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How Fractional Charge on an Electron in the Momentum Space is Quantized?

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Abstract

With our conjecture on charge quantization (quantum dipole moment in a momentum space) and using Fractional Fourier Transform (FRFT) analysis on Hermite Polynomials (usually used for quantum oscillators), we obtained energy profiles (eigenfunctions) for fractional quantum states on the continuously changing surface of the electron. The charge on an electron as a physical constant and a single entity is degenerate because it always resides on the surface. The charge is fractionally quantized in momentum space. The continuous charging surface of the electron is due to competition between the centrifugal and electodynamic potentials. The fractional quantized states of charges in the momentum space are the manifestations of gyroscopic constants, $\frac{g^2}{\hbar c}(0.2 - 0.8)$; twisting and twigging of energy profiles (quantum electrodynamic behavior), oscillatory behavior of energy associated with degeneracy and indeed the position of fractional quanta in terms of rotational vector, $\alpha(t, \omega)$ in complex plane.

Keywords: Fractional Fourier Transform, Fractional Charge Quantization, Hermite polynomials

1. Introduction

We attempted to decipher a new idea based on fractional charge quantization on an electron by using Fractional Fourier Transform. The charge on an electron, being a physical constant and single entity, is fractionally quantized due to momentum impact by photons.

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When a photon collides with an electron, the morphology of a bounded electron is changed due to inelastic collision. The electron quanta is stretched, as a consequence of which, the wave length, λ increases and the frequency, ν decreases, by maintaining, $c = \lambda\nu$. The oscillatory frequency of the bounded electron decreases on the hypothetical wall of the electron string. This hypothetical wall behaves like an adiabatic wall. Due to overwhelming centrifugal potential as compare to electrodynamic potential, the electron quanta string is self -----twisted and twigged (swirling effect). The charge on electron quanta is distributed on twigs (sub quanta). These twigs are beaded sub-quanta on an electron string. The charge on electron, which is a single entity and constant, is distributed on these sub quanta (twigs) and hence the fractional charge quantization. Remember that each of these sub-quanta on an electron has an integrated oscillatory effect (discovered in this paper), .i.e.,2^{n_f}, where $0.1 \leq n_f \leq 0.9$, and are beaded on an electron quanta string. The momentum impact of a photon on a bounded electron causes stretching. This stretching is a manifestation of quantum mechanical scattering (inelastic scattering) which hold true for Compton and photoelectric effects, too. The stretching, twisting and twigging holds true for quantized particles, but not for free particles as the case is for Compton and photoelectric effects. That is why Eisenstein of Caltech (USA), on the basis of experimental results, considered quasi particle nature of bounded electron due to its morphological changes.

2. **Theory**

The quantum dipole moments lead to charge quantization [1, 2, 3, 4]

$$x = hq \qquad (1)$$

Where x is the quantum dipole moment, q the charge and h is the Planck's constant (quantum action)

The matter energy such as of an electron exists in the form of transverse wave. This energy is oscillatory (quantum action). and configures a space called a wave packet or "quanta". We consider that the charge of an electron is treated as its density which is not only smeared on the surface but also inside the volume despite the fact that charges always reside on the surface. With momentum impact, the electron quanta is first stretched, twisted and then twigged. We envisage the electron like a flexible ball, the surface of which would vary continuously due to competing centrifugal and electrodynamic potential. The coupling constant $\frac{e^2}{\hbar c} \sim \frac{1}{137}$on an electron is overwhelmed by the gyroscopic constant, $\frac{g^2}{\hbar c}(0.2 - 0.8)$ due to pronounced centrifugal potential. This causes the charge on an electron to become degenerate and fractionally quantized on its surface and hence the charge quantization. The depth of the quantum well of an electron is equivalent to its radius. With fractional charge quantization, the envelope of energy associated with an electron (electron quanta) is twisted and twigged to smear the density (charge) of energy in its fractional components, as a consequences of which, the fractional charges float on their respective broken "quanta" only on the surface. Each of the broken sub-quanta is woven in a string due to whirling and swirling effects (electro weak interaction) on an electron. These broken sub-quanta are degenerate fractional charged quantized states in the momentum space. Each of sub-quanta would have the oscillatory behavior.

