

## Basic Rules of Aristotelian logic and Induction

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### Abstract

We study comprehensive analysis of the inductive arguments done by various researchers. These researchers have surveyed the basic concepts in logic such as statement, argument, premise, conclusion, deductive argument, inductive argument, primitive terms and axioms. The fundamental laws of Aristotelian logic and Hume's critique of induction are briefly discussed. We argue that all the basic rules of Aristotelian logic are inductively obtained generalized abstract statements. Existence of undefined terms is also analyzed. We conclude that every argument based on Aristotelian logic is a result of induction.

**Keywords:** Aristotelian logic; Empirical World; Reasoning; Argument; Empiricism; Rationalism; Axiom; Undefined Terms.

### 1. Introduction

Logic is applied in the fields of science, mathematics and philosophy. The basic concepts, statements, arguments, theorems and applications of logic are discussed in this study. Since this logic is supposed to be introduced by Aristotle, it is called Aristotelian logical system.

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The mathematicians, scientists, philosophers and logicians have applied logic to arrive at conclusions regarding knowledge. Reasoning in this logical system is the process in which the conclusions are obtained through what are known as valid arguments. Western logic as well as science was originated in ancient Egypt. This logic and science was further developed in Greece. The word "logic" was derived from the word "logos" in the Greek language. The meaning of the word "logos" contains reason, science, language and relation [1]. However, In the New Testament, it is stated to mean the words, more explicitly the "Word of God" [2]. The mathematicians apply deductive reasoning to derive the conclusions called theorems in mathematics from statements known as axioms. Inductive reasoning is used in other subject areas such as physics, biology and chemistry in addition to deductive reasoning [3]. It is said that logic cannot be considered as an empirical subject like biology or physics but it has systematic methods of proof as in mathematics. This work is limited to two-fold two-valued logic. Hence, the other logics are not considered and we are restricted from comparing Aristotelian logic with other logics.

## **2. Preliminary Concepts**

The following definitions of concepts are from various authors as stated in the literature.

### **2.1 Statement**

A sentence that is capable of being true or false is defined as a statement. It is also called declarative sentence [3]. Any statement is considered to be either true or false but not both nor neither, and hence this logical system is called a two-valued logical system.

### **2.2 Argument, Premise and Conclusion**

An argument is a sequence of statements which are arranged in some order to acquire a statement as a conclusion. The other statements which are reinforced to arrive at the conclusion are considered as premises of the argument [3]. The premises provide the evidence to determine the conclusion.

### **2.3 Reasoning**

Reasoning is the process of using existing knowledge to draw conclusions, make predictions, or construct explanations [4]. Three methods of reasoning are the deductive, inductive, and abductive approaches.

### **2.4 Deductive Argument**

An argument is defined as a deductive argument if whenever all the premises are true the conclusion is true [3].

### **2.5 Inductive Argument**

An inductive argument is one whose conclusion is not necessarily relative to the premises; there is a certain probability that the conclusion is true if the premises are true, but there is also a probability that the conclusion is false even if the premises are true [3].

Example for inductive argument,

Suppose that we observe hundreds of crows from the universal set of crows as a sample. We can see that the first crow is black, the second crow is black, third crow is black and so on and finally the hundredth crow is also black. Therefore, it could be concluded that "all the crows are black" in the universal set of crows.

### **2.6 Abductive Argument**

Abductive argument typically begins with an incomplete set of observations and proceeds to the likeliest possible explanation for the set [4]. Abductive reasoning yields the kind of daily decision-making that does its best with the information at hand, which often is incomplete [4]. A medical diagnosis is an application of abductive reasoning: given this set of symptoms, what is the diagnosis that would best explain most of them? Likewise, when jurors hear evidence in a criminal case, they must consider whether the prosecution or the defense has the best explanation to cover all the points of evidence. While there may be no certainty about their verdict, since there may exist additional evidence that was not admitted in the case, they make their best guess based on what they know [4]. While cogent inductive reasoning requires that the evidence that might shed light on the subject be fairly complete, whether positive or negative, abductive reasoning is characterized by lack of completeness, either in the evidence, or in the explanation, or both [4]. A patient may be unconscious or fail to report every symptom, for example, resulting in incomplete evidence, or a doctor may arrive at a diagnosis that fails to explain several of the symptoms. Still, he must reach the best diagnosis he can [4]. However, abduction is not confined to single individual or set of observations, which is often incomplete. Abduction is more general as in the case of induction, but it is not a generalization as in induction. It is the art of arriving at a plausible explanation of a given set of observations. While induction is not an explanation, but a generalization, abduction is a general explanation for a set of incomplete observations. The abductive process can be creative, intuitive, and even revolutionary. Einstein's work, for example, was not just inductive and deductive, but involved a creative leap of imagination and visualization that scarcely seemed warranted by the mere observation of moving trains and falling elevators [4]. In fact, so much of Einstein's work was done as a "thought experiment" (for he never experimentally dropped elevators), that some of his peers discredited it as too fanciful [4]. Nevertheless, he appears to have been right. Until now his remarkable conclusions about space-time continue to be verified experimentally [4].

