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# Application of Robust Estimation Methods for Detecting and Removing Gross Errors from Close-Range Photogrammetric Data

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**Abstract**

Systematic, random and gross errors are considered the main problems facing those working in the photo-triangulation processes. The influence of systematic errors on photo-measurements may include lens distortion, film deformation, refraction and other distortions. Usually, these types of errors can be solved by the calibration process. Meanwhile, the traditional least-squares method was used to adjust photogrammetric data in order to solve the problems of random errors. In case observations contain gross errors, the reliability of least-squares estimates is strongly affected. In this paper, two independent mathematical models (photo-variant self-calibration and robust estimation) are combined for solving and processing the problems of systematic and gross errors in one step. Also, this paper investigated the effectiveness of robust estimation models on solving gross errors in close-range photogrammetric data sets that require photo bundle adjustment solution. The results of investigation indicate that all robust methods have the advantage of detecting and removing gross errors over the least squares method especially in cases of observation contains large-sized errors. Moreover, the Modified M-estimator (IGGIII) method has the best performance and accuracy. Furthermore, gross error was also revealed in the residuals.

***Keywords:*** Gross Errors; Least-square; Robust Estimation Methods; Close-Range Photogrammetry; Photo Bundle adjustment; Photo-Variant Self-Calibration.

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## 1. Introduction

Robust estimation methods were used in different surveying applications such as triangulation and leveling networks for the first time in 1964 [7,10]. Subsequently, several weight functions were developed for the robust estimators. Although most of these weight functions have no theoretical basis, weight functions were empirical [1,2,3,4]. The weight functions could be selected for robust estimates if only their values are less and residuals are larger. This is due to the measures of robustness for robust estimates being not unique [5]. Based on the least squares estimate, this research studies the robustness, defines a measurement for it and deduces the corresponding robust estimate. Robust estimation methods are able to simultaneous parameter estimation and outlier elimination during the estimation process [6]. If observation equations contain additional parameters to model the effect of systematic errors, then the use of iteratively reweighted least squares with an appropriately chosen estimation gives us a tool for simultaneous treatment of all errors [6,14]. The main objective of this research is to study the effect of gross errors on close-range photogrammetric data and the ability to detect and remove this type of errors using a combination model from photo-variant self-calibration and robust estimation.

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**2. Solving Gross Errors**

Close-range applications do not produce large data sets as in aerial triangulation. The failed process acquires data affected by gross errors. Data acquired with a non-metric camera for close-range applications should use a photo-variant bundle solution. Almost, the measurements are acquired with gross errors. Robust estimation methods are then suitable for use in adjusting these measurements. A robustfied bundle adjustment procedure has been developed along this direction [8]. This method showed the exact values of gross errors in the residuals. On the other hand, classical least-squares method distributed these gross errors to other un-affected measurements [9]. This research discusses the combination of photo-variant self-calibration and different robust estimation methods for processing close-range measurements.

**3. Image-Variant Parameters of Interior Orientation**

The photogrammetric data is influenced by systematic errors which may be lens distortion, film deformation, refraction, etc. The values of these types of errors can be modeled and determined from the camera calibration processes in close-range photogrammetry. Metric cameras have stable interior geometry over a period of time. The parameters solved by calibration are always carried as constant from photograph to another. An advanced data processing model allows for the distortions to vary from a photograph to another. This process is known as photo-variant solution [16]. The use of non-metric camera for close-range photogrammetry has been improved by many researchers [12,17,18]. Nowadays, digital consumer cameras of high-resolution are vastly available and used in close-range photogrammetry. The mechanical construction of this type of cameras oftentimes does not achieve the demands of close-range photogrammetry, so they have to be modeled sufficiently. A camera modeling is called image-variant parameters of interior orientation should be applied. Significant improvements of object accuracy have been achieved with respect to standard calibration techniques based on self-calibrating bundle adjustment [18].

