TOKYO METROPOLITAN UNIVERSITY

On the Optimal Funding under the Financial Regulations

by

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Abstract

This thesis studies the cash management strategy for banks subject to various regulations, such as the liquidity coverage ratio (LCR), the leverage ratio, and the capital ratio. We build models that incorporate the stochastic processes into cash outflows and inflows, and lead to an adequate liquidity buffer over the regulatory requirements in the light of the risk tolerance of a bank. As a special case, we treat the net cash outflows as the geometric Brownian motion, and obtain an optimal liquidity buffer analytically. For general cases, we treat cash outflows and inflows as stochastic processes and develop an approximated liquidity buffer by employing Monte-Carlo simulation. Moreover, we obtain workable targets for both the LCR and the capital requirement by incorporating a regulatory cost function.

Keywords: financial regulation, LCR, liquidity risk, funding strategy, liquidity buffer, capital requirement, leverage ratio

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Chapter 1

Introduction

The recent regulatory reforms make banks puzzled about how to control the balance sheets seeking profitability. We hereby clarify about the puzzle and explain about contributions of our study.

1.1 Background

We clarify issues for banks caused by the recent regulatory reforms and the requirement of the risk governance.

1.1.1 Regulatory Reforms

In the late 2000's, the global financial crisis occurred. One of these backdrops was the insufficient liquidity risk management. Liquidity is the ability of a bank to fund increases in assets and meet obligations as they come due, without incurring unacceptable losses. The fundamental role of banks in the maturity transformation of short-term deposits into long-term loans makes banks vulnerable to liquidity risk as shown in Figure1.1. After the crisis, starting from the Washington Summit in November 2008, several regulatory reforms including enhancement of liquidity risk management had been discussed to prevent such affairs from reoccurring. Then, new instruments for supervising the liquidity risk were suggested. One is the liquidity coverage ratio (LCR) and the other is the net stable funding ratio (NSFR). These are collectively called the liquidity regulation. The LCR is to prepare for the short-term liquidity needs, and the NSFR is to have their balance sheets stabilize. In other words, the liquidity regulation performs a role of boosting the funding stability in both the short and long terms. However, the NSFR has not been implemented in most countries and we would like to conduct a simplistic model that can be easy to figure out. Thus, we deal with only the LCR in this thesis.

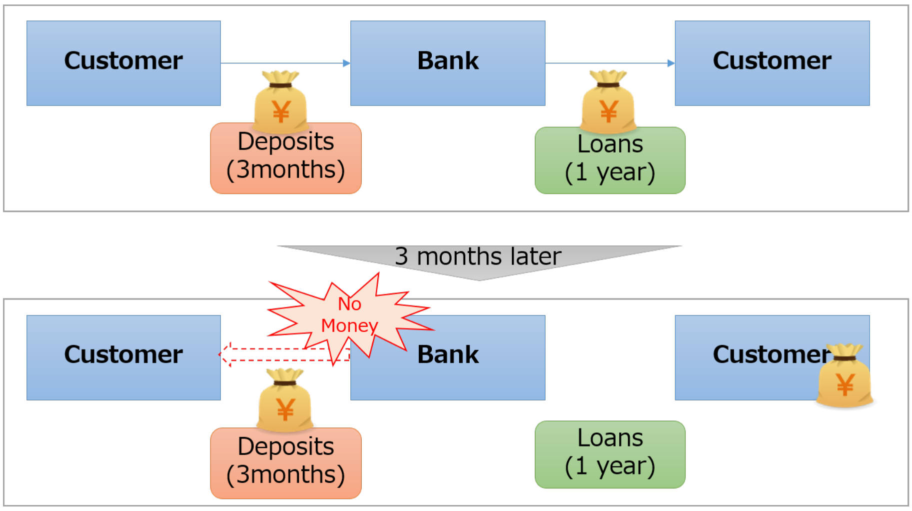

Figure 1.1: Liquidity Risk

On the other hand, not only the liquidity, a new indicator, namely the leverage ratio, was also implemented as one of the financial regulations in an effort of restricting the volume of non-risk-based assets. Under the regulation, it is difficult to increase the funds surplus, even if a bank tries to remedy their LCR. In fact, banks have dilemma between the regulations. Furthermore, existing framework for controlling the risk-based asset was also strengthened. As a result of such regulatory reforms, banks are struggling how to develop reasonable strategies for designing their balance-sheet structures. Therefore, we build an approach that can find optimal targets for each regulatory indicator and help to design a rational balance-sheet structure.

1.1.2 Risk Appetite Framework

The crisis experience has also expanded the recognition that is essential to enhance the discipline of each bank. In November 2013, the Financial Stability Board (FSB) revealed a principle that will help to facilitate a common understanding between supervisors and financial firms and to narrow any gaps between supervisory expectations and practices of banks. Following the principle, banks have established their own risk appetite frameworks, and coordinated some indicators that are able to control their earning, liquidity risk and credit risk and so on in accordance with their risk tolerance. Most banks employ the regulatory indicators as their risk appetite indicators.

For Japanese major banks, funding sustainability for the foreign currency comes to a critical issue. They have increased their assets in foreign currency in an effort of securing their benefits, while a domestic market encounters the lowest interest rate. However, they hardly develop the retail business, so they raise funds by the legal entities and interbank markets, namely the wholesale funding. These main funding sources are wholesale funding [1], which are classified as less-stability in the liquidity regulation. In fact, their businesses in foreign currency are more vulnerable than those in domestic currency. For that reason, they adopt the indicators controlling their liquidity risk in foreign currency and strive to maintain the stability for their funding.

However, it is difficult to determine numerical targets for the indicators that is complementary with the risk tolerance of each bank, though the regulatory levels have been already given. Banks need methods how to determine the rational targets convincingly. Therefore, we conduct a method that can obtain the targets complying with the regulatory requirements and satisfying the profitability reasonably.

1.2 Literature

There are some papers studying the LCR.Keister and Bech[1] incorporate a liquidity requirement by the LCR into an economic model for analyzing the process of implementing monetary policy. They study the interaction that may arise between liquidity regulation and monetary policy implementation. Balasubramanyan and VanHoose[2] formulate a profit of a bank and verify how the LCR impacts the balance sheet of the bank and market interest rate. Cetina and Gleason[3] compare the difference between Basel and U.S. rule on the LCR and show its metrics for understanding clearly. Like these, most of them deal with the LCR impacts from the view point of the financial authorities. There are few studies which refer directly to the cash management strategies of banks.

1.3 Structure of this thesis

The road map for the rest of the thesis is as follows. Chapter 2 illustrates ideas of the recent regulatory reforms for both liquidity and capital requirement. Additionally, we confirm the meanings of each indicators. Chapter 3 assembles models that can elicit the optimal buffer over the regulatory requirement in the LCR that is worth with the risk tolerance of a bank by treating the LCR as a stochastic process. We develop the models to be more realistic by integrating variations of the amount of the deposit, asset, and both. Finally, we introduce the workable method of obtaining rational targets in the liquidity and the capital requirement for risk appetite indicators of a bank regarding the liquidity and capital by incorporating a function representing regulatory costs. Chapter 4 shows the numerical examples so as to clarify the relationship between the models. Furthermore, we execute the comparative statics with the intention to develop an

[1] "Wholesale funding" means liabilities and general obligations that are raised from non-natural persons (i.e., legal entities).

optimal cash management strategies, and achieves optimal targets numerically in both the liquidity and capital requirements. Chapter 5 concludes our study and discusses future works that are able to be more favorable and easy-to-use.

1.4 Contribution

We facilitate a new favorable scheme that banks can accomplish decision-making rationally by using simple models with consideration of tangled regulatory requirement. The risk appetite framework became a pillar of management in the banking sector. Therefore, most banks agonize over how to determine targets as their business plans in line with their risk tolerance. Our study helps to resolve their affair.

Besides, we acquire a useful suggestion that cash managements in banks ought to become more effective by exercising comparative statics. We expect our study would come to find solutions for a lot of issues in banks.

Chapter 2

Financial Regulations

In this chapter, we illustrate the regulatory reforms in the banking sector.

2.1 Liquidity Regulation

Before the crisis, the framework of liquidity risk management had been insufficient for both banks and regulators. Hence, most banks especially in Europe and U.S. went into the lack of liquidity during the crisis. With this experience, Basel committee on banking supervision (BCBS) issued “Principles for Sound Liquidity Risk Management and Supervision” in September 2008 and required the enhancement of liquidity risk management both banks and supervisors. Besides, members of G20 committed themselves to take action to build a stronger, more globally consistent, supervisory and regulatory framework for the future financial sector at the London Summit in April 2009. In accordance with these commitments, BCBS issued a consultative document “International framework for liquidity risk measurement, standards and monitoring” in December 2009 and the liquidity coverage ratio (LCR) and the net stable funding ratio (NSFR) were suggested. One year later, it published Basel III package “A global regulatory framework for more resilient banks and banking systems”. After that, the working group on BCBS discussed the revision points on the LCR through executing quantitative impact studies (QIS) for 2 years and approved these revisions in December 2012. After the group of governors and head of supervision (GHOS) endorsed these, BCBS published the finalized rule “The Liquidity Coverage Ratio and liquidity risk monitoring tool” in January 2013. For the NSFR, the working group had analyzed whether the indicator has ability of finding problem banks. However, even at sufficient level banks, it turned out that there was a little collapse after the crisis. Although there were some bumps and detours, BCBS issued a consultative document revised that was recalibrated to focus on the riskier types of funding profile employed by banks while improving alignment with the LCR and so on in January 2014. After that, the detail of the NSFR was finalized in October, and published "Basel III: the net stable funding ratio".

2.1.1 Liquidity Coverage Ratio

The concept of the LCR is to ensure that a bank has an adequate high-quality liquid asset (HQLA) buffer to meet its fund shortage under a liquidity stress situation during 30 calendar days. Namely,

$$\frac{\text{stock of HQLA}}{\text{Total net cash outflows over the next 30 calendar days}} \geq 100\%. \tag{2.1}$$

Many countries already implemented the LCR in 2015. The minimum requirement was set at 60% and rise in equal annual steps to reach 100% in 2019 to prevent material disruption to the orderly strengthening of banking systems or the ongoing financing of economic activity.

Assets can be considered to be HQLA if they can be easily and immediately converted into cash at little or no loss of value. HQLA is divided into 3 categories: level 1 assets, level 2A assets, and level 2B assets. Level 1 assets are limited to coins, banknotes, central bank reserves, marketable securities assigned a 0% risk-weight, and securities in the bank's home country where sovereign has a non-0% risk weight. Level 2 assets can be included in the stock of HQLA, subject to the requirement that they comprise no more than 40% of the overall stock after haircuts have been applied. Also, the haircut is applied to the current market value of each asset. Level 2A assets are limited to securities assigned a 20% risk-weight and corporate bonds have a credit rating at least AA-. Level 2B assets are limited to equities that have a credit rating at A+$\sim$BBB- and residential mortgage backed securities (RMBS). These can be included in the stock of HQLA, subject to the requirement that they comprise no more than 15% of the overall stock after haircuts have been applied.

Total net cash outflows are defined as the difference between the total expected cash outflows and total expected cash inflows in the specified stress scenario for the subsequent 30 calendar days. Total expected cash outflows are calculated by multiplying the maturing amount among 30 calendar days by the run-off rate according as the types of liability. For example, retail deposits that are fully insured by an effective deposit insurance scheme can be regarded as "stable", then these run-off rate is 3%. Not insured retail deposits are regarded as "less stable", so these run-off rate is 10%. Also, the expected cash outflows include the draw-down from off -balance commitments.[1]

Total expected inflows are calculated by multiplying the maturing amount among 30 calendar days by the rate based on the expected collection by the categories of contractual receivables up to an aggregate cap of 75% of total expected cash outflows.

[1] Commitment line means explicit contractual agreements or obligations to extend funds at a future date to retail or wholesale counter parties.

The LCR have to be reported to supervisors at least monthly. In stress situations, the frequency of reporting increases to weekly or even daily at the discretion of the supervisor. Also, banks should also notify supervisors immediately if their LCR would fall. Hence, banks are expected to maintain the sufficient capacities for calculation.
In Japan, banks disclose their LCR on a quarterly basis that is the daily average for 3 months and the LCR is suggested to be used on an ongoing basis to help monitoring and controlling risk. Therefore, most banks calculate their LCR on a daily basis.

2.1.2 Net Stable Funding Ratio

The concept of the NSFR is to ensure that banks maintain a stable funding profile in relation to their on- and off-balance sheet activities. Thus, the regulators expect that the NSFR reduces the likelihood that a funding disruption of a bank lead to broader systemic stress. The NSFR is defined as

$$\frac{\text{Available amount of stable funding}}{\text{Required amount of stable funding}} \geq 100\%. \tag{2.2}$$

"Available amount of stable funding" is characterized as the portion of capital and liabilities expected to be reliable over the time horizon which extends to one year. "Required amount of stable funding" is calculated based on the liquidity characteristics and residual maturities of the various assets held by that institution as well as those of its off-balance sheet exposures. Initially, the NSFR had been supposed to become a standard by 2018. However, the timing of implementation has not decided yet in most countries. BCBS recognized that each country can decide by their discretion how to calculate about derivatives in October 2017, and published a paper that prompts each country to transit the new standard in December 2017. In that reason, the implementation of the NSFR would facilitate.

