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Mixture and Non-Mixture Bayesian Hierarchical Study of Seizure Count Data Using New Generalized Poisson Model

Jamal A. Al-saleh[a*], Satish K. Agarwal[b]

[a]*Department of Statistics and Operation Research, Faculty of Science, Kuwait University, P.O. Box 5969, Safat, Kuwait*

[b]*Department of Mathematics, College of Science, University of Bahrain, P.O. Box 32038, Bahrain*

[a]*Email: jamal.alsaleh@ku.edu.kw; alsalehj1@yahoo.com*

[b]*Email: skagarwal@uob.edu.bh; satishagarwal123@yahoo.com*

Abstract

In this paper Bayesian methods is performed on a medical trial Seizure count data set by introducing the new three parameter generalized Poisson model GPM(α,β,λ) as an alternative model to the standard Poisson model SPM(λ) which is considered on an earlier work for the generalized linear mixed model. The new model is developed by introducing two more parameters α and β called indicator parameters. The main advantage of an indicator parameter is that it gives the new Poisson model the mixture (when α>0,β=1,2) and non-mixture (when α=0) options. Another feature of proposed new model is that it generalize the posterior of the parameters to predict the behavior of the Seizure counts data, in agreement with generalized linear mixed model. Unlike earlier authors, who confined and limited their work only on standard Poisson model SPM(λ), to analyze the counts data in generalized linear mixed model, which make the new model more resilience and litheness. The parameters of the new model will be estimated using Bayesian approach that serves as a subtle tool for model selection and identification. An illustration is provided using the Seizure count data. The posterior summaries using Markov Chain Monte Carlo (MCMC) Gibbs sampling approach are presented for the new model for different values of the parameters. The study of the estimated parameters would help the users to have more prospect and clarity about the role of the new model. It is found that using proposed new model in generalized linear mixed model has more resiliency than standard Poisson model considered earlier. The proposed model is fully adaptive to the available data and gives scientists another option for modeling the data.

Key words: Bayesian predictions; generalized Poisson model; generalized posterior; Gibbs sampling; Hierarchical model; Markov Chain Monte Carlo.

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* Corresponding author.

1. Introduction

Multiple problems in applied statistical data counts research hinder the usage and application of standard Poisson model, and preferably circulate to a more generalized and extended setting of the Poisson model. This claim is well established in literature, see for example the work of [3,4,5,6,7,8,9].

The main contribution of this paper is to introduce an efficient and more resilience computational Bayesian approach by introducing a new generalized statistical model with three parameters called generalized Poisson model GPM(α,β,λ) and compare it using clinical trial counts data analysis, with the standard Poisson model SPM(λ) implemented in [[2] model III]. The role of new parameters in the new generalized model as indicator parameters is to select, identify the more fit, more realistic, and genuine model for the data, and study the problem involving generalized linear mixed model of uncertain events in more extensive and comprehensive setting. In real life situation, problems involving new generalized and extended statistical model based prediction are well suited using the Bayesian methodology [see [10,11,12,13]]. The motivation of this work is to explore these new statistical models which can be implemented to provide an adequate fit for the real data than well-known available models. The Markov chain Monte Carlo (MCMC) Gibbs sampling methods are used to simulate direct draws from the new statistical models of interest. In section 2, we have proposed new generalized linear mixed model that considers the new generalized Poisson model GPM(α,β,λ) and introduce some of its properties. In section 3, we have developed the procedure to estimate the parameters of the generalized linear mixed model involving the new generalized Poisson model using Bayesian methodology. The Bayesian estimates of the parameters are obtained using Markov Chain Monte Carlo (MCMC) simulation technique based on the assumption that priors are independent, The generalized posterior analysis is performed and estimated. We have examined the issue of model compatibility with the work of [[2] model III] using new predictive results. A real medical trial Seizure count data set [see [1]] are analyzed for illustrating the application and the proposed Bayesian approach.

2. The model

In this section a new three parameter generalized Poisson model GPM(α,β,λ) is introduced with probability function

$$(2.1)\quad f_{\alpha,\beta,\lambda}(x) = \left[\frac{1}{C_{\alpha,\beta,\lambda}}\right] \lambda^x \left[\frac{1+\alpha x}{1+\alpha\lambda}\right]^{\beta} \frac{e^{-\lambda}}{x!},\ \ \lambda>0,\ \alpha\geq 0,\ \beta=1,2,\ \text{and } x= 0,1,2,..,$$

where

$$C_{\alpha,\beta,\lambda} = \sum_{x=0}^{\infty} \lambda^x \left[\frac{1+\alpha x}{1+\alpha\lambda}\right]^{\beta} \frac{e^{-\lambda}}{x!}.$$