***3.1. Camera Parameters Model (Image-variant parameters)***

Camera parameters of interior orientation are applied for all images of photogrammetric projects. Parameters of distortion are normally defined by the photo's principal point. So, the equation of the standard observation given by the following equations:

$$x - x_o + \Delta x_p = -f \frac{m_{11}(X_i - X_o) + m_{12}(Y_i - Y_o) + m_{13}(Z_i - Z_o)}{m_{31}(X_i - X_o) + m_{32}(Y_i - Y_o) + m_{33}(Z_i - Z_o)}$$

$$y - y_o + \Delta x_p = -f \frac{m_{21}(X_i - X_o) + m_{22}(Y_i - Y_o) + m_{23}(Z_i - Z_o)}{m_{31}(X_i - X_o) + m_{32}(Y_i - Y_o) + m_{33}(Z_i - Z_o)}$$

where $\Delta x_p$ , $\Delta y_p$ are offsets from the principle point to the centre of image frame, $f$ is the nominal camera focal length, ($X_o$ , $Y_o$ , $Z_o$) are the ground coordinates of the projection centre, ($X_i$ , $Y_i$ , $Z_i$) are the ground coordinates of point $i$ and $m_{11}$ to $m_{33}$ are elements of rotation matrix.

## 4. Robust Estimation Methods

Usually, the influence of systematic errors on photo- measurements can be solved by calibration reports. The conventional least squares method leads to unbiased estimates with minimum variance if the model is entirely correct [17]. But in case observations contain gross errors or single-point displacement, the outcome is no longer correct. These type of errors or displacements contaminate the estimates of parameters and are distributed over the whole residual vector. Hence, it is extremely difficult and sometimes impossible to locate blunders by screening the residuals. This disadvantage can be avoided, if a robust estimation method is applied. Robust estimates are not influenced by blunders as long as the majority of points conform to the modeled trend [13]. The mathematical models of the robust estimation methods used in this research are summarized in table (1).

**Table 1:** Models of robust estimation methods and their weight functions

| Robust method | Weight function | Critical values | Reference |
|---|---|---|---|
| **R-Estimators** | $Min \sum_{i=1}^{n} \alpha(R_i) \frac{v_i}{\sqrt{Q_{ii}}}$ where $R_i$ is the rank of weighted residuals and $\alpha$ is the score function | 1.5 $s_o$ : 2 $s_o$ | **[1]** |
| **S-Estimator** | $b = \frac{1}{n} \sum_{i=1}^{n} p\left(\frac{v_i}{\delta_o}\right)$ in which $\delta_o$ is the posteriori scale factor given by the robust estimator | 1.5 $s_o$ : 2 $s_o$ | **[21]** |
| **Danish** | $w_i \begin{cases} \exp(-v_i^2/c^2) & \lvert v_i \rvert > c \\ 1 & \lvert v_i \rvert \le c \end{cases}$ | 1.5 $s_o$ : 2 $s_o$ | **[3]** |
| **Andrews M-estimators** | $w_i \begin{cases} \sin(v_i/c)/(v_i/c) & \lvert v_i \rvert \le c\pi \\ 0 & \lvert v_i \rvert > c\pi \end{cases}$ | 1.5 $s_o$: 2 $s_o$ | **[2]** |
| **Modified M-estimator (IGGIII)** | $w_i \begin{cases} 1 & \lvert v_i \rvert \le c_o \\ \frac{c_o}{\lvert v_i \rvert} & c_o < \lvert v_i \rvert \le c_1 \\ 0 & \lvert v_i \rvert > c_1 \end{cases}$ | $c_o$ = 2.0 : 3.0<br>$c_1$ = 4.5 : 8.5 | **[20]** |
| **Huber** | $w_i \begin{cases} 1 & \lvert v_i \rvert \le c \\ c/\lvert v_i \rvert & \lvert v_i \rvert > c \end{cases}$ | 1.5 $s_o$: 2 $s_o$ | **[11]** |

