Golinski's Speed Reducer Problem Revisited Using Genetic Algorithm

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Abstract

There are many constraints that constitute the Gear Box Design. Golinski's Speed reducer problem is studied and the weight of the Gear box is minimized. This is done by considering certain design variables of the Gear box, applying Optimization algorithms that provide results which agree upon a resulting weight that is minimum satisfying the concerned design variables.

Keywords: Gear box; Design Optimization; Aircraft; Propeller shaft; Genetic Algorithm; Sensitivity Analysis; NASA.

1. Introduction

There are many cases of Multidisciplinary Optimization problems that have been set as bench mark problems. Golinski's Speed reducer is one of the most studied bench mark problems of the NASA Langley Multidiscliplinary design Optimization (MDO) test suite. It was modeled by Golinski to minimize the weight of the Gear box. This gear box can be used in a light airplane between the engine and the propeller for allowing each of them to rotate at its efficient speed.

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2. Problem statement

Minimize the weight of gearbox although satisfying a number of constraints constituted by gear and shaft design.

3. Literature Review

Many researchers have worked on the Golinski's speed reducer problem with different approaches. Some of which are highlighted as follows.

Tapabrata Ray [1] worked on this problem. He used Particle Swarm Optimization and reported a feasible solution as all constraints were satisfied.

Kuang and his colleagues [2] worked on this problem using Taguchi search method. He reported the solution of the problem but the solution is not feasible because it pertains constraint violation.

Rao [10] worked on this problem using sequential search method. He reported the solution of this problem but the solution is not feasible because it pertains two constraint violations.

Similarly prior to the above mentioned works, other researchers also proposed solutions. For example Li and Papalambros [3], Azarm and Li [4], and the MDO test suite of NASA also presented solutions to this problem. However all of them were not feasible solutions.

 Other optimization approaches have also been applied by many researchers to find feasible solutions. For example: Rania Hassan, Babak Cohenim and Olivier de Weck [5] applied Particle Swarm Optimization and Genetic algorithm Optimization to this problem and presented a comparison of results obtained. Luis Vicente Santana-Quintero and Carlos A. Coello Coello [6] applied their algorithm (based on Differential evolution) on this problem and studied the results. Xiaoping Du [7] treated this problem as a probabilistic design problem. He applied Sequential optimization and Reliability assessment method to this problem and assessed the results.

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4. Problem formulation

There are seven design variables in this problem (as shown in Figure 1):

x_1 = Width of the Gear face,

x_2 = Teeth module

x_3 = Number of teeth of pinion

x_4 = Length of shaft 1 between bearings

x_5 = Length of shaft 2 between bearings

x_6 = Diameter of shaft 1

x_7 = Diameter of shaft 2

The variable bounds for the problem are as follows:

$2.6 \leq x_1 \leq 3.6$

$0.7 \leq x_2 \leq 0.8$

$17 \leq x_3 \leq 28$

$7.3 \leq x_4 \leq 8.3$

$7.3 \leq x_5 \leq 8.3$

$2.9 \leq x_6 \leq 3.9$

$5.0 \leq x_7 \leq 5.9$

These parameters form a constrained optimization problem given in Equation 1),

This problem is subject to the following constraints:

These constraints are symbolized as $G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9, G_{24}, G_{25}$ respectively. Constraints G_{10} and G_{11} are side constraints of x_1, constraints G_{12} and G_{13} are side constraints of x_2, G_{14} and G_{15} are side constraints of x_3, G_{16} and G_{17} are side constraints of x_4, G_{18} and G_{19} are side constraints of x_5, G_{20} and G_{21} are side constraints of x_6 and the constraints G_{22} and G_{23} are side constraints of x_7.

$$27x_1^{-1}x_2^{-2}x_3^{-1} \leq 1$$

$$397.5x_1^{-1}x_2^{-2}x_3^{-2} \leq 1$$

$$1.93x_2^{-1}x_3^{-1}x_4^3 x_6^{-4} \leq 1$$

$$1.93x_2^{-1}x_3^{-1}x_5^3 x_7^{-4} \leq 1$$

$$\left[(745x_4 x_2^{-1}x_3^{-1})^2 + 16.9 \times 10^6\right]^{\frac{1}{2}} / \left[110.0x_6^3\right] \leq 1$$

$$\left[(745x_5 x_2^{-1}x_3^{-1})^2 + 157.5 \times 10^6\right]^{\frac{1}{2}} / \left[85.0x_7^3\right] \leq 1$$

$$x_2 x_3 / 40 \leq 1.0$$

$$5x_2 / x_1 \leq 1.0$$

$$x_1 / 12x_2 \leq 1.0$$

$$(1.5x_6 + 1.9)x_4^{-1} \leq 1$$

$$(1.1x_7 + 1.9)x_5^{-1} \leq 1$$

5. Optimization algorithms

5.1 Genetic Algorithm

Genetic algorithm has been used to find a feasible solution of this problem. Genetic algorithm is a search algorithm developed by John Holland in 1970. GA (Genetic Algorithm) is based on the Darwinian theory of evolution, "Survival of the fittest ". GA are search algorithms that imitate natural selection and natural genetic behavior. They combine survivors of the fittest among structures, with structured yet randomized information exchange to form a search algorithm.

