal-to-noise ratio (SNR). The experimental results show that the spectrum peak of the output signal of cascade stochastic resonance system is 5.70 times that of the traditional model. The output error rate of cascaded nonlinear system model can be reduced by 92.31% compared to the traditional model when the input signal to noise ratio is -7dB. Consequently, the output signal of the system is more likely to be detected and the accuracy can be greatly improved.

**Keywords:** stochastic resonance; 2DPSK signal; bit error rate; cascaded bistable system; signal-to-noise ratio.

## 1. Introduction

The transmission of digital signal requires high efficiency of signal transmission and the low bit error rate of transmission. 2DPSK signal even contents this requirement. The principle of transmitting signal is to transmit digital signal through the relative phase change of multiple front and rear symbols, so it is also called binary relative phase shift keying. Compared with traditional binary amplitude keying (2ASK) and binary frequency shift keying (2FSK), 2DPSK system's anti-noise performance and channel band efficiency are better than amplitude keying (ASK) and frequency shift keying (FSK), and the system have excellent stability when the channel changes.

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With the invention of 2DPSK method, it overcame the difficulty that the carrier phase of the traditional PSK method is not clear in the coherent demodulation process [1], it is a digital phase modulation system with a wide range of applications and easy to complete, because of its high efficiency and practicability, it has been diffused in the field of high-speed digital transmission. The traditional demodulation methods of 2DPSK signal have two methods, the one is coherent demodulation (polarity comparison method) and the other is differential coherent demodulation (phase comparison method) [2]. However, the two traditional methods require high signal-to-noise ratio (SNR) of signal, so it is not easy to realize the demodulation operation of weak 2DPSK signal. The stochastic resonance was first discovered by Benzi and his colleagues [3], for answering the climate of the earth caused by the influence of the period of the glacier. The development of the application is used in the signal detection, the advantage of the random resonance is also extremely significant, the resonance of the signal and the noise is generated by the cooperative help of the nonlinear system, the transfer of the high-frequency noise energy to the low-frequency signal is achieved, the noise energy is suppressed, the signal energy is enhanced, the signal-to-noise ratio is improved, and the periodic characteristic of the weak signal in the noise environment is highlighted. In order to achieve the noise reduction effect better, the cascaded bistable stochastic resonance system (CBSSRS)[4] is invented, CBSSRS's principle is to transfer the energy of noise to the required signal, so that the filtering effect is more significant, to achieve the purpose of weak signal extraction in the background of strong noise. Random resonance is proposed and applied to various industries, such as machine touch[5], image processing[6], electronic circuit[7], signal detection[8], and etc. But most of them are the application of the stochastic resonance in the analog signal processing, and the stochastic resonance theory used in the digital signal processing is a relatively small number. Previous authors have studied 2FSK digital signal processing, and 2PSK signal processing, and have achieved good results. There are relatively few studies on 2DPSK. In this paper, a method for demodulating weak 2DPSK signal is created on the basis of cascaded stochastic resonance theory. By means of studied the influence of the non-linear demodulation method on the error rate of the system, and compare the new method for detecting the weak 2DPSK signal and the traditional random resonance, and the difference between the error rate of the time domain, the frequency domain and the system output is observed when the signal-to-noise ratio is below 0 dB, and then found the advantages of random resonance in digital signal processing

## 2. Cascaded bistable stochastic resonance system model for 2DPSK signals

The simplest bistable system is  $dx/dt = ax - bx^3$  ( $a > 0, b > 0$ ), the corresponding potential function<sup>[9]</sup> is  $U(x) = -ax^2/2 + bx^4/4$ , and a cascaded bistable system is same as a series circuit that connects the two bistable systems to each other like a resistor. The cascaded bistable system acts as a nonlinear receiver in the 2DPSK signal demodulation process under the strong noise background, the Langevin equation<sup>[10]</sup> in accordance with the first and second order bistable systems is:

$$\frac{dx_1}{dt} = a_1x_1(t) - b_1x_1^3(t) + s(t) + n(t) \quad (1)$$

$$\frac{dx_2}{dt} = a_2x_2(t) - b_2x_2^3(t) + x_1(t) \quad (2)$$

The output signal of the first stage bistable system  $U_1(x)$  of the above equation is  $x_1(t)$ , it also acts as the input signal in the second level bistable system  $U_2(x)$ . On the other hand,  $x_2(t)$  is the output signal of the second level of the cascade bistable system.  $a_1$ 、 $b_1$ 、 $a_2$ 、 $b_2$  in the equation are the system parameters in the first and second bistable systems, requires simultaneous assurance  $a_1$ 、 $b_1$  and  $a_2$ 、 $b_2$  are each greater than zero.  $s(t)$  in(1)represents the required 2DPSK signal. According to the principle of differential coding:  $k(t) = g(t) \oplus k(t - T_s)$ ,  $g(t)$ ,  $k(t)$  represents the absolute code and relative code in the transmission digital symbol, the relative code  $k(t)$  is derived from the differential coding  $g(t)$  of the absolute code, and then the relative code  $k(t)$  obtained by the difference is modulated to obtain the 2DPSK signal needed by the system. The equation of the obtained signal in the period of  $T_s$  is as follows:

$$s(t) = A \cos[w_c t + k(t)\pi], k(t) = 1 \text{ or } 0 \quad (3)$$

$s(t)$  can also be expressed as:

$$s(t) = \begin{cases} u_{1T}(t), & k(t)=0 \\ u_{0T}(t)=-u_{1T}(t), & k(t)=1 \end{cases} \quad (4)$$

Of which:

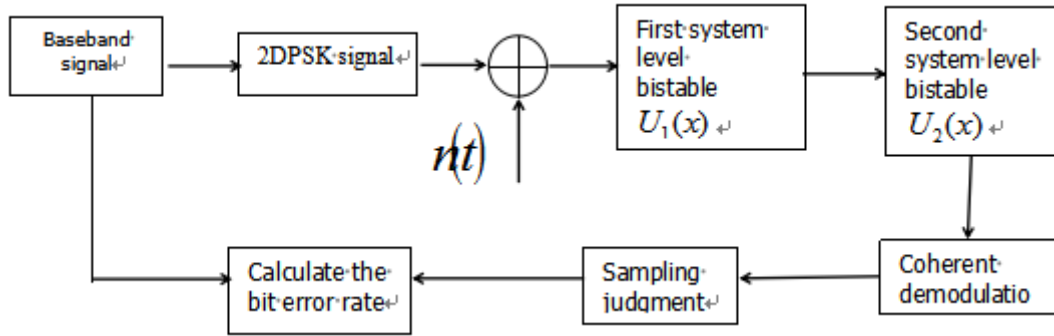
$$u_{1T}(t) = \begin{cases} A \cos w_c t, & 0 < t < T_s \\ 0, & \text{other} \end{cases} \quad (5)$$

$n(t)$  is additive white Gaussian noise which intensity is  $D$ , the correlation functions are as follows:

$$\langle n(t) \rangle = 0 \quad (6)$$

$$\langle n(t)n(s) \rangle = 2D\delta(t-s) \quad (7)$$

The system model of cascaded bistable stochastic resonance receiving 2DPSK signal is as shown in Figure 1:



**Figure 1:** The cascaded bistable stochastic resonance 2DPSK signal processing model

### 3. Theoretical asymptotic bit error rate of 2DPSK signals

The one-dimensional effective FPK equation<sup>[11]</sup> of stochastic processes corresponding to stochastic resonance systems is as follows:

$$\frac{\partial \rho(x, t | A_i)}{\partial t} = -\frac{\partial}{\partial x} [C(x)\rho(x, t | A_i)] + \frac{\partial^2}{\partial x^2} [B(x)\rho(x, t | A_i)] \quad (8)$$

$$B(x) = Dx^2 + \beta, \quad C(x) = (\beta + D)x - b_2x^3 + A_i \quad (9)$$

The bistable system acts as a signal receiver in the whole system, when the appropriate response speed of the system is satisfied, the output signal can be regarded as a constant A in the first order retention sampling time interval, and regard constant A as a value far less than 1, in the absence of noise input, the particle energy is not enough to support the transition in the potential well, and the weak signal energy can only satisfy the periodic motion in a potential well. If the frequency  $f_c$  coincidence of the carrier signal is much less than 1 at the same time, the equilibrium state of the system can be realized in one period, the asymptotically steady-state solution<sup>[12]</sup> of the above FPK equation is as follows:

$$\rho(x | A_i) = \frac{c_0}{B(x)} \exp[\phi(x)] \quad (10)$$

$$\phi(x) = -\frac{1}{D} \left[ \frac{b_2}{2} x^2 - \frac{1}{2} \left( a_2 + D + \frac{\beta b_2}{D} \right) \ln \left( \frac{D}{\beta} x^2 + 1 \right) - A_i \sqrt{\frac{D}{\beta}} \arctan \left( \sqrt{\frac{D}{\beta}} x \right) \right] \quad (11)$$

$C_0$  is obtained by the normalization condition of probability density. The key to realize coherent demodulation of digital signal is to recover a coherent carrier which is strictly synchronous with the modulation carrier. After coherent demodulation (the process of recovering from the adjusted signal carrying information), the waveform

amplitude transmitted into the sampling decision can be roughly divided into two categories, A and -A. each waveform amplitude has the corresponding asymptotically steady-state solution. Owing to 2PSK signal and 2DPSK signal are similar to the sequence of a group of inverted signals from waveform, consequently, the system error rate of coherent demodulation is used for 2DPSK signal  $P'e$ , on the premise of outputting bit error rate  $p_e$  in 2PSK signal system, just consider the effect of code inverse converter on bit error rate. For 2PSK signal, it can be analyzed according to the optimal decision threshold that when the probability of inputting "1" symbol is the same as that of inputting "0" symbol, that is  $p(1) = p(0)$ , the optimal decision threshold is 0. The probability of entering a "1" symbol and misjudging it as a "0" symbol can be expressed as:

$$p(A_0 | A_1) = \int_{-\infty}^0 \rho(x | A_1) dx \quad (12)$$

The probability of entering a "0" symbol and misjudging it as a "1" symbol could be expressed as:

$$p(A_1 | A_0) = \int_0^{+\infty} \rho(x | A_0) dx \quad (13)$$

Integrated from (12) and (13), the synthetic bit error rate(BER) of the system for coherent demodulation of 2PSK signal is obtained and it can be expressed as follows:

$$p_e = p(A_1)p(A_0 | A_1) + p(A_0)p(A_1 | A_0) \quad (14)$$

Moreover, according to the symmetry of probability density distribution:

$$p(A_0 | A_1) = p(A_1 | A_0) \quad (15)$$

Because of  $p(1) = p(0) = \frac{1}{2}$ , then

$$Pe = \frac{1}{2} [p(A_0 / A_1) + p(A_1 / A_0)] = p(A_0 / A_1) = p(A_1 / A_0) \quad (16)$$

Owing to the influence factors of the code inverse converter on the error rate, the relationship<sup>[13]</sup> between the system error rate  $P'e$  of the coherent demodulation of the 2DPSK signal and the output bit error rate  $Pe$  of the 2PSK signal system is as follows:

$$\begin{aligned} P_e' &= 2(1 - P_e)^2 (P_e + P_e^2 + \dots + P_e^n + \dots) \\ &= 2(1 - P_e)^2 P_e (1 + P_e + P_e^2 + \dots + P_e^n + \dots) \end{aligned} \quad (17)$$