2.2 Capital Requirement

In Basel III, there are 2 types of indicators requiring the capital. One is the capital ratio, and the other is the leverage ratio. The capital ratio is a traditional indicator, and it intends to prevent banks from investing risky assets excessively. On the other hand, the leverage ratio aims to restrict investment for both risky and non-risky assets.

2.2.1 Capital Ratio

Capital requirement for banks was firstly implemented in 1988 as Basel I to enforce soundness for international banking system and mitigate compliant of interbank competition. The calculation method of the risk-weighted assets was refined in Basel II that was finalized in 2004. After the financial crisis, some reforms had been developed as Basel III. The minimum requirement level of the common equity raises to 4.5% of risk-weighted assets from 2.0% and the capital conservation buffer (2.5%) is introduced. Banks are imposed to constraint on discretionary distributions of a bank when banks fall into the buffer range. Furthermore, banks are imposed countercyclical buffer ranging within a range of 0-2.5% when authorities judge that credit growth is resulting in an unacceptable buildup of systematic risk. For global systemically important banks (G-SIBs), they are obliged additional loss absorbency requirements ranging from 1% to 2.5%, depending on a systemic importance of a bank.

On the other hand, the calculation method of risk-weighted assets was also refined in Basel III. It strengthened the capital treatment for certain complex securitizations, trading activities, and counter party credit risk and so on. The capital ratio is defined as

$$\text{Capital Ratio} = \frac{\text{Regulatory Capital}}{\text{risk-weighted asset}}. \tag{2.3}$$

2.2.2 Leverage Ratio

A non-risk based leverage ratio that includes off-balance sheet exposures serves as a backstop to the risk-based capital requirement, namely the capital ratio. BCBS published a document regarding the leverage ratio in 2010 at the same time as the liquidity regulation. The monitoring process in the parallel run period from January 2013 to January 2017 was started. Also, banks began to disclose their leverage ratio since 2015. GHOS agreed the minimum required level (3%) in January 2016, and BCBS published a consultative document that to be revised the design in this year. In December 2017, additional surcharge for G-SIBs were determined. Under the existing definition for the exposure, this regulation will become pillar I in 2018. The revised definition will be effective in 2022 and the surcharge for G-SIBs will be implemented in 2022. The leverage ratio is defined as

$$\text{Leverage Ratio} = \frac{\text{Tier1 Capital}}{\text{On- and Off- balance sheet exposures}}. \tag{2.4}$$

The denominator is "non-risk" based, so it does not multiply by the risk weight. That is, it contains even the central bank reserves and the government bond that risk weight is 0%.

2.3 G-SIBs surcharge

G20 Leaders called on the FSB to propose measures to address the systemic and moral hazard risks associated with systemically important financial institutions (SIFIs) at the Pittsburgh Summit in 2009. SIFIs are institutions of such size, market importance and interconnectedness that their distress or failure would cause significant dislocation in the financial system and adverse economic consequences. The "too-big-to-fail" (TBTF) problem arises when the threatened failure of a SIFI leaves public authorities with no option but to bail it out using public funds to avoid financial instability and economic damage. To address the TBTF issues, the framework for additional loss absorbency, for increased supervisory intensity, for more effective resolution mechanisms, and for stronger financial market infrastructure had been discussed.

The global systemic importance of banks (G-SIBs) and insures (G-SIIs) are included in the G-SIFIs. FSB publishes the list of G-SIBs and G-SIIs every year by assessing the systemically importance. In November 2017, it selects 30 banks as G-SIBs. For the G-SIBs, a new strengthened capital regime requiring additional going-concern loss absorption capacity was suggested. As noted before, the G-SIBs are imposed to increase their common equity capital against both their risk-weighted assets and their non-risk based assets according to the systemically importance.

Chapter 3

Models

This chapter builds models that can elicit the optimal buffer over the regulatory requirement in the LCR.

3.1 Setting and Objective

Figure 3.1 illustrates the typical balance sheet of a commercial bank.
The bank owns the high-quality liquid assets (HQLA) because of allowing enough funds for daily cash management and adhering to the liquidity regulation. Also, it invests risky assets such as loans in order to meet the demand of its customer and secure its profits. On the other hand, it raises funds by deposits and corporate bonds to maintain the ability of funding for assets increasing and meeting obligations as they come due. In general, the maturities of deposits are shorter than ones of corporate bonds. Hence, a funding cost of the corporate bond is more expensive than one of the deposit. The capital fulfills a role of stabilizing the cash management because it is perpetual, in addition to regulatory requirement. Each item of the balance sheet is denoted by the character in the parenthesis.[1]

As noted in Equation (2.1), the LCR is a division of the HQLA amount and total net cash outflows. The maturities and total amounts of each items affect the cash flows, so we incorporate the total expected cash flows in the next 30 days and the total amounts of each item into our models. $X_{(.)}$ stands for the total amount, and $x_{(.)}$ stands for the total expected cash flows in the next 30 days stress period. For example, total amount of a deposit is denoted by X_D and the total expected cash outflows form the deposit is denoted by x_D. Figure 3.2 illustrates the relationship between the total amount and the total expected cash outflows. Each bar indicates the amount maturing on each day. The summation of 30 bars with the outflow ratio is x_D. The summation of all bars is X_D.

[1] HQLA is denoted by "S" from "surplus".

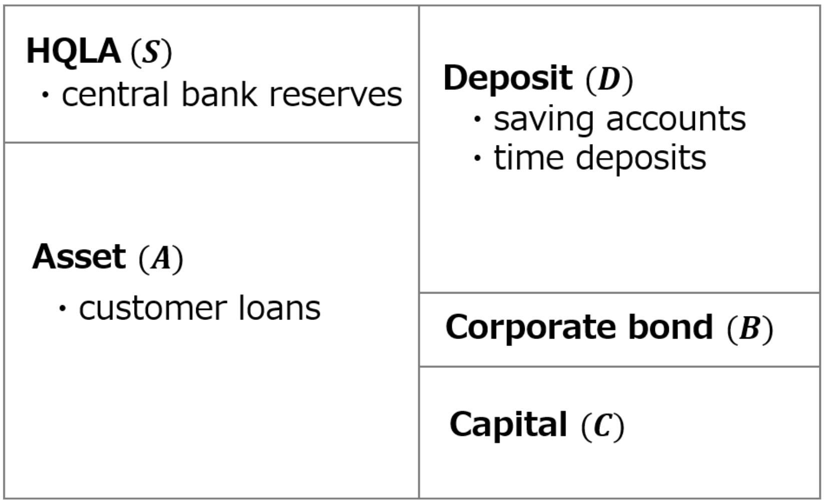

Figure 3.1: Balance Sheet

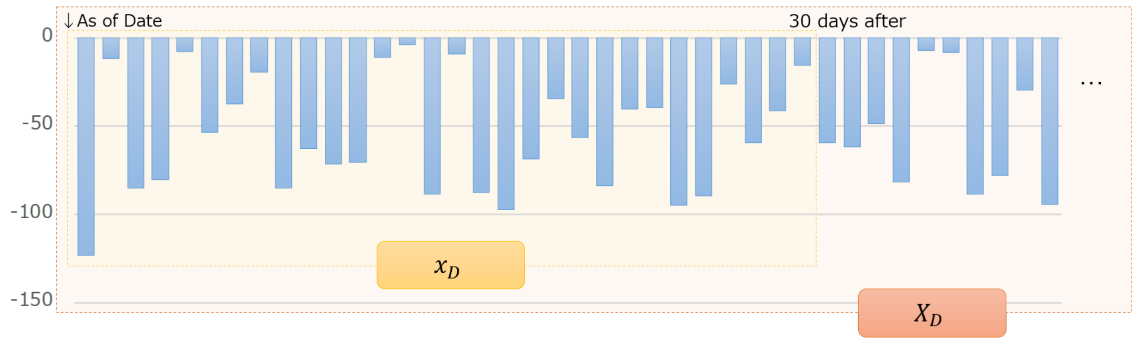

Figure 3.2: Relationship between the total amount, X_D, and the expected cash outflows, x_D, of deposits

Using these setting, the LCR can be written as

$$\begin{aligned} LCR(t) &= \frac{X_S(t)}{N(t)}, && (3.1) \\ N(t) &:= x_{out}(t) - \min\left[x_{in}(t), x_{out}(t) \times 75\%\right] \\ &= \max\left[x_{out}(t) - x_{in}(t), x_{out}(t) \times 25\%\right], && (3.2) \\ x_{out} &:= x_D + x_B + \epsilon, \\ x_{in} &:= x_A, \end{aligned}$$

where ϵ stands for the expected cash outflows from drawing down by commitment lines and additional collateral needs and so on.

Then, we consider a cash management plan during period T. In general, T is determined on the frequency of corporate bond issuances, because the original term

of ordinary corporate bonds is longer than 30 calendar days. Hence, the bank can increase the HQLA without affecting its LCR.
For seeking an effective cash management plan, we need to revolve both the regulations and the profitability. Banks are supervised by the authorities, so they have to avoid infringing the regulatory requirement. Thus, we consider a plan that restrain the breach probability to $(1-\zeta)$. An acceptable breach probability should be determined to meet a risk tolerance of each bank.

Also, as mentioned before, they have to adhere to not only the LCR, but also the leverage ratio that is not easy to increase the HQLA. Banks also do not want to hold the excess HQLA in order to pursue their profitability, because the return of the HQLA is lower than one of the risky assets and sometimes lower than their borrowing rate. Therefore, banks want to find a buffer of the HQLA satisfying

$$P(\beta) := \mathbb{P}\left[\min_{0\leq t\leq T} LCR(t) < \phi\right] = 1-\zeta, \tag{3.3}$$

where

$$LCR(0) = \frac{X_S(0)}{N(0)} = \phi + \beta.$$

Here, ϕ stands for the regulatory requirement level in the LCR, and β stands for the bank's voluntary buffer over the regulation. If a bank treats the LCR as its risk appetite indicator, $(\phi + \beta)$ can be a target that meets its risk tolerance.
Equation (3.3) represents the breach probability of the LCR by using the first-passage-time model. Figure 3.3 shows a sample path of the LCR. It supposes that a bank prepares the total amount of HQLA appropriate for $\beta = 20\%$ at $t = 0$, and its authority sets a regulatory requirement as 100%. An infringing occurs when the minimum level of the LCR during T is less than the regulatory requirement. Thus, we can obtain the breach probability by running a lot of paths of the LCR.
Also, we know the starting point $N(0)$ at $t = 0$, hence denote it by "y", thus the staring position of HQLA is written by

$$X_S(0) = (\phi + \beta)y. \tag{3.4}$$

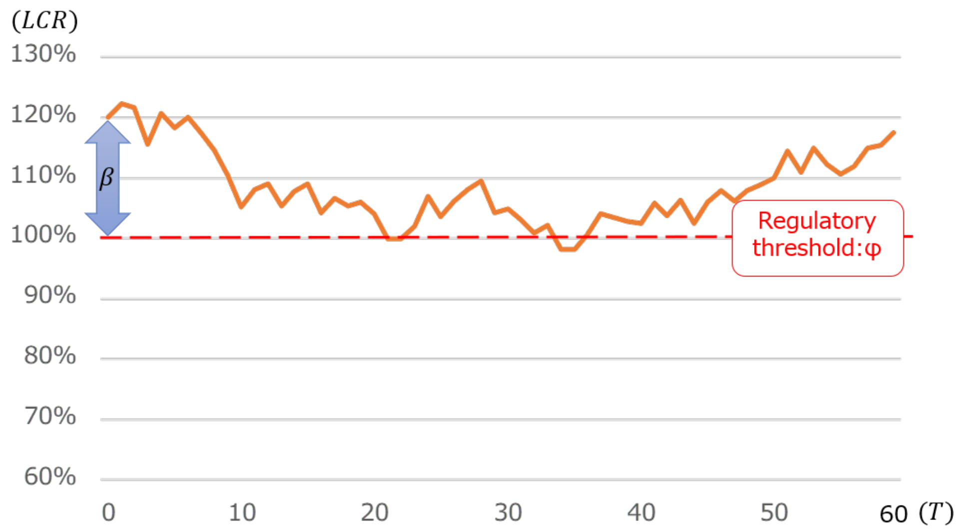

Figure 3.3: Sample Path of the LCR

3.2 Analytic Model

We consider a model that can obtain an analytical solution, i.e., an optimal liquidity buffer over the regulatory requirement.

3.2.1 Additional Setting

Additional settings are needed to obtain an analytical solution. At first, we consider a setting for cash flows. We treat the total net cash outflows, $N(t)$, as a stochastic process represented by

$$\frac{dN(t)}{N(t)} = \mu_N dt + \sigma_N dz(t), \quad N(0) =: y, \tag{3.5}$$

where μ_N and σ_N are constant. Actually, $N(t)$ is the larger of the difference between the total expected cash outflows and inflows and 25% of cash outflows, as noted by Equation (3.2). Thus, it is not realistic in general, though it works effectively when the levels of the total expected cash outflows and inflows are quite different, and when we suppose that a bank can not fully control its maturing amount.

In Japan, banks are imposed to disclose their daily-average of the LCR on a quarterly basis. (see figure A.1 in Appendix A). Therefore, they have enough data that can calibrate the parameters of Equation (3.5).

