TOKYO METROPOLITAN UNIVERSITY

Regulatory Policy to Mitigate Potential Risks Arising from Contingent Convertibles

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Abstract

A Contingent Convertible (CoCo) bond is an instrument that converts into equity or suffers a write-down when the issuing bank is in financial distress. In practice, a trigger event of CoCo takes place when the capital ratio of the bank falls to the pre-defined level or when the national authority declares a trigger at its discretion. The aims of this study are to model CoCos having such triggers and to find effective regulatory policies to handle them. A model for banks issuing CoCos is built within the framework of a structural-default approach. The trigger mechanisms are expressed in a first passage time model and in a stochastic intensity model. CoCo investors are also included in our model as CoCos are designed to enhance the bank's resilience while shifting its risks to the investors. In the numerical example, we show that effective regulatory policy, which is intended to mitigate both banks' and investors' default risks, changes according to the correlation between banks and investors and the impact of the trigger event on the bank asset value process.

Keywords: Contingent convertible, structural approach, accounting trigger, regulatory trigger, systemic risk, regulatory policy

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Chapter 1

Introduction

A Contingent Convertible (CoCo) is a new type of financial instrument that came along with a series of regulatory reforms initiated after the global financial crisis. A CoCo automatically converts into equity or suffers a write-down when a bank has faced a financial difficulty. Unlike other major financial instruments, the design of CoCo heavily depends on regulator's intention. Thus, regulator's behavior is a key factor that should be taken into account when studying this instrument. Before going into details of a CoCo, first we explain the regulatory background that brought about CoCos to appear in the financial market.

1.1 Background

Since the global financial crisis, which began in 2007, one of the agenda for financial regulators has been how to build a resilient financial system. To date, various regulations have been proposed and implemented in this area at both global and jurisdictional levels.

The most fundamental piece of the regulatory reform has been a review of the Basel regulatory framework. The Basel framework provides a global standard with regard to bank capital regulation. It lays out a minimum requirement for bank's capital ratio to ensure that a bank has enough capital to be regarded as solvent. The framework was first agreed in July 1988 under the leadership of the Basel Committee on Banking Supervision (BCBS) – an international organization made up of banking supervisors. Since then, a number of improvements have been made in accordance with the development of financial markets. However, the global financial crisis revealed that the former framework, so called Basel II, was insufficient to address risks which was said to be the root cause of the crisis, such as liquidity risks and off-balance sheet financing. In addition, criticism has been made on massive bail-outs employed in the course of restructuring of large banks. This, so called "too-big-to-fail" problem, highlighted the importance of enhanced capital regulations on systemically important financial institutions (SIFIs).

The new framework, called Basel III, intends to address these shortcomings. The Basel III regulations had initially been agreed upon in July 2011 and were finalized in December 2017. Besides adding new regulations to counter with the shortcomings, regulators discussed enhancement of existing capital requirements. Compared to the former framework, the new Basel III packages require banks, especially SIFIs, to hold higher quality of capital, i.e., loss-absorbing capacity, to enable recovery and resolution without using taxpayers' money. Basically, higher loss-absorbing capacity can be attained by issuing more common stocks or by gaining more profits. However, without a surprise, it is quite costly for banks to raise additional equity or realize profits in a severe market condition. Regulators searched for other tools that enable recapitalization of a bank in the course of crisis. To this end, a CoCo was designed, i.e., a CoCo is cheaper tool for a bank to issue and is loss-absorbable amid financial distress.

1.2 What is CoCo

So far, a clear-cut and uniform definition for a CoCo has not been developed. This paper defines a CoCo as an instrument that converts into equity or suffers a write-down on a going-concern basis when a pre-defined trigger conditions are met.[1] The term "going-concern" is explained in the later section. Accordingly, a CoCo has both bond and equity features. At first, a CoCo pays periodical coupons just like a normal bond. However, as soon as pre-defined trigger conditions are met, it automatically absorb losses by equity-conversion or write-down of its face value.

Three players are involved in the CoCo market – a bank, an investor and a regulator. A bank issues a CoCo in order to adhere to the capital regulation. At the trigger moment, the bank's capital ratio increases by virtue of loss absorption of the CoCo. An investor buys a CoCo as it is often an attractive instrument with a high coupon rate, but may suffer a loss when the CoCo is triggered. A regulator engages in designing of a CoCo by providing guidelines for banks and sometimes for investors. Another important engagement that can be done by the regulator is to determine when to trigger CoCos – so-called "regulatory trigger" – which is described in the later subsection.

As outlined above, there are two defining features for a CoCo: (1) a loss-absorption mechanism and (2) a trigger mechanism.

[1]Some exclude write-down type instruments from the coverage of CoCo. On the other hand, some do not use the term "CoCo" to name an instrument that has the same features as the CoCo discussed in this paper.

1.2.1 Loss-Absorption Mechanism

When a CoCo is triggered, the CoCo absorbs losses either by equity conversion or write-down of the principal.

Equity Conversion

A CoCo investor receives a certain number of shares pursuant to the face value it has. In other words, the CoCo investor pays a certain price, namely the conversion price, to purchase the number of shares. In many cases, the conversion price is offered by the bank at its initial issuance. Some CoCos have a fixed conversion price, for example, the conversion price equals to the share price at the issue date of the CoCo. Other CoCos have a floating conversion price such as the conversion price set equal to the share price at the trigger date.

At the trigger moment, stock dilution may occur depending on the conversion price. For current shareholders, a higher conversion price would be preferable as smaller number of new shares is generated at the conversion. Some banks offer a floored conversion price to avoid excessive dilution to take place at the trigger moment.

Write-down

In some circumstances, a write-down may be a preferred choice to enable loss absorption. For instance, if we think of a bank who issues non-listed shares, it is difficult to define conditions with regard to equity conversion including the conversion price.

Percentage of the haircut, or the write-down ratio, can either be fixed or floating. In fixed cases, some CoCos may suffer a full write-down at the trigger moment, while others may be partial. In floating cases, a write-down ratio is determined depending on how severe the financial condition is at the trigger moment. For example, some CoCos are designed to suffer a write-down to the extent that is necessary to recover the minimum capital ratio.

1.2.2 Trigger Mechanism

According to De Spegeleer and Schoutens (2012), three types of triggers can be considered: (1) market trigger, (2) accounting trigger and (3) regulatory trigger. A CoCo can have one or more triggers and its trigger mechanism is usually defined in the contract document.

Market Trigger

A trigger event is expected to happen when the issuer is in financial distress. Thus, when designing a trigger mechanism, it is natural to think of using some indicators related to the bank's solvency and define: "when this indicator reaches an insolvency threshold, the CoCo is triggered." A market observable number such as a share price or a CDS spread can be a candidate for such an indicator since they are considered forward-looking data which reflect issuer's future risk. However, a market-trigger CoCo may be vulnerable as the trigger timing can be intentionally changed by market manipulation.

Accounting Trigger

Instead of market-based numbers, an accounting ratio can also be used as an indicator that reflects issuer's solvency. Common Equity Tier 1 (CET 1) ratio defined in the Basel III framework is an example for such an indicator. By using such regulatory capital ratio, a CoCo can be designed consistently with the existing regulatory framework; however, we should be aware that the ratio is not always accessible – they are calculated periodically with some delay. In addition, from a CoCo investor's point of view, the ratio might not be transparent as the ratio-calculation method is not disclosed to public in detail.

Regulatory Trigger

This trigger, sometimes called as non-viability trigger or point-of-non-viability (PONV) trigger, takes a different approach from the above-mentioned triggers. If a CoCo has a regulatory trigger, a regulator responsible for the oversight of the CoCo issuer has discretion over when to trigger the CoCo. That means, regulators may exercise the right to trigger CoCos when they conclude that it is necessary to do so in order to maintain resilient financial system.

While some argue that the existence of this trigger reduces transparency of the trigger mechanism, recently issued CoCos tend to have a regulatory trigger to enable flexible capital recovery led by authorities.

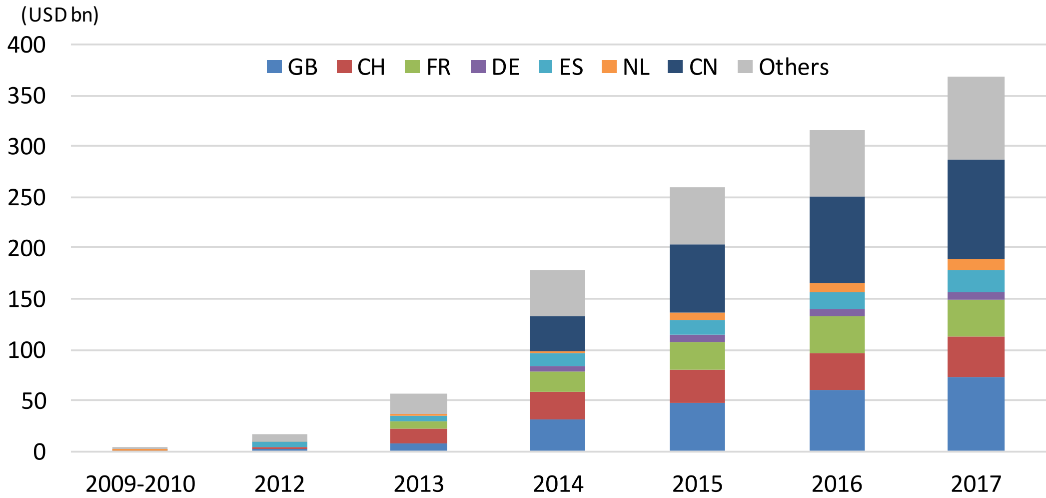
1.3 CoCos in Real Markets

The first CoCo was issued by Lloyds Banking Group in December 2009. Only one bank, the Rabobank in the Netherlands, followed Lloyds and issued CoCos in 2010. CoCo market expanded after the publication of the Basel III documents and other announcements made by national authorities.[2] Issuance of CoCo has peaked in 2014 and the cumulative amount of CoCo issuance has reached more than $350 billion as of

[2]For example, the European Banking Authority provided a term sheet and the Office of the Superintendent of Financial Institutions Canada published guidelines with respect to the design of CoCos in 2011.

Figure 1.1: Cumulative Amount of CoCo Issuance

The data show the cumulative amount of issuance of instruments which are labeled as "Contingent Convertible" in Bloomberg. Jurisdictional classification is applied based on the bond domicile (GB: the United Kingdom except the British Overseas Territories (e.g., Cayman Islands), CH: Switzerland, FR: France, DE: Germany, ES: Spain, NL: the Netherlands, CN: the People's Republic of China). Source: Bloomberg.



2017 Q3. Many CoCos have their domiciles in European countries such as the United Kingdom, Switzerland and Germany. CoCo issuance by Chinese banks has also been increasing.

