

Numerical Study of Kermack-Mckendrik SIR Model to Predict the Outbreak of Ebola Virus Diseases Using Euler and Fourth Order Runge-Kutta Methods

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Abstract

Mathematical Modeling has emerged as a vital tool for understanding the dynamics of the spread of many infectious diseases, one amongst is Ebola virus. The main focus of this paper is to model mathematically the transmission dynamics of Ebola virus. For this purpose we tend to use basic SIR model of Ebola Virus to predict the outbreak of the diseases. As we cannot fully solve the 3 basic equations of SIR model with a certain formula solution, we introduce Euler and fourth-order Runge-Kutta methods (RK4). These two proposed strategies are quite efficient and practically well suited for solving initial value problem (IVP) for ordinary differential equations (ODE). We discuss the numerical comparisons between Euler method and Runge-Kutta methods and also discuss regarding their performances with the actual data. The population that we used for this model had roughly a similar number of individuals as the number was living in Republic of Liberia during 2014.

Keywords: Ebola; Outbreak; Evolution; Mathematical Modeling.

1. Introduction

Mathematical models are a powerful tool for investigating human infectious diseases, such as Ebola virus, contributing to the understanding of the dynamics of disease and providing useful predictions about the potential transmission of a disease and the effectiveness of possible control measures, which can provide valuable information for public health policy makers.

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The Ebola virus disease was first discovered in 1976 in the present Democratic Republic of Congo [1]. Since then, there have been many outbreaks; with the greatest was 2014 outbreak [2] which has spread through many countries. According to Bekoe (2015), from the first confirmed case recorded on 23 March 2014 which more than 18 months, at least 11,312 people have been reported died from the disease in six countries; Liberia, Sierra Leone, Guinea, Nigeria, Mali and US. Since the virus keeps spreading through contacts and the mortality rate of 0.7[3], it is needed to understand the patterns and epidemiology of the disease. By these conditions, mathematical model of the outbreaks of Ebola virus can be helpful as it is a platform for understanding the behavior of a dynamical system. The objectives are to understand better the mathematical dynamics of an infected population when an outbreak occurs. Another goal is to use mathematical modeling to examine and to analyze the viral dynamics of the Ebola virus. To model this outbreak, the systems of differential equations are used. To study the known data, several distinct models will be used and each model is different depends on the parameters acquired. I decided to model the Ebola Epidemics in Liberia in 2014 and compare their spread using an SIR model.

1.1 Objectives

The purposes of this research are

1. To apply SIR model to predict the outbreaks of Ebola virus.
2. To determine the effect of the initial number of infectives of the population.
3. Compare with real data and fit with model.

1.2 Scope

In the proposal study, we will only investigate the outbreaks of Ebola virus by applying the mathematical model. The data has been collected and it covers the area in the continent of Africa, on Liberia that was recorded by CDC [4]. The calculation will be done by using tools of MATLAB software.

2. Methodology

2.1 Formulation of SIR model

The SIR model is used to illustrate the transfer of the epidemic through the interaction of the following three different variables:

S = Number of people

that are susceptible to Ebola

I = Number of people infected with Ebola

R = Number of people recovered

from Ebola with total immunity

It makes sense to assume that a fixed population of N people, whereby there are no births and deaths by natural cause i.e.

$$N = S + I + R \quad [5]$$

This is because the population is fixed and therefore, there are only three compartments in which the population may fit into. Thus, the total of the number of people susceptible infected and recovered is equivalent to the total population. The assumption that N is fixed, with no births or deaths, makes sense given 60 days, although it is a simplification.

These variables change over time, so we will define the variable $t =$ time in days. We will set $t = 0$ at the start of August 2014.

The model uses two parameters β and γ with $\beta, \gamma > 0$.

Given these parameters, the model uses 3 differential equations.

The rate of change of the number of people susceptible to the disease over time

$$\frac{dS}{dt} = -\beta IS \quad (1)$$

The rate of change of the number of people recovered over time

$$\frac{dR}{dt} = \gamma I \quad (2)$$

The rate of change of the number of people infected.

$$\frac{dI}{dt} = \beta IS - \gamma I \quad (3)$$

Parameterization of the model

In order to calculate β (the rate of infection) and γ (the rate of recovery), it helps to define two more parameters.

$D =$ Duration of disease for those recovered

$M =$ Mortality rate for those who die per day

(0.7 for Ebola)

This leads to two further equations.

The rate at which the disease is spread

$$\gamma = \frac{1}{D} \quad [6] \quad (4)$$

The infection rate of the disease

$$\beta = \frac{M}{S} \quad [7] \quad (5)$$

2.2 Transformation of Runge-Kutta Equations for SIR modeling

RK4 is one of the classic methods for numerical integration of ODE models.

Consider the following initial problem of ODE

$$\frac{dy}{dt} = f(t, y)$$

$$y(t_0) = y_0$$

Where $y(t)$ is the unknown function (scalar or vector) which I would like to approximate.

The Iterative formula of RK4 method for solving ODE is as follows

$$k_1 = hf(t_n, y_n)$$

$$k_2 = hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(t_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2\right)$$

$$k_4 = hf(t_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

For simplicity, here we use the simplest SIR model to examine whether the RK4 method has been implemented correctly. The SIR model is defined as follows

$$\frac{dS}{dt} = -\beta IS$$

$$\frac{dI}{dt} = \beta IS - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

where $S(t)$ is the number of susceptible people in the population at time t , $I(t)$ is the number of infectious people at time t , $R(t)$ is the number of recovered people at time t , β is the transmission rate, γ represents the recovery rate, and

$N = S(t) + I(t) + R(t)$ is the fixed population.