The basic GA is composed of a fitness function, a selection technique, a reproduction (cross over) and mutation operators with fixed probabilities. A genetic loop is formed which is shown in the figure [2].

The complete functional loop of Genetic Algorithm is shown in the figure [3].

There are advantages of using GA for this problem are that GA is an intelligent random search method it searches in a feasible search area. The structure of functionality of GA allows a broader search in an area with feasible solutions.

5.2 Fmincon

FMINCON is a constrained function minimization method. It searches for the best solution in a space around the starting point hence it searches a local minima. Therefore it has been used as a hybrid function for this problem by providing it with the function value obtained by GA, as an initial or starting value to further enhance the solution obtained by GA.

The optimization flow diagram of the hybrid optimization approach is shown in Figure [4].

5.3 Fminmax

This function starts with an initial estimate and minimizes the largest function value (this large value is the worst case) of a set of multi-variable functions. Since this problem consists of multivariable functions, so FMINMAX is used to find a feasible solution.

6. Discussion and Comparison of results

In this work, the results of Rao, Kuang et.al NASA test suite and Ray are analyzed. Before starting optimization, the sensitivity analysis using Latin Hypercube sampling is done. After optimization, the robustness of obtained results is checked. The pre-optimization sensitivity and robustness of result is compared to those of Rao, Kuang and Ray.

Table 1: Starting variables for Optimization

x1	x2	x3	x4	x5	x6	x7	Fval
2.87	0.73	18.73	7.86	7.76	3.04	5.18	3087

Genetic Algorithm Optimization tool Conditions

Population Type	Vector Size	Creation Function	Scaling	Selection
Double	20	Constraint Dependent	Rank	Stochastic Uniform
Mutation	**Crossover**	**Migration**	**Initial Penalty**	**Factor**
Constraint Dependent	Scattered	Forward	10	100

7. Pre-optimization sensitivity analysis

The pre-optimization sensitivity analysis is done using Latin Hypercube Sampling with 300 samples. It was observed that x_1 was the most effective variable among all design variable.

On the basis of the sensitivity result, the six samples which gave minimum constraint values were sorted out and their average value was taken. This average value was set as initial guess for optimization. This value is 3087.

The initial variables are changed now, these are shown in table [1] and are set as starting variables for the optimization. The Genetic algorithm is hybridized with FMINCON. The results are shown in tables [2] and [3].

8. Robustness of Solution

The solution obtained 3040.63 kg is perturbed +1%. Then again LHS (Latin Hypercube Sampling) is used. Same is done for the results of Rao, Kuang et.al NASA test suite and Tapabrata Ray. The variation in function values and sum of constraints is noticed for all results. The result of the comparison is shown as in table [4].

9. Conclusion

The Speed Reducer problem is revisited to minimize the weight of the Gearbox using Optimization techniques. The Genetic Algorithm is utilized and then it is hybridized by FMINCON and FMINMAX separately to get the best results. The following conclusions can be drawn from this work:

1. The table [4] clearly shows that the result obtained in the present work is feasible and robust than the other results.
2. The variations and deviations in function values is an acceptable one provided that all constraints are satisfied. The product of function value and Constraint value is a good indicator of feasibility of results.
3. The obtained value can be further minimized by a different approach. Particle swarm optimization is an attractive option for future work on this problem.
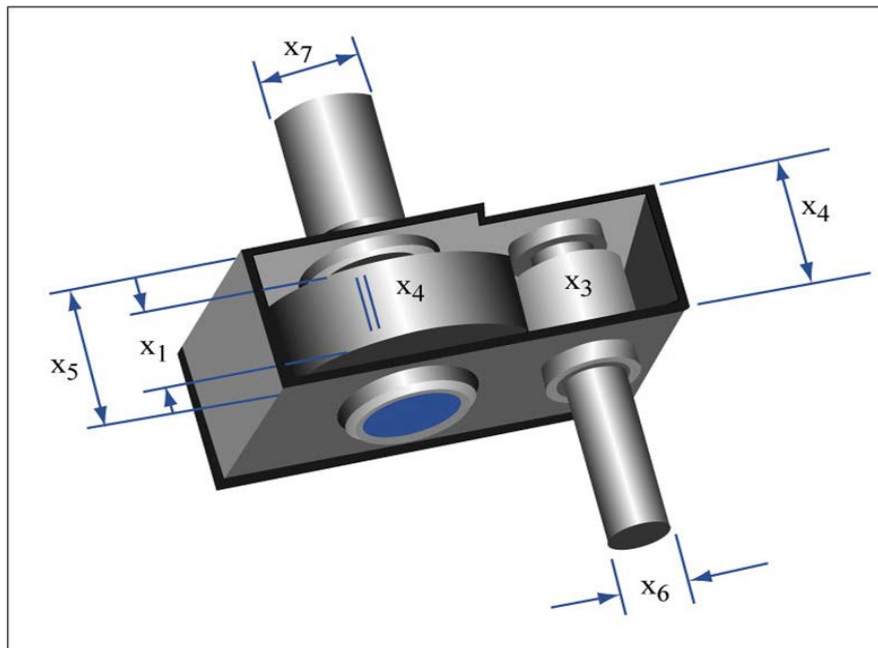