When α =0, the mean in (2.1) is E[x]=λ, and when α >0, β=1, we have $C_{\alpha,1,\lambda}$=1, and the nean is

$$(2.2)\quad E[x]= \frac{\alpha\lambda}{1+\alpha\lambda}+\lambda$$

The mean when α >0, β=2, is

$$(2.3)\quad E[x]= \frac{(\alpha(1+2\lambda)+2)\alpha\lambda}{1+\lambda(1+\lambda)\alpha^2+2\alpha\lambda}+\lambda.$$

In some particular cases the parameters α, and β of model (2.1) can be seen as providing not only an extra flexibility to the probability function, but also helps to express probability distribution as an exact form of mixture of probability distributions under certain conditions. We should emphasize that eqn (2.1) can be reduced to standard Poisson model ($f_{0,\beta,\lambda}(x)=Poisson(\lambda)$).

The generalized linear mixed model (model III) considered by [2] is generalized by using the new probability function (2.1) when α≥ 0, and β =1, 2. For distinctness, the model is explained through the data from [1] concerning seizure counts in a randomized medical trial of anti-conversant therapy in epilepsy. For ready reference, the data is reproduced in Table A (see Appendix) which shows the seizure counts for 59 hospitalized patients. The covariates are treatment (0=Placebo, 1=Progabide drug), 8-week baseline seizure counts, and age in years. While considering the model, we used the same transformation which [2] considered in their (model III). For example, "Base" in the data set is transformed to log(Base/4), Age to log(Age), and the treatment times log(Base/4) where their interaction is included. To test the new model (2.1), we also considered the random effects for both individual subjects $SS1_j$ and also subject by visit random effects SS_{jk} variability within subjects. V4 is an indicator variable for the 4th visit. The model considered below leads to a Markov chain that is highly correlated with poor convergence properties. In order to overcome this poor convergence property, each covariate is standardized about its mean to ensure approximate prior independence between the regression coefficients as shown below:

(i) SPMM : Standard Poisson Mixed Model [see [2] model III**]**

(1) $y_{jk} \sim (\lambda_{jk})^{x_{jk}} \frac{e^{-\lambda_{jk}}}{x_{jk}!}$,

(2) $log(\lambda_{jk}) = C_0 + C_{Base}\, log\left[\frac{Base_j}{4}\right] + C_{Trt}\, Trt_j + C_{BT} Trt_j\, log\left[\frac{Base_j}{4}\right] + C_{Age}\, Age_j + C_{V4} V4 + S\,S1_j + SS_{jk}$

(3) $SS1_j \sim Normal(0, t_{b1})$

(4) $SS_{jk} \sim Normal(0, t_b)$

(ii) GPMM 1: Generalized Poisson Mixed Model 1

(1) $y_{jk} \sim (\lambda_{jk})^{x_{jk}} \left[\frac{1+\alpha_{jk}x_{jk}}{1+\alpha_{jk}x_{jk}}\right] \frac{e^{-\lambda_{jk}}}{x_{jk}!}$,

(2) $log(\frac{\alpha_{jk}\lambda_{jk}}{1+\alpha_{jk}\lambda_{jk}}+\lambda_{jk}) = C_0 + C_{Base}\, log\left[\frac{Base_j}{4}\right] + C_{Trt}\, Trt_j + C_{BT} Trt_j\, log\left[\frac{Base_j}{4}\right] + C_{Age}\, Age_j + C_{V4} V4 + SS1_j + SS_{jk}$

(3) $SS1_j \sim Normal(0, TS1)$

(4) $SS_{jk} \sim Normal(0, TS)$

(iii) GPMM 2: Generalized Poisson Mixed Model 2

(1) $y_{jk} \sim (\lambda_{jk})^{x_{jk}} \left[\frac{1+\alpha_{jk}x_{jk}}{1+\alpha_{jk}\lambda_{jk}}\right]^2 \frac{e^{-\lambda_{jk}}}{x_{jk}!}$,

(2) $log\left[\left[\frac{(\alpha_{jk}(1+2\lambda_{jk})+2)\alpha_{jk}\lambda_{jk}}{1+\lambda_{jk}(1+\lambda_{jk})\alpha_{jk}{}^2+2\lambda_{jk}\alpha_{jk}}\right] + \lambda_{jk}\right]$

$= C_0 + C_{Base}\, log\left[\frac{Base_j}{4}\right] + C_{Trt}\, Trt_j + C_{BT} Trt_j\, log\left[\frac{Base_j}{4}\right] + C_{Age}\, Age_j + C_{V4} V_4 + SS1_j + SS_{jk}$

(3) $SS1_j \sim Normal(0, TS1)$

(4) $SS_{jk} \sim Normal(0, TS)$

We should emphasis that all coefficients and precisions of model (i)-(iii) are given independent "non-informative" priors.