According to the general iterative formula, the iterative formulas for $S(t)$, $I(t)$ and $R(t)$ of SIR model can be written out

$$S_{n+1} = S_n + \frac{\Delta t}{6} (k_1^S + 2k_2^S + 2k_3^S + k_4^S)$$

$$k_1^S = f(t_n, S_n, I_n) = -\beta S_n I_n$$

$$\begin{aligned} k_2^S &= f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right) \\ &= -\beta \left(S_n + \frac{k_1^S \Delta t}{2}\right) \left(I_n + \frac{k_1^I \Delta t}{2}\right) \end{aligned}$$

$$\begin{aligned} k_3^S &= f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right) \\ &= -\beta \left(S_n + \frac{k_2^S \Delta t}{2}\right) \left(I_n + \frac{k_2^I \Delta t}{2}\right) \end{aligned}$$

$$\begin{aligned} k_4^S &= f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t) \\ &= -\beta (S_n + k_3^S \Delta t) (I_n + k_3^I \Delta t) \end{aligned}$$

$$I_{n+1} = I_n + \frac{\Delta t}{6} (k_1^I + 2k_2^I + 2k_3^I + k_4^I)$$

$$k_1^I = f(t_n, S_n, I_n) = \beta S_n I_n - \gamma I_n$$

$$k_2^I = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right) = \beta \left(S_n + \frac{k_1^S \Delta t}{2}\right) \left(I_n + \frac{k_1^I \Delta t}{2}\right) - \left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_3^I = f\left(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$= \beta \left(S_n + \frac{k_2^S \Delta t}{2} \right) \left(I_n + \frac{k_2^I \Delta t}{2} \right) - \left(I_n + \frac{k_2^I \Delta t}{2} \right)$$

$$k_4^I = f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t)$$

$$= \beta (S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t) - (I_n + k_3^I \Delta t)$$

$$R_{n+1} = R_n + \frac{\Delta t}{6} (k_1^R + 2k_2^R + 2k_3^R + k_4^R)$$

$$k_1^R = f(t_n, I_n) = \gamma I_n$$

$$k_2^R = f\left(t_n + \frac{\Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}\right) = \gamma \left(I_n + \frac{k_1^I \Delta t}{2}\right)$$

$$k_3^R = f\left(t_n + \frac{\Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}\right) = \gamma \left(I_n + \frac{k_2^I \Delta t}{2}\right)$$

$$k_4^R = f(t_n + \Delta t, I_n + k_3^I \Delta t) = \gamma (I_n + k_3^I \Delta t)$$

2.3 Transformation of Euler Equations for SIR modeling

If we have a "slope formula," i.e., a way to calculate dy/dt at any point (t, y) , then we can generate a sequence of y -values,

$$y_0, y_1, y_2, y_3 \dots \dots \dots$$

By starting from a given y_0 and computing each *rise* as *slope* x *run*. That is,

$$y_{n+1} = y_n + slope_n \Delta t$$

where Δt is a suitably small step size in the time domain.

It really doesn't matter in this calculation if the slope formula happens to depend not just on t and y but on other variables, say x and z -- as long as we know how x and z are related to t and y . If x and z happen to be other dependent variables in a system of differential equations, I can generate values of x and z in the same way.

Of course, for the SIR model, we want the dependent variable names to be S , I and R . Thus we have three Euler formulas of the form

$$S_{n+1} = S_n + slope_n \Delta t$$

$$I_{n+1} = I_n + slope_n \Delta t$$

$$R_{n+1} = R_n + slope_n \Delta t$$

More specifically, given the SIR equations,

$$\frac{dS}{dt} = -\beta IS ; \frac{dI}{dt} = \beta IS - \gamma I; \quad \frac{dR}{dt} = \gamma I$$

The Euler formulas become

$$S_{n+1} = S_n - \beta I_n S_n \Delta t$$

$$I_{n+1} = I_n + (\beta I_n S_n - \gamma I_n) \Delta t$$

$$R_{n+1} = R_n + \gamma I_n \Delta t$$

To calculate something from these formulas, we must have explicit values for β , γ , $S(0)$, $I(0)$, $R(0)$ and Δt .

3. Application, Comparison and Result Discussion

If we now take the example of the Ebola outbreak in Liberia 2014, we can assign the parameters with the following values. The total population of Liberia, $N = 4294000$ [8], and according to data from WHO, the number of people infected, $I = 391$ [9] and the number of people dead is 227 [9]. Seeing as R includes the number of people who have received permanent immunity, this includes those who have died as they have permanent immunity, in addition to those who have recovered with permanent immunity.

$$\text{Therefore, number of people recovered } R = 227 + (0.3 \times 391) = 344$$

We will now use this data to provide the parameters with the following values.

$$N = 4294000; \quad I = 391; \quad R = 344$$

$$\text{Therefore, } S = N - I + R = 4294000 - (344 + 391) = 4293265$$

The duration of the disease ranges from 2 to 21 days [10], therefore we could roughly estimate the duration of the disease at the midpoint, i.e. 11 (approx.) days.

$$D = 11; \quad \gamma = \frac{1}{11} = 0.09$$

According to WHO, the mortality rate of Ebola is 0.7 [3] and the number of people susceptible is 4293265.