### **2.7 Undefined Terms**

Concepts that are not defined in terms of previously defined concepts are called undefined terms [6]. Undefined terms are also known as primitive term [5]. For example, the undefined terms in Euclidean Geometry are the point and the line, and the undefined terms in the real number system and system of integers are, 0 and 1, which are abstract from empirical observations. The notions of the point, line 0 and 1 have been drilled into us, so that we consider those objects to exist as real objects.

### **2.8 Axiom**

An axiom is a premise or starting point of an argument. As classically conceived, an axiom is a premise so

evident as to be accepted without controversy as a self-evident truth [6]. Axioms are also known as postulates [6].

### **3. Aristotelian logic**

Aristotle identified that every science could be described as a system starting from a finite set of basic rules called axioms and undefined terms to derive results, as in Euclidean Geometry. The basic rules to derive the results from the axioms are the same for every science.

#### **3.1 Concepts and Basic Rules of Aristotelian logic**

The concepts and the basic rules of Aristotelian logic can be expressed as follows.

##### **3.1.1 Subject**

A subject is typically an individual entity, for example a dog, a table, a woman or a group such as a set of numbers, a group of people etc. can be considered as subjects [7].

##### **3.1.2 Predicate**

A predicate is the property or attribute or mode of existence of the subject [7]. A given subject possesses or does not possess a given predicate. The color (property) of the box (subject) is red. Some women (subject) in the world are kind (predicate).

#### **3.2 Basic Rules of Aristotelian logic**

##### **3.2.1 The Rule of Identity**

Everything is what it is. In symbolic form it can be expressed as  $A$  is  $A$  [7]. This implies that an object called  $A$  exists. Modern formulation of the above rule is  $\forall x(A(x) = A(x))$  [7].

##### **3.2.2 The Rule of Non-Contradiction**

It is impossible for a subject both to be and not to be at the same time. A given predicate cannot both belong and not belong to a given subject in a given respect at a given time [7]. This implies that contradictions do not exist. That is,  $A$  and  $\neg A$  cannot be at the same time. Modern formulation of the above rule is  $\neg\exists x(A(x) \wedge \neg A(x))$  [7].

##### **3.2.3 The Rule of Either-Or**

Everything must either be or not be. A predicate either belongs or does not belong to a given subject in given respect at a given time [7]. That is either  $A$  or  $\neg A$ . Modern formulation of the above rule is  $\forall x(A(x) \vee \neg A(x))$  [7].

#### **4. Empiricism**

Empiricism is the school of philosophy which states that sense experience is primary in gaining knowledge and mental faculty is secondary [8].

#### **5. Rationalism**

Rationalism is the school of philosophy which states that mental faculty is primary in gaining knowledge, and sense experiences is secondary [9].

#### **6. Hume's Critique of Induction**

Consider the following example as a critique of induction. The Britons had come to the conclusion by induction that "All swans are white" [10]. However, when they went to Australia they came across black swans. Therefore, the generalized statement that had been obtained by induction "All swans are white" [10] was falsified. The famous empiricist David Hume argued that the generalized statements cannot be justified by empirical observations. The statement "All swans are white" [10] implies that a swan that would be born in the future would also be white. However, this implication is not empirical. Therefore, David Hume stated that the induction argument is not empirically valid. As per Hume scientific knowledge depends heavily on inductive method, the foundation of scientific knowledge is vulnerable. Therefore, Hume's criticism of the method of induction appeared to shatter the foundation of scientific knowledge. However, the scientists continued with the method of induction, as thought by Hume, as it was very useful to scientific knowledge. Hume acknowledged the importance of the method of induction to scientific knowledge despite its invalidity in logic. Hume himself had no solution to this dilemma. He admitted that it is human nature to make generalized statements from a finite number of similar observations.

#### **7. Empirical World**

We may define the world that we conceive and perceive as empirical world.

#### **8. Inductively Obtained Generalized Abstract Concepts and Statements**

We may define the inductively obtained generalized abstract concepts and statements as follows:

If a general concept is created by the mental faculty through observing finite number of similar observations in empirical world, then it is defined as an inductively obtained generalized abstract concept.