3. Bayesian updating prediction data analysis

A realistic Bayesian model for the Seizure count data is to suggest the following hierarchical model:

(a) At the first stage we assume that the count data follow the SPM, GPM 1, and GPM 2, respectively.

(b) At the second stage we assume the following prior specification for the parameter *α~exponential(0.1)*, and also we assume the following prior specifications

C0 ~ Normal(0.0,1.0E-5)

CBase ~ Normal(0.0,1.0E-5)

C_{Trt} *~ Normal(0.0,1.0E-5);*

CBT ~ Normal(0.0,1.0E-5)

CAge ~ Normal(0.0,1.0E-5)

CV4 ~Normal(0.0,1.0E-5)

SS1 ~ Gamma(1.0E-4,1.0E-4); where SS1 $= \frac{1}{\sqrt{TS1}}$

SS ~ Gamma(1.0E-4,1.0E-4); where SS $= \frac{1}{\sqrt{TS}}$

A Markov Chain Monte Carlo (MCMC) Gibbs sampling approach implemented in using OPENBUGS[@] computer software can give an analysis of estimates of each parameter.

A burn in of 1000 updates followed by a further 20k updates is implemented. The table 3.1, represent the coefficient estimates for SPMM, the table 3.2, represent the coefficient estimates for GPMM 1, where the table 3.3, represent the coefficient estimates for GPMM 2, along with standard deviation, mean and MC error.

	Table 3.1: Bayesian summary for α=0, Model SPMM			**Table 3.2: Bayesian summary for α>0, β=1, Model GPMM 1**			**Table 3.3: Bayesian summary forα>0, β=2, Model GPMM 2**		
	Mean	SD	MC error	Mean	SD	MC error	Mean	SD	MC error
CAge	0.4657	0.3684	0.01396	0.2941	0.484	0.04712	-0.6236	0.1239	0.01227
CBT	0.3385	0.2151	0.01208	0.1293	0.1821	0.01776	-0.1736	0.1839	0.01876
CBase	0.8786	0.1462	0.008372	0.9292	0.2052	0.01994	1.198	0.1542	0.01504
CTrt	-0.9357	0.4251	0.02097	-0.6574	0.3822	0.03727	-0.1222	0.3136	0.03208
CV4	-0.1022	0.0869	0.001852	-0.1292	0.09721	0.008823	-0.1712	0.09761	0.009652
C0	-1.332	1.248	0.04975	0.4442	1.765	0.1721	2.862	0.3045	0.03032
SS	0.3647	0.0561	0.002633	0.35	0.05668	0.004067	0.447	0.07737	0.006636
SS1	0.4989	0.0730	0.002762	0.5969	0.1159	0.008981	0.6556	0.08867	0.004404