Therefore, β (the rate of infection) = $\frac{0.7}{4293265} = 1.63 \times 10^{-7}$

In order to use the SIR model to predict the evolution of the disease, it would be helpful if we could solve the system of differential equations.

Unfortunately, we cannot completely solve these equations with an explicit formula solution.

Therefore, we will use numerical approaches. We will use Euler method and fourth order Runge-Kutta method to extract the solution.

3.1 Euler Method

For each day, we will calculate the values of S , I and R using

1. $S_{n+1} = S_n - \beta I_n S_n$
2. $I_{n+1} = I_n + (\beta I_n S_n - \gamma I_n)$
3. $R_{n+1} = R_n + \gamma I_n$

We take the initial values as

$$S_0 = 4293265; \quad I_0 = 391; \quad R_0 = 344; \quad \gamma = 0.09 \quad ; \quad \beta = 1.63 \times 10^{-7};$$

We will do this explicitly for the transition from $t = 0$ to $t = 1$. Using equations 1, 2 and 3, the following values for S , I and R can be calculated.

$$\begin{aligned} S_1 &= S_0 - \beta I_0 S_0 \times \Delta t = 4293265 - (1.63 \times 10^{-7} \times 391 \times 4293265) \times 1 \\ &= 4292991.377 \approx 4292991 \end{aligned}$$

$$\begin{aligned} I_1 &= I_0 + (\beta I_0 S_0 - \gamma I_0) \times \Delta t = 391 + (1.63 \times 10^{-7} \times 391 \times 4293265 - 0.09 \times 391) \times 1 \\ &= 629.4326582 \approx 630 \end{aligned}$$

$$\begin{aligned} R_1 &= R_0 + \gamma I_0 \\ &= 344 + 0.09 \times 391 \\ &= 379.19 \approx 379 \end{aligned}$$

Here we use MATLAB to evaluate S , I , R over a two month period. **Table-3.1** shows the result.

Table 3.1: Euler Method

| Day | S | I | R | S+I+R |
|-----|---------|---------|---------|---------|
| 01 | 4293265 | 391 | 344 | 4294000 |
| 02 | 4292991 | 630 | 379 | 4294000 |
| 03 | 4292550 | 1014 | 436 | 4294000 |
| 04 | 4291842 | 1630 | 528 | 4294000 |
| 05 | 4290700 | 2625 | 675 | 4294000 |
| 06 | 4288865 | 4225 | 910 | 4294000 |
| 07 | 4285912 | 6798 | 1290 | 4294000 |
| 08 | 4281162 | 10936 | 1902 | 4294000 |
| 09 | 4273530 | 17584 | 2886 | 4294000 |
| 10 | 4261283 | 28248 | 4469 | 4294000 |
| 11 | 4241662 | 45327 | 7011 | 4294000 |
| 12 | 4210323 | 72586 | 11091 | 4294000 |
| 13 | 4160509 | 115868 | 17623 | 4294000 |
| 14 | 4081931 | 184017 | 28052 | 4294000 |
| 15 | 3959494 | 289892 | 44614 | 4294000 |
| 16 | 3772399 | 450898 | 70703 | 4294000 |
| 17 | 3495141 | 687575 | 111284 | 4294000 |
| 18 | 3103424 | 1017410 | 173166 | 4294000 |
| 19 | 2588759 | 1440508 | 264733 | 4294000 |
| 20 | 1980912 | 1918710 | 394378 | 4294000 |
| 21 | 1361382 | 2365556 | 567062 | 4294000 |
| 22 | 836453 | 2677585 | 779962 | 4294000 |
| 23 | 471386 | 2801669 | 1020945 | 4294000 |
| 24 | 256117 | 2764788 | 1273095 | 4294000 |
| 25 | 140695 | 2631379 | 1521926 | 4294000 |
| 26 | 80349 | 2454901 | 1758750 | 4294000 |
| 27 | 48198 | 2266111 | 1979691 | 4294000 |
| 28 | 30394 | 2079964 | 2183642 | 4294000 |
| 29 | 20090 | 1903072 | 2370838 | 4294000 |
| 30 | 13858 | 1738028 | 2542114 | 4294000 |
| 31 | 9932 | 1585531 | 2698537 | 4294000 |
| 32 | 7365 | 1445400 | 2841235 | 4294000 |
| 33 | 5630 | 1317049 | 2971321 | 4294000 |
| 34 | 4422 | 1199723 | 3089855 | 4294000 |
| 35 | 3557 | 1092613 | 3197830 | 4294000 |