Almeida [5] defines the fractional Fourier transform (FRFT) of a function $x(t)$, with angle α (t is the time and u is the frequency) as

$$\mathcal{F}_\alpha[x(t)] = X_\alpha(u) = \int_{-\infty}^{\infty} x(t)\, K_\alpha(t,u)dt$$

$$=\begin{cases} \sqrt{\frac{1-j\cot\alpha}{2\pi}}\, e^{j\frac{u^2}{2}\cot\alpha} \int_{-\infty}^{\infty} x(t) e^{j\frac{t^2}{2}\cot\alpha - jut\csc\alpha} dt & if\ \alpha\ \ is\ not\ a\ multiple\ of\ \pi \\ x(t) & if\ \alpha\ \ is\ not\ a\ multiple\ of\ 2\pi \\ x(-t) & if\ \alpha+\pi\ \ is\ not\ a\ multiple\ of\ 2\pi \end{cases} \quad (2)$$

In our case we consider a function $f(x)$ and frequency $u = \omega$

$$f(x) = H_{n_f}(x) exp\left(-\frac{x^2}{2}\right) \quad (3)$$

With equation (1) $\quad x \equiv hq, \quad 0.1 \le n_f \le 0.9 \quad$ (4)

We know that the FRFT of $f(x)$ is given as follows [5]

$$\mathcal{F}_\alpha[f(x)] = e^{in_f\alpha} H_{n_f}(x)\exp(-\frac{x^2}{2}) \quad (5)$$

Using eq (4) on eq (3), we have

$$f(x) = H_{n_f}(hq) exp\left[\left(-\frac{hq}{\sqrt{2}}\right)^2\right] \quad (6)$$

Using eq (5)

$$\mathcal{F}_\alpha\left[H_{n_f}(hq) exp\left[\left(-\frac{hq}{\sqrt{2}}\right)^2\right]\right] = e^{in_f\alpha} H_{n_f}(hq)\exp\left[\left(-\frac{hq}{\sqrt{2}}\right)^2\right] \quad (7)$$

Where α is the angle of rotation in the complex plane (t,ω) With changing surface and indeed the shape of an electron, α is also changed in (t,ω) coordinates , $\omega = 2\pi\nu$. We are dealing with fractional quantum oscillators and hence with fractional charge distributions so that Fourier transform (FT) should not enter in our analysis. For this purpose we set $\alpha \neq \frac{\pi}{2}$. With $\alpha = 1, \left(\alpha = \frac{a\pi}{2}\right)$ we get the FRFT to change into FT. The FRFT analysis is a time frequency distribution and an extension of the classical FT. Considering the Schrodinger's equation for oscillatory quanta of fractional charges on the continuous changing surface of the electron [6, 7, 8].

$$-\frac{h^2}{2\mu}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2}kx^2\psi(x) = E\psi(x) \quad (8)$$

Where $\omega = \left(\frac{k}{\mu}\right)^{\frac{1}{2}}$, k the restoring constant μ the reduced mass of an electron. Using the dimensionless variable

$$\xi = \alpha x, \alpha = \left(\frac{\mu k}{\hbar^2}\right)^{\frac{1}{4}} = \left(\frac{\mu\omega}{\hbar}\right)^{\frac{1}{2}} \quad (9)$$

We shall replace dimensionless, α with rotation vector in coordinates, (t, ω). Substitution of eq (9) in eq (8), we have

$$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \xi^2)\psi(\xi) = 0 \quad (10)$$

Where λ is also dimensionless, but considered as binding energy of the quantum system. For large $|\xi|$, it is readily verified that the eigenfunctions $\psi(\xi) = \xi^P e^{\pm\xi^2}$ exists. The asymptotic analysis provides us an indication for valid solutions to eq (10) and for all ξ having the form ξ^P where P is the polynomial. Thus

$$\psi(\xi) = e^{-\frac{\xi^2}{2}} H(\xi) \quad (11)$$

Using eq (11) in eq (10), we have Hermite equation

$$\frac{d^2H}{d\xi^2} - 2\xi\frac{dH}{d\xi} + (\lambda - 1)H = 0 \quad (12)$$

We assume a solution to eq (12) in the form of a finite polynomial

$$H(\xi) = \sum_{s=0}^{N} a_s \xi^{2s}; s \geq 0 \quad (13)$$

Using eq (13) in eq (12), we obtain a recursion formula to reproducing the shape of the energy profiles oscillating within the momentum quantized space

$$\sigma_{s+2=\frac{2s+1-\lambda}{(s+2)(s+1)}a_s,\ s\geq 0} \quad (14)$$

For an upper cut off and the coefficients so that the polynomial equation (14) is not an infinite series, we have to insert a condition

$$\lambda = 2n_f + 1, 0.1 \leq n_f \leq 0.9 \ \Rightarrow 1.2 \leq \lambda \leq 2.8 \quad (15)$$