Where $s_o$ is the a priori standard error of the unit weight given by:

$med\{\lvert\sqrt{p_i v_i}\rvert\}/0.6745,$

$v_i$ is the residual of observation, *med* is the median, $p_i$ is the weight of observation, $c_i$ is the constant critical value given by: $c_i = s_o \sqrt{Q_{vv}} . \sqrt{p_{ii}} . t_{f,1-\alpha/2}$ in which $Q_{vv}$ is the cofactor matrix of the residuals, $p$ is the weight matrix of the observations, *f* is the degree of freedom, $\alpha$ is the significance level, and *t* represents t-table. So, the critical value can be calculated as follows: $c = \sum_{i=1}^{n} c_i / n$ in which *n* is the number of observations.

## 5. Numerical Example

Cannon *digital IXUS 990 IS*, digital compact camera equipped with 5x zoom lens and has been calibrated at the widest view of its zoom lenses. It was also calibrated with a lens equivalent to 35 mm film format. A building was photographed with one stereo-pair by the target camera for the calibration purpose. The distance between the target camera and the photographed building was 15 m. the distance between the two projection centers was 3 m. The control and check points have been marked on the acquired image. The coordinates of the control and check points have been measured from the stereo-model. The differences between their coordinates derived from ground surveying technique and those defined by the photogrammetric process have been computed. A photogrammetric software is called BUNDLEH Lite [15,19] has been used for the process of non-metric camera calibration. Four values of gross errors: zero, 5 mm, 10 mm and 20 mm were added to one coordinate of an image point. The different robust methods in table (1) were used to process the data in turn while the photo-variant self-calibration mode was activated in the adjustment.

## 6. Results and Analysis

Results of the test are tabulated in tables (2), (3), (4) and (5) with the Root Mean Square Errors (RMSE) of check points when zero, 5 mm, 10 mm and 20 mm blunders were introduced to a point coordinates obtained from traditional least-squares and different robust estimation methods respectively.

**Table 2:** Root Mean Square Error (RMSE) of check points in case of zero gross error using different robust methods

| **Method of adjustment** | **RMSE** | | |
|---|---|---|---|
| | **X (mm)** | **Y (mm)** | **Z (mm)** |
| **Least Squares** | 0.28 | 0.25 | 0.79 |
| **R-Estimators** | 0.25 | 0.27 | 0.92 |
| **S-Estimator** | 0.22 | 0.23 | 0.75 |
| **Danish** | 0.12 | 0.14 | 0.83 |
| **Andrews** | 0.20 | 0.24 | 0.69 |
| **M-estimators** | | | |
| **Modified M-estimator (IGGIII)** | 0.16 | 0.15 | 0.50 |
| **Huber** | 0.36 | 0.41 | 0.94 |

**Table 3:** Root Mean Square Error (RMSE) of check points in case of 5 mm gross error using different robust methods

| Method of adjustment | RMSE | | |
|---|---|---|---|
| | X (mm) | Y (mm) | Z (mm) |
| **Least Squares** | 12.15 | 10.16 | 99.79 |
| **R-Estimators** | 0.35 | 0.38 | 1.03 |
| **S-Estimator** | 0.33 | 0.34 | 1.26 |
| **Danish** | 0.23 | 0.25 | 1.01 |
| **Andrews M-estimators** | 0.31 | 0.35 | 1.34 |
| **Modified M-estimator (IGGIII)** | 0.27 | 0.26 | 1.03 |
| **Huber** | 0.48 | 0.52 | 1.56 |

