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Simulation of Parallel Beam CT Reconstruction Algorithm Based on Grating Phase Contrast Imaging

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Abstract

X-ray phase contrast imaging has higher resolution in the imaging of light element materials than absorption imaging, and is increasingly used in many fields such as medicine, biology and materials science.Grating phase contrast imaging stands out among many phase contrast imaging methods due to its unique advantages.This paper briefly introduces the filtering back projection reconstruction algorithm of the traditional parallel beam CT, and then introduces the Hilbert filter back projection algorithm suitable for the parallel beam CT of the grating phase contrast. Finally, the simulation of the parallel beam CT reconstruction algorithm based on the grating phase contrast imaging is carried out.Through the detailed introduction of the realization principle of traditional CT imaging and phase contrast CT imaging, it can be seen that there is no essential difference between the two implementation principles.The reconstruction algorithm of traditional CT imaging is mature, and then only needs to be slightly improved to be suitable for CT reconstruction based on grating phase contrast imaging.

Keywords: Parallel beam CT; Grating phase contrast imaging; Filtered back projection; Hilbert filter back projection.

1. Introduction

Conventional X-ray imaging is based on the absorption principle of X-rays by substances. Since some substances composed of light elements absorb X-rays weakly, high contrast images cannot be obtained by conventional X-ray imaging techniques.When X-rays pass through a substance.

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not only the intensity is attenuated but the phase also changes.The interaction of X-rays with matter can be expressed by a complex refractive index, ie: $1-\delta+i\beta$ [1], Where δ is the phase factor, β is the absorption factor.In hard X-rays, the phase factor of a substance composed of light elements is usually 1000 to 100,000 times that of the absorption factor[2], so that a higher resolution phase contrast pattern can be obtained by obtaining the phase change amount of the X-rays after passing through the light element material.

At present, the main phase contrast imaging methods include interference imaging[3~5], coaxial imaging[6~10], diffraction enhanced imaging[11~13], and raster imaging[14~18].The first three imaging methods must use a coherent light source, and the imaging field of view is small, thus limiting its promotion and development.In 2006, F.Pfeiffer realized the application of common X-ray source in grating imaging by using Talbot-Lau effect[19], which reduced the requirement of grating imaging for light source coherence, improved imaging field of view, and made it practical.Due to its unique advantages, the grating imaging method has become a hot topic in recent years. At the same time, the phase extraction method based on the grating imaging method and the CT imaging reconstruction algorithm have also made many breakthroughs.Huang and his colleagues used Hilbert Filtered Back-Projection Algorithm (HFBP) to directly reconstruct the phase information distribution of the object through the obtained phase first-order differential information[20].The HFBP algorithm was first used for CT reconstruction of diffraction-enhanced imaging. Since the diffraction-enhanced imaging method and the grating phase contrast imaging method are both the first-order differential information of the projection, the CT reconstruction algorithm based on diffraction-enhanced imaging is also suitable for the grating imaging method.

The HFBP algorithm draws on the filtered back-projection (FBP) of traditional CT. At the same time, based on the HFBP algorithm, a series of CT reconstruction algorithms suitable for grating phase contrast imaging are developed by using the ideas and methods of traditional CT.Therefore, the FBP reconstruction algorithm in traditional CT and the HFBP algorithm in phase contrast CT are particularly important.This article will briefly introduce the related content of parallel beam absorption CT and parallel beam phase contrast CT based on grating imaging.Finally, the simulation experiment of HFBP algorithm based on grating imaging method is carried out.