With dimensionless eigen value, $\lambda = \frac{2E}{\hbar\omega}$ and measuring the variation of electron radius, .i.e., depth of the quantum well in units of $\left(\frac{\hbar}{\mu\omega}\right)^{\frac{1}{2}}$ where $\omega^2 = \frac{k}{\omega}$, we can ascertain that

$$E_{n_f} = \left(n_f + \tfrac{1}{2}\right)\hbar\omega = \left(n_f + \tfrac{1}{2}\right)\hbar\nu;\ \hbar = \frac{h}{2\pi},\ n = 0 \quad (16)$$

Using eq (16) with the collection of even and odd cases, the physically acceptable solution of eq (10) corresponding to eigenvalues (eq(16)) are given by

$$\psi_{n_f}(\xi) = e^{-\frac{\xi^2}{2}} H_{n_f}(\xi); \quad 0.1 \leq n_f \leq 0.9 \qquad (17)$$

where the function $H_{n_f}(\xi)$ are polynomials of order n_f. Moreover, the polynomials $H_{n_f}(\xi)$ satisfy the Hermite equation (eq(12)) with $\lambda = 2n_f + 1$, [eq(15)].

$$\frac{d^2 H_{n_f}}{d\xi^2} - 2\xi \frac{dH_{n_f}}{d\xi} + 2nH_{n_f} = 0, \quad 0.1 \leq n_f \leq 0.9 \qquad (18)$$

Where the function $H_{n_f}(\xi)$ are Hermite polynomial. Their constant is traditionally chosen so that the highest power of ξ appear with the coefficients of 2^{n_f} in $H_{n_f}(\xi)$.

$$H_{n_f}(\xi) = 2^{n_f}\ ; 0.1 \leq n_f \leq 0.9 \qquad (19)$$

Eq (19) is consist with the following definition of Hermite polynomials

$$H_{n_f}(\xi) = (-1)^{n_f} e^{\xi^2} \frac{d^{n_f}}{d\xi^{n_f}} e^{-\xi^2} \qquad (20)$$

$$H_{n_f}(\xi) = e^{\frac{\xi^2}{2}} (\xi - \frac{d}{d\xi})^{n_f} e^{-\frac{\xi^2}{2}} \qquad (21)$$

Eq (20) and (21) show fractional exponents of $\frac{d}{d\xi}$ and $\left(\xi - \frac{d}{d\xi}\right)$ and can be dealt either with Heaviside approximation or Lypanov exponents for attractors. The Lypanov exponents for equations (20) and (21) will preferably show the behavior of attracting the fractional quantum states in the momentum space with a string, .i.e., a quantum wire.

We calculate the values of the Hermite polynomials from eqs (19), (20), and (21) as shown below

$H_o(hq) = 1$ for $n = 0$ and $H_1(hq) = 2$ for $n = 1, \xi \equiv hq$ now for $0.1 \leq n_f \leq 0.9$

$H_{0.1}(hq) = 1.072, H_{0.2}(hq) = 1.149, H_{0.3}(hq) = 1.231, H_{0.4}(hq) = 1.319, H_{0.5}(hq) = 1.414,$

$H_{0.6}(hq) = 1.516, H_{0.7}(hq) = 1.624, H_{0.8}(hq) = 1.741, H_{0.9}(hq) = 1.8666.$ (22)

The generating function is given by the following relations

$$G(hq, s) = e^{-s^2 + 2shq} \sum_{n=0}^{\infty} \frac{H_{n_f}(hq) s^n}{n!} \qquad (23)$$

The above relation says that if the function e^{-s^2+2shq} is expanded in a power series in s, the coefficients of successive powers of s are just $\frac{1}{n!}$ times the Hermite polynomial, H_{n_f} . By using eq (23) and $n = 0$, we can prove that the Hermite polynomials satisfy the recursion relations by the following relations:

$$H_{n_f+1}(hq) - 2hqH_{n_f}(hq) + 2n_f H_{n_f-1}(hq) = 0$$

$$\frac{dH_{n_f}(hq)}{dhq} = 2n_f H_{n_f-1}(hq) = 0 \quad (24)$$

Using eq (24) for each of the fractional discrete and distinct values of E_{n_f} given by $\lambda_f = 2n_f + 1, 0.1 \leq n_f \leq 0.9$, there is only one physically acceptable solution for eigenfunctions having oscillatory behavior in fractionally quantized momentum space, .i.e.,

$$\psi_{n_f}(hq) = N_{n_f} e^{-\alpha^2\left(\frac{hq}{\sqrt{2}}\right)^2} H_{n_f}(\alpha hq) \quad (25)$$