Then, we consider a setting for the total amounts. At $t = 0$, a bank prepares the initial volume of the HQLA, $X_S(0)$, satisfying Equation (3.4) funded by the corporate bond.[2] The amount of the HQLA, asset, deposit, corporate bond, and capital, $(X_S, X_A, X_D, X_B, X_C)$, are constant during T.

By using these additional settings, Equation (3.3) can be rewritten by

$$P(\beta) := \mathbb{P}\left[\max_{0\le t\le T} N(t) < \frac{(\phi+\beta)y}{\phi}\right] = 1-\zeta, \tag{3.6}$$

where

$$X_S, X_A, X_D, X_B, X_C \ : \ \textit{constant at } t \in (0,T).$$

3.2.2 The Solution

Equation (3.5) can be rewritten by using $N(0) = y$ as

$$N(t) \quad = y \exp\left\{\left(\mu_N - \tfrac{1}{2}\sigma_N^2\right)t + \sigma_N z_N(t)\right\}.$$

Also, the bracket of Equation (3.6) is

$$\max_{0\le t<T}\left(\mu_N - \frac{1}{2}\sigma_N^2\right)t + \sigma_N z_N(t) > \log\left(\frac{\phi+\beta}{\phi}\right). \tag{3.7}$$

According to Shreve[11], we obtain the distribution of Equation (3.6) as below by using the reflection principal:[3]

$$P(\beta) \quad = \quad 1 - N\left(\frac{m(\beta)-\alpha T}{\sqrt{T}}\right) + \mathrm{e}^{2\alpha m(\beta)} N\left(\frac{-m(\beta)-\alpha T}{\sqrt{T}}\right) = 1-\zeta, \tag{3.9}$$

where

$$\begin{cases} \alpha & = \mu_N - \frac{1}{2}\sigma_N^2, \\ m(\beta) & = \log\left(\frac{\phi+\beta}{\phi}\right). \end{cases}$$

[2] An original term of the corporate bond is longer than 30 days, so it does not affect the bank's LCR position.

[3] Brownian motion and distribution of its maximum: Define,

$$\begin{aligned} \widehat{W(t)} &= \alpha t + W(t) \\ \widehat{M(t)} &= \max_{0\le t\le T} \widehat{W(t)} \end{aligned}$$

where W is Brownian motion and α is constant number. Then,

$$P\left(\widehat{M(t)} \le m\right) = N\left(\frac{m-\alpha T}{\sqrt{T}}\right) - \mathrm{e}^{2\alpha m} N\left(\frac{-m-\alpha T}{\sqrt{T}}\right), \ m \ge 0. \tag{3.8}$$

Therefore, the optimal β satisfies

$$N\left(\frac{m(\beta)-\alpha T}{\sqrt{T}}\right)-\mathrm{e}^{2\alpha m(\beta)}N\left(\frac{-m(\beta)-\alpha T}{\sqrt{T}}\right)=\zeta. \tag{3.10}$$

We can easily obtain an adequate buffer over the regulatory requirement in the LCR by estimating each parameter from historical data of each bank, and by determining the period for its cash management plan and the acceptable breach probability.

3.2.3 Extension to the Multi-Period

By Equation (3.9), the breach probability is independent of the starting point of the total net cash outflows, y. Thus, it is easy to extend to the multi-period problem. It is useful when a bank determines the period for its cash management plan, T. Then, we consider a business period; $\mathcal{T} := mT$. $\mathcal{T}$ depends on the term of a business plan, so one or three years is common in the Japanese banks. Also, the initial volume of the HQLA on each T is supposed to be adjusted by the sufficient long-term funding such as corporate bond.
Hence, the breach probability during $\mathcal{T}$ is

$$\mathcal{P}(\beta,T)=1-\left(1-P(\beta,T)\right)^{m}. \tag{3.11}$$

From Equation (3.11), we find that if β increases with an fixed T, then P decreases. On the other hand, if T increases with an fixed β, then P increases. Therefore, we can obtain an optimal β and T by considering an adequate cost function, though we do not refer in this thesis.

3.2.4 Summary

We can obtain an analytical solution from this model, and it is easy to extend to the multi-period problem. However, it is not realistic because the floor of the total net cash outflows is not incorporate. Additionally, this model is difficult to extend to the multi-currency problem. In general, banks operate their cash management by currency. Thus, it is reasonable setting that the total net cash outflows of each currency follow the geometric Brownian motion respectively. However, the LCR is measured by the summation of all currencies.[4] Hence, the LCR is the summation of the geometric Brownian motions. It is difficult to treat it when developing an analytical solution. Therefore, this is the cons of the analytic model.[5]

[4] Some countries impose the currency base LCR in addition to the all currencies base LCR.

[5] If seeking an analytical solution, the moment-matching method is useful. There are a lot of methods for pricing the basket options and the volume-weighted average price (VWAP) options. For example, Brigo et al[12] study the approximation of the basket options and Funahashi and Kijima[13] study the VWAP options.

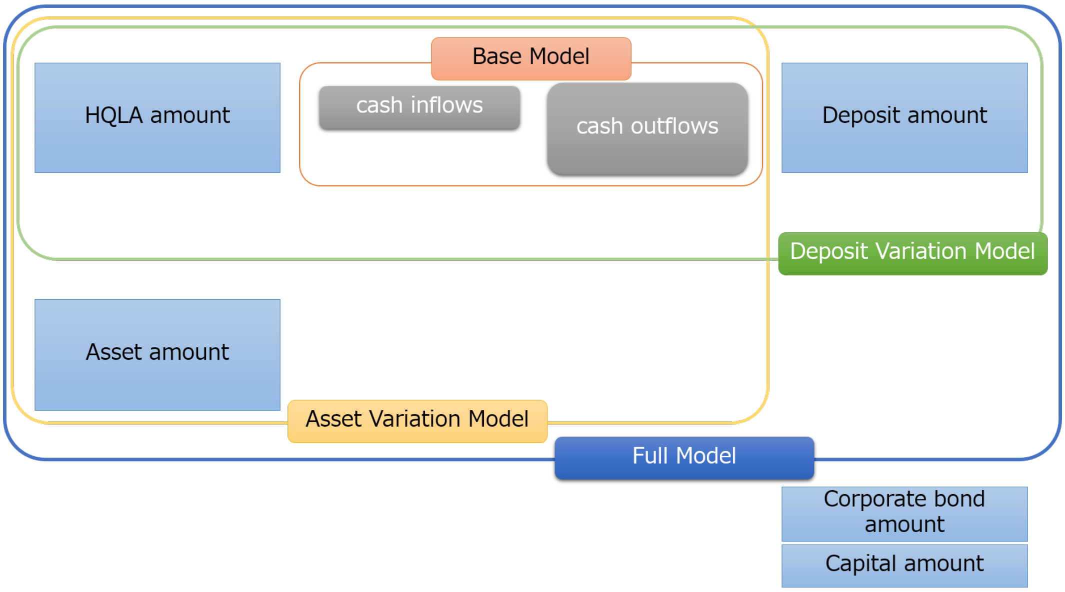

Figure 3.4: Overview of the Simulation Models

3.3 Simulation Model

We need to introduce other models to overcome the negative point of the analytic model. Figure 3.4 shows an overview of the models to be explained. At the first model, we treat the total expected cash outflows and inflows as stochastic processes respectively. The second model extends the first model, and incorporates the variation of the deposit amount. The third model involves the variation of the asset amount, instead of the deposit amount variation. The Last model integrates both the variations of the deposit and the asset amount. The amount of HQLA varies according to the movement of other lending or borrowing amount.

3.3.1 Base Model

At first, we consider the most simplistic model. In this model, we treat the total expected cash outflows and inflows as stochastic processes respectively in order to represent a floor rule.

Banks try to make its cash management stabilize by smoothing their maturing amounts. However, they can not control their whole positions, because they get a deal with other banks or customers. Only major banks are easy to control because they have more authority for the negotiation of the condition. Hence, it is a reasonable setting that their total expected cash outflows and inflows follow the mean-reversion processes.[6] However, it is difficult for most banks, especially foreign banks to control their maturing amount constantly. Therefore, we treat as the diffusion processes.

Other settings are the same as the analytic model. Prepare the initial volume of HQLA ($X_S(0)$) satisfying Equation (3.3) and the amount of HQLA, asset, deposit, corporate bond, and capital, $(X_S, X_A, X_D, X_B, X_C)$, are constant during T.

$$\begin{bmatrix} dx_{out}(t)/x_{out}(t) \\ dx_{in}(t)/x_{in}(t) \end{bmatrix} = \begin{bmatrix} \mu_{out} \\ \mu_{in} \end{bmatrix} dt + \Omega \begin{bmatrix} dz_{out}(t) \\ dz_{in}(t) \end{bmatrix}, \tag{3.12}$$

where

$$\Omega \cdot \Omega' = \begin{bmatrix} \sigma_{out}^2 & \rho_{out,in}\sigma_{out}\sigma_{in} \\ \rho_{out,in}\sigma_{out}\sigma_{in} & \sigma_{in}^2 \end{bmatrix}, \tag{3.13}$$

$$x_{out}(0) = x_{out}, \quad x_{in}(0) = x_{in}.$$

In this model, total net cash outflows are the subtraction of the geometric Brownian motions. Appendix C introduces the approximation method about it.

3.3.2 Deposit Variation Model

We incorporate a deposit variation to make more realistic. Figure 3.5 is the transition of the deposit amount in Japanese major banks. The deposit amount varies stochastically. Hence, we treat the deposit amount, X_D, as a stochastic process in addition to the total expected cash outflows and inflows. An increment of the deposit amount means a boost of the bank's capability for investment. Hence, it can increase either risky assets or non-risky assets. This model supposes that the bank would invest to non-risky assets, namely HQLA in order to avoid the infringing of the regulatory requirement. Conversely, when the deposit amount decreases, the bank would reimburse it to depositor by using HQLA, because most assets have not yet matured. Equation (3.14) represents this setting. Also, prepare the initial volume of HQLA, $X_S(0)$, satisfying Equation (3.3), and the amount of asset, corporate bond and capital, (X_A, X_B, X_C), are constant during T.

$$dX_S(t) = dX_D(t), \tag{3.14}$$

$$\begin{bmatrix} dX_D(t)/X_D(t) \\ dx_{out}(t)/x_{out}(t) \\ dx_{in}(t)/x_{in}(t) \end{bmatrix} = \begin{bmatrix} \mu_D \\ \mu_{out} \\ \mu_{in} \end{bmatrix} dt + \Omega \begin{bmatrix} dz_D(t) \\ dz_{out}(t) \\ dz_{in}(t) \end{bmatrix}, \tag{3.15}$$

[6] We roughly refer this setting in Appendix B.

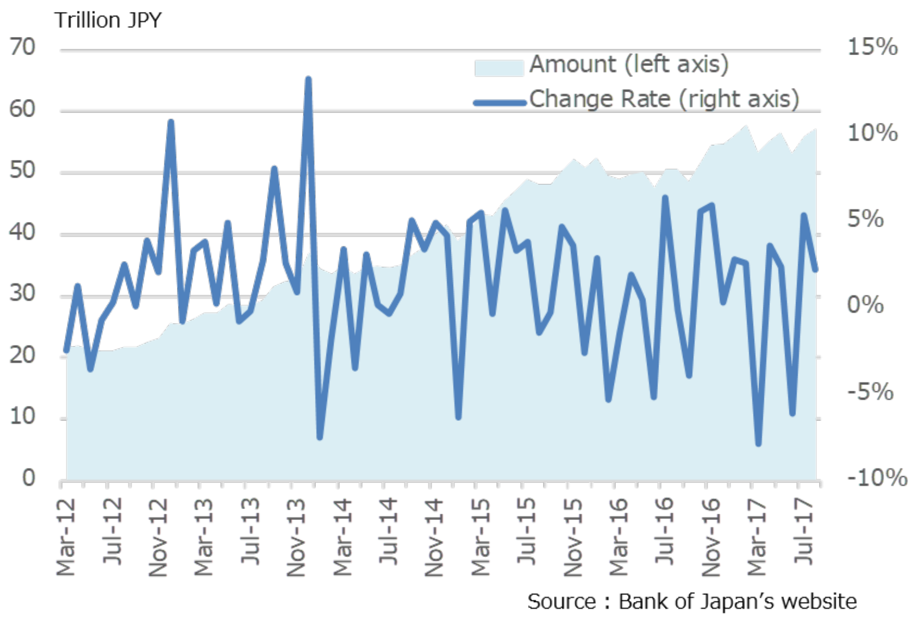

Figure 3.5: Monthly Deposit Variation in Japanese major banks

where

$$\Omega \cdot \Omega' = \begin{bmatrix} \sigma_D^2 & \rho_{D,out}\sigma_D\sigma_{out} & \rho_{D,in}\sigma_D\sigma_{in} \\ \rho_{D,out}\sigma_D\sigma_{out} & \sigma_{out}^2\rho_{out,in}\sigma_{out}\sigma_{in} & \\ \rho_{D,in}\sigma_D\sigma_{in} & \rho_{out,in}\sigma_{out}\sigma_{in} & \sigma_{in}^2 \end{bmatrix}, \tag{3.16}$$

$$X_D(0) = X_D, \quad x_{out}(0) = x_{out}, \quad x_{in}(0) = x_{in}.$$

When calibrating these parameters, we have to care about the positive definiteness for covariance matrix in Equation (3.16). Also, Figure A.2 in Appendix A is the relationship between the deposit amount and its total expected cash outflows in Japanese major banks. It is not enough data, but the fraction of the deposit amount and its cash outflows is roughly constant. Therefore, the correlation of these, $\rho_{D,out}$, can be presumed to be positive.