Figure 1: The Speed Reducer labelled with the Design variables

Table 2: Strategies employed and Resulting Function values

SR#	Strategy Changes	x1	x2	x3	x4	x5	x6	x7	FUNCTION VALUE
1	DEFAULT OPTIONS	3.5	1	17	7.3	7.71843216	3.3502512	5.2866879	3040.824577
2	POPULATION SIZE=100	3.5	1	17	7.3	7.71531991	3.35021467	5.28665446	3040.629933
3	SELECTION=REMAINDER	3.5	1	17	7.3	7.71733339	3.35027333	5.28666176	3040.706076
4	SELECTION=TOURNAMENT	3.5	1	17	7.3	7.71723781	3.35021469	5.28665532	3040.684092
5	SELECTION=ROULETTE	3.5	1	17	7.3	7.71659944	3.35024189	5.28665572	3040.673446
6	MIGRATION=BOTH	3.5	1	17	7.3	7.72648597	3.35021467	5.28665852	3040.944651
7	REPRODUCTION:CO=57.26%	3.5	1	17	7.3	7.71541222	3.35021467	5.2866545	3040.632527
8	REPRODUCTION:CO=50.42%	3.5	1	17	7.300004	7.71537178	3.35021466	5.28666058	3040.638033
9	HYBRID FUNC: FMINCON	3.5	1	17	7.3	7.71531991	3.35021467	5.28665446	3040.629928

Table 3: Effect on Constraints

Cons1	Cons2	Cons3	Cons4	Cons5	Cons6	Cons7	Con8	Con9	Con10	Con11	Sum of cons
-0.0754	-0.2194	-0.1682	-0.9074	0.1284	-0.0387	-0.6892	0.0822	-0.615	-0.1705	-0.0198	**-2.6931**
-0.2088	-0.3254	-0.1879	-0.9041	0.2003	0.0671	-0.6677	0.1541	-0.639	-0.1606	-0.0135	**-2.6854**
-0.1401	-0.2628	-0.186	-0.8961	0.172	-0.0063	-0.6741	0.1965	-0.6518	-0.1643	-0.0596	**-2.6725**
0.0116	-0.1617	-0.3489	-0.9046	-0.0188	0.0654	-0.6874	0.16	-0.6408	-0.1517	0.0083	**-2.6688**
-0.0832	-0.4006	-0.3473	-0.9176	0.1755	0.0352	-0.6007	0.3639	-0.6945	-0.1579	-0.0354	**-2.6626**
-0.1384	-0.4044	-0.24	-0.9064	0.2199	0.022	-0.609	0.3461	-0.6905	-0.1813	-0.0736	**-2.6556**
-0.1322	-0.3318	-0.2615	-0.9079	0.0984	0.0821	-0.6373	0.3425	-0.6896	-0.1891	-0.0232	**-2.6496**
-0.1913	-0.3682	-0.2779	-0.9204	0.2243	0.0241	-0.6256	0.4161	-0.7058	-0.153	-0.007	**-2.5855**
-0.0839	-0.3327	-0.3823	-0.9193	0.1014	0.0862	-0.6231	0.4236	-0.7073	-0.1492	0.009	**-2.5776**
-0.0245	-0.2432	-0.2001	-0.9192	0.0821	0.0296	-0.6679	0.1764	-0.6458	-0.1921	0.0302	**-2.5743**
-0.1469	-0.4634	-0.1727	-0.9309	0.2297	0.0719	-0.5804	0.3636	-0.6944	-0.22	0.0203	**-2.5233**
-0.107	-0.292	-0.1632	-0.9161	0.2334	0.0314	-0.6625	0.1803	-0.647	-0.1636	0.0126	**-2.4936**
0.0168	-0.1602	-0.3403	-0.889	0.074	0.0832	-0.6783	0.2623	-0.6699	-0.1318	-0.0444	**-2.4778**
-0.0468	-0.1754	-0.1756	-0.9317	0.1329	-0.0473	-0.6712	0.3861	-0.6994	-0.182	0.0609	**-2.3495**