**Table 4:** Root Mean Square Error (RMSE) of check points in case of 10 mm gross error using different robust methods

| Method of adjustment | RMSE | | |
|---|---|---|---|
| | X (mm) | Y (mm) | Z (mm) |
| **Least Squares** | 25.15 | 30.16 | 134.45 |
| **R-Estimators** | 0.33 | 0.41 | 1.12 |
| **S-Estimator** | 0.36 | 0.36 | 1.34 |
| **Danish** | 0.28 | 0.29 | 1.11 |
| **Andrews M-estimators** | 0.37 | 0.38 | 1.35 |
| **Modified M-estimator (IGGIII)** | 0.29 | 0.29 | 1.12 |
| **Huber** | 0.52 | 0.55 | 1.58 |

**Table 5:** Root Mean Square Error (RMSE) of check points in case of 20 mm gross error using different robust methods

| Method of adjustment | RMSE | | |
|---|---|---|---|
| | X (mm) | Y (mm) | Z (mm) |
| **Least Squares** | 32.15 | 42.16 | 134.45 |
| **R-Estimators** | 0.35 | 0.43 | 1.45 |
| **S-Estimator** | 0.38 | 0.38 | 1.55 |
| **Danish** | 0.31 | 0.31 | 1.23 |
| **Andrews M-estimators** | 0.38 | 0.40 | 1.56 |
| **Modified M-estimator (IGGIII)** | 0.30 | 0.32 | 1.34 |
| **Huber** | 0.54 | 0.56 | 1.59 |

Statistical tests have been performed for detecting and decreasing the effect of gross errors and then the adjustment should be performed again. Least-squares method was carried out with blunder-free measurements as a reference adjustment. Six robust estimation methods were used to adjust the same measurements after adding blunders to a point coordinates.

It is easy to notice that the effect of zero or small-sized gross errors on the adjusted coordinates was minimal and can be neglected as shown in table (2). On the other hand, large-sized gross errors breakdown the adjusted coordinates completely as shown in tables (3), (4) and (5). Furthermore, gross errors were also included in the residuals as shown in tables (6) and (7). The results were not sufficiently accurate in case of using least-squares method for adjusting close-range photogrammetric data that contains blunder values of 10 mm and 20 mm, while there was an improvement in accuracy when robust estimators were used.

The Modified M-estimator (IGGIII), which is one of the robust estimation methods, gives better accuracy than other robust methods (see tables (3), (4) and (5)). From table (3), it can be noticed that the gross error was forbade from share in the solution, then the accuracy of the solution obtained earlier from the least squares method can be improved, provided sound geometry is still maintained. Table (8) shows that gross error was detected in the residual. The Modified M-estimator (IGGIII) method performance was the best for X, Y and Z coordinates without iterations.

Huber's estimation method has shown the least accurate results in both plane and height coordinates accuracy with few iterations. In case of large-sized blunders (10 mm and 20 mm), the Modified M-estimator (IGGIII) method has the best performance and accuracy without iterations.

**Table 6:** Robust Estimator for detecting gross errors of selected points on images in case of use 5 mm error

| Point-id | $V_X$ (mm) | Weight | $V_Y$ (mm) | Weight |
|---|---|---|---|---|
| **1** | 0.0004 | 0.685 | 0.0008 | 0.000 |
| **2** | 0.0000 | 1.000 | 0.0001 | 1.000 |
| **3** | -0.0009 | 1.000 | 0.0002 | 1.000 |
| **4** | 0.0006 | 1.000 | 0.0003 | 1.000 |
| **5** | 0.0005 | 1.000 | 0.0002 | 1.000 |
| **6** | 0.0007 | 1.000 | 0.0001 | 1.000 |
| **7** | 0.0003 | 1.000 | 0.0002 | 1.000 |
| **8** | 0.0003 | 1.000 | 0.0001 | 1.000 |
| **9** | 0.0003 | 1.000 | 0.0001 | 1.000 |
| **10** | 0.0029 | 0.000 | 0.0009 | 0.000 |
| **11** | 0.0003 | 1.000 | 0.0003 | 1.000 |
| **12** | 0.0004 | 1.000 | 0.0001 | 1.000 |
| **13** | 0.0002 | 1.000 | 0.0002 | 1.000 |
| **14** | -0.0005 | 1.000 | 0.0002 | 1.000 |
| **15** | 0.0002 | 1.000 | 0.0002 | 1.000 |