3.3.3 Asset Variation Model

At third, we consider the asset variation instead of the deposit variation. Figure 3.6 is the transition of the asset amount in Japanese major banks. The asset amount varies stochastically as the same as the deposit amount. Hence, we treat the asset amount, X_A, as a stochastic process in addition to the total expected cash outflows and inflows. An increment of the asset amount makes the bank's capability for investment depressed. Thus, the bank has to decrease non-risky assets or increase the deposit.

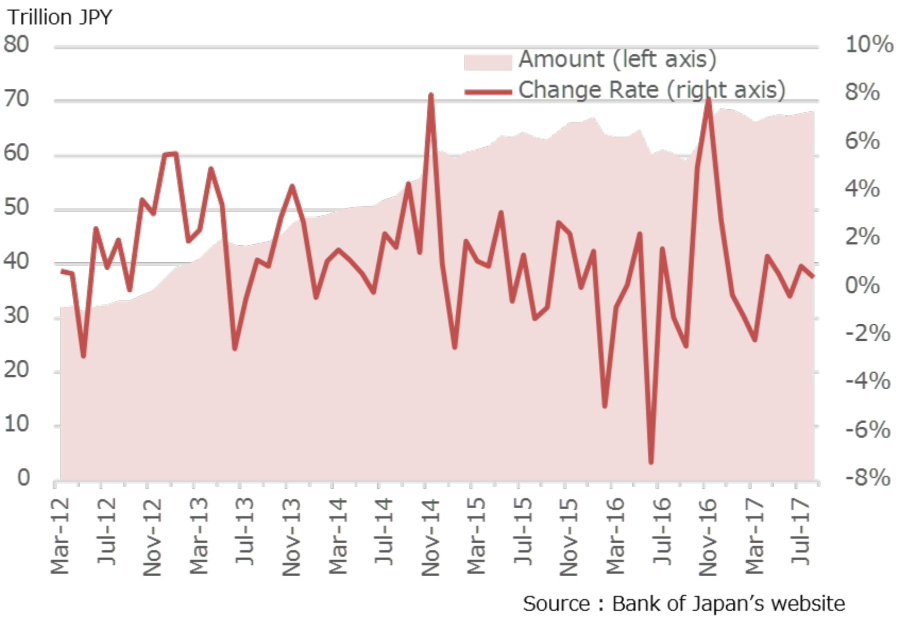

Figure 3.6: Monthly Asset Variation in Japanese major banks

This model supposes that a bank would shift funds from non-risky assets, namely HQLA, because there is not always possibility to fund by the deposit over the 30 calendar days. Conversely, when the asset amount decreases, HQLA amount increases. Equation (3.17) represents this setting. Also, prepare the initial volume of HQLA, $X_S(0)$, satisfying Equation (3.3), the amount of deposit, corporate bond and capital, (X_D, X_B, X_C), are constant during T.

$$dX_S(t) = -dX_A(t), \tag{3.17}$$

$$\begin{bmatrix} dX_A(t)/X_A(t) \\ dx_{out}(t)/x_{out}(t) \\ dx_{in}(t)/x_{in}(t) \end{bmatrix} = \begin{bmatrix} \mu_A \\ \mu_{out} \\ \mu_{in} \end{bmatrix} dt + \Omega \begin{bmatrix} dz_A(t) \\ dz_{out}(t) \\ dz_{in}(t) \end{bmatrix}, \tag{3.18}$$

where

$$\Omega \cdot \Omega' = \begin{bmatrix} \sigma_A^2 & \rho_{A,out}\sigma_A\sigma_{out} & \rho_{A,in}\sigma_A\sigma_{in} \\ \rho_{A,out}\sigma_A\sigma_{out} & \sigma_{out}^2 & \rho_{out,in}\sigma_{out}\sigma_{in} \\ \rho_{A,in}\sigma_A\sigma_{in} & \rho_{out,in}\sigma_{out}\sigma_{in} & \sigma_{in}^2 \end{bmatrix}, \tag{3.19}$$

$$X_A(0) = X_A, \quad x_{out}(0) = x_{out}, \quad x_{in}(0) = x_{in}.$$

When calibrating these parameters, we have to care about the positive definiteness for covariance matrix in Equation (3.19). Also, Figure A.3 in Appendix A is the relationship between the asset amount and its total expected cash inflows in Japanese major banks. There is not enough data, but the fraction of the asset amount and its cash inflows is roughly constant. Therefore, the correlation of these, $\rho_{A,in}$, can be presumed to be positive.

3.3.4 Full Model

Finally, we consider a full model to make it more realistic. It treats both the asset and the deposit amounts, (X_A, X_D), as stochastic processes. Also, we consider the upper limit of the asset amount by revolving the regulation for capital requirement against the risk-weighted-assets amount, that is the capital ratio.

We define a voluntary capital ratio, ψ; it can be a target of the risk appetite indicator, and a factor converted non-risk-based assets amount into risk-weighted-asset amount, ω.[7] That is to say, the upper limit of the asset amount, $X_A^{\max}$, satisfies

$$\psi = \frac{X_C}{X_A^{\max} \cdot \omega} \quad \Leftrightarrow \quad X_A^{\max} = \frac{X_C}{\psi\omega}. \tag{3.20}$$

The asset amount normally follows the geometric Brownian motion. However, when it reaches the barrier level, in the next step, it can either remain or move downward. Hence, the asset amount has a retaining barrier.

Similar to the previous models, HQLA varies according to the movement of the asset and deposit amounts. However, when increasing the deposit amount, a bank uses this fund to invest risky assets according to correlation between the deposit and the asset amount to the extent that does not exceed the voluntary capital ratio. If it can not invest risky assets, HQLA amount increases. Conversely, when decreasing the deposit amount, a bank collects their assets according to correlation between the deposit and the asset amounts, or uses their HQLA for reimbursing the deposit. Figure 3.7 shows the change rate of the asset and the deposit amounts. In general, the correlation between the asset and the deposit amounts is positive. Equation (3.21) represents these settings. Also, prepare the initial volume of HQLA, $X_S(0)$, satisfying Equation (3.3), the amount of corporate bond and capital, (X_B, X_C), are constant during T.

$$dX_S(t) = dX_D(t) - dX_A(t), \tag{3.21}$$

$$\begin{bmatrix} dX_D(t)/X_D(t) \\ dX_A(t)/X_A(t) \\ dx_{out}(t)/x_{out}(t) \\ dx_{in}(t)/x_{in}(t) \end{bmatrix} = \begin{bmatrix} \mu_D \\ \mu_A \\ \mu_{out} \\ \mu_{in} \end{bmatrix} dt + \Omega \begin{bmatrix} dz_D(t) \\ dz_A(t) \\ dz_{out}(t) \\ dz_{in}(t) \end{bmatrix}, \tag{3.22}$$

[7] As Figure A.4 in Appendix A shows, the factor is almost constant

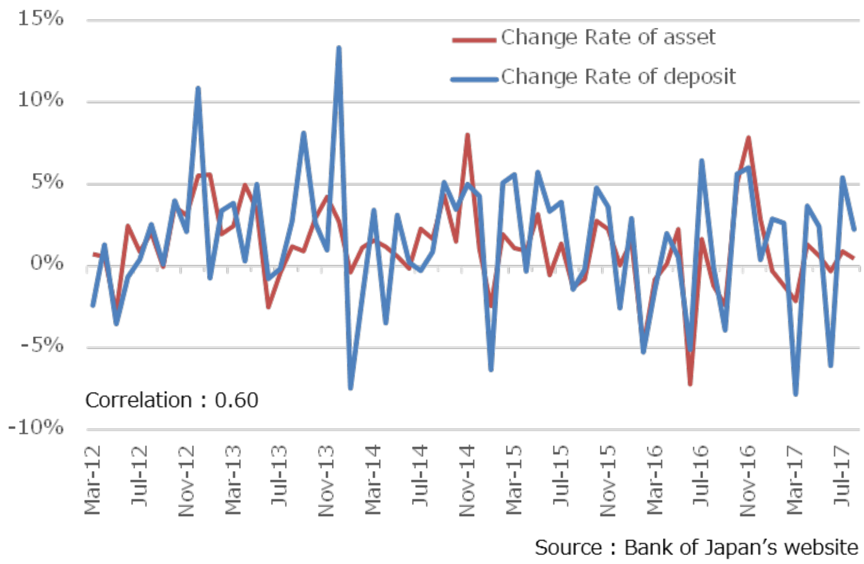

Figure 3.7: Monthly Deposit and Asset Variation in Japanese major banks

where

$$\Omega \cdot \Omega' = \begin{bmatrix} \sigma_D^2 & \rho_{D,A}\sigma_D\sigma_A & \rho_{D,out}\sigma_D\sigma_{out} & \rho_{D,in}\sigma_D\sigma_{in} \\ \rho_{D,A}\sigma_D\sigma_A & \sigma_A^2 & \rho_{A,out}\sigma_A\sigma_{out} & \rho_{A,in}\sigma_A\sigma_{in} \\ \rho_{D,out}\sigma_D\sigma_{out} & \rho_{A,out}\sigma_A\sigma_{out} & \sigma_{out}^2 & \rho_{out,in}\sigma_{out}\sigma_{in} \\ \rho_{D,in}\sigma_D\sigma_{in} & \rho_{A,in}\sigma_A\sigma_{in} & \rho_{out,in}\sigma_{out}\sigma_{in} & \sigma_{in}^2 \end{bmatrix}, \quad (3.23)$$

$$X_D(0) = X_D,\ X_A(0) = X_A,\ x_{out}(0) = x_{out},\ x_{in}(0) = x_{in}.$$

When calibrating these parameters, we have to care about the positive definiteness for covariance matrix in Equation (3.23). As noted before, the correlation between the deposit and the asset amount, and the correlation between the deposit amount and the total expected cash outflows, and the correlation between the asset amount and the total expected cash inflows can be presumed to be positive.

3.3.5 Optimal Voluntary Buffer Levels

For obtaining a more effective cash management strategy, we consider optimal voluntary levels of both liquidity and capital regulations. In the full model, if the voluntary capital buffer (ψ) is high, it is difficult for the bank to invest more risky assets, so it increases HQLA. Hence, opportunity loss would occur, while the voluntary liquidity buffer, β, would decrease because the breach probability for the regulatory requirement for the LCR would become low by the sufficient HQLA. On the other hand, if ψ is low, it is easy for the bank to invest more risky assets. However, the breach probability for

the LCR would become high. The bank prepares more excess HQLA, thus the funding cost for HQLA would become expensive. Therefore, we can find the optimal level by considering an appropriate cost functions for opportunity loss and funding cost.
We define opportunity loss as

$$\sum_{t=0}^{T} s\left(\mathbb{E}\left[\tilde{X}_A(t) - X_A(t)\right]\right) =: f_{OL}, \tag{3.24}$$

where $\tilde{X}$ is the asset amount without a retaining barrier, and s is a loan spread.

On the other hand, when the voluntary buffer for the LCR is high, the bank has to acquire a lot of HQLA, so its funding cost increases.
Also, we define funding cost as

$$c \cdot \{X_S(0) - X_S^*(0)\} =: f_{FC}, \tag{3.25}$$

where c is a borrowing spread, and $X_S^*(0)$ is the regulatory required amount.

Regulatory cost is defined as the summation of opportunity loss and funding cost, i.e.,

$$f_{RC} = f_{OL} + f_{FC}. \tag{3.26}$$

Chapter 4

Numerical Example

The optimal cash management strategy depends on the characteristics of a bank's existing balance-sheet structure. We confirm the relationship between the simulation models, and examine the effects of some parameters. The parameters are set according to Japanese major banks with noise as shown in Table 4.1. In actual situation, as noted in Chapter 2, most banks calculate the LCR daily basis, though not disclose daily figure, for responding the supervisor's requirements and controlling own liquidity risk. Hence, it is easy to calibrate the parameters.

4.1 Relationship between the Simulation Models

Figure 4.1 shows the relationship between simulation models defined in Chapter 3. This shows the optimal liquidity buffer, β, satisfying Equation (3.6) to meet each breach probability for the LCR, $(1-\zeta)$. The base model (orange line) positions between the deposit variation model (green line) and the asset variation model (yellow line). That is because the deposit variation is positive effect for the amount of HQLA, while the asset variation is negative effect. The amounts of the deposit and the asset tend to increase by positive drift, μ_D, μ_A. Hence, the deposit variation model requires less liquidity buffer, while the asset variation model requires more it. The full model (blue line) positions between the base model (orange line) and the deposit variation model (green line). It is according to the level of the retaining barrier, however the correlation between the deposit amount and the total expected cash outflows, $\rho_{D,out}$, is higher than the correlation between the asset amount and the total expected cash inflows, $\rho_{A,in}$, and additionally the amounts of the deposit and the asset diffuse as the same, $\mu_D = \mu_A$, $\sigma_D = \sigma_A$. Therefore, the amount of HQLA in the full model is easier to increase than one in the asset variation model.