Because the bit error rate  $P_e$  is always less than 1 and the  $n$  value is approximately infinite, the following formula is valid:

$$(1 + P_e + P_e^2 + \dots + P_e^n + \dots) = \frac{1}{1 - P_e} \tag{18}$$

The formula (19) is substituted into the formula (18):

$$\begin{aligned} P_e' &= 2(1 - P_e)P_e \\ &= 2[1 - p(A_0 / A_1)]p(A_0 / A_1) \end{aligned} \tag{19}$$

#### 4. Numerical simulation experiment analysis

##### 4.1 Simulation model analysis

The simulation model is established by using the Simulink module library in MATLAB, and it is assumed that the channel characteristic is constant parameter channel, the channel noise is additive Gaussian white noise, and only the interference of Gaussian white noise to the reception of the signal is considered.

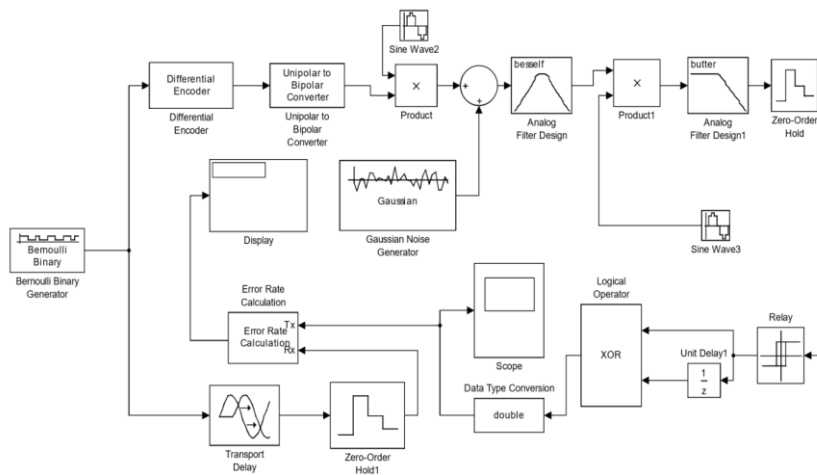


Figure 2: The simulation model of traditional coherent demodulation of 2DPSK signal