Table 1.1 shows some examples of CoCos in real markets. According to Avdjiev et al. (2017), equity-conversion type CoCos were dominant in early years, but write-down type CoCos have become majority recently.

Compared to other major financial instruments, the size of CoCo market is still small. However, the term CoCo was widely noticed in early 2016, when a well-known large banking group in Europe, who had issued CoCos, revealed a huge amount of loss in its statement. As seen in Figure 1.2, the CoCo market experienced a sort of "crush" at that time. People worried whether the bank had enough strength to afford high-yield CoCos and questioned whether the risk of CoCos, including ones issued by other banks, had been evaluated appropriately. Unfortunately, it may be quite challenging to find a convincing answer to this question since no CoCo has experienced a trigger as of today. However, recent growth in CoCo markets and the realized crush emphasize the importance of market participants, as well as regulators, to properly understand and address risks arising from CoCos.

Table 1.1: CoCo examples

(a) Equity Conversion

	Lloyds	Credit Suisse	HSBC
Issue Size	£7bn (32 series)	\$2 bn	\$2.25 bn
Issue Date	December 1, 2009	February 24, 2011	March 24, 2015
Maturity	10-20 years	30 years	perpetual
Coupon	6.385-16.125%	7.875%	6.375%
Callable	None	Yes	Yes
Conversion Price	fixed	floored	fixed
Accounting Trigger	5% Core Tier 1 (Basel II)	7% CET 1	7% CET 1
Regulatory Trigger	None	Yes	Yes

(b) Write-down

	Rabobank	Deutsche Bank	Mitsubishi UFJ FG
Issue Size	€1.25 bn	€17.5 bn	¥10bn
Issue Date	March 12, 2010	May 27, 2014	May 23, 2015
Maturity	10 years	perpetual	perpetual
Coupon	6.875%	6.000%	2.700%
Callable	None	Yes	Yes
Write-down ratio	partial (75%)	Equal to the amount required to recover minimum CET1 ratio	Full or partial depending on the condition at the trigger
Accounting Trigger	5% CET 1	5.125% CET 1	5.125% CET 1
Regulatory Trigger	None	Yes	Yes

Source: De Spiegeleer and Schoutens (2012), Prospectus

1.4 CoCos and Regulation

Owing to the loss-absorption mechanism it has, a CoCo is regarded as regulatory capital. Regulatory capital defined in the Basel III framework consists of multiple categories – CET 1, Additional Tier 1, Tier 2 and other buffers. To be brief, Tier 1 and Tier 2 capitals are considered "going-concern" and "gone-concern," respectively.

In the regulatory context, the term going-concern is often used to emphasize that an instrument is expected to absorb losses in order to continue business operation. On the other hand, there exists a similar but different, so called gone-concern instruments, which are designed to absorb losses upon entry into resolution. This thesis deals with a going-concern type CoCo.[3] In other words, the trigger level is set equal to or higher

[3]The Financial Stability Board (2011) requires a regulator to have powers to bail-in an un-triggered CoCo upon entry into resolution. Although we focus on a going-concern feature of a CoCo, in a nutshell, sooner or later, a CoCo is expected to function as a loss-absorbing instrument.

Figure 1.2: Markit iBoxx USD Contingent Convertible Liquid Developed Markets AT1 Index

The index has been calculated daily and rebalanced monthly since December 31, 2013. It is composed of contingent convertibles which are eligible as Additional Tier 1 under the Basel III regulation. As of December 2017, 46 CoCos issued by 21 banking groups are included. Source: Bloomberg, Markit.

than the regulatory minimum to ensure business continuity.

Taking into account that a CoCo is a going-concern instrument, it could belong to Tier 1 capital, but it is not always the case. According to the Basel documents, requirements to be counted as Additional Tier 1 include without limitation:

- The issuer has full discretion over payment of a coupon or a dividend.
- The instrument has perpetual maturity and the issuer does not have an incentive to redeem early, including step-up in coupons.
- The instrument could be callable but requires national authority's approval in advance to exercise the call.

If a CoCo satisfies all the requirements, then it can be classified as Additional Tier 1 capital.[4] If otherwise, it is labeled as Tier 2 capital.

[4]Only "pure" capital, which is categorized as "capital" in accounting statements, e.g., common equity and undistributed profits, can be labeled as CET 1 capital. Thus, a CoCo is not a CET 1 instrument.

1.5 Literature

The idea of CoCo was introduced by academic studies such as Flannery (2015), Duffie (2009) and the Squam Lake Working Group[5] (2009). Regulators such as Bernanke (2009), Dudley (2009) and Haldane (2011) also endorsed to study CoCos by stating that contingent capital would be an option to enhance capital regulation. Since then, there have been many studies on CoCos, especially on valuation using diverse approaches.

Some build their models within the framework of a structural approach, which models bank's balance-sheet dynamics and is attributed to Merton (1974). Glasserman and Nouri (2012) derive a closed-form solution for CoCos with a capital-ratio trigger by modeling the bank's asset value process as a geometric Brownian motion. Buergi (2013) proposes a method to price Tier 1 ratio based CoCos by assuming a linear relationship between Tier 1 ratio and disclosed capital ratio. Albul, Jaffee, and Tchistyi (2015) apply Leland (1994) to find an optimal capital structure of a bank issuing CoCo.

De Spiegeleer and Schoutens (2012) propose a credit derivatives approach and an equity derivatives approach to price a CoCo. The former is a straightforward approach that applies a reduced-form model to express the trigger intensity, similarly to the default intensity which is often handled in pricing of credit derivatives. The latter attempts to price a CoCo by using existing barrier-option pricing methods to evaluate cashflow streams that is unique to a CoCo – equity purchase at the trigger and cancellation of coupon payments after the trigger. Closed-form solutions are available in both approaches since they consider a stock price, which follows the simple Black-Scholes model, as the trigger indicator.

Chung and Kwok (2015) include a regulatory trigger into their scope. They consider a joint process of stock price and capital ratio, and then apply the structural approach to express an accounting trigger and the reduced-form approach to a regulatory trigger.

Although various models are proposed to evaluate CoCos, there seems to be a mismatch between academic CoCos and actual CoCos from two aspects. First, many studies deal with a single-trigger CoCo, which is often a market trigger or an accounting trigger CoCo, despite the fact that the major trigger mechanism is the combination of an accounting trigger and a regulatory trigger. Some studies include a regulatory trigger into their coverage; however, it may not be sufficient to do an analysis from a regulator's perspective. Second, continuous-time modeling is often used for the sake of simple calculation although accounting numbers are only available at discrete time (e.g., at the end of each quarter).

[5] The Squam Lake Group is a group of fifteen academics who offer guidance on the reform of financial regulation.

1.6 Aims of the Study

Aims of the study are to provide a model that can fill in the gap caused by the mismatch and to answer the question that regulators might come up with: "when is the best timing for the regulators to trigger CoCos in practice?" In this thesis, we first model a bank issuing CoCo with an accounting trigger and a regulatory trigger by using a structural-default approach, and then find an optimal capital structure for the bank. We also model a CoCo investor with the structural-default approach and define regulator's problem. Finally, we look for effective regulatory policy to deal with such CoCos, based on default probabilities of the bank and the investor.

Our contribution can be summarized into three points, all of these issues have not been much addressed in other studies. First, we propose a comprehensive model that includes a CoCo investor and a regulator in addition to a bank. By including an investor into scope, risk contagion, or systemic risks, can also be examined in our model. Second, we deal with a CoCo that has the trigger mechanism consistent with CoCos in real markets – a combination of an accounting trigger and a regulatory trigger. Finally, we conduct an analysis from regulator's perspective and suggest effective regulatory policy to handle such CoCo.

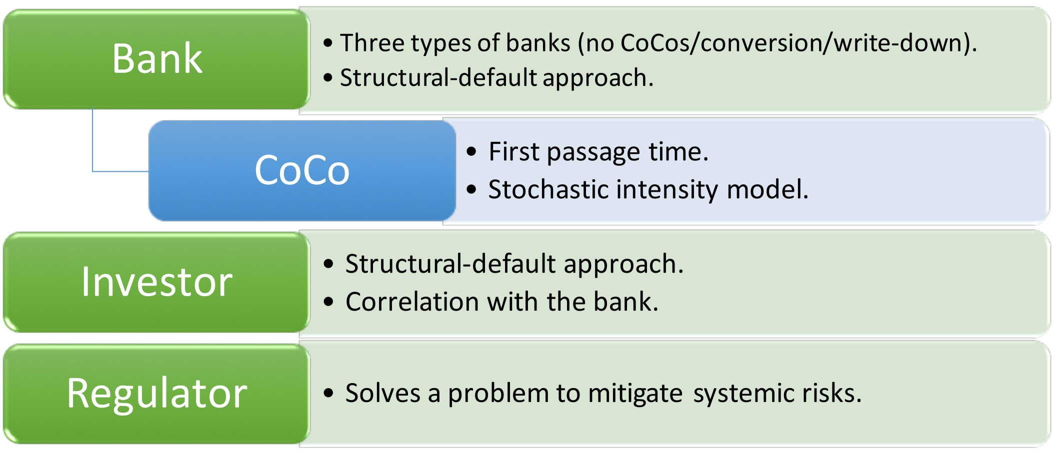
This thesis is organized as follows. Chapter 2 is a model description. Numerical examples using the model are provided in Chapter 3. In Chapter 4, we summarize our work and derive effective regulatory policy implicated from the numerical examples.

Chapter 2

The Model Setup

We build a model for each stakeholder – a bank, a CoCo investor and a regulator – respectively. Figure 2.1 provides an overview of our model. A bank model is constructed within the framework of a structural-default approach, following the idea of studies such as Merton (1974) and Leland (1994). We assume that the CoCo has two types of triggers, i.e., accounting trigger and regulatory trigger. An accounting trigger is expressed in a first-passage-time model, and it happens at periodic disclosure dates. On the other hand, a regulatory trigger happens upon regulator's decision, and it is expressed in both first-passage-time and stochastic intensity model. A CoCo investor, which we consider a financial institution, is also modeled by the structural-default approach and has correlation with the bank. In addition, we assume that a regulator solves a problem to mitigate systemic risks, i.e., default probabilities of both bank and investor.

Figure 2.1: Model Overview



2.1 Bank

We consider three types of banks, whose assets are financed by deposits, equity, and a bond or a CoCo:

- **Type 0 Bank** issues a normal subordinated bond (no trigger mechanism).
- **Type 1 Bank** issues a CoCo which converts into equity when triggered.
- **Type 2 Bank** issues a CoCo which suffers a write-down when triggered.