Deposit	X_D	JPY 190 trillion
	μ_D	0.0001
	σ_D	0.001
Asset	X_A	JPY 170 trillion
	μ_A	0.0001
	σ_A	0.001
	ω	0.5
outflows	x_{out}	JPY 49 trillion
	μ_{out}	0.003
	σ_{out}	0.015
inflows	x_{in}	JPY 10 trillion
	μ_{in}	0.001
	σ_{in}	0.06
Capital	X_C	JPY 13 trillion
Cor	$\rho_{D,A}$	0.8
	$\rho_{D,out}$	0.8
	$\rho_{A,in}$	0.5
	$\rho_{out,in}$	0.5
	$\rho_{D,in}$	0.3
	$\rho_{A,out}$	0.5
Threshold	ψ	15.2%
	ϕ	100%
Term	T	60 business days

Table 4.1: Parameters

4.2 Comparative Statics

The optimal liquidity buffer depends on the characteristics of the structures of the balance-sheet and cash flows. We examine the effect of some parameters in order to consider the optimal cash management strategy, when the breach probability is 10%.

4.2.1 Impact of Volatilities in the Base Model

We observe the impact of the volatilities of the expected cash outflows and inflows, $(\sigma_{out}, \sigma_{in})$, in the base model.

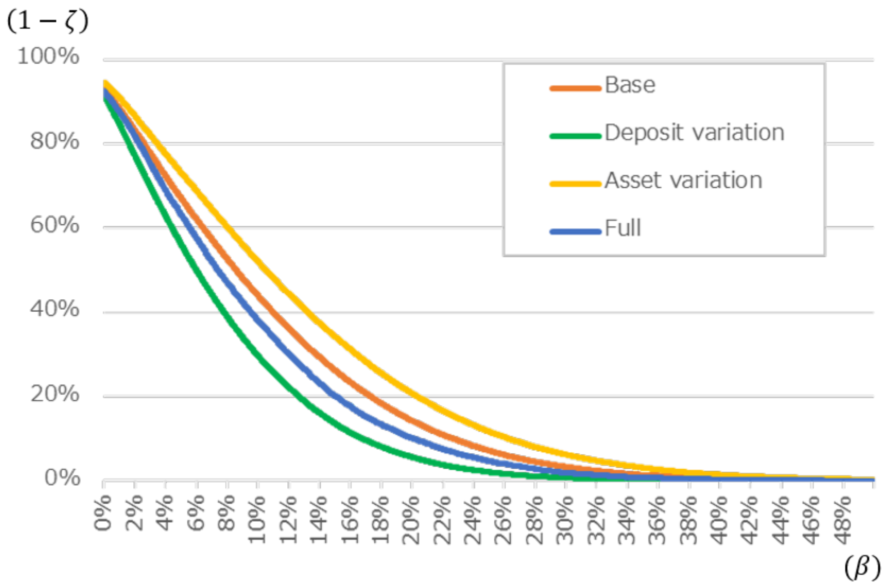

Figure 4.1: LCR Breach Probability vs Optimal Liquidity Buffer

Volatility of the Total Expected Cash Outflows

Figure 4.2 shows the relationship between the volatility of the total expected cash outflows, σ_{out}, and the its optimal liquidity buffer, β, by the correlation between the total expected cash outflows and inflows, $\rho_{out,in}$. It shows that there is a minimum level for the optimal liquidity buffer when $\rho_{out,in}$ is positive. This is because the total net cash outflows, the denominator of the LCR, are modeled by subtraction of the 2 Brownian motions. Thus, the bank is able to minimize the liquidity buffer by operating with a few volatilities for the total expected cash outflows. In other words, banks can reduce the liquidity buffer by controlling their maturing amount of the deposit per day flexibly, instead of trying to smooth their maturing amount.

Volatility of the Total Expected Cash Inflows

Figure 4.3 shows the relationship between the volatility of the total expected cash inflows, σ_{in}, and the its optimal liquidity buffer, β, by the correlation between the total expected cash outflows and inflows, $\rho_{out,in}$. There is a minimum level for the optimal liquidity buffer when $\rho_{out,in}$ is positive, as the same as the volatility of the total expected cash outflows. Therefore, banks can reduce the liquidity buffer by controlling their maturing amount of the asset per day flexibly, instead of trying to smooth their maturing amount, not only one of the deposit.

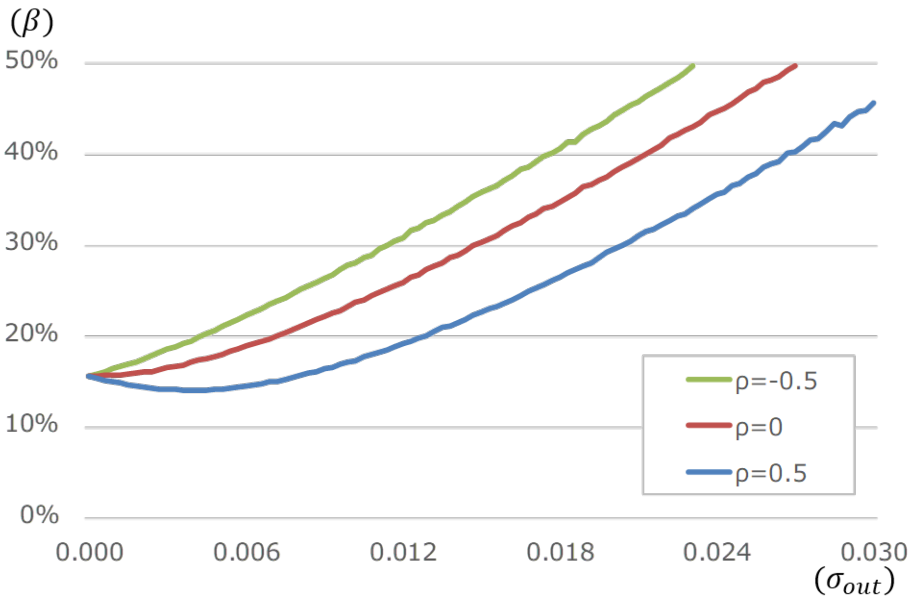

Figure 4.2: Optimal Liquidity Buffer vs Volatility of Cash Outflows

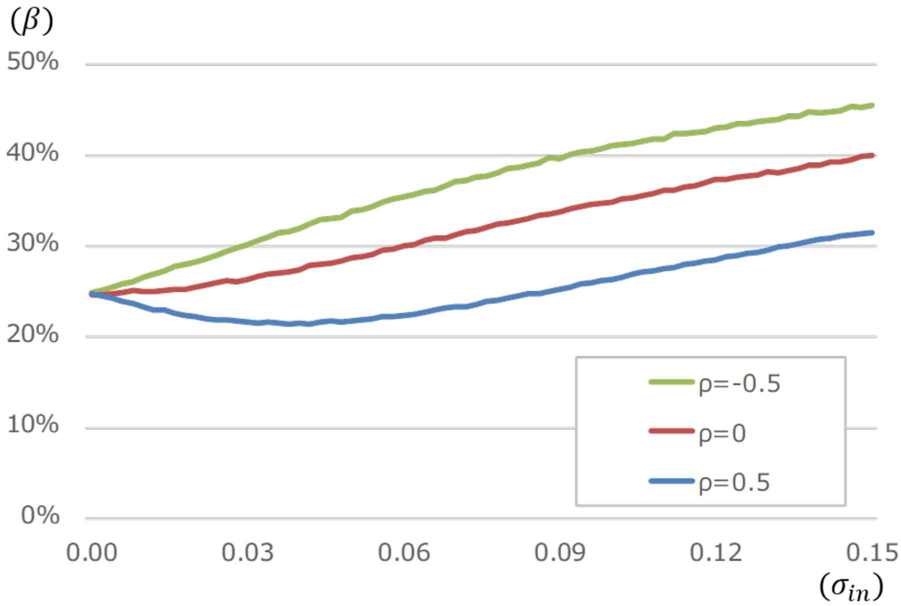

Figure 4.3: Optimal Liquidity Buffer vs Volatility of Cash Inflows

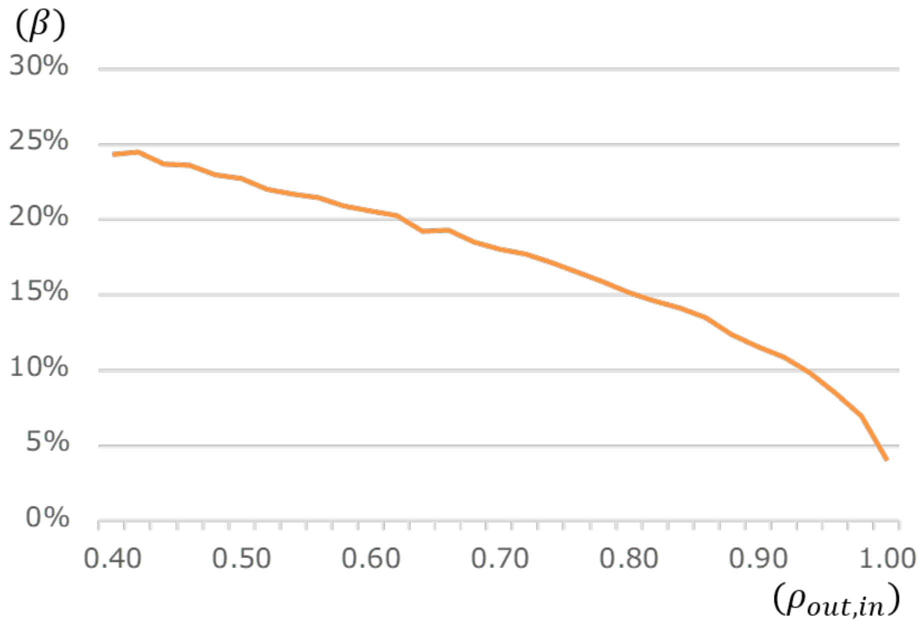

Figure 4.4: Optimal Liquidity Buffer vs Correlation

4.2.2 Impact of Correlation in the Base Model

Figure 4.4 shows the optimal liquidity buffer, β by the correlation between the total expected cash outflows and inflows, $\rho_{out,in}$. We can imagine easily that $\rho_{out,in}$ affects the level of the optimal liquidity buffer. As we can surmise, the higher the correlation is, the lower the optimal liquidity buffer is. However, the reduction of the buffer accelerates at a high correlation level. Hence, the optimal liquidity buffer increases significantly, even if the correlation decreases slightly. Therefore, we can find that banks need to monitor the correlation levels carefully.

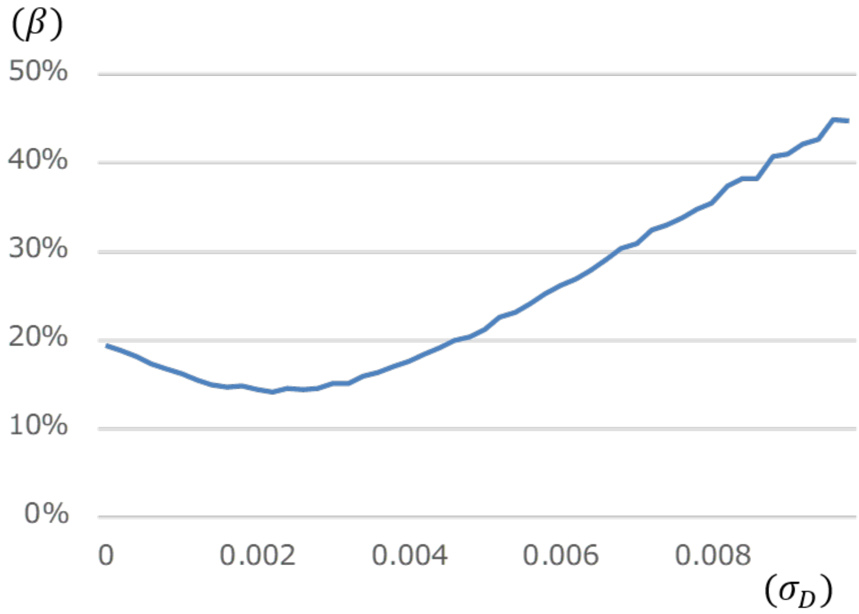

Figure 4.5: Optimal Liquidity buffer vs Volatility of the Deposit Amount

4.2.3 Impact of Volatility in the Deposit Variation Model

The volatility of the deposit amount, σ_D, is important for the cash management in the banks, because they can not fully control their deposits. Hence, we observe the effect of the volatility of the deposit amount in the deposit variation model.

Figure 4.5 shows the optimal liquidity buffer by the volatility of the deposit amount. It shows there is a minimum level, so banks can suppress their excess funds at the minimum point, that is with a few volatilities. This is because the LCR is modeled by subtraction of the Brownian motions in the deposit model. Therefore, banks should try to control their deposit amount well-flexibly in order to accomplish the effective cash management.