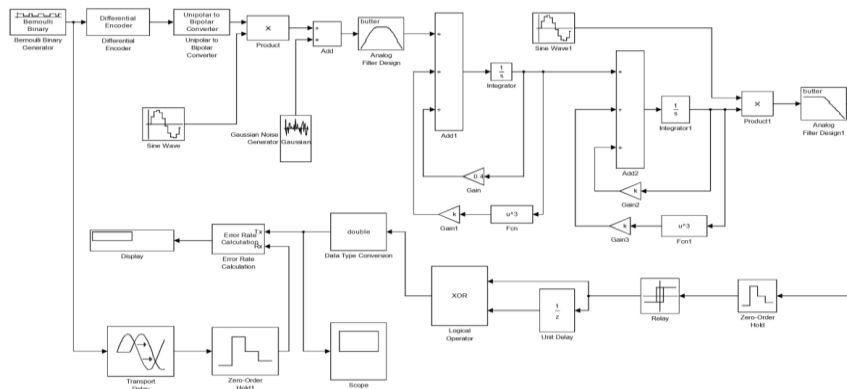


Figure 3: The simulation model of cascaded stochastic resonance

coherent demodulation of 2DPSK signal

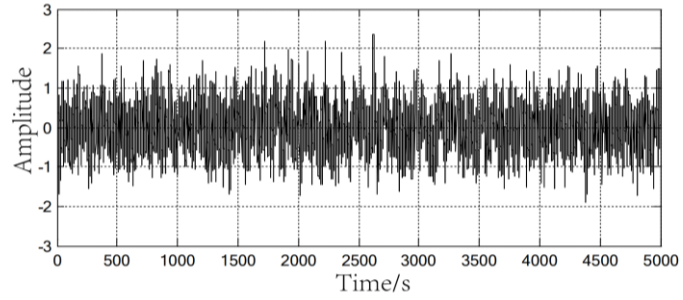
Fig.2 is a conventional 2DPSK signal coherent demodulation simulation model, and Fig.3 is the added 2DPSK random resonance coherent demodulation simulation model of the cascaded bistable nonlinear system. The sine wave signal generator (Sinusoidal Wave Signal Generator) module creates a sine wave signal  $f_c$  with a frequency period of 0.01 Hz. The binary sequence oscillator module outputs a unipolar binary baseband signal and then enters the two modules of an incremental differential encoder and an analog-to-digital single-policy converter, the unipolar binary code element output by the binary sequence oscillator is converted into a bipolar binary code element, and the 2DPSK signal is output by the multiplication of the sine signal. The noise is produced by the Digital White Noise Generator module and then the resulting 2DPSK signal is added to the noise and then passed through the non-linear receiving module. In the coherent demodulation process, a coherent carrier wave in-phase with the modulation carrier is first manufactured, and then multiplied by an output signal, and a low-pass filter is used for differential decoding and sampling judgment. The baseband signal restored after the decision and the binary baseband signal at the transmitting end are used by the error rate calculation module to compare and calculate the bit error rate.

#### **4.2 Time domain and frequency domain analysis of output signal**

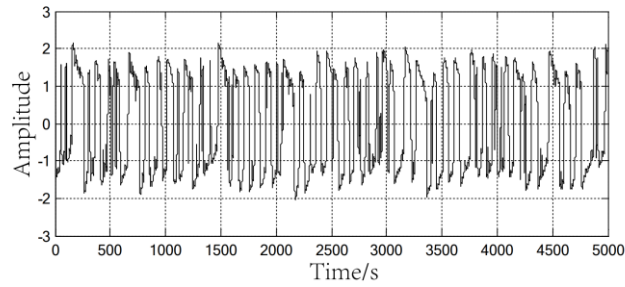
Simulation models in Fig.2 and Fig.3 are adopted respectively to conduct coherent demodulation for 2DPSK signals with amplitude  $A=0.3$  and input signal-to-noise ratio of -8db, the parameters in Fig.3 model are obtained by the adaptive system<sup>[14]</sup>. Fig.4 shows the time domain diagram of the system response outputted by the two simulation models. The signal can be observed by Fig.4(a) when the traditional stochastic resonance is used. It is found that the influence of noise is obvious, and the signal is covered by noise; in Fig.4(b), the signal output by concatenated bistable random resonance model has less burr in time domain, and the signal has periodicity. It can be seen that the mixing of 2DPSK signal and noise through a new cascade stochastic resonance nonlinear bistable system triggers the stochastic resonance phenomenon, that is, part of the noise energy in the system is absorbed by the signal, which reduces the interference effect of noise to the signal, and achieves the purpose of noise suppression more effectively through the cascade stochastic resonance system, and then improves the detection ability of weak signal in the background of strong noise.

through simulation models in Figures 2 and 3

The system output spectrum graph of the two signal random resonance coherent demodulation simulation models of figures 2 and 3 is shown in Fig.5. the carrier frequency amplitude of the 2DPSK signal for the first simulation model is as shown in Fig.5(a) up to 19094, and the second type of 2DPSK signal carrier frequency amplitude in Fig.5(b) is 108917. Comparing the maximum output spectrum of Fig.5(a) and 5(b), it can be observed that the maximum value of the output spectrum obtained through the novel cascaded stochastic resonance model in Fig.3 is 5.704 times as much as the maximum value of the output spectrum through the traditional simulation model. The above phenomenon proves that the signal stochastic resonance nonlinear coherence demodulation method can improve the accuracy of carrier frequency detection compared with the traditional linear coherence demodulation method.