2.1.1 General Framework

First, we illustrate a general framework that can be applied to all three banks, following the idea of Harding et al. (2013) who apply Leland (1994) to obtain firm value of a bank. We assume that each bank has the same deposit-interest rate d, which is constant and continuously paid to depositors per instantaneous time.[1] Depositors are protected by the government deposit-insurance scheme and thus the deposits are considered "default-free." Given a fixed risk-free rate r, value of the deposits $D(t)$ (which must be equal to the face value $\bar{D}$) is given as

$$D(t) = \bar{D} = \int_0^\infty \mathrm{e}^{-rt} d \, \mathrm{d}t = \frac{d}{r}. \tag{2.1}$$

Similarly, face value $\bar{C}$ of the bond which pays continuous coupon c is equal to

$$\bar{C} = \int_0^\infty \mathrm{e}^{-rt} c \, \mathrm{d}t = \frac{c}{r}. \tag{2.2}$$

The bank assets $V(t)$ follow geometric Brownian motion under the risk-neutral measure $\mathbb{Q}$, i.e.,

$$\mathrm{d}V(t) = rV(t)\mathrm{d}t + \sigma V(t)\mathrm{d}z^{\mathbb{Q}}(t) \quad \text{under } \mathbb{Q}, \tag{2.3}$$

where $V(0) = x$. We also consider the process under the physical measure $\mathbb{P}$ as regulators are concerned with the "actual" bank-default probabilities. Suppose $z^{\mathbb{P}}(t) = -\frac{\mu - r}{\sigma} t + z^{\mathbb{Q}}(t)$, it follows that

$$\mathrm{d}V(t) = \mu V(t)\mathrm{d}t + \sigma V(t)\mathrm{d}z^{\mathbb{P}}(t) \quad \text{under } \mathbb{P}. \tag{2.4}$$

As banks are subject to the capital regulation and required to maintain minimum capital levels, it is natural to assume that their default conditions are associated with

[1] Leland (1994) assume that the firm finances the net cost of the debt payment by issuing additional equity. Although this assumption is not practical, we follow Leland (1994) to keep matters analytically tractable.

the capital ratio. Thus, we assume that a bank enters into resolution when its capital ratio falls to a certain fixed level $\chi_D \in [0, 1)$. Let $Eq(t)$ be the value of equity disclosed on the balance sheet, then the capital ratio is equivalent to $Eq(t)/V(t)$.[2] Therefore, the default time τ_D can be expressed in a first passage time, i.e.,

$$\tau_D = \inf\left\{t \geq 0;\ \ \frac{Eq(t)}{V(t)} \leq \chi_D\right\}. \tag{2.5}$$

We assume that a bank can issue a CoCo which has an accounting trigger and a regulatory trigger. An accounting-trigger condition can be defined similarly to the default condition by using the capital ratio $Eq(t)/V(t)$. If we assume a continuous-time accounting trigger, i.e., a trigger may happen at any instantaneous time, the trigger time denoted as τ_A^c can be expressed as

$$\tau_A^c = \inf\left\{t \geq 0;\ \frac{Eq(t)}{V(t)} \leq \chi_A\right\}, \tag{2.6}$$

where $\chi_A \in (\chi_D, 1)$ is the trigger level. The condition $\chi_A > \chi_D$ must be satisfied to ensure going-concern loss absorption. Otherwise, the bank enters into resolution before the trigger event to take place.

In practice, however, accounting numbers are only disclosed periodically, for instance, quarterly or annually. Thus, we need to think of a discrete-time accounting trigger, i.e., triggers can only happen at periodic disclosure dates.[3] Assume that $T_n,\ n \in \mathbb{N}$ is a calculation date of the capital ratio, the trigger time τ_A is defined as

$$\tau_A = \inf\left\{T_n;\ n \in \mathbb{N},\ \frac{Eq(T_n)}{V(T_n)} \leq \chi_A\right\}. \tag{2.7}$$

Next, we define a model for a regulatory trigger. The regulatory trigger happens depending on the regulator's discretion, but of course it does not come haphazardly; a regulator would exercise the right to trigger the CoCo when some sort of "insolvency sign" is found. Thus, we need to think of a model that is associated with some signs in order to express the regulatory trigger. The sign could be induced by either (1) factors

[2]To be precise, $V(t)$ is the "market" asset value and it should not be equal to the "book-value" of the assets. However, according to Buergi (2013), the book-value of assets can be approximated by its market value when the financial condition is severe, as accounting standards support "the conservatism principle." Under the principle, the lower of cost or market rule is applied, thus market value is more likely to be disclosed during the stress period.

[3]Banks may calculate the amount of regulatory capital more often when its financial condition become severe, but it may still take certain time to obtain confirmed numbers at the group consolidated level when we consider a SIFI, who is operating globally. Thus, it is more realistic to assume the discrete-time accounting trigger rather than the continuous one.

other than the asset value (e.g., liquidity of assets) [4] or (2) the asset value itself.

The first one can be modeled by applying a stochastic intensity model, following the idea of Chung and Kwok (2015);

$$\tau_{R_1} = \inf \Big\{ t \geq 0;\ \int_0^t h(V(u))\mathrm{d}u \geq X \Big\}, \tag{2.8}$$

where h is a trigger-intensity function and X is an exponentially distributed random variable independent of the Brownian motion $z^{\mathbb{Q}}(t)$. Given an appropriate function h, the above formula makes τ_{R_1} to follow a Cox process. Hereinafter, the expectation at time t conditional on $V(0) = x$ under the measure $\mathbb{Q}$ is denoted as $\mathbb{E}^{\mathbb{Q}}_{x,t}[\cdot]$ for brevity (t could be omitted when $t = 0$). Under the Cox process, the probability of the trigger to take place within some risk horizon t is given by

$$\mathbb{Q}\{\tau_{R_1} < t\} = \mathbb{Q}\left\{ \int_0^t h(V(s))\mathrm{d}s \geq X \right\} = 1 - \mathbb{E}^{\mathbb{Q}}_x \left[\mathrm{e}^{-\int_0^t h(V(s))\mathrm{d}s} \right]. \tag{2.9}$$

Although any function that satisfies $h(t) \geq 0$ and $\int_0^\infty h(t)\mathrm{d}t = \infty$ works as an intensity function, here we suppose that h is a non-increasing function with regard to the asset value $V(t)$. The rationale behind this is that the factor generating the insolvency sign should have negative correlation with the asset value. To be more specific, when the market condition become severe, market liquidity decreases and pushes up the trigger-intensity; at that time, the asset value $V(t)$ is also in the decreasing phase. Thus, if we look at the relationship between h and $V(t)$, we can say that they should have negative correlation.

The second regulatory trigger is modeled by the first passage time, i.e.,

$$\tau_{R_2} = \inf \Big\{ t \geq 0;\ V(t) + \epsilon \leq V_R \Big\}, \tag{2.10}$$

where ϵ is a Gaussian noise with 0 mean and the standard deviation being given by σ_ϵ. The noise indicates that regulators are monitoring the approximate asset value as the exact amount of $V(t)$ cannot be obtained at time t.

[4] OSFI (2011) announces the criteria that the OSFI (the Superintendent) may consider when they trigger a CoCo issued by a deposit-taking institution (DTI) under its supervision. The criteria include but not limited to:

- "Whether the assets of the DTI are, in the opinion of the Superintendent, sufficient to provide adequate protection to the DTI's depositors and creditors."
- "Whether the DTI has lost the confidence of depositors or other creditors and the public. This may be characterized by ongoing increased difficulty in obtaining or rolling over short-term funding."
- "Whether the DTI's regulatory capital has, in the opinion of the Superintendent, reached a level, or is eroding in a manner, that may detrimentally affect its depositors and creditors."

The second trigger defined in (2.10) is a key element in our model, because it enables an analysis from the regulator's perspective. Although (2.8) is one reasonable way to express a regulatory trigger, it is totally "stochastic" because X is a random variable. On the other hand, (2.10) is considered a "controllable" trigger as it is possible to set the level of V_R in alignment with the regulator's strategies. For example, if we suppose a relatively low V^R, it indicates that the regulator is not willing to trigger the CoCo unless the bank faces undoubtedly severe financial condition.

Having set all the necessary triggers, the trigger time τ_T can be obtained by

$$\tau_T = \min\{\tau_A, \tau_{R_1}, \tau_{R_2}\}. \tag{2.11}$$

Given the fact that even "a possibility of a trigger" had a considerable impact on market volatilities in early 2016, we should make a presumption that "an actual trigger" may have even larger impact. To express this, we assume that the trigger event changes the volatility of the bank asset-value process, i.e.,

$$\mathrm{d}V(t) = rV(t)dt + \sigma V(t)\mathrm{d}z^{\mathbb{Q}}(t), \qquad t \in [0, \tau_T), \tag{2.12}$$

$$\mathrm{d}V(t) = rV(t)dt + \sigma_T V(t)\mathrm{d}z^{\mathbb{Q}}(t), \qquad t \in [\tau_T, \infty). \tag{2.13}$$

It is reasonable to assume that the trigger makes the asset process more volatile, i.e., $\sigma < \sigma_T$. The reason behind this assumption is as follows; as the fact, a trigger of CoCo delivers the information that the bank has been facing a financial difficulty. Although its capital ratio recovers by the trigger, taking into account that less historical evidence is available to be convinced that the bank can fully turn around, it is more likely that the trigger of CoCo weakens the bank's credibility among market participants. The bank may suffer a higher funding costs due to less credibility, and forced to do some riskier investments to attain higher expected returns to meet the costs, which results in larger asset volatility.

Having provided all the necessary conditions, finally we define the total value of the bank, denoted as $v(x)$,

$$v(x) = x - B(x) + F(x) + I(x), \tag{2.14}$$

where B, F and I stand for bankruptcy costs, franchise value and insurance benefits, respectively. Specific expressions for each term are provided in the following subsections, as they differ according to the type of the bond issued.

2.1.2 Type 0 Bank (Subordinated Bond)

Type 0 Bank issues a normal subordinated bond, not a CoCo (thus, for the time being, the above trigger mechanism can be ignored). Its capital structure (i.e., deposits, the bond and equity on the credit side) does not change throughout its life. Hence, $V(t) = \bar{D} + \bar{C} + Eq(t)$ must be satisfied for $\forall t \leq \tau_D$.