| | | | | |
|----|------|--------|---------|---------|
| 36 | 2923 | 994911 | 3296166 | 4294000 |
| 37 | 2449 | 905843 | 3385708 | 4294000 |
| 38 | 2088 | 824679 | 3467233 | 4294000 |
| 39 | 1807 | 750738 | 3541455 | 4294000 |
| 40 | 1586 | 683393 | 3609021 | 4294000 |
| 41 | 1409 | 622065 | 3670526 | 4294000 |
| 42 | 1266 | 566222 | 3726512 | 4294000 |
| 43 | 1150 | 515378 | 3777472 | 4294000 |
| 44 | 1053 | 469091 | 3823856 | 4294000 |
| 45 | 972 | 426953 | 3866075 | 4294000 |
| 46 | 905 | 388595 | 3904500 | 4294000 |
| 47 | 847 | 353679 | 3939474 | 4294000 |
| 48 | 799 | 321896 | 3971305 | 4294000 |
| 49 | 757 | 292968 | 4000275 | 4294000 |
| 50 | 720 | 266637 | 4026643 | 4294000 |
| 51 | 689 | 242671 | 4050640 | 4294000 |
| 52 | 662 | 220858 | 4072480 | 4294000 |
| 53 | 638 | 201004 | 4092358 | 4294000 |
| 54 | 617 | 182935 | 4110448 | 4294000 |
| 55 | 599 | 166489 | 4126912 | 4294000 |
| 56 | 583 | 151521 | 4141896 | 4294000 |
| 57 | 568 | 137899 | 4155533 | 4294000 |
| 58 | 555 | 125501 | 4167944 | 4294000 |
| 59 | 544 | 114217 | 4179239 | 4294000 |
| 60 | 533 | 103948 | 4189519 | 4294000 |

3.2 Runge-Kutta (RK4) Method

For each day, we will calculate the values of S , I and R using

$$k_1^S = -\beta S_n I_n$$

$$k_1^I = \beta S_n I_n - \gamma I_n$$

$$k_1^R = \gamma I_n$$

$$k_2^S = -\beta \left(S_n + \frac{k_1^S \Delta t}{2} \right) \left(I_n + \frac{k_1^I \Delta t}{2} \right)$$

$$k_2^I = \beta \left(S_n + \frac{k_1^S \Delta t}{2} \right) \left(I_n + \frac{k_1^I \Delta t}{2} \right) - \left(I_n + \frac{k_1^I \Delta t}{2} \right)$$

$$k_2^R = \gamma \left(I_n + \frac{k_1^I \Delta t}{2} \right)$$

$$k_3^S = -\beta \left(S_n + \frac{k_2^S \Delta t}{2} \right) \left(I_n + \frac{k_2^I \Delta t}{2} \right)$$

$$k_3^I = \beta \left(S_n + \frac{k_2^S \Delta t}{2} \right) \left(I_n + \frac{k_2^I \Delta t}{2} \right) - \left(I_n + \frac{k_2^I \Delta t}{2} \right)$$

$$k_3^R = \gamma \left(I_n + \frac{k_2^I \Delta t}{2} \right)$$

$$k_4^S = -\beta (S_n + k_3^S \Delta t) (I_n + k_3^I \Delta t)$$

$$k_4^I = \beta (S_n + k_3^S \Delta t) (I_n + k_3^I \Delta t) - (I_n + k_3^I \Delta t)$$

$$k_4^R = \gamma (I_n + k_3^I \Delta t)$$

$$S_{n+1} = S_n + \frac{\Delta t}{6} (k_1^S + 2k_2^S + 2k_3^S + k_4^S)$$

$$I_{n+1} = I_n + \frac{\Delta t}{6} (k_1^I + 2k_2^I + 2k_3^I + k_4^I)$$

$$R_{n+1} = R_n + \frac{\Delta t}{6} (k_1^R + 2k_2^R + 2k_3^R + k_4^R)$$

We take the initial values as

$$S_0 = 4293265 ; \quad I_0 = 391; \quad R_0 = 344 ; \quad \gamma = 0.09; \quad \beta = 1.63 \times 10^{-7}$$

We will do this explicitly for the transition from t = 0 to t = 1. Using those equations the following values for S, I and R can be calculated.

$$k_1^S = -\beta S_0 I_0$$

$$= -1.63 \times 10^{-7} \times 4293265 \times 391 = -273.622658245000$$

$$k_1^I = \beta S_0 I_0 - \gamma I_0 = 1.63 \times 10^{-7} \times 4293265 \times 391 - 0.09 \times 391$$

$$= 238.432658245000$$

$$k_1^R = \gamma I_0 = 0.09 \times 391 = 35.1900000000000$$

$$\begin{aligned} k_2^S &= -\beta \left(S_0 + \frac{k_1^S \Delta t}{2} \right) \left(I_0 + \frac{k_1^I \Delta t}{2} \right) \\ &= -1.63 \times 10^{-7} \times \left(4293265 - \frac{273.622658245000}{2} \right) \times \left(391 + \frac{238.432658245000}{2} \right) \\ &= -357.039129094785 \end{aligned}$$

$$\begin{aligned} k_2^I &= \beta \left(S_0 + \frac{k_1^S \Delta t}{2} \right) \left(I_0 + \frac{k_1^I \Delta t}{2} \right) - \left(I_0 + \frac{k_1^I \Delta t}{2} \right) \\ &= -1.63 \times 10^{-7} \times \left(4293265 + \frac{-273.622658245000}{2} \right) \times \left(391 + \frac{238.432658245000}{2} \right) \\ &\quad - \left(391 + \frac{238.432658245000}{2} \right) = 311.119659473760 \end{aligned}$$

$$k_2^R = \gamma \left(I_0 + \frac{k_1^I \Delta t}{2} \right) = 0.09 \times \left(391 + \frac{238.432658245000}{2} \right) = 45.9194696210250$$

$$\begin{aligned} k_3^S &= -\beta \left(S_0 + \frac{k_2^S \Delta t}{2} \right) \left(I_0 + \frac{k_2^I \Delta t}{2} \right) \\ &= -1.63 \times 10^{-7} \times \left(4293265 + \frac{-357.039129094785}{2} \right) \times \left(391 + \frac{311.119659473760}{2} \right) \\ &= -382.467864374178 \end{aligned}$$