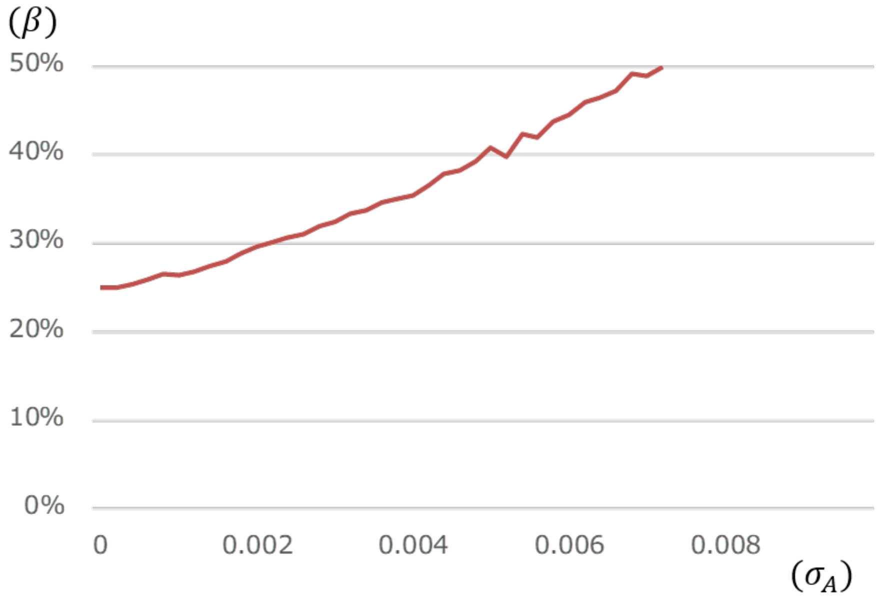

Figure 4.6: Optimal Liquidity Buffer vs Volatility of the Asset Amount

4.2.4 Impact of Volatility in the Asset Variation Model

We observe the effect of the volatility of the asset amount, σ_A, in order to contrast it with the volatility of the deposit amount.

Figure 4.6 shows the optimal liquidity buffer by the volatility of the asset amounts. Apart from the volatility of the deposit amount, it is monotonically increasing. This difference is caused by the fact that the LCR is modeled by subtraction of the Brownian motions in the deposit variation model, while the LCR is modeled by the summation of the Brownian motions. Obviously, banks should suppress their volatility of the asset amount in order to avoid holding HQLA excessively.

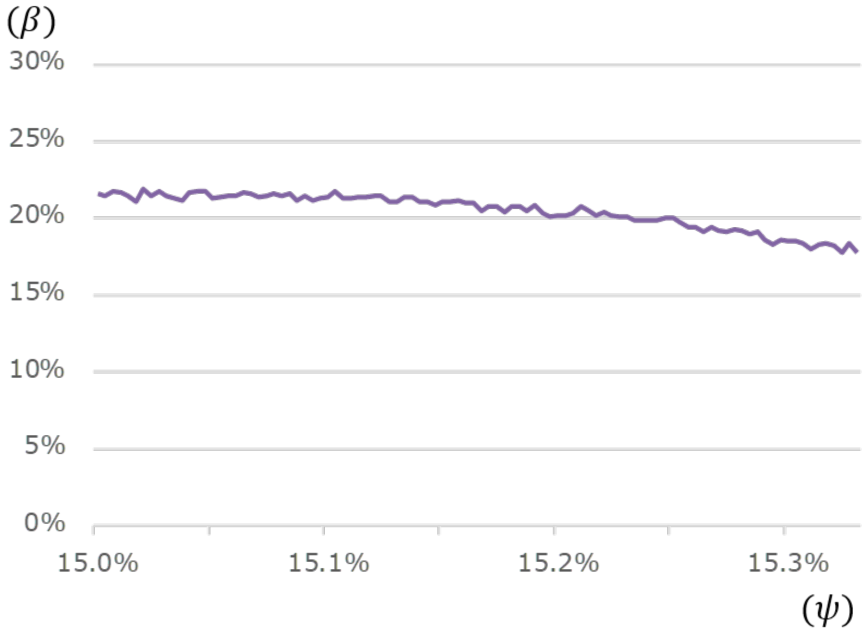

Figure 4.7: Required Buffer vs Voluntary Capital Ratio

4.2.5 Impact of Capital Ratio in the Full Model

Banks mostly concern about the relationship between the thresholds of both the capital ratio for the risk-weighted assets (ψ) and the LCR, in order to determine their effective business plan.

Figure 4.7 shows the optimal liquidity buffer for adhering to the LCR by the capital ratio. The higher the capital ratio is, the lower the liquidity buffer is. This is because a bank increases its HQLA in order not to be able to invest risky assets, when capital trigger is high. On the other hand, it decreases its HQLA in order to invest more risky assets when the capital trigger is low. Therefore, we can find the optimal levels for both liquidity and capital regulations by considering an appropriate cost function (see the next section).

Loan Spread	s	50 bps
Borrowing Spread	c	2 bps

Table 4.2: Parameters

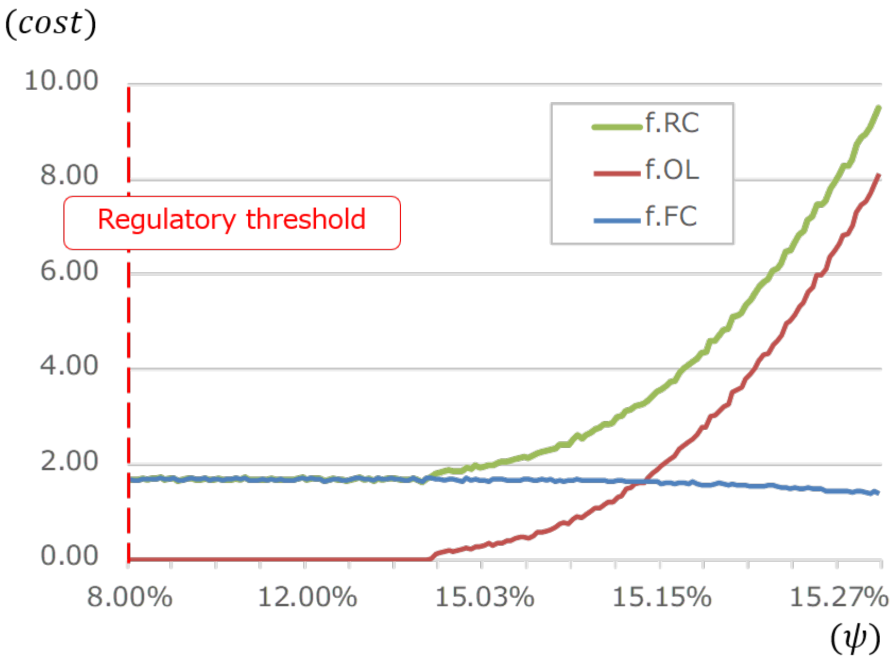

Figure 4.8: Regulatory Cost vs Capital Ratio

4.3 Optimal Levels

We construct the rough cost function in the previous chapter. In this section, we introduce the method of obtaining the optimal capital and liquidity target by setting additional parameters form Japanese major banks with noise, as in Table 4.2.

Figure 4.8 shows the regulatory cost by the capital ratio, ψ. The lowest level of the total cost (green line) is the optimal, hence the optimal capital target is 8%, though it is not concave. Also, the optimal liquidity buffer is 22%, when the capital target is 8%. Japanese banks should increase their risky asset so as to secure their profits according to this model. This result is consistent with the fact that the cost of maintaining the HQLA is quite low, because of low policy rate in Japanese market.

4.4 Summary

Banks usually try to keep their deposit amount so as to smooth their maturing amount per day for the stable cash management. The comparative statics show that they should operate with well-flexibly, that is with a few volatilities for the deposit amount and the total expected cash outflows and inflows, in order to achieve to reduce their regulatory cost. Also, banks need to monitor the correlation between the total expected cash outflows and inflows, especially when it is at high level, because of preventing from surging the breach probability for the LCR.

Besides, banks can find the optimal target for both the liquidity and capital regulations which is useful for determining the target of the risk appetite indicator, by incorporating an adequate function representing the regulatory cost.

Chapter 5

Conclusion

After the financial crisis, a lot of regulatory reforms had been implemented. Banks are struggling how to develop reasonable designs for their balance sheets under the regulations. Also, they have endeavored to control their profitability, risk, and so on by using the risk appetite indicators, though it is difficult to determine the targets.

Our study can obtain the optimal target for not only the LCR, but also the capital ratio. It is very useful for determining the rational business plan, that is the target of the risk appetite indicators. Besides, we can find that banks should operate their cash management, as far as the controls of the deposit amount and the maturing amount of the asset and deposit, with well-flexibly, not fully stable. These operations enable banks to reduce their regulatory cost for the LCR.

Our study is an initial stage for addressing the above problems, so we use only the most simplistic model. Therefore, we discuss about future works that can obtain more realistic and convenient method so as to make use of it in actual operation. At first, we would like to extend the model to multi-currency. In actual situation, banks control their funds by currency. Hence, we would represent each currency as a stochastic process by using the approximation methods such as the pricing methods in the basket options. Also, the approximation methods are useful for our model because of obtaining targets in a timely manner. Besides, we take in more adequate cost functions reflecting the realistic situation are needed.

Appendix A

Performance of Japanese Banks

A.1 Total net cash outflows on the LCR

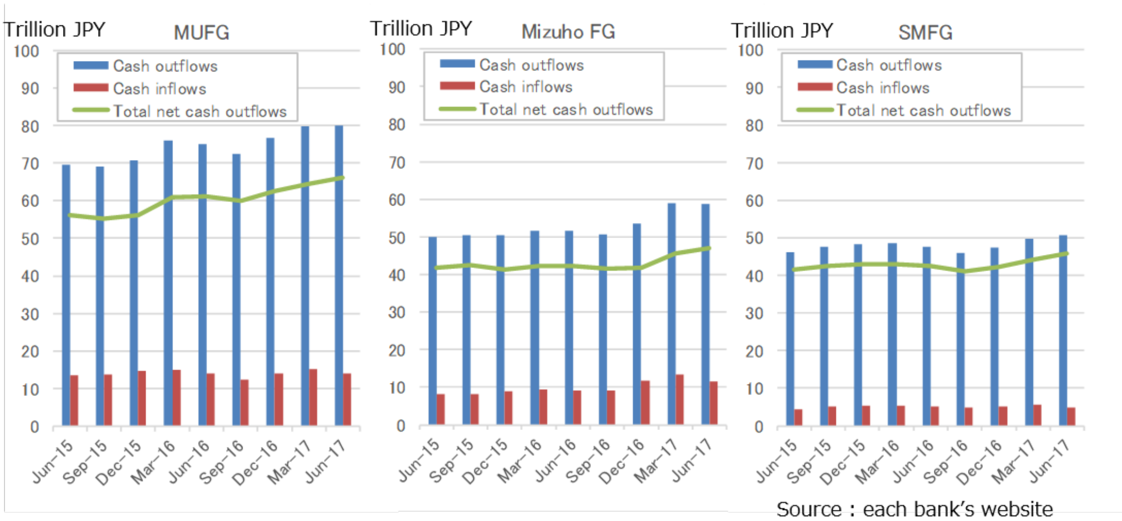

Figure A.1: Quarterly average in Japanese major banks.

A.2 Deposit amount and Total Expected Cash Outflows

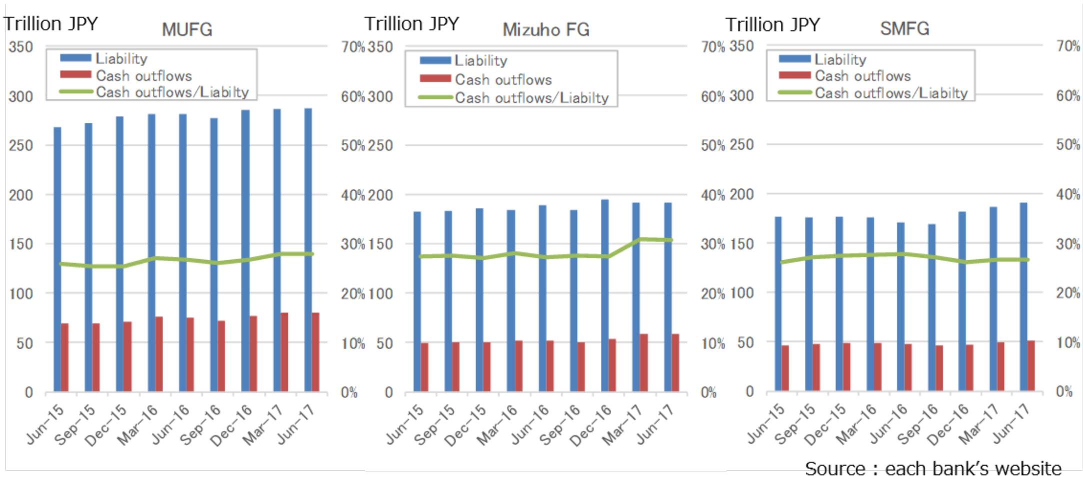

Figure A.2: Quarterly average in Japanese major banks.

A.3 Asset amount and Total Expected Cash Inflows

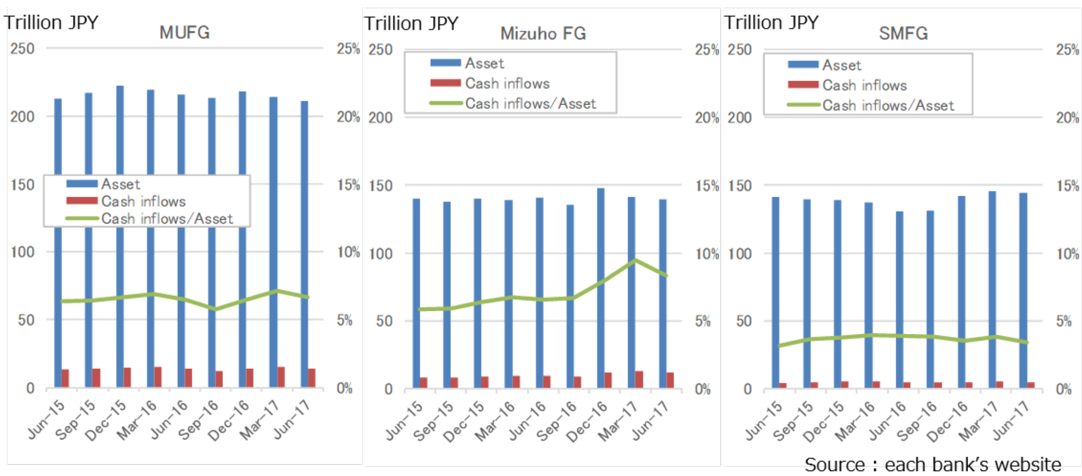

Figure A.3: Quarterly average in Japanese major banks.