Hence, Equation (2.5) can be rewritten as

$$\tau_D = \inf\{t \geq 0; \quad V(t) \leq V_D\}, \quad \text{where} \;\; V_D = \frac{\bar{D}+\bar{C}}{1-\chi_D}. \tag{2.15}$$

When a bankruptcy occurs, the bank enters a liquidation process which obliges the bank to pay bankruptcy costs (e.g., judicial costs). Here we assume that the bankruptcy costs are proportionate $\alpha \in (0,1)$ of the asset value at time τ_D, thus equal to αV_D. Hence, $B(x)$ is expressed as

$$B(x) = \alpha V_D \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right]. \tag{2.16}$$

As long as the bank is solvent, the bank benefits from the debt financing (e.g., tax benefit) and we call this the franchise value. We assume that the benefit is proportional to the payment generated from the debt financing, which is equal to $d+c$. Given the proportional factor $\delta \in (0,1)$, the franchise value $F(x)$ become

$$F(x) = \mathbb{E}_x^{\mathbb{Q}}\left[\int_0^{\tau_D} \mathrm{e}^{-rt}\delta(d+c)\mathrm{d}t\right] = \delta(\bar{D}+\bar{C})\left(1-\mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right]\right). \tag{2.17}$$

The residual assets after the bankruptcy, $(1-\alpha)V_D$, is allocated to the depositors. If not sufficient to compensate all the deposits $\bar{D}$, the gap is refunded from the deposit-insurance fund. Hence, the value protected by the deposit-insurance scheme equals to

$$\max\left\{\bar{D}-(1-\alpha)V_D, 0\right\}. \tag{2.18}$$

In practice, it is reasonable to assume $\max\left\{\bar{D}-(1-\alpha)V_D, 0\right\} \neq 0$, otherwise it is meaningless to think of a deposit-insurance scheme. Thus, under the assumption that the value of the protection is not zero, the insurance benefits $I(x)$ become

$$I(x) = (\bar{D}-(1-\alpha)V_D)\mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right]. \tag{2.19}$$

Given τ_D defined in Equation (2.5), we can prove that

$$\mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right] = \left(\frac{x}{V_D}\right)^{-\gamma} \tag{2.20}$$

where $\gamma = 2r/\sigma^2$. The proof is provided in Appendix A. Hence, by collecting above, the firm value of Type 0 Bank at $t=0$ can be expressed analytically as

$$v(x) = x + \delta(\bar{D}+\bar{C})\left(1-\left(\frac{x}{V_D}\right)^{-\gamma}\right) + (\bar{D}-V_D)\left(\frac{x}{V_D}\right)^{-\gamma}. \tag{2.21}$$

Next, we evaluate the market value of the bond. Recall that we are considering a subordinated bond, the residual assets after the bankruptcy are reimbursed to depositors first and then to the bond holders, if any. Thus, the value of recovery equals to

$$\min\{\bar{C}, \max\{(1-\alpha)V_D - \bar{D}, 0\}\}. \tag{2.22}$$

In practice, though, it is realistic to suppose that the value of recovery is usually 0. If otherwise, it implies that there exists plenty amount of assets left at the time of default, thus no need to put deposit-insurance scheme to practical use. With no recovery, the value of the bond $C(V(t))$ is expressed as

$$C(V(t)) = \mathbb{E}^{\mathbb{Q}}_{x,t}\left[\int_t^{\tau_D} \mathrm{e}^{-rs} c \; \mathrm{d}s\right] = \bar{C}\left(1 - \mathbb{E}^{\mathbb{Q}}_{x,t}\left[\mathrm{e}^{-r(\tau_D - t)}\right]\right), \tag{2.23}$$

which indicates from Equation (2.20) that $C(V(t))$ also has an analytic expression.

2.1.3 Type 1 Bank (Equity-Conversion CoCo)

We now consider Type 1 Bank, who issues an equity-conversion type CoCo. We assume that the full amount of the CoCo converts into equity, the value of which is equal to $\lambda\bar{C}$, $\lambda \in \mathbb{R}+$. Thus, λ can be interpreted as a conversion ratio. For example, in the case of $\lambda = 0.5$, it indicates that the CoCo investor receives some stocks at the trigger moment, but value of which only equals to the half of the face value of the original CoCo.

Hence, the market value of the CoCo is given by

$$\begin{aligned} C(V(t)) &= \mathbb{E}^{\mathbb{Q}}_{x,t}\left[\int_t^{\tau_T} \mathrm{e}^{-rs} c \; \mathrm{d}s + \mathrm{e}^{-r\tau_T}\lambda\bar{C}\right] \\ &= \bar{C}(1 - \mathbb{E}^{\mathbb{Q}}_{x,t}\left[\mathrm{e}^{-r(\tau_T - t)}\right]) + \lambda\bar{C}\cdot\mathbb{E}^{\mathbb{Q}}_{x,t}\left[\mathrm{e}^{-r(\tau_T - t)}\right]. \end{aligned} \tag{2.24}$$

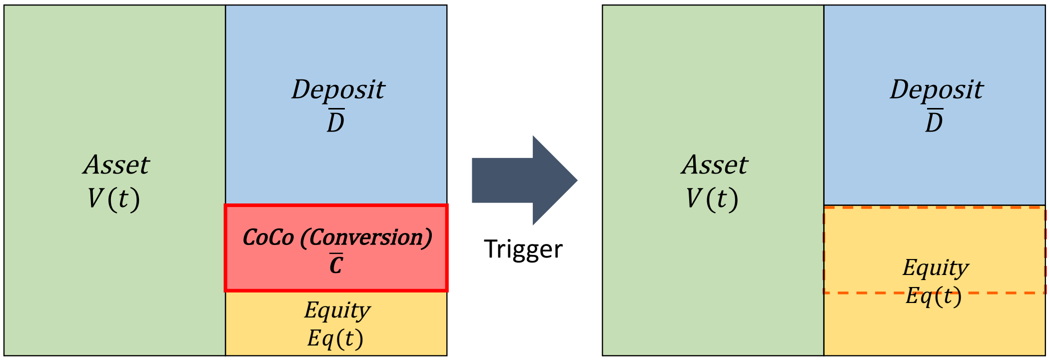
As shown in Figure 2.2, the capital structure of the bank changes as a result of the trigger. Consequently, the balance sheet condition for Type 1 Bank is $V(t) = \bar{D} + \bar{C} + Eq(t)$ if $t \le \tau_T$, and $V(t) = \bar{D} + Eq(t)$ otherwise. From (2.5) and (2.7), we obtain

$$\tau_D = \inf\{t \ge 0; \quad V(t) \le V_D\}, \quad \text{where} \quad V_D = \frac{\bar{D}}{1-\chi_D}, \tag{2.25}$$

$$\tau_A = \inf\{T_n; n \in \mathbb{N}, V(T_n) \le V_A\}, \quad \text{where} \quad V_A = \frac{\bar{D}+\bar{C}}{1-\chi_A}. \tag{2.26}$$

The bankruptcy costs and the insurance benefits are the same as those of Type 0 Bank, defined as (2.16) and (2.19), respectively. On the other hand, the amount of payment

Figure 2.2: Capital Structure of Type 1 Bank



generated from the debt financing is reduced from $d + c$ to d after the conversion; thus the expression for $F(x)$ differs, i.e.,

$$\begin{aligned} F(x) &= \mathbb{E}_x^{\mathbb{Q}}\left[\int_0^{\tau_D} \mathrm{e}^{-rs}\delta d \ \mathrm{d}s + \int_0^{\tau_T} \mathrm{e}^{-rs}\delta c \ \mathrm{d}s\right] \\ &= \delta\bar{D}\left(1 - \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right]\right) + \delta\bar{C}\left(1 - \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_T}\right]\right). \end{aligned} \tag{2.27}$$

Now let's consider how we can calculate the firm value $v(x)$ in the case of Type 1 Bank. If we assume that the CoCo only has the continuous-time accounting trigger, a closed-form solution is available, provided that

$$\mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_T}\right] = \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_A^c}\right] = \left(\frac{x}{V_A}\right)^{-\gamma}, \quad t \le \tau_A^c, \tag{2.28}$$

and

$$\begin{aligned} \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right] &= \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_A^c}\right] \cdot \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r(\tau_D - \tau_A^c)}\right] \\ &= \left(\frac{x}{V_A}\right)^{-\gamma}\left(\frac{V_A}{V_D}\right)^{-\gamma_T}, \quad t \le \tau_A^c \end{aligned} \tag{2.29}$$

where $\gamma_T = 2r/(\sigma^T)^2$. The proof is provided in Appendix A. The second equation in (2.29) follows from the strong Markov property of the Brownian motion.

Unfortunately, if we consider the case of the trigger defined in (2.11), which we aim to discuss in this thesis, the story is not as simple as above – an analytic solution is not available. Instead, we can formulate an integral equation with regard to $v(x)$, which is

$$\begin{aligned} v(x) = \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-rT}v(V(T))1_{\{\tau_T > T\}}\right] &+ \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-rT}v_T(V(T))1_{\{\tau_R > T, \tau_A = T\}}\right] \\ &+ \mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_R}v_T(V(\tau_R))1_{\{\tau_R < T\}}\right], \end{aligned} \tag{2.30}$$

where $\tau_R = \min\{\tau_{R_1}, \tau_{R_2}\}$ and $v_T(V(t))$ is the firm value calculated under the asset-value process after the trigger as defined in Equation (2.13).

Here we provide a narrative description on how this equation can be constructed. If we consider only one period of time until the next capital-ratio evaluation date denoted as T, one of the following three cases would happen:

1. The bank experiences no trigger until T, thus the bank structure is unchanged and the firm value at T equals to $v(V(T))$.
2. An accounting trigger takes place at T, while regulatory triggers have not emerged. In this case, the firm value at T is expressed as $v_T(V(T))$ because the asset-value process changes to (2.13).
3. A regulatory trigger happens somewhere in $[0, T)$, resulting in the firm-value function to change from v to v_T starting at τ_R.

Figure 2.3 shows an image of the above description. By taking expectation and discounting each of the three terms, we obtain the current firm value $v(x)$.

The function $v_T(V(t))$ can be obtained explicitly. Suppose that $V(t) = y$. It follows that

$$v_T(y) = y + \delta\bar{D}\left(1 - \left(\frac{y}{V_D}\right)^{-\gamma_T}\right) + (\bar{D} - V_D)\left(\frac{y}{V_D}\right)^{-\gamma_T}. \tag{2.31}$$

It appears that the above equation is quite similar to (2.21), the firm value of Type 0 Bank. It is not surprising because Type 1 Bank after conversion and Type 0 Bank share the same features – the asset-value process following geometric Brownian motion (no volatility change expected in the future) and the default condition expressed in the first passage time (expressions of V_D are different, though).