$$\begin{aligned} k_3^I &= \beta \left(S_0 + \frac{k_2^S \Delta t}{2} \right) \left(I_0 + \frac{k_2^I \Delta t}{2} \right) - \left(I_0 + \frac{k_2^I \Delta t}{2} \right) \\ &= 1.63 \times 10^{-7} \times \left(4293265 + \frac{-357.039129094785}{2} \right) \times \left(391 + \frac{311.119659473760}{2} \right) \\ &\quad - \left(391 + \frac{311.119659473760}{2} \right) \\ &= 333.277479697859 \end{aligned}$$

$$k_3^R = \gamma \left(I_0 + \frac{k_2^I \Delta t}{2} \right) = 0.09 \times \left(391 + \frac{311.119659473760}{2} \right) = 49.1903846763192$$

$$k_4^S = -\beta (S_0 + k_3^S \Delta t) (I_0 + k_3^I \Delta t)$$

$$= -1.63 \times 10^{-7} \times (4293265 - 382.467864374178) \times (391 + 333.277479697859)$$

$$= -506.805816985307$$

$$k_4^I = \beta(S_0 + k_3^S \Delta t)(I_0 + k_3^I \Delta t) - (I_0 + k_3^I \Delta t)$$

$$= 1.63 \times 10^{-7} \times (4293265 - 382.467864374178) \times (391 + 333.277479697859) \\ - (391 + 333.277479697859)$$

$$= 441.62084381250$$

$$k_4^R = \gamma(I_0 + k_3^I \Delta t)$$

$$= 0.09 \times (391 + 333.277479697859) = 65.1849731728073$$

$$\therefore S_1 = S_0 + \frac{\Delta t}{6} (k_1^S + 2k_2^S + 2k_3^S + k_4^S)$$

$$= 4293265 + \frac{1}{6} (-273.622658245000 - 2 \times 357.039129094785 - 2 \times 382.467864374178 \\ - 506.805816985307)$$

$$= 4292888.42625631 \approx 4292888$$

$$\therefore I_1 = I_0 + \frac{\Delta t}{6} (k_1^I + 2k_2^I + 2k_3^I + k_4^I)$$

$$= 391 + \frac{1}{6} (238.432658245000 + 311.119659473760 + 333.277479697859 + 441.62084381250)$$

$$= 719.141296733456 \approx 719$$

$$\therefore R_1 = R_0 + \frac{\Delta t}{6} (k_1^R + 2k_2^R + 2k_3^R + k_4^R)$$

$$= 344 + \frac{1}{6} (35.1900000000000 + 45.9194696210250 + 49.1903846763192 + 65.1849731728073)$$

$$= 392.432446961249 \approx 393$$

Here we use MATLAB to evaluate S, I, R over a two month period. **Table-3.2** shows the result.

Table 3.2

| RK4 Method | | | | |
|-------------------|----------|----------|----------|--------------|
| Day | S | I | R | S+I+R |
| 01 | 4293265 | 391 | 344 | 4294000 |
| 02 | 4292888 | 719 | 393 | 4294000 |
| 03 | 4292196 | 1322 | 482 | 4294000 |
| 04 | 4290923 | 2432 | 645 | 4294000 |
| 05 | 4288583 | 4471 | 946 | 4294000 |
| 06 | 4284286 | 8214 | 1500 | 4294000 |
| 07 | 4276407 | 15077 | 2516 | 4294000 |
| 08 | 4261994 | 27626 | 4380 | 4294000 |
| 09 | 4235753 | 50457 | 7790 | 4294000 |
| 10 | 4188376 | 91623 | 14001 | 4294000 |
| 11 | 4104130 | 164649 | 25221 | 4294000 |
| 12 | 3958283 | 290515 | 45202 | 4294000 |
| 13 | 3717118 | 496967 | 79915 | 4294000 |
| 14 | 3346977 | 809182 | 137841 | 4294000 |
| 15 | 2838838 | 1226389 | 228773 | 4294000 |
| 16 | 2237427 | 1696343 | 360230 | 4294000 |
| 17 | 1636735 | 2124419 | 532846 | 4294000 |
| 18 | 1127150 | 2428029 | 738821 | 4294000 |
| 19 | 747957 | 2580728 | 965315 | 4294000 |
| 20 | 489487 | 2605010 | 1199503 | 4294000 |
| 21 | 321556 | 2540850 | 1431594 | 4294000 |
| 22 | 214459 | 2424226 | 1655315 | 4294000 |
| 23 | 146134 | 2280695 | 1867171 | 4294000 |
| 24 | 102037 | 2126431 | 2065532 | 4294000 |
| 25 | 73073 | 1971024 | 2249903 | 4294000 |
| 26 | 53657 | 1819893 | 2420450 | 4294000 |
| 27 | 40360 | 1675938 | 2577702 | 4294000 |
| 28 | 31056 | 1540566 | 2722378 | 4294000 |
| 29 | 24413 | 1414309 | 2855278 | 4294000 |
| 30 | 19575 | 1297197 | 2977228 | 4294000 |
| 31 | 15987 | 1188973 | 3089040 | 4294000 |
| 32 | 13279 | 1089223 | 3191496 | 4294000 |
| 33 | 11204 | 997456 | 3285340 | 4294000 |
| 34 | 9589 | 913148 | 3371263 | 4294000 |