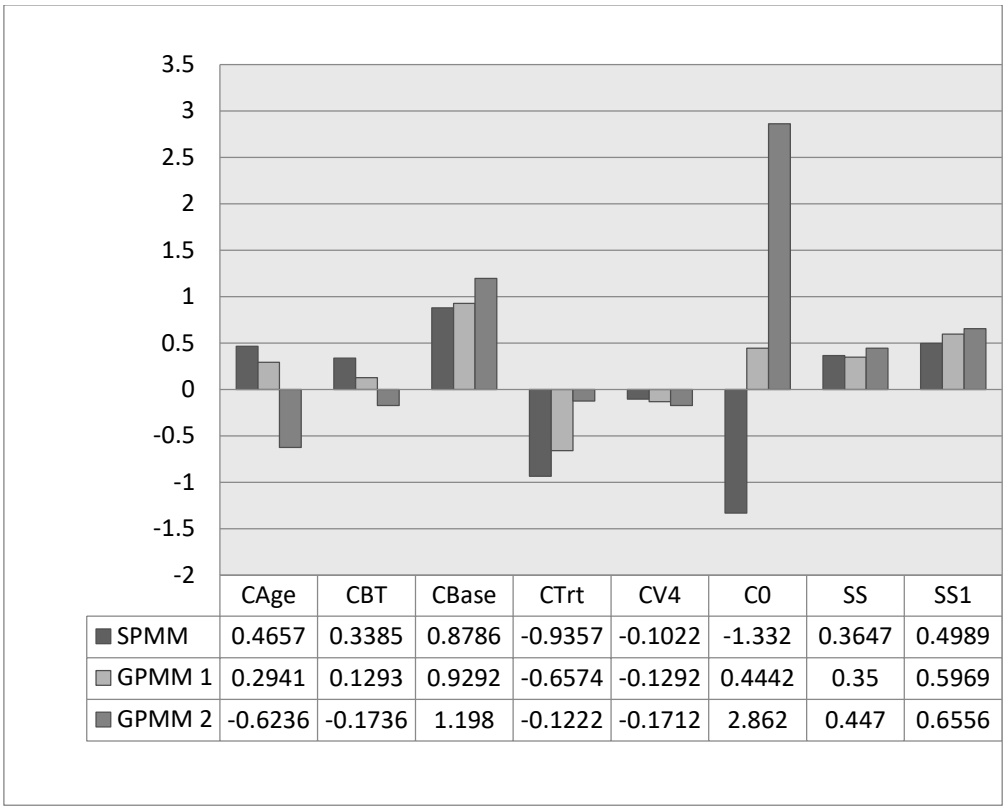

Figure 1: Comparisons of Seizure count data estimates between models SPMM, GPMM 1, and GPMM 2

Table 3.4: Bayesian summary estimates for α, whe $\alpha_{jk} \geq 0$, β=1 Model GPMM 1

j	$\hat{\alpha}_{j1}$	$\hat{\alpha}_{j2}$	$\hat{\alpha}_{j3}$	$\hat{\alpha}_{j4}$
1	0.8823	1.08	1.132	1.062
2	1.087	0.8857	1.078	1.066
3	1.082	0.8871	0.9573	0.7573
4	0.9502	0.9375	1.199	0.9295
5	1.499	0.7103	1.275	0.5273
6	1.104	1.373	0.8225	0.8517
7	0.7375	0.9802	0.9626	1.148
8	0.4216	1.021	0.9892	1.376
9	1.093	1.014	0.9765	1.045
10	0.4931	0.5497	1.039	1.087
11	0.543	1.223	1.688	0.5752
12	0.6814	1.177	0.9981	1.257
13	1.086	1.052	0.8331	1.233
14	1.34	1.118	0.9119	0.6844
15	0.9234	0.5582	1.339	1.297
16	0.5096	1.069	1.063	0.9632
17	0.943	0.9274	1.007	1.004
18	0.811	1.033	1.041	0.8966
19	1.171	0.9419	1.26	0.9103
20	1.16	1.037	0.8249	0.6609
21	1.1	1.016	1.105	0.9467
22	1.086	0.9936	1.086	0.9526
23	1.2	1.137	1.105	0.8738
24	0.926	0.6431	1.481	0.8669
25	1.54	1.19	0.28	1.05
26	1.07	1.135	1.079	1.09
27	1.032	1.159	0.9099	1.094
28	0.9833	0.8421	1.013	0.9546
29	0.9596	0.759	1.094	1.127
30	0.9239	1.031	0.8298	1.214
31	0.9021	0.89	0.9791	0.9116
32	1.04	0.7122	1.145	1.007
33	1.243	0.8326	0.7402	1.042
34	0.962	1.084	1.231	1.046
35	0.6823	0.9127	0.8105	0.8421
36	0.9982	1.047	0.7897	1.036
37	1.099	0.8714	0.954	0.876
38	1.407	1.019	1.018	0.9441
39	1.286	0.3608	1.471	1.111
40	1.034	1.074	1.108	0.895
41	0.9065	1.059	0.8691	0.9005
42	0.8442	0.93	0.9706	1.025
43	1.293	1.062	0.555	0.8473
44	0.7172	1.129	1.345	0.7959
45	0.4504	1.157	1.273	1.121
46	1.13	1.102	1.04	0.9358
47	1.141	0.8311	0.9854	0.9184
48	1.087	1.099	0.9017	0.8808
49	0.5053	1.041	0.874	0.9479
50	0.9708	1.093	1.211	0.9566
51	0.8744	1.04	1.193	0.9136
52	1.213	1.073	1.224	0.8332
53	0.8828	1.294	0.5057	1.063
54	0.7711	1.117	0.9973	0.983
55	1.086	0.8802	0.9749	1.059
56	1.644	0.3578	0.4772	1.012
57	1.092	0.9596	0.9162	1.123
58	0.9181	0.909	0.9127	0.904
59	1.142	0.9079	1.001	1.093