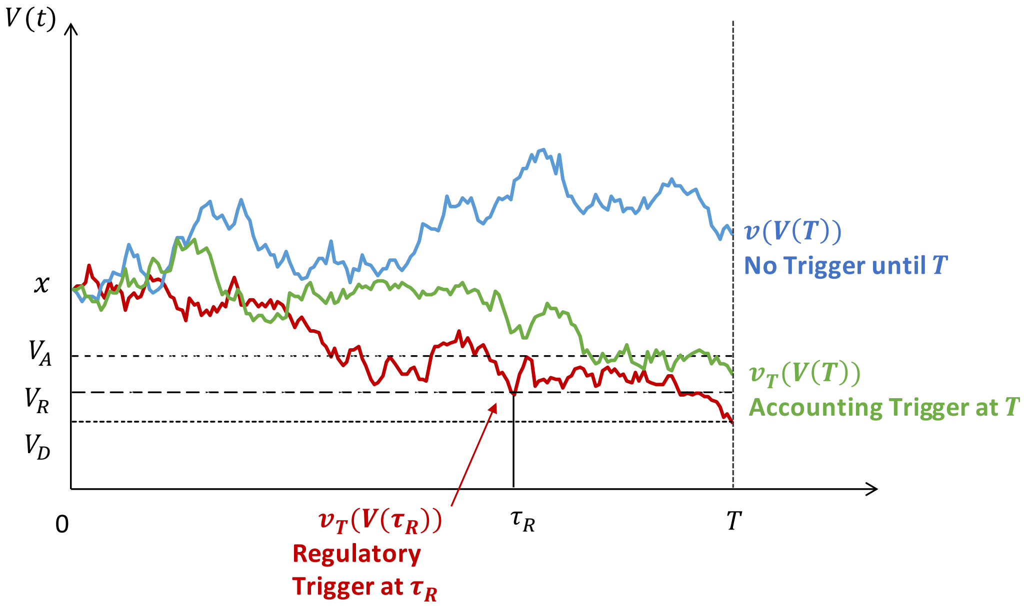
To solve (2.30), numerical calculation is available for some parts, yet not fully applicable. We can use Monte-Carlo simulation to make up for the residual calculation; however, it is not straightforward as it involves rare-event simulation. More details are provided in Appendix B.

2.1.4 Type 2 Bank (Write-down CoCo)

Type 2 Bank issues a write-down type CoCo. At the trigger event, a fraction $\psi \in [0, 1]$ of the CoCo suffers a write-down as shown in Figure 2.4. We assume that a part of the CoCo that has not been written-down stays in its balance sheet as a normal subordinated bond.[5] Accordingly, the balance-sheet condition for Type 2 Bank is

[5] In practice, the issuer may redeem the part of CoCo that has not suffered a write-down rather than keeping them on its balance sheet as a normal bond. However, early redemption has the same impact as the equity conversion in the sense that they both provide a certain fixed value to the investor at the trigger. Thus, no redemption is assumed in our model.

Figure 2.3: Firm-Value Calculation



$V(t) = \bar{D} + \bar{C} + Eq(t)$ if $t \leq \tau_T$, and $V(t) = \bar{D} + (1-\psi)\bar{C} + Eq(t)$ otherwise, which derives

$$\tau_D = \inf\{t \geq 0; \quad V(t) \leq V_D\}, \quad \text{where} \quad V_D = \frac{\bar{D} + (1-\psi)\bar{C}}{1-\chi_D}, \tag{2.32}$$

$$\tau_A = \inf\{T_n; n \in \mathbb{N}, V(T_n) \leq V_A\}, \quad \text{where} \quad V_A = \frac{\bar{D} + \bar{C}}{1-\chi_A}. \tag{2.33}$$

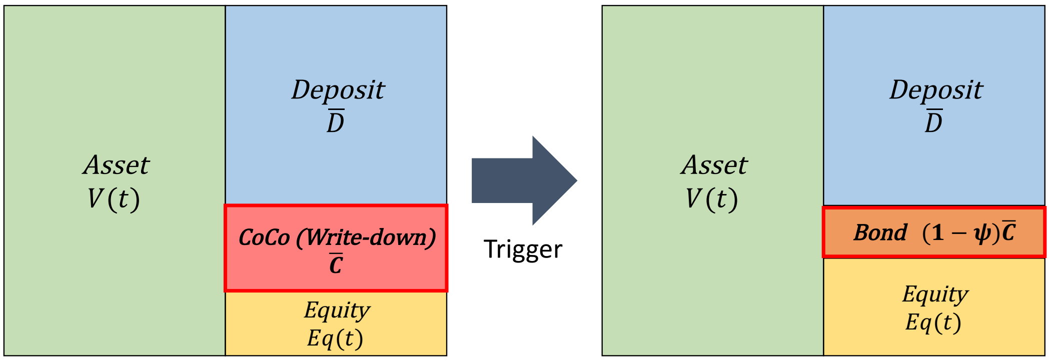
The market value of the write-down type CoCo is given by

$$\begin{aligned} C(V(t)) &= \mathbb{E}^{\mathbb{Q}}_{x,t}\left[\int_t^{\tau_D} \mathrm{e}^{-rs}(1-\psi)c \,\mathrm{d}s + \int_t^{\tau_T} \mathrm{e}^{-rs}\psi c \,\mathrm{d}s\right] \\ &= (1-\psi)\bar{C}(1-\mathbb{E}^{\mathbb{Q}}_{x,t}\left[\mathrm{e}^{-r(\tau_D-t)}\right]) + \psi\bar{C}\left(1-\mathbb{E}^{\mathbb{Q}}_{x,t}\left[\mathrm{e}^{-r(\tau_T-t)}\right]\right). \end{aligned} \tag{2.34}$$

If we consider the case of $\psi = 0$, we see that (2.34) becomes (2.23), which is not surprising as $\psi = 0$ indicates no write-down at the trigger event thus the bond can be considered a normal subordinated bond. On the other hand, $\psi = 1$ makes (2.34) to be the same as (2.24) given $\lambda = 0$. It implies that the write-down type CoCo takes a middle position between the subordinated bond and the equity-conversion type CoCo.

The bankruptcy costs and the insurance benefits are the same as those of the other banks. As for the franchise value, bear in mind that the amount of debt payment has

Figure 2.4: Capital Structure of Type 2 Bank



changed at time τ_T, $F(x)$ is given as

$$\begin{aligned} F(x) &= \mathbb{E}_x^{\mathbb{Q}}\left[\int_0^{\tau_D} \mathrm{e}^{-rs}\delta(d+(1-\psi)c)\ \mathrm{d}s + \int_0^{\tau_T} \mathrm{e}^{-rs}\delta\psi c\ \mathrm{d}s\right] \\ &= \delta(\bar{D}+(1-\psi)\bar{C})\left(1-\mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right]\right) + \delta\psi\bar{C}\left(1-\mathbb{E}_x^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_T}\right]\right). \end{aligned} \tag{2.35}$$

2.2 CoCo Investor

If we assume that a CoCo investor is a financial institution, the investor should also be included in the analysis scope as regulators are concerned with its behavior to prevent systemic risks. In the same way as banks, we apply the structural-default approach to model a CoCo investor who possesses a certain fraction $\phi \in [0,1]$ of a bond/CoCo issued by the bank.

Figure 2.5 shows the capital structure of the investor. As indicated, the investor has the certain amount of the bond or CoCo issued by the bank, $\phi C(t)$, and the other assets denoted as $\widetilde{Y}(t)$. In the case of an investor who buys the bond issued by Type 0 Bank, $\widetilde{Y}(t)$ follows

$$\mathrm{d}\widetilde{Y}(t) = \phi c 1_{\{t<\tau_D\}} dt + \widetilde{\mu}\widetilde{Y}(t)\mathrm{d}t + \widetilde{\sigma}\widetilde{Y}(t)\mathrm{d}\widetilde{z}^{\mathbb{P}}(t). \tag{2.36}$$

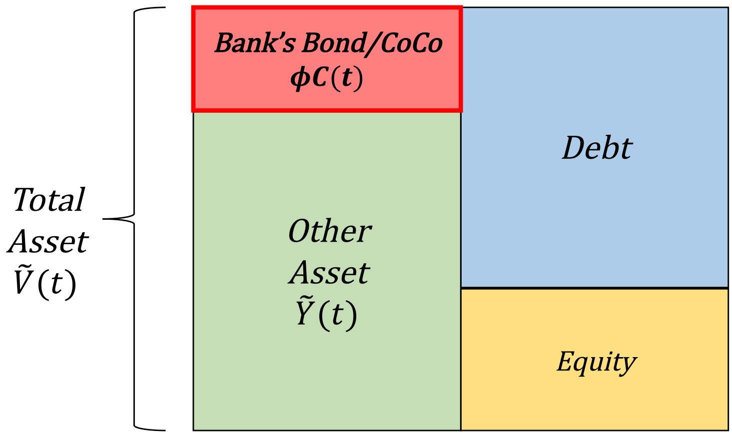
where 1_A is an indicator function (equals to 1 if A is true and 0 otherwise) and $\widetilde{z}^{\mathbb{P}}(t)$ is a Brownian motion under the measure $\mathbb{P}$, which has correlation ρ with $z(t)$; thus

$$\mathrm{d}z^{\mathbb{P}}(t)\mathrm{d}\widetilde{z}^{\mathbb{P}}(t) = \rho\ \mathrm{d}t. \tag{2.37}$$

For an investor who buys a CoCo issued by Type 1 Bank, $\widetilde{Y}(t)$ follows

$$\mathrm{d}\widetilde{Y}(t) = \phi c 1_{\{t<\tau_T\}}\mathrm{d}t + \phi\lambda\bar{C}1_{\{t=\tau_T\}} + \widetilde{\mu}\widetilde{Y}(t)\mathrm{d}t + \widetilde{\sigma}\widetilde{Y}(t)\mathrm{d}\widetilde{z}^{\mathbb{P}}(t). \tag{2.38}$$

Figure 2.5: Capital Structure of Investor



Finally, the process of $\widetilde{Y}(t)$ for an investor having Type 2 Bank CoCo is

$$\mathrm{d}\widetilde{Y}(t) = \phi c 1_{\{t<\tau_T\}}\mathrm{d}t + \phi(1-\psi)c 1_{\{\tau_T \le t < \tau_D\}}\mathrm{d}t + \widetilde{\mu}\widetilde{Y}(t)\mathrm{d}t + \widetilde{\sigma}\widetilde{Y}(t)\mathrm{d}\widetilde{z}^{\mathbb{P}}(t). \tag{2.39}$$

The idea here is that $\widetilde{Y}(t)$ basically follows a geometric Brownian motion – the last two terms on the RHS of (2.36), (2.38) and (2.39), – and relevant income generated from the bond or the CoCo (e.g., coupon, amount of stocks received at the conversion) is added to the process. For example, in the case of Type 1 Bank, the first and the second terms on the RHS of (2.38) indicate that the investor earns coupon ϕc until the trigger moment and receives shares which have the value equals to $\phi\lambda\bar{C}$ at the trigger moment.