| | | | | |
|----|------|--------|---------|---------|
| 35 | 8316 | 835770 | 3449914 | 4294000 |
| 36 | 7300 | 764806 | 3521894 | 4294000 |
| 37 | 6479 | 699764 | 3587757 | 4294000 |
| 38 | 5809 | 640176 | 3648015 | 4294000 |
| 39 | 5257 | 585604 | 3703139 | 4294000 |
| 40 | 4798 | 535640 | 3753562 | 4294000 |
| 41 | 4414 | 489905 | 3799681 | 4294000 |
| 42 | 4089 | 448050 | 3841861 | 4294000 |
| 43 | 3813 | 409750 | 3880437 | 4294000 |
| 44 | 3577 | 374709 | 3915714 | 4294000 |
| 45 | 3374 | 342652 | 3947974 | 4294000 |
| 46 | 3199 | 313328 | 3977473 | 4294000 |
| 47 | 3046 | 286506 | 4004448 | 4294000 |
| 48 | 2913 | 261974 | 4029113 | 4294000 |
| 49 | 2796 | 239538 | 4051666 | 4294000 |
| 50 | 2694 | 219019 | 4072287 | 4294000 |
| 51 | 2603 | 200255 | 4091142 | 4294000 |
| 52 | 2523 | 183095 | 4108382 | 4294000 |
| 53 | 2452 | 167404 | 4124144 | 4294000 |
| 54 | 2389 | 153057 | 4138554 | 4294000 |
| 55 | 2333 | 139937 | 4151730 | 4294000 |
| 56 | 2283 | 127941 | 4163776 | 4294000 |
| 57 | 2238 | 116972 | 4174790 | 4294000 |
| 58 | 2197 | 106943 | 4184860 | 4294000 |
| 59 | 2161 | 97773 | 4194066 | 4294000 |
| 60 | 2128 | 89389 | 4202483 | 4294000 |

Figure 3.1-3.3 shows the comparison between Euler and RK-4 method in the cases of infectives, recovered and susceptibles respectively.

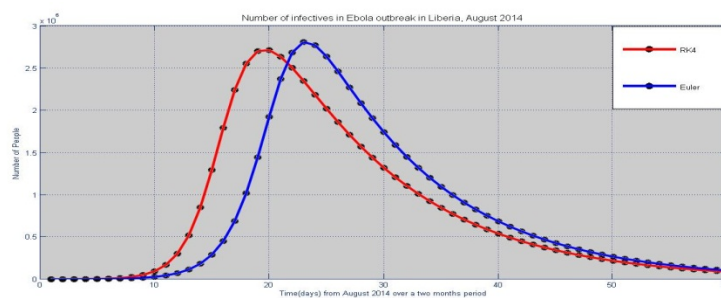


Figure 3.1

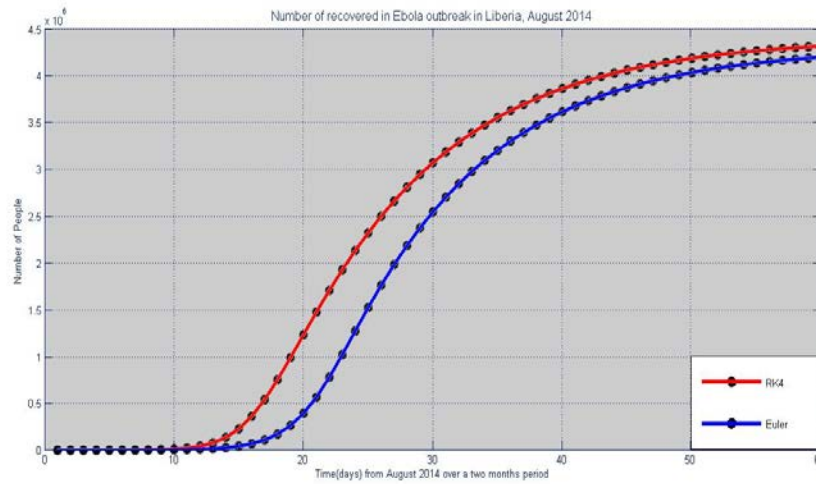


Figure 3.2

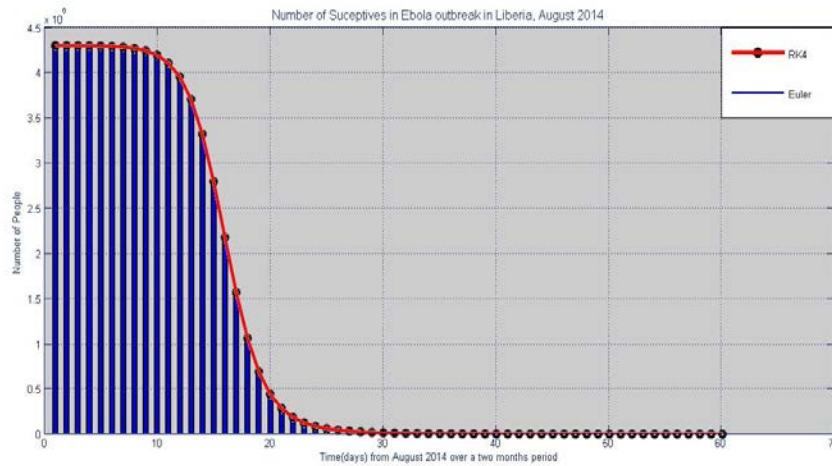


Figure 3.3

For predict the evolution, we only discuss about the infectives.