Table 3.5: Bayesian summary estimates for α, whe $\alpha_{jk} \geq 0$, β=2 Model GPMM 2

j	$\hat{\alpha}_{j1}$	$\hat{\alpha}_{j2}$	$\hat{\alpha}_{j3}$	$\hat{\alpha}_{j4}$
1	1.031	1.202	1.215	1.21
2	1.209	1.018	1.259	1.208
3	1.278	1.123	0.6939	0.9935
4	1.089	1.113	1.207	1.115
5	1.408	0.8496	1.343	0.6483
6	1.17	1.342	0.9277	0.9533
7	0.8745	1.137	0.6919	1.305
8	0.5043	0.9518	0.9845	1.236
9	1.148	1.086	1.068	1.128
10	0.5782	0.6268	1.102	0.6778
11	0.6367	1.15	1.546	0.629
12	0.7762	1.188	1.057	1.307
13	1.184	1.177	0.9624	1.283
14	1.263	1.122	0.943	0.7398
15	0.9914	0.7048	1.344	1.288
16	0.6389	0.6469	0.6708	1.05
17	0.6931	0.7021	1.235	1.243
18	0.833	1.097	1.025	0.9002
19	1.175	1.065	1.286	1.059
20	1.259	0.6647	1.005	0.8706
21	1.214	1.135	1.209	1.155
22	1.26	1.162	1.247	1.139
23	1.285	1.243	1.227	1.041
24	1.018	0.7938	1.435	0.9764
25	1.4	1.175	0.3752	0.9974
26	1.359	1.246	1.312	1.294
27	1.21	1.199	1.106	1.312
28	1.039	0.9325	1.072	0.9867
29	1.048	0.8666	1.191	1.173
30	0.9511	1.057	0.911	1.251
31	0.6809	1.141	1.268	0.6667
32	1.224	0.922	1.183	1.173
33	1.29	1.024	0.9364	1.175
34	1.156	1.217	1.224	1.188
35	0.7561	0.9618	0.8775	0.8733
36	1.093	1.197	0.917	1.162
37	1.276	1.16	0.6969	1.103
38	1.388	1.091	1.074	1.017
39	1.271	0.4717	1.384	1.188
40	1.365	1.341	1.358	0.7597
41	0.7086	1.28	1.131	0.712
42	1.011	1.15	0.6643	1.22
43	1.197	1.012	0.6524	0.8668
44	0.8456	1.231	1.332	0.9322
45	0.5273	1.148	1.232	1.086
46	1.258	1.281	1.317	1.259
47	1.224	0.9063	1.045	0.9752
48	1.354	1.342	0.7529	0.7762
49	0.7273	0.8665	0.7551	0.8251
50	1.143	1.273	1.317	1.124
51	1.002	1.141	1.199	1.022
52	1.199	1.232	1.186	1.05
53	0.9191	1.234	0.5913	1.006
54	0.9328	1.222	1.147	0.6903
55	1.246	1.038	1.155	1.244
56	1.336	0.4808	0.5539	1.086
57	1.275	1.257	0.6867	1.259
58	0.7632	0.7869	0.8097	0.8158
59	1.215	1.083	1.198	1.303

We should emphasizes that the estimates in (Tables 3.1-3.5 and Figure 1) give more information about the behavior of seizure counts data than that of [2] who considered SPMM only in their work. This can be easily seen, by comparing the above results with their reported estimates, and they are: CAge= 0.47 +/- 0.35, CBT= 0.34 +/- 0.21, CBase = 0.86 +/- 0.13, CTrT= -0.93 +/- 0.40, CV4= -0.10 +/- 0.90, C0 = -1.27 +/- 1.2, SS1 = 0.48 +/- 0.06, and SS = 0.36+/-0.04.

Examination of the above simulations yields the following observations:

1. The posterior mean of the estimate CAge of models SPMM, GPMM 1, and GPMM 2 are 0.4657, 0.2941, and -0.6236, respectively. There is a clear and substantial shift of the posterior mean to the left. The posterior standard deviation (SD) is 0.3684, 0.484 and 0.1239, respectively, and hence a decrease in posterior SD. Comparison of the MC error for SPMM, GPMM 1 and GPMM 2 shows that the MC error are about the same.
2. The posterior mean of the estimate CBt of models SPMM, GPMM 1, and GPMM 2 are 0.3385, 0.1293, and -0.1736 respectively. There is a clear and substantial shift of the posterior mean to the left. The posterior standard deviation (SD) is 0.1462, 0.1821 and 0.1839, respectively, and hence about the same result in posterior SD. Comparison of the MC error for SPMM, GPMM 1 and GPMM 2 shows also, that the MC error are about the same.
3. The posterior mean of the estimate CBase of models SPMM, GPMM 1, and GPMM 2 are 0.8786, 0.9292 and 1.198, respectively. There is a slight shift of the posterior mean to the right. Comparison of the posterior standard deviation (SD) and the MC error for SPMM, GPMM 1 and GPMM 2 shows that they are about the same.
4. The posterior mean of the estimate CTrt of models SPMM, GPMM 1, and GPMM 2 are -0.9357, -0.6574 and -0.1222, respectively. There is a clear and substantial shift of the posterior mean to the right. The posterior standard deviation (SD) is 1.248, 1.765 and 0.3045, respectively, and hence a decrease in posterior SD. Comparison of the posterior standard deviation (SD) and the MC error for SPMM, GPMM 1 and GPMM 2 shows that they are about the same.
5. The posterior mean of the estimate CV4 of models SPMM, GPMM 1, and GPMM 2 are -0.1022, -0.1292 and -0.1712, respectively. There is a slight shift of the posterior mean to the lift. The posterior standard deviation (SD) is 1.248, 1.765 and 0.3045, respectively, and hence a decrease in posterior SD. Comparison of the posterior standard deviation (SD) and the MC error for SPMM, GPMM 1 and GPMM 2 shows that they are about the same.
6. The posterior mean of the estimate C0 of models SPMM, GPMM I, and GPMM 2 are -1.332, 0.4442 and 2.862, respectively. There is a clear and substantial shift of the posterior mean to the right. The posterior standard deviation (SD) is 1.248, 1.765 and 0.3045, respectively, and hence a decrease in posterior SD. Comparison of the MC error for SPMM, GPMM I and GPMM 2 shows that the MC error are about the same.
7. The posterior mean of the estimate SS of models SPMM, GPMM I, and GPMM 2 are 0.3647, 0.35 and 0.447, respectively. There is a slight shift of the posterior mean to the right. Comparison of the posterior standard deviation (SD) and MC error for SPMM, GPMM I and GPMM 2 shows that the MC error are about the same.

8. The posterior mean of the estimate SS1 of models SPMM, GPMM I, and GPMM 2 are -1.332, 0.4442 and 2.862, respectively. There is a clear and substantial shift of the posterior mean to the right. The posterior standard deviation (SD) is 1.248, 1.765 and 0.3045, respectively, and hence a decrease in posterior SD. Comparison of the MC error for SPMM, GPMM I and GPMM 2 shows that the MC error are about the same.
9. The posterior mean of the estimate α of models GPMM 1 (table 3.4) vary between (0.4216, 1.54) in the first two weeks of treatments, for second two weeks of treatments it vary between (0.3608, 1.373), for third two weeks of treatments it vary between (0.28, 1.47), and for fourth two weeks of treatments it vary between (0.5273, 1.376). This indicate that the seizure counts data are mixing in the generalized linear mixed model GPMM 1. These findings do not sport the work done by [2] using SPM(λ). We also note the following embodiment: (i) patient (No. 8) is an interesting subject, where for the first two weeks of treatments, he/she has the lowest estimated value α at 0.4216 (with high number of seizure counts at 40 counts), at the second two weeks of treatments, the estimate increased to 1.021 (number of seizure counts decreased to 20 counts), at the third two weeks of treatments, he/she has the estimate at 0.8982 (number of seizure counts increased by one count to 21 counts), and at the fourth two weeks of treatments, he/she has the estimate at 1.376 which is the highest estimate in the fourth two week treatments group (with a drop in the number of seizure counts to 12 counts). This maybe, related to the factor effect of either baseline data on the number of epileptic seizures which is the next highest at 52 counts, and/or to his/her age at 42 years old (which is the highest in the age group). (ii) patient (No. 25) where for the first two weeks of treatments, he/she has the highest estimated value α at 1.54 (with number of seizure counts at 18 counts), at the second two weeks of treatments, the estimate is 1.19 (number of seizure counts increased to 24 counts), at the third two weeks of treatments, he/she has the lowest estimate of the group at 0.28 (number of seizure counts jumped at 76 counts), and at the fourth two weeks of treatments, he/she has the estimate at 1.05 (with a drop in the number of seizure counts to 25 counts). This may be, related to the factor effect of either baseline data on the number of epileptic seizures which is high at 55 counts, and/or to his/her age at 30 years old . (iii) patient (No. 39) where for the first two weeks of treatments, he/she has the estimated value α at 1.286 (with low number of seizure counts at 4 counts), at the second two weeks of treatments, he/she has the lowest estimate at 0.368 (number of seizure counts increased to 18 counts), at the third two weeks of treatments, he/she has the highest estimate of the group at 1.471 (number of seizure counts dropped to 2 counts), and at the fourth two weeks of treatments, he/she has the estimate at 1.111 (with a slight increase in the number of seizure counts to 5 counts). This maybe, related to the factor effect of either baseline data on the number of epileptic seizures which is at 41 counts, and/or to the type of drug treatment (Progabide drug), and/or to his/her age at 22 years old.
10. The posterior mean of the estimate α of models GPMM 2 (table 3.5) vary between (0.5043,1.408) in the first two weeks of treatments, for second two weeks of treatments it vary between (00.4717,1.341), for third two weeks of treatments it vary between (0.5539,1.546), and for fourth two weeks of treatments it vary between (0.629,1.312). Which as indicated earlier for GPMM 1 the seizure counts data are mixing in the generalized linear mixed model GPMM 2 and hence, they do not sport the work done by [2] Breslow and Clayton (1993) using SPM(λ). We also note the following embodiment: (i)