Let us denote the investor's total assets by $\widetilde{V}(t)$. Taking into account that the asset side of the investor changes at the event of trigger or default of the bank, $\widetilde{V}(t)$ is equivalent to

$$\text{Type 0 Bank investor:} \quad \widetilde{V}(t) = \begin{cases} \widetilde{Y}(t) + \phi C(V(t)), & 0 \le t < \tau_D, \\ \widetilde{Y}(t), & \tau_D \le t, \end{cases} \tag{2.40}$$

$$\text{Type 1 Bank investor:} \quad \widetilde{V}(t) = \begin{cases} \widetilde{Y}(t) + \phi C(V(t)), & 0 \le t < \tau_T, \\ \widetilde{Y}(t), & \tau_T \le t, \end{cases} \tag{2.41}$$

$$\text{Type 2 Bank investor:} \quad \widetilde{V}(t) = \begin{cases} \widetilde{Y}(t) + \phi C(V(t)), & 0 \le t < \tau_T, \\ \widetilde{Y}(t) + \phi(1-\psi)C(V(t)), & \tau_T \le t < \tau_D, \\ \widetilde{Y}(t), & \tau_D \le t. \end{cases} \tag{2.42}$$

Provided that the investor is also a financial institution, it is reasonable to assume that its default condition is also fixed by some kind of regulation; we define the investor's default time $\widetilde{\tau}_D$ as

$$\widetilde{\tau}_D = \inf\{t \geq 0; \widetilde{V}(t) \leq \widetilde{\chi}_D\} \tag{2.43}$$

where $\widetilde{\chi}_D$ is an exogenous constant indicating a default barrier.

2.3 Regulator

A regulator has the right to trigger a CoCo issued by the bank under its supervision. Specific criteria on when and how the regulator determines the trigger is not made public. However, we can define some reasonable "mechanism" to determine the regulatory strategy, by paying attention to the fact that regulators act to prevent systemic risks.

It may be possible to express the systemic risks in several ways, e.g., joint default-probabilities of the bank and the investor. In this study, however, we put more focus on the bank's default probability compared to the investor's. Namely, we define the regulator's problem as

$$\min_{V_R, \bar{c}} \quad \mathbb{P}\{\tau_D < \mathcal{T}\} \tag{2.44}$$

$$\text{subject to} \;\; \mathbb{P}\{\widetilde{\tau}_D < \mathcal{T} \mid \tau_T < \widetilde{\tau}_D, \tau_T < \mathcal{T}\} < \bar{p}, \tag{2.45}$$

where $\mathcal{T}$ is a risk horizon set by the regulator. Explanations for $\bar{c}$ and $\bar{p}$ are provided later in this chapter.

(2.45) is the investor-default probability conditional on a trigger event to happen before default of the investor. This implies that the regulator would not allow investor-default probability to become higher than a certain level, denoted as $\bar{p}$. Hence, $\bar{p}$ can be interpreted as a "breaking point" of a stable financial system, i.e., the financial system is no longer resilient if the investor-default probability becomes higher than the threshold $\bar{p}$.

The problem indicates that regulator's main objective is to mitigate bank-default risks, but investor's default risk is also taken into account as a constraint. To begin with, a CoCo is invented to enhance the resilience of the bank; thus it is reasonable to assume that the regulator triggers the CoCo if necessary to recover the bank capital. However, we should not consider bank default risks "only" because an eventual goal that regulators bear in mind is the resilient financial system as a whole, which should include CoCo investors. Thus, the regulator is also responsible for preventing investors from suffering an unendurable loss due to the CoCo.

We assume that the regulator can control the level of V_R defined in (2.10). In addition, we add another controllable parameter, which is an issuance limit $\bar{c}$. The issuance limit is a regulation imposed to banks; under the limit, banks are not allowed to issue a CoCo if its coupon c does not fall within the range of $c \in [0, \bar{c}]$.[6]

[6] In practice, it may be difficult for regulators to directly restrict the level of coupons. However, there exist some regulations that indirectly limits the amount of CoCo issuance. For example, a minimum requirement for CET 1 ratio works as a restriction against CoCo issuance because CoCos are not included in the CET 1 capital.

Chapter 3

Numerical Example

Having defined the model setup as explained in the previous chapter, next we show numerical examples to examine how the regulator's problem can be "solved." Note that we are not trying to reach to the explicit solution, but rather attempt to obtain implication on how the regulator's strategy (i.e., choice of V_R and $\bar{c}$) affects the behavior of a bank and an investor.

The procedure for the numerical experiment is as follows: (1) find an optimal capital structure of a bank that maximizes its firm value, (2) evaluate default probabilities of both bank and investor by Monte-Carlo simulation, and finally (3) examine the result and search for effective regulatory policy to mitigate default risks.

3.1 Optimal Capital Structure

We assume that a bank determines the coupon c by maximizing its firm value. Thus, we first need to evaluate the firm value for each bank, and it can be done by solving the integral equation (2.30). However, what we should bear in mind is that regulator's strategy is always kept strictly confidential, in other words, conditions with regard to τ_R is unknown to banks. Hence, it is unrealistic to suppose that the bank finds an optimal capital structure under the model setup which includes the regulatory trigger.

One alternative solution is to ignore the regulatory trigger and assume $\tau_T = \tau_A$ (a discrete-time accounting trigger) is always the case, instead of the combination with the regulatory trigger. However, banks know that the regulatory trigger do exists, and it makes τ_T to happen at any time. Accordingly, it is more acceptable to proceed the firm value calculation with an assumption that $\tau_T = \tau_A^c$, i.e., a continuous-time accounting trigger. In this case, the firm value can be expressed analytically, thus it is not difficult to find an optimal c that maximizes the firm value.

Table 3.1 shows parameters we use in our numerical example, which are chosen to be as realistic as possible. For example, $\bar{D} = d/r = 70$ where $x = 100$ indicates that

Table 3.1: Parameters for Bank

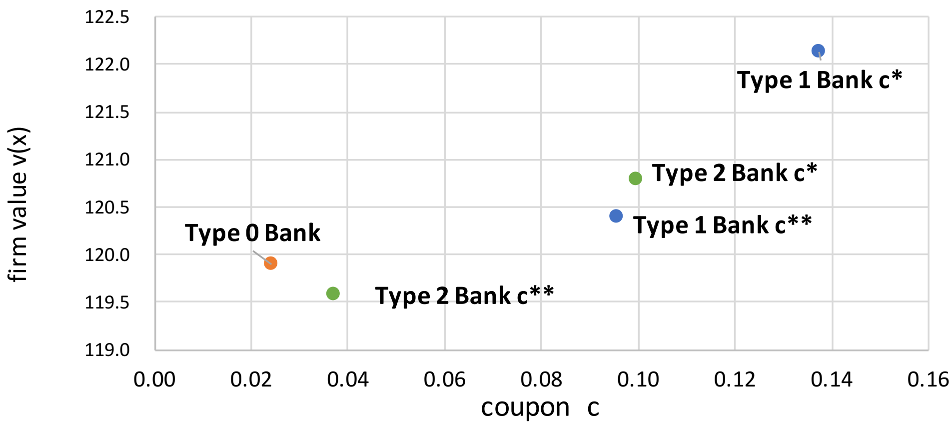
α	δ	μ	σ	σ_T	r	d	χ_A	χ_D	ψ	λ	x
0.2	0.3	0.005	0.05	0.05/0.07	0.01	0.7	0.07	0	0.5	0.5	100

70% of the bank assets is financed by deposits, which is an acceptable approximation for Japanese Mega Banks.[1]

It is difficult to calibrate the asset volatility σ directly from the market data, as $V(t)$ is not observable.[2] Instead, some approximation methods are proposed as discussed in Buergi (2013), and we use the number provided in this thesis for the level of σ. As for the volatility after the trigger, σ_T, we prepare two levels to check its impact on the firm value.

Let c^* and c^{**} be an optimal coupon assuming that $\sigma = \sigma_T = 0.05$ and $\sigma < \sigma_T = 0.07$, respectively. Optimal capital structures under these parameters are shown in Figure 3.1. It indicates that Type 1 Bank – equity-conversion CoCo – can attain the highest firm value, followed by Type 2 Bank. In addition, we see that the increase in σ_T has negative effect to the firm value in both banks; in the case of $\sigma_T = 0.07$, banks decide to issue smaller value of CoCos compared to the other case, resulting in the smaller firm value. As to Type 2 Bank, the firm value after the optimization under the higher σ_T become even smaller than that of Type 0 Bank.

Figure 3.1: Optimal Capital Structure



[1]According to the disclosure reports, percentages of deposits (including certificate of deposits) are 60%, 65% and 66% for Mitsubishi UFJ FG, Sumitomo Mitsui FG and Mizuho FG, respectively, as of September 2017.

[2]Zhou (2001) suggests that the asset volatility can be approximated by the stock-price volatility in the case of low leveraged firms, however, unfortunately it is not the case for banks.

Comparative Statics

Although it may be a digression from the main analysis, it might be interesting to do comparative statics with regard to parameters that define CoCo features. Having set the basic parameters as shown in Table 3.1, we check the impact of σ_T, χ_A, ψ, and λ on the firm value by changing one of these parameters in turns.

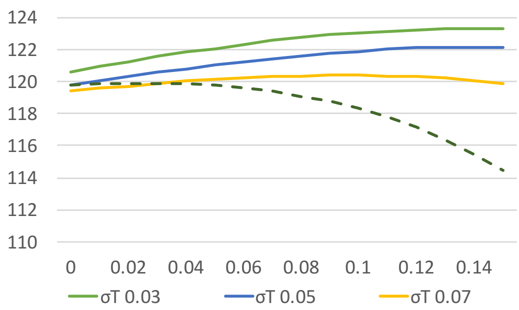
Figure 3.2 shows the results of the comparative statics, which can be summarized as:

- Volatility after trigger, σ_T, has negative impact on the firm value, i.e., the firm value become higher if we assume lower σ_T.
- The impact of the accounting-trigger level χ^A is also negative, however, not to the extent of σ_T.
- The impact of the write-down ratio ψ on the firm value is positive, i.e., the firm value become higher if we assume more write-down at the trigger.
- A conversion ratio λ is independent of the firm value, as it only affects the cashflow of the CoCo investor.
- In many cases, CoCos improve the firm value compared to Type 0 Bank.