Each strategies show that initially, the number of individuals infected will increase steeply, however, over an extended period of time, the numbers eventually decrease. This happens simultaneously because the range of individuals recovered will increase because as those infected decreases, they're being transferred into the recovered class. The main reason is owing to the raised awareness of the disease leasng to additional medical support being given in order to assist combat the transmission of the disease. Furthermore, an increased awareness ends up in additional folks being conscious of strategies of protection. The steep increase within the beginning of the primary fifteen days is possibly to flow from to the good uncertainty that lied with Ebola allowing a greater rate of transmission. The peak of every graph illustrates the utmost number of individuals ever to be infected and once this point; there's a transition whereby the numbers decrease.

3.3 Comparing the model to actual data

However, in order for the model to be valid and allow informing government policy, it obviously needs to correspond fairly close to reality.

The **Table-3.3.1** below compares the data collected from the SIR model (using Euler and RK4) for the number of people infected and the real life data of the number of people infected.

Table 3.3.1

| Time(Days) | I -Actual | I- Euler | I -RK4 | Time(Days) | I -Actual | I- Euler | I -RK4 |
|------------|-----------|----------|---------|------------|-----------|----------|--------|
| 1 | 391 | 391 | 391 | 35 | 1871 | 1092613 | 835770 |
| 2 | 486 | 630 | 719 | 37 | 2046 | 905843 | 699764 |
| 6 | 554 | 4225 | 8214 | 41 | 2081 | 622065 | 489905 |
| 10 | 599 | 28248 | 91623 | 45 | 2407 | 426953 | 342652 |
| 11 | 670 | 45327 | 164649 | 47 | 2710 | 353679 | 286506 |
| 13 | 786 | 115868 | 496967 | 51 | 3022 | 242671 | 200255 |
| 17 | 834 | 687575 | 2124419 | 53 | 3280 | 201004 | 167404 |
| 19 | 972 | 1440508 | 2580728 | 55 | 3458 | 166489 | 139937 |
| 20 | 1082 | 1918710 | 2605010 | 60 | 3696 | 103948 | 89389 |
| 26 | 1378 | 2454901 | 1819893 | | | | |

Using the value of **Table-3.3.1** we can plot a graph using MATLAB which compares the model data to the actual data for the number of people infected (**Figure-3.4**).

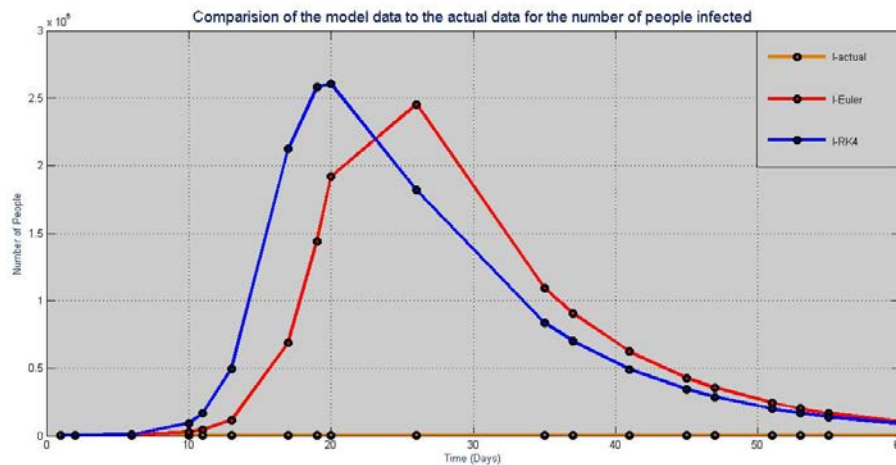


Figure 3.4

As the graph demonstrates, the real data does not correspond very well to the data received from the model. Although the actual data may seem to follow a straight like graph, this is untrue as it is only depicted in this

manner due to the limitations on the axis of the graph. The difference between the real life data and the data from the model is so vast that the straight line looks like a graph of $y = 0$. Therefore, we decided to plot it separately (**Figure-3.5**).

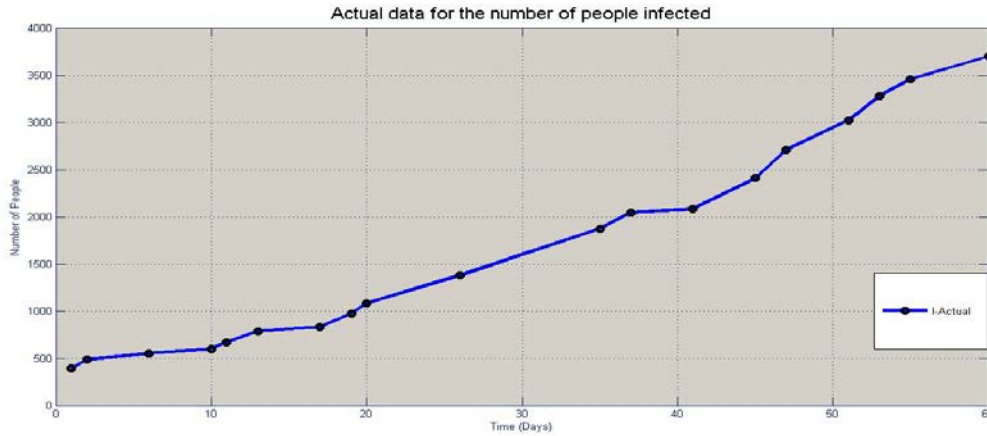


Figure 3.5

3.4 Curve Fitting with Actual Data

The model has significantly overestimated the number of individuals who can become infected with Ebola fever. This is often because of the many limitations that the model presents. One of the main limitations includes the incorrect beta and gamma values that were calculated. Once fixing the beta and gamma values, we were able to find another gamma value that resulted in similar values to the real data. Here is that the graph to indicate this, with the suitable gamma value of 0.66. The **Table-3.4.1** below compares the data collected from the fitted model for the number of people infected and the real life data of the number of people infected.