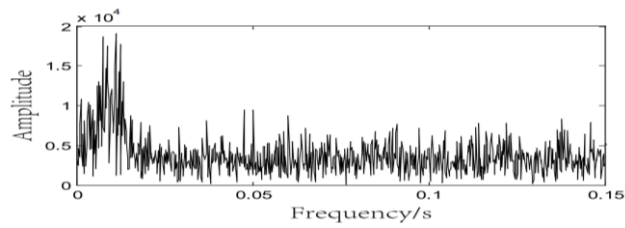


(a) The time domain waveform of the simulation model in Figure. 2

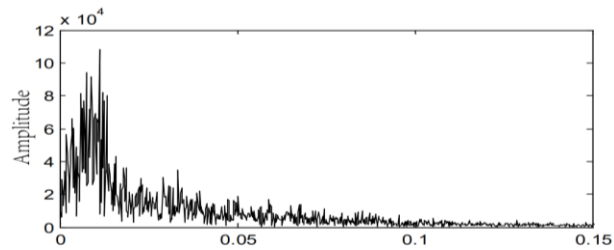


(b) The time domain waveform of the simulation model in Figure. 3

**Figure 4:** The time domain waveform of 2DPSK signal



(a) The spectrogram of the simulation model in Fig. 2



(b) The spectrogram of the simulation model in Figure. 3

**Figure 5:** The spectrogram of 2DPSK signal through simulation models in Figure. 2 and 3



### 4.3 Bit error rate analysis

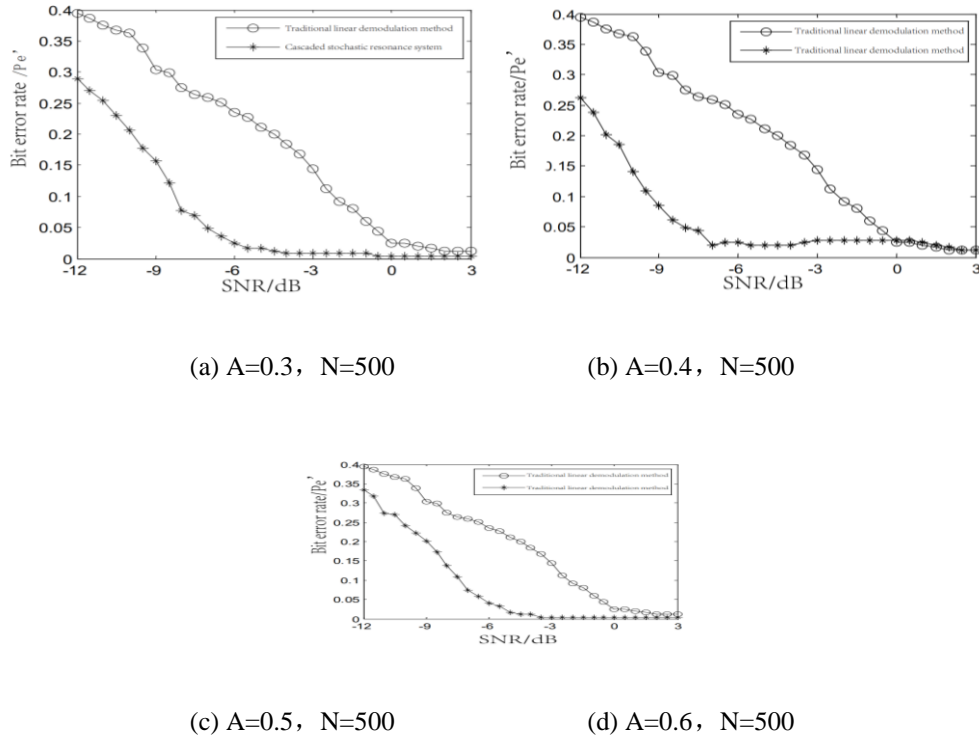
Two kinds of coherent demodulation simulation model experiments in Fig.2 and 3, using MATLAB software to realize the simulation. For the 2DPSK signal, the frequency of the carrier signal is  $f_c = 0.01\text{Hz}$ , the period of the binary baseband signal number is  $T = 100s$ , and the number of code elements in the numerical experiment is  $N = 500$ . The range of input signal-to-noise ratio (SNR) is -12dB to 3dB.