2. parallel beam absorption CT

The Austrian mathematician Radon proposed reconstructing images through projection and became the theoretical basis of CT technology. Therefore, acquiring projection data is an important part of CT technology.The projection of the absorption CT is the line integral of the attenuation coefficient μ of the substance passing through the object path in the projection direction, which can be expressed as:

$$p=\int g(x,y)dl \tag{1}$$

Where l is the ray propagation path, $g(x,y)$ is the attenuation coefficient of the object to the ray, and p is the resulting projection.The Fourier transform of the projection is equal to the central section of the Fourier transform of the object, which is the famous central slice theorem.Based on the projection back projection and the center slice theorem, a Filtered Back-Projection (FBP) or Convolution Back-Projection (CBP) algorithm for traditional absorption CT is developed. The FBP algorithm can be expressed as:

$$g(x,y)=\int_0^{\pi}\int_{-\infty}^{\infty}P_\theta(w)e^{j2\pi w(x\cos\theta+y\sin\theta)}\,|w|\,dw\,d\theta \qquad (2)$$

Where $g(x,y)$ is the attenuation coefficient distribution function of the object to be reconstructed, and $P_\theta(w)$ is the Fourier transform of the projection obtained by the X-ray of the direction angle θ passing through the object.The FBP algorithm first obtains the projection data of the ray source rotated 180° around the object, then selects the appropriate filter to filter the projection data, and finally backprojects the filtered projection back to complete the image reconstruction.

3. parallel beam phase contrast CT

The phase shift after the X-ray passes through the object can be expressed as[21]:

$$\Phi=\frac{2\pi}{\lambda}\int\delta(x,y,z)dl \qquad (3)$$

Where l is the ray propagation path.It can be known from equations (1) and (3) that the phase shift after X-ray passing through the object has the same physical meaning as the intensity attenuation.Therefore, as long as the phase shift information is obtained, the FBP or CBP algorithm of the traditional CT can be directly used to directly reconstruct the distribution of the phase shift factor of the object.However, the grating phase contrast imaging method obtains the first-order differential information of the phase shift, which can be expressed as[22]:

$$\Delta\theta\approx\frac{1}{\mathrm{k}}\nabla\Phi \qquad (4)$$

Where k is the wave number of the X-ray.Since the obtained phase shift information does not satisfy the rotation invariance, the CT reconstruction cannot be directly performed using the FBP or CBP algorithm.

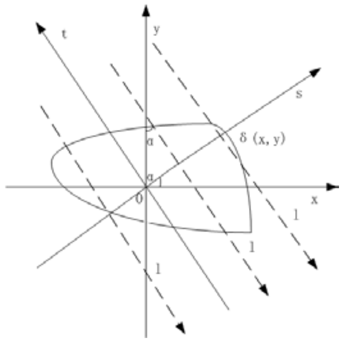

Figure 1: Schematic diagram of parallel beam scanning

To simplify the calculation, it is assumed that the phase shift factor of the object to be reconstructed is only related to the x-y coordinate axis, regardless of the z axis.Figure 1 is a schematic diagram of parallel beam scanning.Where x-y represents the coordinate system of the object, s-t represents the rotation coordinate, l is the projection direction, a is the angle between the l and the y axis, and the angle between the s axis and the x axis of the rotation coordinate is also a.Since the grating phase contrast obtains the first-order differential information of the phase shift and is perpendicular to the projection direction.When the X-ray projection direction is as shown in Figure 1, the combination of equations (3) and (4) can be obtained:

$$\Delta\theta = \int \frac{\partial\delta(x,y)}{\partial s} dl \tag{5}$$

Since x changes with the projection direction and does not satisfy the rotation invariance, the partial shift of the phase shift factor to the x and y axes satisfies the rotation invariance, so that:

$$\frac{\partial\delta(x,y)}{\partial s} = \cos\alpha \frac{\partial\delta(x,y)}{\partial x} + \sin\alpha \frac{\partial\delta(x,y)}{\partial y} \tag{6}$$