A.4 Conversion Factor for Risk-weighted Asset

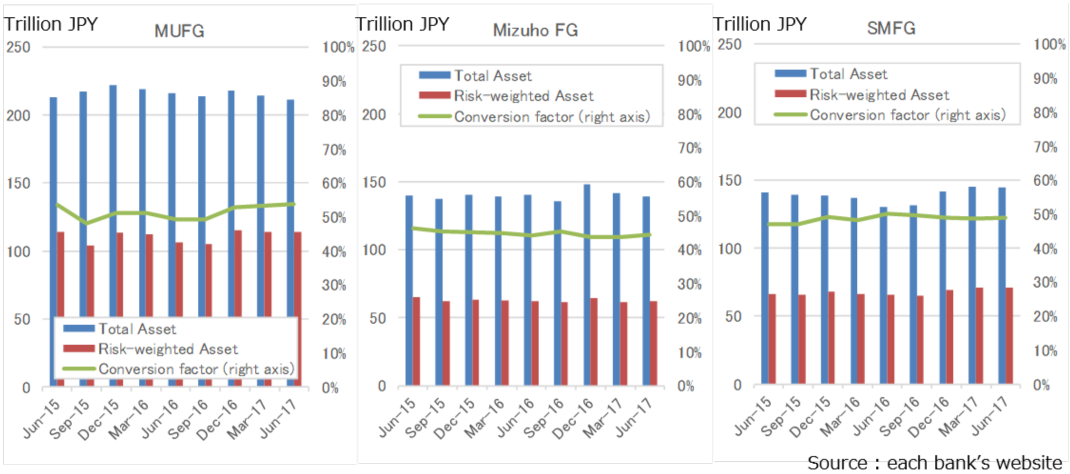

Figure A.4: Quarterly disclosure in Japanese major banks.

Appendix B

Mean-Reversion Model

For the largest banks, the assumption that treat cash outflows and inflows as diffusion processes is too conservative, because they can handle their maturing amounts of assets and deposits easier. They try to smooth their maturing amounts due to make their funding stable. Thus, it is natural assumption that cash flows in the 30 days stress period calculated by their maturing amount follow the mean-reversion process.

B.1 Setting

Total expected cash outflows, $\tilde{x}_{out}$, follow

$$d\tilde{x}_{out}(t) = a_{out}(\tilde{\mu}_{out} - \tilde{x}_{out}(t))dt + \tilde{\sigma}_{out} d\tilde{z}_{out}(t), \quad \tilde{x}_{out}(0) = \tilde{x}_{out}. \tag{B.1}$$

Also, cash inflows, $\tilde{x}_{in}$, follow

$$d\tilde{x}_{in}(t) = a_{in}(\tilde{\mu}_{in} - \tilde{x}_{in}(t))dt + \tilde{\sigma}_{in} d\tilde{z}_{in}(t), \quad \tilde{x}_{in}(0) = \tilde{x}_{in}, \tag{B.2}$$

where

$$d\tilde{z}_{out}(t) d\tilde{z}_{in}(t) = \tilde{\rho} dt, \quad \rho \geq 0. \tag{B.3}$$

Our objective is to find the adequate liquidity buffer, β, satisfying Equation (3.3). We can obtain an optimal buffer by running the paths following the above processes and calculating the total net cash outflows.

B.2 Numerical Example

We confirm the difference between the base model and mean-reversion model by using a numerical example.

Parameters

TableB.1 shows the parameters which are set according to Japanese banks with noise. These parameters also can be calibrated, because most banks have the LCR data on a daily basis.

outflows	a_{out}	0.03
	$\tilde{x}_{out}$	JPY 49 trillion
	$\tilde{\mu}_{out}$	50
	$\tilde{\sigma}_{out}$	1.4
inflows	a_{in}	0.03
	$\tilde{x}_{in}$	JPY 4.9(10) trillion
	$\tilde{\mu}_{in}$	5
	$\tilde{\sigma}_{in}$	0.5
Cor	$\rho_{out,in}$	0.5
Trigger	ϕ	100%
Term	T	60 business days

Table B.1: Parameters

Numerical Result

Figure B.1 shows the optimal liquidity buffer, β, by the correlation between the total expected cash outflows and inflows, $\tilde{\rho}_{out,in}$. As the same as the base model (see Figure 4.4), the higher correlation is, the lower the optimal liquidity buffer is. However, the reduction of the buffer does not accelerate, apart from the base model. Therefore, the monitoring for the correlation level is not necessary in the major banks that can control their maturing amount sufficiently.

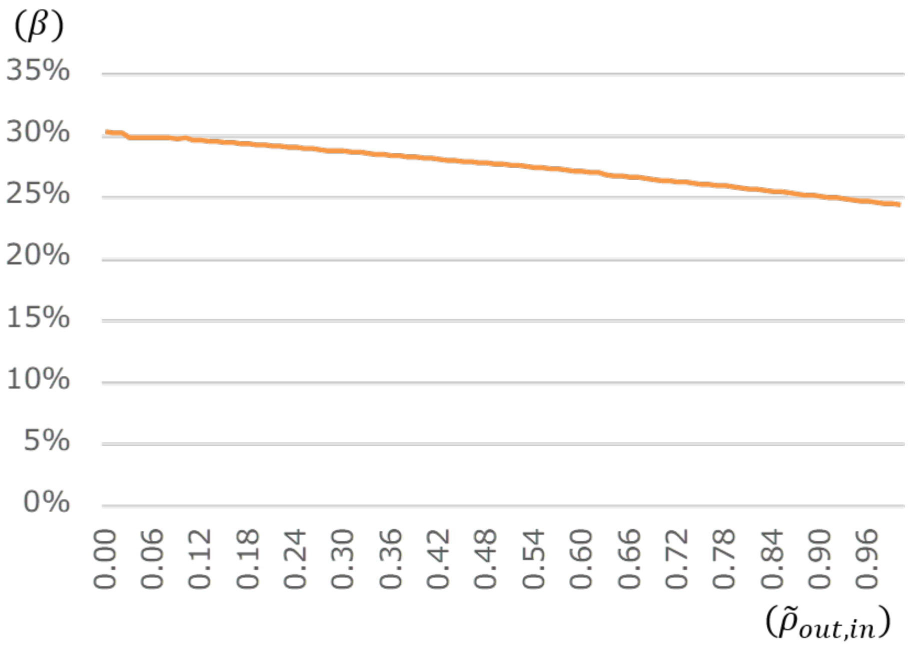

Figure B.1: Optimal Liquidity Buffer vs Correlation

Appendix C

Moment-Matching Method

Approximation method

We consider the approximation based on the method in Kijima[14]; $(x_{out}(t), x_{out}(t) - x_{in}(t)) \approx (x_{out}(t), \hat{x}(t))$. We suppose $(x_{out}(t), \hat{x}(t))$ follows the quadratic log-normal distribution. We find the parameters of $\hat{x}(t)$ by following approach.

We approximate

$$E[x_{out}(t) - x_{in}(t)] = E[\hat{x}(t)], \tag{C.1}$$

$$E[(x_{out}(t) - x_{in}(t))^2] = E[\hat{x}(t)^2], \tag{C.2}$$

$$E[x_{out}(t) \cdot (x_{out}(t) - x_{in}(t))] = E[x_{out}(t)\hat{x}(t)], \tag{C.3}$$

where

$$\hat{x}(t) = \hat{x} \exp\left\{\left(\hat{\mu}(t) - \frac{1}{2}\hat{\sigma}(t)^2\right) t + \hat{\sigma}(t)\hat{z}(t)\right\}, \tag{C.4}$$

$$dx_{out}(t)d\hat{x}(t) = \hat{\rho}(t)dt. \tag{C.5}$$

Here,

$$\begin{aligned} E[\hat{x}(t)] &= \hat{x} \exp\left(\hat{\mu}(t)t - \frac{1}{2}\hat{\sigma}(t)^2 t\right) E[\exp(\hat{\sigma}(t)\hat{z}(t))] \\ &= \hat{x} \exp\left(\hat{\mu}(t)t\right), \qquad \text{(C.6)} \\ E[\hat{x}(t)^2] &= \hat{x}^2 \exp\left(2\hat{\mu}(t)t - \hat{\sigma}(t)^2 t\right) E[\exp(2\hat{\sigma}(t)\hat{z}(t))] \\ &= \hat{x}^2 \exp(2\hat{\mu}(t)t + \hat{\sigma}(t)^2 t), \qquad \text{(C.7)} \\ E[x_{out}(t)\hat{x}(t)] &= x^{out}\hat{x} \exp\left\{\left(\mu_{out} + \hat{\mu}(t) - \frac{1}{2}\sigma_{out}^2 - \frac{1}{2}\hat{\sigma}(t)^2\right) t\right\} \\ &\qquad \cdot E\left[\exp(\sigma_{out} z_{out}(t) + \hat{\sigma}(t)\hat{z}(t))\right] \\ &= x_{out}\hat{x} \exp\left(\mu_{out} t + \hat{\mu}(t)t + \hat{\rho}(t)\sigma_{out}\hat{\sigma}(t)t\right). \qquad \text{(C.8)} \end{aligned}$$

On the other hand,

$$\begin{aligned}
E\left[x_{out}(t)-x_{in}(t)\right] &= x_{out}\exp\left\{\mu_{out}t-\frac{1}{2}\sigma_{out}^2 t\right\}E\left[\exp(\sigma_{out}z_{out}(t))\right] \\
&\quad -x_{in}\exp\left\{\mu_{in}t-\frac{1}{2}\sigma_{in}^2 t\right\}E\left[\exp(\sigma_{in}z_{in}(t))\right] \\
&= x_{out}\exp(\mu_{out}t)-x_{in}\exp(\mu_{in}t), \qquad \text{(C.9)}
\end{aligned}$$

$$\begin{aligned}
E\left[(x_{out}(t)-x_{in}(t))^2\right] &= E\left[x_{out}(t)^2+x_{in}(t)^2-2x_{out}(t)x_{in}(t)\right] \\
&= E\left[x_{out}(t)^2\right]+E\left[x_{in}(t)^2\right]-2E\left[x_{out}(t)x_{in}(t)\right], \qquad \text{(C.10)}
\end{aligned}$$

where

$$\begin{aligned}
E\left[x_{out}(t)^2\right] &= x_{out}^2\exp\left\{2\mu_{out}t+\sigma_{out}^2 t\right\}, \\
E\left[x_{in}(t)^2\right] &= x_{in}^2\exp\left\{2\mu_{in}t+\sigma_{in}^2 t\right\}, \\
E\left[x_{out}(t)x_{in}(t)\right] &= x_{out}x_{in}\exp\left\{(\mu_{out}+\mu_{in})t-\frac{1}{2}\sigma_{out}^2 t-\frac{1}{2}\sigma_{in}^2 t\right\} \\
&\qquad E\cdot\left[\exp\{\sigma_{out}z_{out}(t)+\sigma_{in}z_{in}(t)\}\right] \\
&= x_{out}x_{in}\exp\left\{(\mu_{out}+\mu_{in})t+\rho_{out,in}\sigma_{out}\sigma_{in}t\right\}.
\end{aligned}$$

Also,

$$E\left[x_{out}(t)\left(x_{out}(t)-x_{in}(t)\right)\right] = E\left[x_{out}(t)^2\right]-E\left[x_{out}(t)x_{in}(t)\right]. \qquad \text{(C.11)}$$

Therefore, by Equation (C.1), $\hat{\mu}_t$ satisfies

$$\begin{aligned}
& E[\hat{x}(t)] && = E[x_{out}(t)-x_{in}(t)] \\
\Leftrightarrow\ & \hat{x}\exp(\hat{\mu}(t)t) && = x_{out}\exp(\mu_{out}t)-x_{in}\exp(\mu_{in}t) \\
\Leftrightarrow\ & \hat{\mu}(t) && = \frac{1}{t}\log\frac{1}{\hat{x}}\left\{x_{out}(\exp(\mu_{out}t)-x_{in}\exp(\mu_{in}t))\right\}. \qquad \text{(C.12)}
\end{aligned}$$