Considering the Hermite generating function and equate the coefficient of equal parts of s and t, the normalized eigenfunctions are given by

$$\psi(x) = \left(\frac{\alpha}{\sqrt{\pi}2^n n!}\right)^{\frac{1}{2}} e^{-\alpha^2\frac{x^2}{2}} H_n(\alpha x) \quad (26)$$

For our case $x \equiv hq$, $0.17 \leq \alpha \leq 1.53$ where $\alpha = \frac{\alpha\pi}{2} \equiv \lambda$ as $\alpha \neq 1$. The rotation vector will show that the position of the fractional quantized momentum space for charge quantization on the varying surface of the electron in terms of radius. Put $n! = 0! = 1$, the exponent n in terms of n_f (fractional exponents) and the subscript n in H with n_f, where $0.1 \leq n_f \leq 0.9$ in eq (26), we have

$$\psi_{n_f}(hq) = \left(\frac{0.17 \leq \alpha \leq 1.53}{\sqrt{\pi}2^{n_f}.1}\right)^{\frac{1}{2}} e^{-\alpha^2\left(\frac{hq}{\sqrt{2}}\right)^2} H_{n_f}(\alpha hq) \quad (27)$$

Using eq (22), .i.e., $H_{n_f}(hq) = H_{n_f}(\alpha hq)$ in eq (27) and putting $n_f = 0.1\ (\alpha = 0.17), n_f = 0.2\ (\alpha = 0.34)$, …,$n_f = 0.9\ (\alpha = 1.53)$. We can reproduce the distribution of the fractional quantized states for charges on the surface of the electron. In other words, we can either reproduce the shape of the fractional charge distribution which are beaded in a string on the surface of the electron or the shape of the garland with beads with fractional charge quantization. The garland with beads (fractional charge quantization with sub quanta) could be envisaged like a quanta wire. For FT representation of eigenfunctions, the eq (26), we put $\alpha = \frac{\pi}{2}, n = 0$ and 1, we shall then have the asymptotic variation of $\psi_n(hq)$ with $x \equiv hq$.

With arbitrary values of x starting from zero (depth of the quantum well of electron) to radius of an electron, $x = r_e(10 - 15\mu m)$ and using eq (26) with $n! = 0! = 1, 2^{0.1 \leq n_f \leq 0.9}, H_{n_f}(\alpha x) = H_{n_f}(hq)$ and $0.17 \leq \alpha \leq 1.53$, we can reproduce the energy profile for each of the fractional states inside the quantum well.

At $x = r_e$ we have the brim of the quantum well. Each of the eigenfunctions for $0.1 \leq n_f \leq 0.9$ will show the whirling profile for energy whereas the fractional change quantization on the surface of an electron is a swirling phenomenon.

On comparison of eq (7) and (27) with condition that $H_{n_f}(hq) = H_{n_f}(\alpha hq)$, we have $e^{-\alpha^2\left(\frac{hq}{\sqrt{2}}\right)^2} \equiv 1$

we find

$$e^{in_f\alpha} = \left(\frac{\alpha}{\sqrt{\pi}2^{n_f}}\right)^{\frac{1}{2}} e^{-\alpha^2} \tag{28}$$

Considering eq (27) with unitary operator

$H_{n_f}(hq) = H_{n_f}(\alpha hq) \equiv U_{op,}$

$UH_{n_f}(\alpha hq)U^T = I_{op}$ With this unitary operator $H_{n_f}(hq) = H_{n_f}(\alpha hq) \equiv I_{op} = 1$

$H_{n_f}(hq)$ Converges to unity because a new space is configured for an electron quanta with twisting and twigging effects. Gaussian like function, .i.e., $e^{-\alpha^2\left(\frac{hq}{\sqrt{2}}\right)^2} \equiv e^{-\alpha^2} = 1$ also converges to unity for a new configured space. The quanta of electron initially existed in the Wiener space, but with twisting and twigging a new space, .i.e., a Wigner space is configured. Wiener space is transformed in to a Wigner space which is a reciprocal space. The reciprocity is a manifestation of hyperbolic space which depends only on operators. Thus

$$e^{in_f\alpha} = \left(\frac{\alpha}{\sqrt{\pi}2^{n_f}}\right)^{\frac{1}{2}} \tag{29}$$

This reciprocal space is a manifestation of reflection under inversion symmetry (orthogonality is maintained).

3. Conclusion

We presented a new thesis about the morphology of a bounded electron which suffers momentum impact, as a consequence of which the electron quanta is first stretched twisted and then twigged. This behavior different from, Compton and photoelectric effects, respectively. Such morphology of a bounded electron quanta is termed as quasi particle. This morphology of an electron can explained Fractional Charge Quantization, Quantum Hall Effect, Giant Magneto Resistance and Quantum Capacitance.

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