-0.0406	-0.1718	-0.051	-0.8893	0.1888	0.0469	-0.6905	0.1587	-0.6404	-0.1874	-0.0442	**-2.3209**
-0.0919	-0.2672	-0.0778	-0.8943	0.2298	0.0281	-0.6443	0.4555	-0.7137	-0.2053	-0.0796	**-2.2608**
0.0259	-0.1681	0.1573	-0.894	0.2082	0.0215	-0.6709	0.3144	-0.683	-0.1659	-0.0589	**-2.2281**
0.0149	-0.2087	-0.0105	-0.9134	0.2119	-0.0004	-0.6552	0.3828	-0.6987	-0.2206	-0.0172	**-2.115**
0.0154	-0.1272	-0.0958	-0.8845	0.1587	0.0615	-0.6702	0.4711	-0.7168	-0.2001	-0.0723	**-2.0601**

Table 4: Robustness of Solution

| Results by	Variation in function value	Variation in constraint violation sum	Product of Fv and Cv	Constra-ints violated	X-optimal	Perturb-ation in X-optimal	Total Variation in X-optimal
Present work (GA+Fminc on) Hybrid and Fminimax	145.6312328	0.139603932	20.33069272	0	3040	1%	4.79%
Rao^3	130.9404375	0.134717201	17.63992921	2	2987.2985	1%	4.38%
Kuanget	143.4972434	0.156281035	22.42589766	1	2876.1176	1%	4.98%
NASA	150.6812575	0.155023719	23.35916893	2	2985.1519	1%	5.04%
Tapabrata Ray	156.2475078	0.15757876	24.62128845	0	2996.2322	1%	5.20%
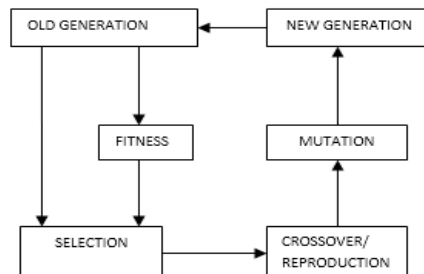
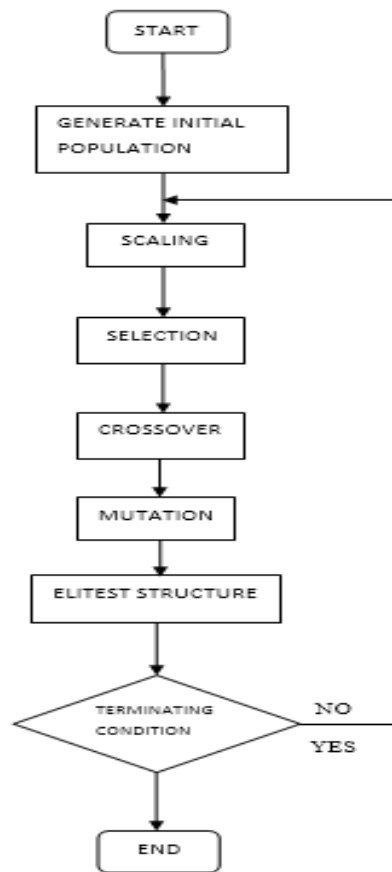**Figure 2:** Basic Genetic Algorithm cycle

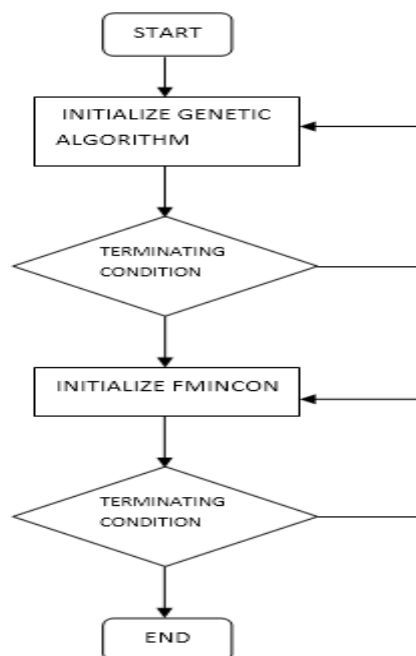**Figure 3:** Complete Genetic Algorithm cycle applied for the Speed reducer problem

Figure 4: Optimization flow diagram with Hybrid optimization approach.

$$f(x) = 0.7854 x_1 x_2^2 (3.3333 x_3^2 + 14.9334 x_3 - 43.0934)$$
$$-1.5079 x_1 (x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3)$$
$$+0.7854(x_4 x_6^2 + x_5 x_7^2)$$

(1)

Acknowledgements

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