By Equation (C.2), $\hat{\sigma}_t$ satisfies

$$\begin{aligned}
& E[\hat{x}(t)^2] && = E\left[(x_{out}(t) - x_{in}(t))^2\right] \\
\Leftrightarrow\ & \hat{x}^2 \exp(2\hat{\mu}(t)t + \hat{\sigma}(t)^2 t) && = E\left[x_{out}(t)^2\right] + E\left[x_{in}(t)^2\right] - 2E\left[x_{out}(t)x_{in}(t)\right] \\
& && = x_{out}^2 \exp(2\mu_{out}t + \sigma_{out}^2 t) \\
& && \quad + x_{in}^2 \exp(2\mu_{in}t + \sigma_{in}^2 t) \\
& && \quad - 2x_{out}x_{in} \exp\left\{(\mu_{out} + \mu_{in})t + \rho_{out,in}\sigma_{out}\sigma_{in}t\right\} \\
\Leftrightarrow\ & \exp(2\hat{\mu}(t)t + \hat{\sigma}(t)^2 t) && = \left(\frac{x_{out}}{\hat{x}}\right)^2 \exp(2\mu_{out}t + (\sigma_{out})^2 t) \\
& && \quad + \left(\frac{x_{in}}{\hat{x}}\right)^2 \exp(2\mu_{in}t + (\sigma_{in})^2 t) \\
& && \quad - 2\left(\frac{x_{out}x_{in}}{\hat{x}^2}\right) \exp\left\{(\mu_{out} + \mu_{in})t + \rho\sigma_{out}\sigma_{in}t\right\} \\
& && =: c_1 + c_2 - 2c_3 \\
\Leftrightarrow\ & \hat{\sigma}_t^2 && = \frac{1}{t}\log(c_1 + c_2 - 2c_3) - 2\hat{\mu}(t) \\
\Leftrightarrow\ & \hat{\sigma}_t && = \left\{\frac{1}{t}\log(c_1 + c_2 - 2c_3) - 2\hat{\mu}(t)\right\}^{\frac{1}{2}}.
\end{aligned} \tag{C.13}$$

Additionally, by Equation(C.3), we obtain

$$\begin{aligned}
& E[x_{out}(t)\hat{x}(t)] && = E\left[x_{out}(t)\left(x_{out}(t) - x_{in}(t)\right)\right] \\
\Leftrightarrow\ & x_{out}\hat{x} \exp\left(\mu_{out}t + \hat{\mu}(t)t + \hat{\rho}(t)\sigma_{out}\hat{\sigma}(t)t\right) && = x_{out}^2 \exp\left(2\mu_{out}t + \sigma_{out}^2 t\right) \\
& && \quad - x_{out}x_{in} \exp\left(\mu_{out}t + \mu_{in}t + \rho_{out,in}\sigma_{out}\sigma_{in}t\right) \\
\Leftrightarrow\ & \exp\left(\mu_{out}t + \hat{\mu}(t)t + \hat{\rho}(t)\sigma_{out}\hat{\sigma}(t)t\right) && = \left(\frac{x_{out}}{\hat{x}}\right) \exp\left(2\mu_{out}t + \sigma_{out}^2 t\right) \\
& && \quad - \left(\frac{x_{in}}{\hat{x}}\right) \exp\left(\mu_{out}t + \mu_{in}t + \rho_{out,in}\sigma_{out}\sigma_{in}t\right) \\
& && =: d_1 - d_2 \\
\Leftrightarrow\ & \left(\mu_{out} + \hat{\mu}(t) + \hat{\rho}(t)\sigma_{out}\hat{\sigma}(t)\right)t && = \log(d_1 - d_2) \\
\Leftrightarrow\ & \hat{\rho}_t && = \frac{1}{\sigma_{out}\hat{\sigma}(t)} \\
& && \quad \cdot \left\{\frac{1}{t}\log(d_1 - d_2) - \mu_{out} - \hat{\mu}(t)\right\}.
\end{aligned} \tag{C.14}$$

outflows	x_{out}	JPY 49 trillion
	μ_{out}	0.0003
	σ_{out}	0.015
inflows	x_{in}	JPY 4.9 trillion
	μ_{in}	0.001
	σ_{in}	0.06
Cor	$\rho_{out,in}$	0.6
Term	T	60 business days

Table C.1: Parameters

Accuracy of the Approximation

We compare the simulated distribution of $(x_t^{out} - x_t^{in})$ and the approximated distribution of $\hat{x}$ because of assessing the approximation. Parameters are set as in Table C.1.

Movement of Parameter

As developed in the previous section, the parameters of the approximated distribution depend on t as shown in Figure C.1, C.2, and C.3.

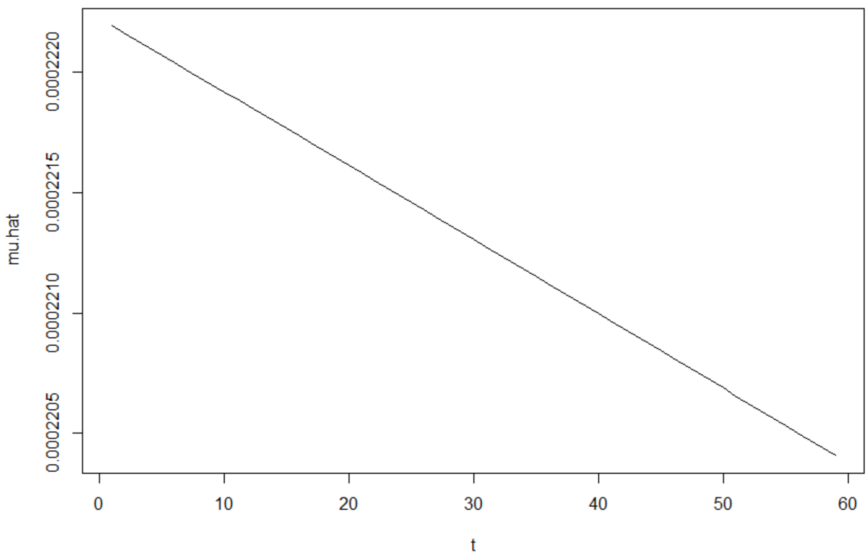

Figure C.1: $\hat{\mu}_t$

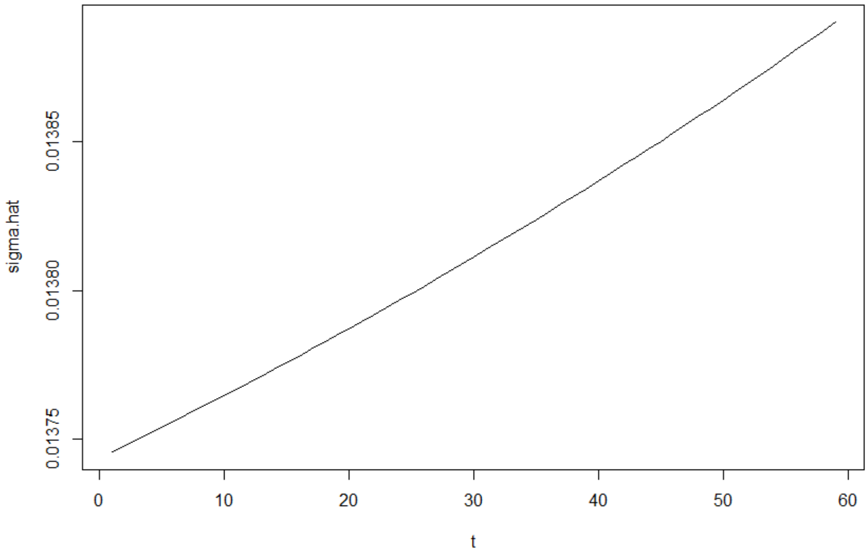

Figure C.2: $\hat{\sigma}_t$

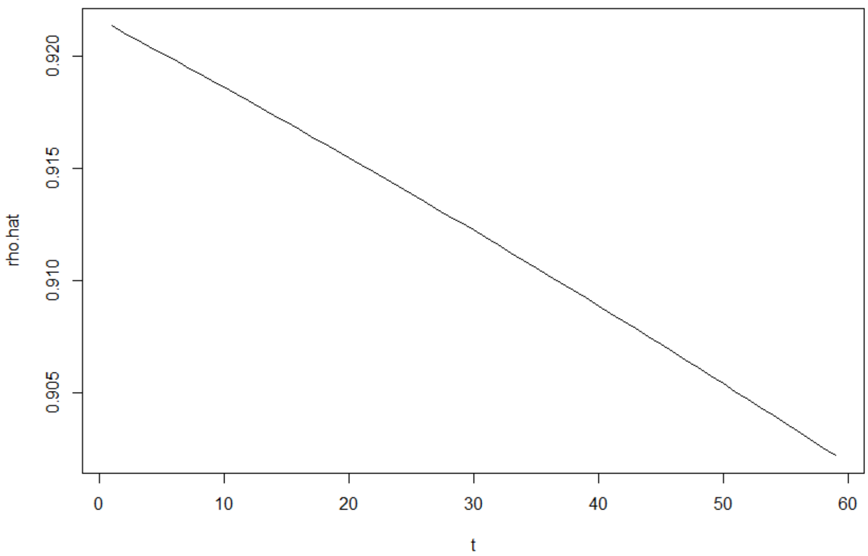

Figure C.3: $\hat{\rho}_t$

Distribution

Figure C.4 and C.5 show the original (black line) and approximated (red line) distributions.

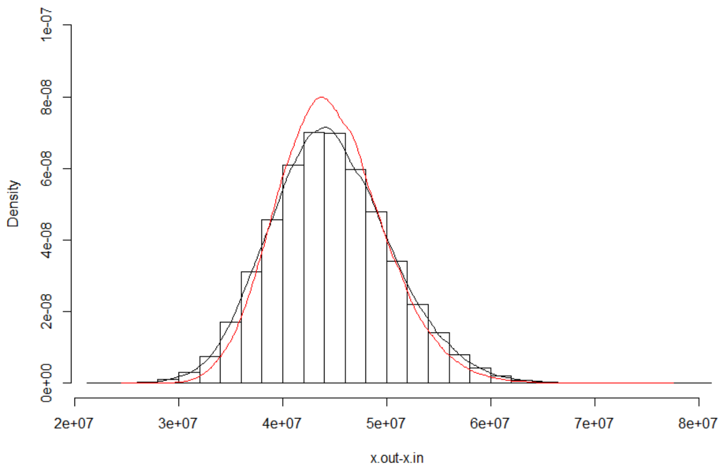

Figure C.4: Distribution of $(x_{out} - x_{in})$ at certain t

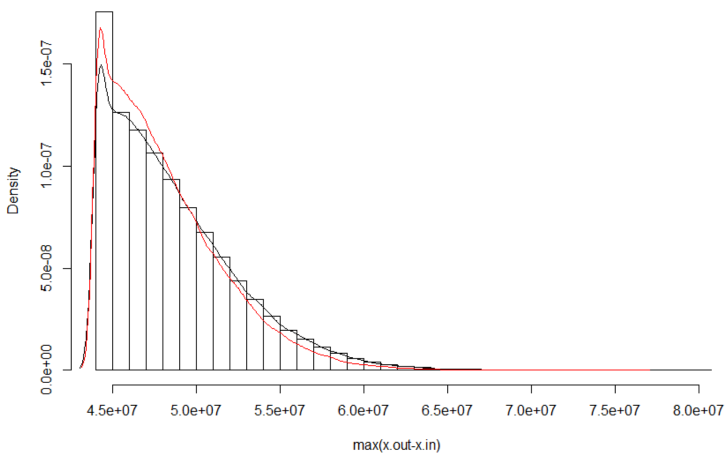

Figure C.5: Distribution of $\max(x_t^{out} - x_t^{in})$

Summary

This method is probable approximation substantially according to Figure C.4 and C.5. However, all parameters of the approximated distribution is time-dependent. (see Equation C.12, C.13, and C.14.) We have to obtain the joint probability density of $(x_{out}, \hat{x})$, when eliciting an optimal liquidity buffer. Thus, it is difficult to treat the time-dependent parameters without simulating. In other words, we need another approach, when obtaining a target in a timely manner.

References

[1] Keister, T. and M. Bech (2012), "On the liquidity coverage ratio and monetary policy implementation", *BIS Quarterly Review*

[2] Balasubramanyan, L. and D. D. VanHoose (2013), "Bank balance sheet dynamics under a regulatory liquidity-coverage-ratio constraint", *Journal of Macroeconomics*, **37**, 53-67

[3] Cetina, J. and K. Gleason (2015), "The Difficult Business of Measuring Bank's Liquidity: Understanding the Liquidity Coverage Ratio", *Working paper*

[4] BCBS (2008), "Principles for Sound Liquidity Risk Management and Supervision"

[5] BCBS (2009), "International framework for liquidity risk measurement, standards and monitoring"

[6] BCBS (2010), "A global regulatory framework for more resilient banks and banking systems"

[7] BCBS (2013), "Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools"

[8] BCBS (2014), "Basel III: the net stable funding ratio"

[9] BCBS (2014), "Basel III leverage ratio framework and disclosure requirements"

[10] BIS (2017), "Governors and Heads of Supervision finalise Basel III reforms"

[11] Shreve, S. E. (2008), *Stochastic Calculus for Finance II: Continuous-Time Models*, Springer Finance (長山いづみ他訳『ファイナンスのための確率解析 II（連続時間モデル）』丸善出版、2012）

[12] Brigo, D. et al (2004), "Approximated moment-matching dynamics for basket-options pricing" *Quantitative Finance*, **4**, **4**, 1-16

[13] Funahashi, H. and M. Kijima (2017), "An analytical approximation for pricing VWAP options", *Quantitative Finance*, **17**, 1119-1133

[14] 木島正明 (1994) 『ファイナンス工学入門 第二部　派生証券の価格付け理論』 日科技連