This phenomenon occurs because the signal will suffer a certain proportion of attenuation when passing through the channel, so in the simulation process, the 2DPSK modulation signal takes not greater than 0.3, 0.4, 0.5, 0.60 signal amplitudes. Observed the two curves shown in figure 6, the circle curve represents the output bit error rate curve of the traditional linear model, and the asterisk curve is the output bit error rate curve of the cascade bistable stochastic resonance system model, it can be seen that the curve change of the traditional linear model is very obvious, while the cascaded stochastic resonance nonlinear system model is much smoother than the traditional linear model.

The BER of the cascaded stochastic resonance nonlinear system model decreases obviously when the input signal-to-noise ratio is lower than 0dB, and the BER of the two models is close when the input SNR is larger than 0dB and less than 3dB when observing the change of transverse coordinates. Compared with the curves in four cases, the effect of figure 6(a) is the most obvious. taking this graph as an example, when the input signal-to-noise ratio is between -12dB and -5.5dB, the BER curve of the cascaded stochastic resonance nonlinear system model decreases rapidly. For example, the bit error rate can be reduced by 92.31% at -7dB. The cascaded bistable system can be regarded as a low-pass filter, the function of the cascade bistable system is to filter the 2DPSK input signal mixed with noise through the cascaded bistable system and transmit the noise energy to the signal in order to improve the input signal-to-noise ratio (SNR) of the subconcatenated system and thus reduce the output bit error rate (BER) of the system.

The above phenomena shows that under the background of strong noise, compared with the traditional linear method, the 2DPSK signal passing through the cascaded bistable stochastic resonance system is more accurate, the bit error rate is lower, the accurate signal is easier to receive, the error signal is avoided, and the signal quality is improved. When the input signal-to-noise ratio (SNR) is controlled from -5.5dB to 3dB, the BER curve of the cascade stochastic resonance nonlinear system model does not fluctuate obviously.

Through the comparison of the four graphs in figure(6), it can be seen that when the input signal-to-noise control is below 0dB, the output bit error rate (BER) of the system can be reduced by adding cascaded stochastic resonance nonlinear system to the receiver, and the effect is very remarkable. Fig.6(b), 6(c), 6(d) is similar to figure 6(a), except that the experimental results obtained at 0.5 and 0.6 are not repeated when the input signal amplitude is  $A = 0.4$ , respectively. The four graphs' laws of fig. 6 are about the same, which shows that the stochastic resonance nonlinear system model is robust and can cope with different signal amplitudes.



**Figure 6:** The comparison of output error rate of 2DPSK signal

through simulation models in Figure. 2 and 3 .

**5. Conclusion**

In this paper, based on the principle of cascaded stochastic resonance, a nonlinear receiver model is established, and the stochastic resonance phenomenon of 2DPSK signal passing through nonlinear cascade bistable system is studied. The experimental results show that the maximum spectrum of the 2DPSK signal output by the nonlinear cascade stochastic resonance system is 5.704 times higher than that of the traditional random resonance model. At the same time, the periodicity of the new system signal can be observed by observing and comparing the time domain diagram of the output of the two systems. It can be proved that the cascaded stochastic resonance nonlinear system method can effectively improve the accuracy of digital carrier signal detection. In addition, it is found that when the input signal-to-noise ratio (SNR) is between -12dB and 3dB, the output bit error rate (BER) of cascaded stochastic resonance nonlinear system model is also much lower than that of the traditional method. When the SNR is -7dB, the BER can be reduced by 92.31%. The experimental results shown that the cascade stochastic resonance method can be used to output 2DPSK signals with better accuracy in the background of strong noise. In the future, it provides a new idea for dealing with more complex noises, such as signals of various types of mixed noise, and lays a certain foundation.

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