If a general statement is created by the mental faculty through observing finite number of similar observations in empirical world, then it is defined as an inductively obtained generalized abstract statement.

#### **9. Inductively Obtained Generalized Abstract Concepts and Statements in Aristotelian logic**

In this section, we discuss the ways by which Aristotle derived the concepts and fundamental laws in

Aristotelian logic from the empirical world.

### ***9.1 Existence of Subjects in Aristotelian logic***

It is important to analyze how formulated basic rules in Aristotelian logic. Could Aristotle (or anybody) observe everything in the world? Actually, nobody could observe everything in the world. Next we analyze how Aristotle created the concepts and the fundamental laws. This implies that objects in the world. Observing finite number of objects and beings in the empirical world such as chairs, dogs, men and pens etc., Aristotle brought them into the one category called subjects. It is clear that the subject is an inductively created abstract concept. Subsequently, Aristotle asserted the statement that "there are subjects." Therefore, the statement of the existence of subjects is an inductively obtained generalized abstract statement. Objects exist in space and time. However, we also observe that objects change during the course of time and/or change position in space. It is impossible to observe a single object during large interval of time what we observe is an object in a given plain in a finite interval of time. The object is assumed to be static during this short interval of time, but invariably it changes in time. Therefore, the existence of objects is an inductive generalization of objects, or subject in the sense of Aristotelian logic, is an inductive generalization of limited observations.

### ***9.2 Existence of Predicates in Aristotelian logic***

Observing the subjects in the empirical world Aristotle identified properties or attributes or modes of existence of the subjects and called them predicates. Hence, the predicate is also an inductively created abstract concept. Next, Aristotle also mentioned that there is a predicate for each subject. The assertion regarding predicates is an inductively obtained generalized abstract statement. What we have said 9.1 of existence of subjects is valid here as well.

### ***9.3 Creation of Fundamental Rules in Aristotelian logic***

In this we analyze how Aristotle formulated the fundamental laws in Aristotelian logic. We identify that all the fundamental laws of predication as inductively obtained generalized abstract statements.

#### ***9.3.1 Deriving the Rule of 'Identity'***

Consider the following examples from the empirical world. "Pen is a pen.", "Chair is a chair." and "Man is a man." etc. After observing only, a finite number of such examples, one can inductively conclude that A is A. Therefore it is clear that after observing a finite number of such identities Aristotle inductively concluded through his mental faculty that A is A. In other words, the above concluded statement is an inductively obtained generalization. Therefore, the statement "A is A" is a generalized abstract statement obtained by the method of induction.

#### ***9.3.2 Deriving the Rule of 'Non-Contradiction'***

Examine the following empirical examples "If something is a 'pen', then it cannot be 'not a pen'.", "If something

is a 'chair', then it cannot be 'not a chair'." and "if one is a 'man', then one cannot be 'not a man' " etc. Using the above finite number of examples one can conclude that A and notA cannot be at the same time. Hence, it can be concluded that the rule of non-contradiction is an inductively obtained generalized abstract statement.

### 9.3.3 Deriving the Rule of 'Either Or'

There are examples from the empirical world such as "If something is a pencil or it is not a pencil.", "If something is a chair or it is not a chair." and "If one is a man or one is not a man." etc. Examining the above examples one can generalize that if something is A either or notA. Hence, the rule of either or is also an inductively obtained generalized abstract statement. Similarly, we may conclude that all the concepts and fundamental laws in Aristotelian logic are inductively arrived abstract concepts and statements respectively.

## 10. Conclusions

Aristotelian logical system was created by means of the examples in the empirical world. All the concepts and rules of Aristotelian logic are inductively obtained generalized abstract concepts and statements respectively. Since all the arguments in science, in mathematics and in philosophy are based on Aristotelian logic, all the arguments are inductively created arguments. In other words, Aristotelian logic was abstracted by using the day to day experiences. Even today, most of the logicians, philosophers, scientists and mathematicians consider that Aristotelian logic is the absolute root of deduction. However, deduction is based on induction as rules of inference are nothing but generalizations of empirical observations. According to empiricism, the sense experience is primary and the mental faculty is secondary in gaining knowledge. According to rationalism, the mental faculty is primary and sense experience is secondary in gaining knowledge. The authors of this paper are of the view, that knowledge is created rather than it is gained or discovered. It is very difficult to identify the primary source of creating knowledge out of the two sources, namely sense experiences and mental faculty. This paper offers a novel criticism of Aristotelian logic and induction, and proposes a new theory of knowledge (epistemology).

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