patient (No. 5), for the first two weeks of treatments, he/she has the highest estimated value α at 1.408 (with low number of seizure counts at 7 counts), at the second two weeks of treatments, the estimate deccreased to 0.8496 (number of seizure counts increased to 18 counts), at the third two weeks of treatments, he/she has the estimate at 1.343 (number of seizure counts decreased 9 counts), and at the fourth two weeks of treatments, he/she has the estimate at 0.6843 (with a jump in the number of seizure counts to 21 counts). This maybe, related to the factor effect of either baseline data on the number of epileptic seizures which is the next high at 66 counts, and/or to his/her age at 22 years old. (ii) patient (No. 11) where for the first two weeks of treatments, he/she has the estimated value α at 0.6367 (with number of seizure counts at 26 counts), at the second two weeks of treatments, the estimate is 1.15 (number of seizure counts decreased to 12 counts), at the third two weeks of treatments, he/she has the highest estimate of the group at 1.516 (number of seizure counts down to 6 counts), and at the fourth two weeks of treatments, he/she has the estimate at 0.629 which is lowest in the group (with an increase in the number of seizure counts to 22 counts). This maybe, related to the factor effect of either baseline data on the number of epileptic seizures which is high at 52 counts, and/or to his/her age at 36 years old. (iii) patient (No. 39) where for the first two weeks of treatments, he/she has the estimated value α at 1.271 (with low number of seizure counts at 4 counts), at the second two weeks of treatments, he/she has the lowest estimate at 0.4717 (number of seizure counts increased to 18 counts), at the third two weeks of treatments, he/she has the estimate at 1.384 (number of seizure counts dropped to 2 counts), and at the fourth two weeks of treatments, he/she has the estimate at 1.188 (with a slight increase in the number of seizure counts to 5 counts). This maybe, related to the factor effect of either baseline data on the number of epileptic seizures which is at 41 counts, and/or to the type of drug treatment (Progabide), and/or to his/her age at 22 years old. (vi) patient (No. 40) where for the first two weeks of treatments, he/she has the estimated value α at 1.365 (with low number of seizure counts at 2 counts), at the second two weeks of treatments, he/she has the highest estimate of the group at 1.341 (number of seizure counts decreased to 1 counts), at the third two weeks of treatments, he/she has the estimate at 1.358 (number of seizure counts stayed at 1 counts), and at the fourth two weeks of treatments, he/she has the estimate at 0.7597 (with a slight decrease in the number of seizure counts to 0 counts). This maybe, related to either baseline data on the number of epileptic seizures which is at low 7 counts, and/or to the type of treatment (Progabide), and/or to his/her age at 28 years old. (v) patient (No. 56) for the first two weeks of treatments, he/she has the estimated value α at 1.336 (with low number of seizure counts at 1 counts), at the second two weeks of treatments, he/she has the estimate at 0.4808 (number of seizure counts increased to 23 counts), at the third two weeks of treatments, he/she has the lowest estimate of the group at 0.5539 (number of seizure counts dropped to 19 counts), and at the fourth two weeks of treatments, he/she has the estimate at 1.086 (with a decrease in the number of seizure counts to 8 counts). This maybe, related to the factor effect of either baseline data on the number of epileptic seizures which is at 22 counts, and/or to the type of drug treatment (Progabide), and/or to his/her age at 26 years old.

In brief, the values of the posterior means of estimates vary to some extent across the results for models SPMM, GPMM 1, and GPMM 2. For a few estimators, the values are similar. However, the differences for CAge, CBt,

CTrt and C0 are dramatic. The difference is clearer in the case when $\alpha>0$ (mixture model) compared to $\alpha=0$ (non-mixture model). Hence we think, in the above illustration, the analysis using the new generalized Poisson model for the Seizure count data seems more successful than the standard Poisson model ($\alpha=0$) considered by [2]. The proposed class of new generalized distributions offers more flexibility for Bayesian methods to choose among the existing classes of distribution models.