Table-3.4.1

| Time(Days) | I -Actual | I- Euler | I -RK4 | Time(Days) | I -Actual | I- Euler | I -RK4 |
|------------|-----------|----------|--------|------------|-----------|----------|--------|
| 1 | 391 | 391 | 391 | 35 | 1871 | 1555 | 1419 |
| 2 | 486 | 408 | 407 | 37 | 2046 | 1679 | 1525 |
| 6 | 554 | 482 | 475 | 41 | 2081 | 1952 | 1758 |
| 10 | 599 | 569 | 555 | 45 | 2407 | 2261 | 2020 |
| 11 | 670 | 593 | 576 | 47 | 2710 | 2430 | 2163 |
| 13 | 786 | 644 | 623 | 51 | 3022 | 2796 | 2472 |
| 17 | 834 | 759 | 726 | 53 | 3280 | 2994 | 2639 |
| 19 | 972 | 824 | 783 | 55 | 3458 | 3201 | 2814 |
| 20 | 1082 | 858 | 813 | 60 | 3696 | 3757 | 3287 |
| 26 | 1378 | 1092 | 1019 | | | | |

Using the value of **Table-3.4.1** we can plot a graph using MATLAB which compares curve fitted data with real data for the number of people infected (**Figure-3.6**).

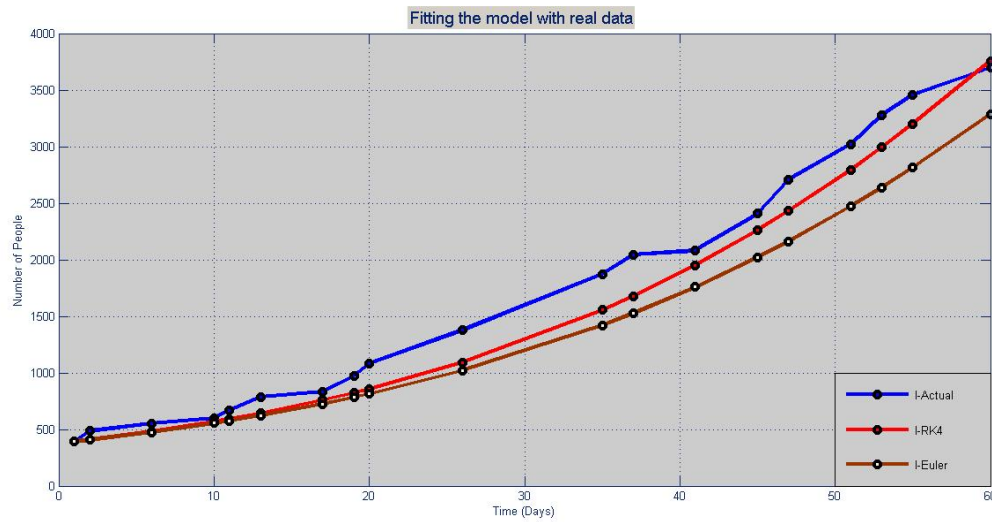


Figure 3.6

3.5 Result Discussion

Although, this doesn't match the graph exactly, it shows a much better correlation of the number of individuals infected. Between Euler and RK4 technique, Rk4 provides better correlation as because Rk4 methods divide its step size more than Euler methods and the local truncation error is less than Euler methods. Therefore, in order to improve the model, many changes should be done, together with altering the gamma value. The value that we eventually used to alter the model, led to being in 2 decimal places. This goes to illustrate the necessary precision required as very little deviance will cause massive changes. This can be because; the gamma is calculated through extreme simplification, leaving great possibilities for more room for errors. Furthermore, it is tough to differentiate between the numbers of people who have died and also the numbers of people who have survived with permanent immunity as they both fall under the same class of being 'recovered'. It is also a fact that rate of recovery is faster than the time scale of birth and death. The infection rate and recovery rate plays a vital role in the model and in this method we find more accurate value of infection rate and recovery rate to fit the model with the actual data collected from CDC.

4. Conclusion

The results obtained from modeling data will lead to completely different views and interpretations. This is due to the unequal distribution of knowledge across the globe whereby in countries like Liberia, there is little access to the statistics which makes it troublesome to form constructive predictions regarding the outbreak. Through our research, we have gained further insight into the uses of mathematical modeling so as to work out the outbreak of diseases similarly as evaluating its flaws. Having chosen Ebola as the diseases of concentration, as it is incredibly relevant to this situation in continent, it has enabled a practical understanding of its rate of

transmission.

5. Recommendations

The Numerical methods give a better prediction if the data collected from an authentic source. This result can help to estimate future predictions of the disease and consequently, it will facilitate to determine practical components like the quantity of beds required in the hospitable, number of vaccination and reallocation costs etc.

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