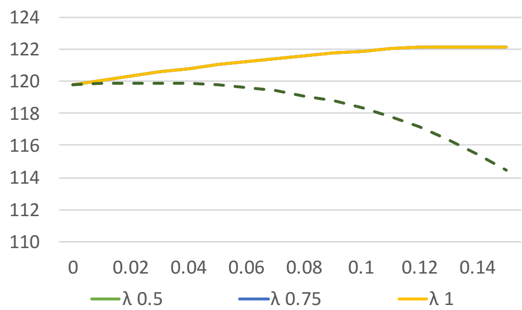
Figure 3.2: Comparative Statics on Firm Value

Each panel shows the relationship between the coupon of CoCo (x-axis) and the firm value (y-axis). For comparison, the result of Type 0 Bank (no CoCo) is displayed in dashed lines. As for σ_T, we test 0.03 (volatility reduction at the trigger), 0.05 (no volatility change, base schenario) and 0.07 (volatility hike at the trigger). In addition, we test 0.05, 0.07 (base schenario) and 0.09 for χ_A and 0.5 (base scenario), 0.75 and 1 for λ and ψ.

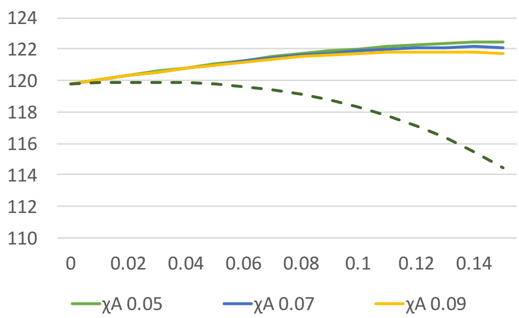
(a) Impact of σ_T (Type 1 Bank)



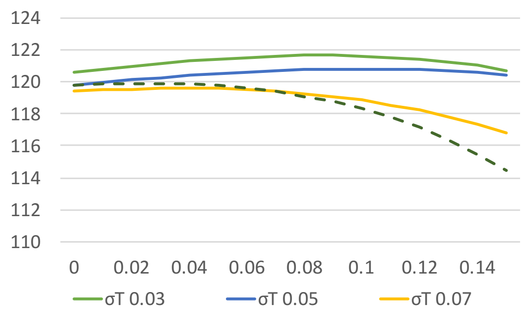
(b) Impact of λ (Type 1 Bank)



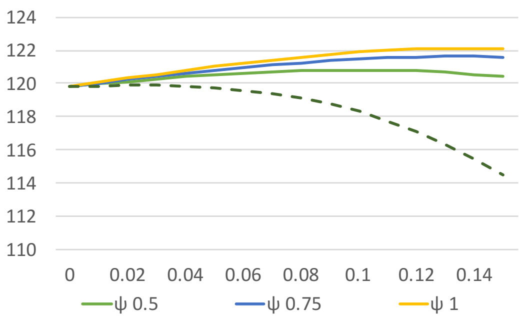
(c) Impact of χ_A (Type 1 Bank)



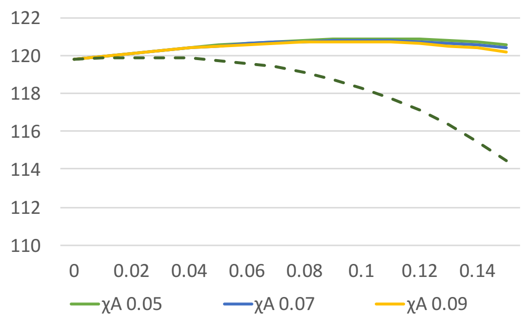
(d) Impact of σ_T (Type 2 Bank)



(e) Impact of ψ (Type 2 Bank)



(f) Impact of χ_A (Type 2 Bank)



3.2 Default Probabilities

Having determined the optimal coupon c for each bank, we are almost ready to carry out simulation to evaluate default probabilities. A few more steps remain to be finished before the simulation.

Parameters for Investors

As for investors, we use parameters provided in Table 3.2.[3] We test three levels of correlation ρ to deepen our analysis, which are $\rho = 0.5, \rho = 0$, and $\rho = -0.5$. If we consider a financial institution as a CoCo investor, the correlation should be positive. Thus, we focus on the case of $\rho = 0.5$ and compare the result to the other cases.

Table 3.2: Parameters for Investor

$\widetilde{\mu}$	$\widetilde{\sigma}$	$\widetilde{x}$	$\widetilde{\chi}_D$	ϕ	ρ
0.005	0.05	100	70	0.1	-0.5/0/0.5

Parameters for Regulators: Risk Horizon and Intensity Function

We assume that the risk horizon $\mathcal{T}$ in (2.44) and (2.45) is set to 10 years while accounting numbers are calculated quarterly, i.e., $T_n = 0.25n, n \in \mathbb{N}$. Actually, "10 years" may be a rather long-term target for regulators. For example, the Federal Reserve Board provides a four-year scenario to conduct a stress test, so-called Comprehensive Capital Analysis and Review (CCAR). We focus on 10-year default probabilities in our study because our main purpose is to assess the impact of trigger in the long run, so that we are able to analyze the subsequent behavior of the investor.

In addition to the risk horizon, we also need to find an "appropriate" intensity function $h(V(t))$ which makes τ_{R_1} to follow a Cox process, as indicated in (2.8). As mentioned earlier, among functions that satisfy $h(V(t)) \geq 0$ and $\int_0^\infty h(V(t))dt = \infty$, we should choose the function that is non-increasing in $V(t)$. Moreover, we should select the function that generates τ_{R_1} in a manner that is likely to happen in the real world, i.e., frequencies of τ_{R_1} and $V(\tau_{R_1})$ should be well-suited to our intuition. Among various possible functions we tested, we adopt

$$h(V(t)) = a\big(\bar{V} - \min(V(t), \bar{V})\big)^2 + b \tag{3.1}$$

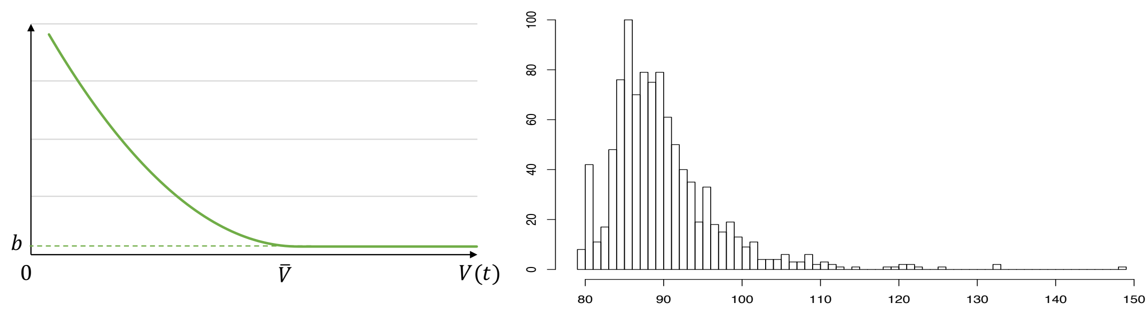
where $a = 7.5 \times 10^{-4}, b = 1.5 \times 10^{-4}$ and $\bar{V} = 90$ as an intensity function. According to simulation, given intensity (3.1), probabilities of the regulatory trigger to happen

[3] $\phi = 0.1$ is applied to the investor possessing Type 2 Bank CoCo. For Type 0 Bank and Type 1 Bank investors, we adjust the amount of ϕ to ensure that the value of CoCo they possess at $t = 0$ would be exactly the same.

Figure 3.3: Intensity Function

Panel (a) shows a graph image of function (3.1). It clearly shows that $h(V(t)) \geq b > 0$, and thus $\int_0^\infty h(V(t))dt = \infty$ is satisfied. It also shows that h is a non-increasing function with regard to $V(t)$. Panel (b) shows the histogram of $V(\tau_R)$ as a result of simulation (100,000 runs). Note that $\tau_R = \tau_{R_1} \wedge \tau_{R_2}$. Thus, it includes the frequencies of $V(\tau_{R_2})$, which is induced by the controllable valuable V_R. The frequency increases as $V(\tau_R)$ decreases, and jumps up around 80, since we have $V_R \approx 81$ in this case.

(a) Graph Image **(b)** Histogram of $V(\tau_R)$



within 10 years become higher than those of the accounting trigger. It is an acceptable result provided that the regulatory trigger could happen anytime while the accounting trigger can occur only once in a quarter. More features with regard to the intensity function (3.1) is provided in Figure 3.3.

Valuation of Non-Analytic Elements

Another preparation that should be done is to calculate two expectations in advance of the main simulation, which are $\mathbb{E}_x^{\mathbb{Q}}[e^{-r\tau_T}]$ and $\mathbb{E}_x^{\mathbb{Q}}[e^{-r\tau_D}]$. As shown in Equations (2.23), (2.24) and (2.34), calculations of these two expectations are necessary to evaluate the value of bond/CoCo, which must be known to obtain investor's asset value $\widetilde{V}(t)$.

However, these expectations cannot be expressed analytically in the case of τ_T defined in (2.11). Hence, we carry out another round of simulation to find explicit functions $\hat{g}_T(x)$ and $\hat{g}_D(x)$ that satisfies $\hat{g}_T(x) \approx \mathbb{E}_x^{\mathbb{Q}}[e^{-r\tau_T}]$ and $\hat{g}_D(x) \approx \mathbb{E}_x^{\mathbb{Q}}[e^{-r\tau_D}]$. Having done the simulation with regard to Type 2 Bank with c^{**}, we get

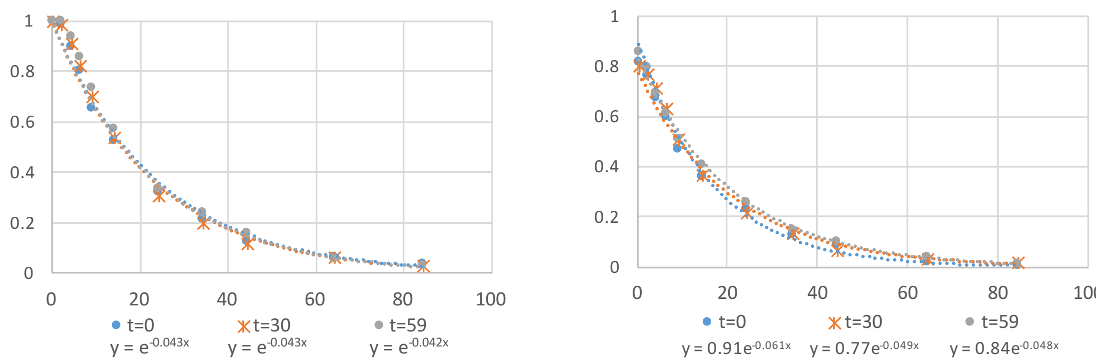
$$\hat{g}_T(x) = e^{-0.043(x-V_A)}, \quad \hat{g}_D(x) = 0.9103\, e^{-0.061(x-V_A)}, \quad x > V_A.$$

Note that approximations could be different if we assume a different starting date t, as shown in Figure 3.4. This is because the probability of τ_A to take place at the next calculation date is time-dependent. For instance, suppose that the next calculation date is approaching tomorrow, and we have $V(t) << V_A$ at that time. Then it is

Figure 3.4: Approximation by Simulation

Under the parameter set provided in Table 3.1, we obtain $V_A = 79.3$ and $V_D = 71.9$ for Type 2 Bank given c^{**}. We test $t = 0$, $t = 30$ and $t = 59$ (days) as a starting date, and $T = 60$ (days) as the next calculation date. The horizontal axes shows $x - V_R$ (difference between the initial asset value and the regulatory threshold), where $V_R = (V_A + V_D)/2 = 76.5$ is assumed. The intercept of $\hat{g}_T(x)$ is adjusted to 1 given that $\mathbb{E}_x^{\mathbb{Q}}[\mathrm{e}^{-r\tau_T}] \to 1$ as $x \to V_R$.