4. Conclusion

In this paper we investigated the impact of having a new three parameter generalized Poisson probability model GPM(α,β,λ) as an alternative model to the standard Poisson model SPM(λ) in (model III) of [2] generalized linear mixed model. We have shown the importance and usefulness of the new GPMM through the Seizure count data set, which are available and used by authors in the past. Another feature of proposed new generalized linear mixed model, is that under Bayesian perspective, it generalize the posterior of the parameters to predict the behavior of the Seizure count data which make the new model more resilience and litheness. Unlike the work of [[2] model III] who confined and limited there work only on standard Poisson model SPM(λ) to analyze the count data in generalized linear mixed model. The present study helps to identify problems involving uncertain events, and gives an efficient computational Bayesian approach with new ways of predicting and measuring behavior.

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5. Appendix

5.1. Description of the data set

The data set in table A taken from [1], represent a placebo-controlled medical randomized clinical trial to 59 epileptics. Patients who are diagnosed with partial seizures were enrolled in a randomized medical trial of the anti-epileptic drug, called progabide. The participants in the study were randomized to either take progabide or a placebo, as an adjuvant to the standard anti-epileptic chemotherapy. The drug progabide has an anti-epileptic function which binds to both $GABA_A$ and $GABA_B$ receptors and is located on the terminals of primary afferent fibers and is the primary inhibitory neurotransmitter in the brain. Activation of the $GABA_B$ receptors retards the influx of calcium ions into the terminals, thereby reducing the evoked release of excitatory amino acids and possibly other transmitters. Prior to receiving treatment, baseline data on the number of epileptic seizures during the preceding eight week interval were recorded. Counts of epileptic seizures during two week intervals before each of four successive post-randomization clinic visits were recorded. Patient ID, Treatment (0=Placebo, 1=Progabide drug), Age, Baseline 8 week seizure count, First two week seizure count, Second two week seizure count, Third two week seizure count, Fourth two week seizure count. A total of five seizure counts were recorded.

Table A

j	y_{j1}	y_{j2}	y_{j3}	y_{j4}	Trt_j	$Base_j$	Age_j
1	5	3	3	3	0	11	31
2	3	5	3	3	0	11	30
3	2	4	0	5	0	6	25
4	4	4	1	4	0	8	36
5	7	18	9	21	0	66	22
6	5	2	8	7	0	27	29
7	6	4	0	2	0	12	31
8	40	20	21	12	0	52	42
9	5	6	6	5	0	23	37
10	14	13	6	0	0	10	28
11	26	12	6	22	0	52	36
12	12	6	8	4	0	33	24
13	4	4	6	2	0	18	23
14	7	9	12	14	0	42	36
15	16	24	10	9	0	87	26
16	11	0	0	5	0	50	26
17	0	0	3	3	0	18	28
18	37	29	28	29	0	111	31
19	3	5	2	5	0	18	32
20	3	0	6	7	0	20	21
21	3	4	3	4	0	12	29
22	3	4	3	4	0	9	21
23	2	3	3	5	0	17	32
24	8	12	2	8	0	28	25
25	18	24	76	25	0	55	30
26	2	1	2	1	0	9	40
27	3	1	4	2	0	10	19
28	13	15	13	12	0	47	22
29	11	14	9	8	1	76	18
30	8	7	9	4	1	38	32
31	0	4	3	0	1	19	20
32	3	6	1	3	1	10	30
33	2	6	7	4	1	19	18
34	4	3	1	3	1	24	24
35	22	17	19	16	1	31	30
36	5	4	7	4	1	14	35
37	2	4	0	4	1	11	27
38	3	7	7	7	1	67	20
39	4	18	2	5	1	41	22
40	2	1	1	0	1	7	28
41	0	2	4	0	1	22	23
42	5	4	0	3	1	13	40
43	11	14	25	15	1	46	33
44	10	5	3	8	1	36	21
45	19	7	6	7	1	38	35
46	1	1	2	3	1	7	25
47	6	10	8	8	1	36	26
48	2	1	0	0	1	11	25
49	102	65	72	63	1	151	22
50	4	3	2	4	1	22	32
51	8	6	5	7	1	41	25
52	1	3	1	5	1	32	35
53	18	11	28	13	1	56	21
54	6	3	4	0	1	24	41
55	3	5	4	3	1	16	32
56	1	23	19	8	1	22	26
57	2	3	0	1	1	25	21
58	0	0	0	0	1	13	36
59	1	4	3	2	1	12	37

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