**Table 7:** Robust Estimator for detecting gross errors of selected points on the images in case of use 10mm error

| Point-id | $V_X$ (mm) | Weight | $V_Y$ (mm) | Weight |
|---|---|---|---|---|
| **1** | 0.0004 | 0.685 | 0.0007 | 0.423 |
| **2** | 0.0000 | 1.000 | -0.0001 | 0.965 |
| **3** | -0.0002 | 0.897 | -0.0007 | 0.453 |
| **4** | 0.0002 | 0.754 | 0.0001 | 0.990 |
| **5** | 0.0002 | 1.875 | 0.0002 | 0.954 |
| **6** | 0.0001 | 0.987 | 0.0001 | 0.989 |
| **7** | 0.0002 | 0.943 | 0.0005 | 0.897 |
| **8** | 0.0001 | 0.989 | 0.0004 | 0.912 |
| **9** | 0.0001 | 0.978 | 0.0003 | 0.876 |
| **10** | 0.0089 | 0.000 | 0.0000 | 1.000 |
| **11** | 0.0003 | 0.764 | 0.0009 | 0.345 |
| **12** | 0.0001 | 0.990 | 0.0007 | 0.453 |
| **13** | 0.0002 | 0.986 | 0.0006 | 0.654 |
| **14** | -0.0005 | 0.987 | 0.0008 | 0.534 |
| **15** | 0.0002 | 0.885 | 0.0005 | 0.867 |

**Table 8:** Robust Estimator for detecting gross errors of selected points on the images in case of use 20mm error

| Point-id | $V_X$ (mm) | Weight | $V_Y$ (mm) | Weight |
|---|---|---|---|---|
| **1** | 0.0005 | 0.355 | 0.0007 | 0.312 |
| **2** | 0.0000 | 1.000 | 0.0000 | 1.000 |
| **3** | -0.0002 | 1.000 | -0.0005 | 0.342 |
| **4** | 0.0001 | 1.000 | 0.0004 | 0.423 |
| **5** | 0.0001 | 1.000 | 0.0001 | 1.000 |
| **6** | 0.0002 | 1.000 | 0.0004 | 0.412 |
| **7** | 0.0002 | 1.000 | 0.0003 | 0.543 |
| **8** | 0.0002 | 1.000 | 0.0001 | 1.000 |
| **9** | 0.0002 | 1.000 | 0.0003 | 0.654 |
| **10** | 15.432 | 0.000 | 0.0006 | 0.353 |
| **11** | 0.0002 | 1.000 | 0.0005 | 0.342 |
| **12** | 0.0002 | 1.000 | 0.0006 | 0.234 |
| **13** | 0.0001 | 1.000 | 0.0004 | 0.542 |
| **14** | -0.0001 | 1.000 | 0.0008 | 0.234 |
| **15** | 0.0002 | 1.000 | 0.0002 | 0.864 |

## 7. Conclusion

The main objective of this paper was to apply different methods of robust estimation with the classical least-square method for detecting and solving the gross errors in close range-photogrammetric data. For this purpose, one stereo-pair terrestrial photos were taken by non-metric digital camera. A calibration process was also carried out using BUNDLEH Lite software. Four values of gross errors: zero, 5 mm, 10 mm and 20 mm were added to one coordinate of an image point. The different robust methods in table (1) were used to process the data in turn while the photo-variant self-calibration mode was activated in the adjustment. The results indicate that the effect of zero or small-sized gross errors on the adjusted coordinates was minimal and can be neglected. Large-sized gross errors breakdown the adjusted coordinates completely. Gross errors were also included in the residuals. The results were not sufficiently accurate in case of using least squares method for adjusting close-range data that contains 10 mm and 20 mm, while there was an improvement in accuracy when robust estimators were used. The Modified M-estimator (IGGIII), which is one of the robust estimation methods, gives better accuracy than other robust methods.