Where α is the rotation factor and is also the angle between the X-ray projection direction and the y-axis.Since $\frac{\partial\delta(x,y)}{\partial x}$ and $\frac{\partial\delta(x,y)}{\partial y}$ does not change with the projection direction, $\frac{\partial\delta(x,y)}{\partial s}$ is determined when the projection direction α is fixed.In the HFBP algorithm, the two-dimensional Fourier transforms of $\frac{\partial\delta(x,y)}{\partial s}$, $\delta(x,y)$, $\frac{\partial\delta(x,y)}{\partial x}$ and $\frac{\partial\delta(x,y)}{\partial y}$ are: $\Theta_s'(u,v)$, $\Theta(u,v)$, $\Theta_x'(u,v)$, $\Theta_y'(u,v)$ .among them:

$$\Theta_x'(u,v) = j2\pi u\Theta(u,v) \tag{7}$$

$$\Theta_y'(u,v) = j2\pi v\Theta(u,v) \tag{8}$$

$$\Theta_s'(u,v) = \Theta_x'(u,v)\cos\alpha + \Theta_y'(u,v)\sin\alpha \tag{9}$$

From equation (7)(8)(9):

$$\Theta_s'(u,v) = j2\pi(u\cos\alpha + v\sin\alpha)\Theta(u,v) \tag{10}$$

Convert the coordinate system (u,v) to the polar coordinate system (ρ,α), and the polar axis coincides with the u-axis, then:

$$\Theta_s'(u,v) = j2\pi\rho\Theta(u,v) \tag{11}$$

From equation (1)(2)(3), it can be seen that the phase shift factor $\delta(x,y)$ can be reconstructed into the FBP algorithm of the absorption CT, which can be expressed by polar (ρ,α) coordinates:

$$\delta(x,y)=\int_0^{\pi}\int_{-\infty}^{\infty}\Theta(u,v)\,|\rho|\,e^{j2\pi\rho(x\cos\alpha+y\sin\alpha)}d\rho \tag{12}$$

Substituting equation (11) into (12) gives:

$$\delta(x,y)=\int_0^{\pi}\int_{-\infty}^{\infty}\Theta_s^{'}(u,v)[-j\,\mathrm{sgn}(p)]e^{j2\pi\rho(x\cos\alpha+y\sin\alpha)}d\rho \tag{13}$$

among them:

$$\mathrm{sgn}(\rho)=\begin{cases}1, & \rho>0\\ -1, & \rho<0\end{cases} \tag{14}$$

It can be known from equation (13) that the first-order differential data of the phase shift is multiplied by $-j\,\mathrm{sgn}(p)$ in frequency, which is equivalent to the Hilbert transform.Therefore, the HFBP algorithm is to obtain the first-order differential information of the phase shift obtained by rotating the ray around the object by 180°, then perform Hilbert filter transformation on the obtained data, and finally backproject the filtered data sback.

4. Simulation experiment

Due to experimental conditions, the projection data of the actual grating phase contrast imaging cannot be obtained.The projection data of the grating phase contrast is obtained by simulation, and the simulation test of HFBP algorithm is carried out.The simulation model selects the Shepp-Logan head model as shown in Figure 2. The image resolution is 256x256, and the image area value represents the phase factor of the object to be reconstructed.The experimental steps are as follows:

- Obtain a gradient in the horizontal direction of the image and a gradient in the vertical direction.
- As shown in Figure 1, when the angle between the projection angle and the y-axis is α, it can be known from equation (6) that the projection data in the grating phase contrast CT is the sum of the product of the horizontal gradient and the vertical gradient of the image and the rotation factor, respectively.
- The projection angle is from 0° to 179°, and the interval is 1°. Step (2) is repeated to obtain the first-order differential information of the projection.
- The projection data is filtered by Hilbert, and the filtered projection data is back-projected back, and the CT reconstruction map shown in Figure 3 is obtained.

Figure 2: Head model original image **Figure 3:** CT reconstruction map

It can be seen from Figure 3 that the first-order differential information of the projection of the phase contrast CT is obtained by simulation, and then the reconstruction of the image is completed very well by using the HFBP algorithm.

5. Conclusion

By comparing the FBP algorithm in the absorption CT with the HFBP algorithm in the phase contrast CT, it can be seen that there is no essential difference between the conventional CT and the phase contrast CT.The traditional CT theory is relatively mature. By referring to the traditional CT reconstruction theory method and the idea of reconstructing the phase factor distribution directly using the phase first-order information in the HFBP algorithm, the traditional CT theory method can be applied to the grating phase contrast CT.

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Author Biographies

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