## References

[1]. Andersen R. (2008). Modern Methods for Robust Regression. Sag Publications, London.

[2]. Andrews D.F (1974). A Robust Method for Multiple Linear Regression. Technometrics, 16.

[3]. Berberan A (1995). Multiple Outlier Detection, A Real Case Study. Survey Review, 33, pp. 41-49.

[4]. Faig, W. and K. Owolabi (1988a). Distributional Robustness in Bundle Adjustment. Proceedings of ACSM-ASPRS Annual Convention, St. Louis.

[5]. Faig W. and K. Owolabi (1988b). The Effect of Image Point Density on Photo-variant and Photo-invariant Bundle Adjustment. International Archives of Photogrammetry and Remote Sensing, XVI(A)3, Comm III.

[6]. Hogg R.V (1979). Statistical Robustness: One View of its use in Application Today. The American Statistician, 33(3).

[7]. Holland P.W and R.E Welsch (1977). Robust Regression Using Iteratively Reweighted Least Squares. Communications in Statistics, A6.

[8]. Huber P.I (1964). Robust Estimation of Location Parameter. Annals of Mathematical Statistics, 35.

[9]. Huber P.I (1972). Robust Statistics: A Review, the 1972 WALD Lecture. Annals of Mathematical Statistics, 43.

[10]. Huber P.I (1973). Robust Regression: Asymptotic, Conjectures and Monte Carlo. Annals of Statistics, 1.

[11]. Huber P.I (1977). Robust Methods of Estimation of Regression Coefficients. Mathematic Operations for stung Statistic, Series Statistics, 8(1).

[12]. Karara H. M and W. Faig (1980). An Expose on Photogrammetric Data Acquisition Systems in Close-range Photogrammetry. International Archives of Photogrammetry, XIV(5).

[13]. Kilpela E (1980). Compensation of Systematic Errors in Bundle Adjustment. International Archives of Photogrammetry, 23(B9).

[14]. Kraus K. (2004). Photogrammetrie, Band 3, Topographische Informations systeme. Dümmler Verlag,

Bonn, pp. 188.

[15]. Kubik K. and D. Merchant (1986). Photogrammetric Work without Blunders. Proceedings American Society of Photogrammetry and Remote Sensing, Fall Convention, Baltimore.

[16]. Luhmann T, Wendt K (2000). Recommendations for an Acceptance and Verification Test of Optical 3-D Measurement Systems. International Archives for Photogrammetry and Remote Sensing, 33(5), pp. 493-499, Amsterdam.

[17]. Mevlut Y (2017). Application of robust estimation in geodesy using the harmony search algorithm. Spatial Science, 63 (13): pp. 1-11, DOI: 10. 1080 / 14498596. 2017. 1341856.

[18]. Sergio B (2007). Global Optimization Solution of Robust Estimation. Journal of Surveying Engineering, 133 (3), DOI: 10.1061/(ASCE)0733-9453(2007)133:3(123).

[19]. Xie L, Jingxue W, Min C. and Han H (2016). An Asymmetric Re-Weighting Method for the precision Combined Bundle Adjustment of Aerial Oblique Images. Journal of Photogrammetry and Remote Sensing, 117, pp. 92-107, DOI: 10.1016/j. isprsjprs. 2016. 03. 017.

[20]. Yang Y, song L, and Xu T (2001). Robust Estimator for the Adjustment of Correlated GPS Networks. First International Symposium on Robust Statistics and Fuzzy Techniques in Geodesy and GIS, March 12-16, Zurich/Switzerland.

[21]. Yohai VJ, Rousseeuw PJ (1984). Robust Regression by Means of S-Estimator. Lecture Notes in Statistics, No. 26, Springer-Verlag, New York, pp. 256-272.