(a) $\hat{g}_T(x) \approx \mathbb{E}_x^{\mathbb{Q}}[\mathrm{e}^{-r\tau^T}]$ **(b)** $\hat{g}_T(x) \approx \mathbb{E}_x^{\mathbb{Q}}[\mathrm{e}^{-r\tau^D}]$



highly likely that the accounting trigger would happen tomorrow, unless we have a dramatic increase in $V(t)$. On the other hand, suppose that $V(t) << V_A$ but we still have enough time until the next calculation date, then the possibility of the asset value to exceed the accounting threshold would be higher than the former case, especially when we have a positive drift.

As such, although we obtain slightly different approximations for different starting dates, we confirm that the difference does not significantly affect the default probabilities of both bank and investor. Thus, we proceed with the approximations obtained from the result of $t = 0$.

The above approximations show the case of Type 2 Bank with c^{**}, i.e., $\sigma_T = 0.07$. We have conducted simulations with all the possible cases and successfully obtained similar approximations.

Controllable Variables

As defined in (2.44), V_R and $\bar{c}$ are supposed to be controllable variables for the regulator. If we could test all the possible choices with respect to these variables, we may be able to find a "solution" to the regulator's problem. However, unfortunately, it requires tremendous computational costs to do so. Hence, we pick up some "sample" quantities for each variable to enable simulation and necessary analysis.

As for the regulatory threshold V_R, we test three levels and name them High(H), Middle(M) and Low(L), respectively.

- **Strategy H :** V_R is determined to satisfy $V_A < V_R$. It indicates that the regulator is "aggressive," as they intend to trigger a CoCo at a relatively "early stage" where the capital ratio is still above the accounting threshold.

- **Strategy M :** V_R is set to meet $V_D << V_R \leq V_A$. Under this strategy, the regulator triggers a CoCo when the capital ratio falls below the accounting threshold yet some buffers remain until it hits the default barrier.

- **Strategy L :** V_R is fixed to be $V_R = V_D + \varepsilon$, where ε is positive but almost zero. In this case, we may say that the regulator is "easy-going," as they wait until the capital ratio to become quite close to the default barrier.

The most feasible strategy to be chosen by regulators would be Strategy M, because the other strategies are impractical from the following reasons. Under Strategy H, the accounting trigger is unworkable because the regulatory trigger is expected to happen in advance.[4] In the case of Strategy L, though loss-absorption of the CoCo enhances the capital ratio, it may not be enough to bring the bank back to the solvency level.

It is important to include extreme cases when we want to get some implications from just a few samples; thus, we analyze Strategy H and Strategy L although it may be unrealistic.

As for the coupon c, we test three quantities as well:

- **Case 1**: $c = c^*$. It indicates that the issuance limit $\bar{c}$ is large enough to allow a bank to choose an optimal c^*. The bank finds an optimal coupon with the assumption that the trigger of CoCo has nothing to do with the asset volatility, i.e., $\sigma = \sigma_T$.

- **Case 2**: $c = c^{**}$. It also indicates that $\bar{c}$ is large enough, but a bank finds an optimal coupon under the assumption that the trigger of CoCo generates the volatility hike, i.e., $\sigma < \sigma_T$.

- **Case 3**: $c = \bar{c}$. Regulator imposes issuance limit $\bar{c} < \min\{c^*, c^{**}\}$ to banks, thus a bank can only issue a bond/CoCo that has a coupon amount upto $\bar{c}$.

Table 3.3 summarizes the amount of c and corresponding $C(x)$ in each case.

Scenarios

As indicated from the comparative statics, σ_T considerably impacts the firm value, which implies that it remarkably affects the default probability of the bank as well.

[4] As defined in (2.10), regulators observe $V(t) + \epsilon$. It indicates that a regulatory trigger is induced by the noise added process, while an accounting trigger is associated with $V(t)$ itself. Thus, the probability of the accounting trigger to happen under Strategy H is not zero.

Table 3.3: Simulation Setup of c and $C(x)$

		Case 1 c^*	Case 2 c^{**}	Case 3 $\bar{c}$
Type 0 Bank	c	0.024		0.020
	$C(x)$	2.23		1.86
Type 1 Bank	c	0.137	0.096	0.020
	$C(x)$	10.77	8.19	1.87
Type 2 Bank	c	0.099	0.037	0.020
	$C(x)$	7.95	3.29	1.81

Thus, as we have done in the previous section, we test two scenarios with regard to σ_T to check its impact.

- **Scenario A**: $\sigma = \sigma_T$. The trigger of CoCo does not affect the volatility of $V(t)$.
- **Scenario B**: $\sigma < \sigma_T$. The volatility hike occurs due to the trigger of CoCo.

It is reasonable to assume Scenario B to happen in practice since the trigger of CoCo may weaken the bank's credibility in current circumstances.

3.3 Regulatory Policy

Under the setup defined in Section 3.2, we conduct a series of simulation to evaluate default probabilities.

Figures 3.5a and 3.5b show default probabilities of banks under measure $\mathbb{P}$. We refer to two consequences that should be worth noting.

First, if we take a look at the result of Scenario A and compare it to that of Scenario B, we can easily find that the default probabilities are higher in Scenario B for Type 1 Bank and Type 2 Bank, to the extent that they are even worth than that of Type 0 Bank. This is because an increase in the asset volatility (while drift remains the same) pushes up the probability of the bank to go bankrupt. It seems trivial yet important because it implies that the impact of possible volatility change should be taken into account when studying a CoCo.

Second, the default probabilities of Type 2 Bank are higher in comparison to Type 1 Bank. It indicates that equity-conversion type CoCo works well to recover a bank facing a financial difficulty. One reason behind this is the difference in the default threshold V_D. As defined in (2.25) and (2.32), V_D is higher for Type 2 Bank because Type 2 Bank has more "debt" in its balance sheet after the trigger.

Figures 3.6a and 3.6b show conditional default probabilities of investors as defined in (2.45). In the positive correlation case, we can see that the probabilities become higher, i.e., CoCo investors may not have enough strength to absorb losses when CoCos

Figure 3.5: 10-Year Default Probability of Banks

The numbers in parentheses ( ) indicate the standardized deviation. 50,000 runs of simulation are conducted for each case.

(a) Scenario A

V^R	Type 0 Bank Case 1 & 2 0.024	Type 0 Bank Case 3 0.02	Type 1 Bank (conversion) Case 1 0.099	Type 1 Bank (conversion) Case 2 0.037	Type 1 Bank (conversion) Case 3 0.02	Type 2 Bank (write-down) Case 1 0.137	Type 2 Bank (write-down) Case 2 0.096	Type 2 Bank (write-down) Case 3 0.02
Strategy H	2.24% (0.14)	2.31% (0.15)	1.21% (0.08)	1.23% (0.09)	1.31% (0.09)	4.20% (0.13)	1.95% (0.09)	1.69% (0.10)
Strategy M			1.34% (0.08)	1.39% (0.09)	1.24% (0.09)	4.05% (0.13)	2.13% (0.10)	1.77% (0.11)
Strategy L			1.30% (0.08)	1.35% (0.09)	1.25% (0.09)	4.29% (0.13)	2.03% (0.10)	1.77% (0.11)

(b) Scenario B

V^R	Type 0 Bank Case 1 & 2 0.024	Type 0 Bank Case 3 0.02	Type 1 Bank (conversion) Case 1 0.099	Type 1 Bank (conversion) Case 2 0.037	Type 1 Bank (conversion) Case 3 0.02	Type 2 Bank (write-down) Case 1 0.137	Type 2 Bank (write-down) Case 2 0.096	Type 2 Bank (write-down) Case 3 0.02
Strategy H	2.24% (0.14)	2.31% (0.15)	3.50% (0.15)	2.65% (0.14)	2.07% (0.10)	6.99% (0.20)	3.46% (0.14)	2.48% (0.13)
Strategy M			4.04% (0.16)	3.10% (0.14)	1.86% (0.09)	6.88% (0.20)	3.04% (0.13)	2.48% (0.13)
Strategy L			4.55% (0.17)	3.47% (0.16)	1.91% (0.09)	7.43% (0.20)	3.35% (0.14)	2.47% (0.13)

are triggered. The trigger event of CoCo takes place when the bank assets $V(t)$ has decreased to an insolvency level; positive correlation indicates that CoCo investor's assets $\widetilde{V}(t)$ is also decreasing and likely to be in a distressed level at the time of trigger.

Next, if we compare the results of Cases 1, 2 and 3 when the positive correlation is assumed, Case 3 (banks subject to the issuance limit) does not seem to be a desirable choice for investors. This is because smaller c induces lower V_A and V_R, which makes trigger events to happen when banks are in severe financial condition. As noted, investors may also be in difficult condition at that time when we assume positive correlation.

Finally, we should also note that Strategy H seems to be a preferable choice compared to the other strategies when we assume positive correlation. The reason is the same as the earlier consequences. Strategy H makes the trigger to happen when $V(t)$ is still at the high level. It follows that the investor's assets is likely to be in solvent level as well. However, Strategy H is not always the best choice if we look at the result of banks; as it induces earlier trigger which results in the premature volatility hike.

Figure 3.6: 10-Year Conditional Default Probability of Investors

The numbers in parentheses ( ) indicate the standardized deviation. 50,000 runs of simulation are conducted for each case. The result of Type 0 Bank investor is not applicable since $\tau_T = \infty$.

(a) Scenario A

ρ	V^R	Investor (Type 1 Bank) Case 1 0.137	Case 2 0.096	Case 3 0.02	Investor (Type 2 Bank) Case 1 0.099	Case 2 0.037	Case 3 0.02
0.5	Strategy H	2.00%	2.46%	3.39%	2.89%	3.89%	4.17%
		(0.09)	(0.13)	(0.13)	(0.05)	(0.05)	(0.16)
	Strategy M	2.73%	3.31%	4.35%	3.73%	4.24%	5.39%
		(0.10)	(0.15)	(0.14)	(0.05)	(0.06)	(0.18)
	Strategy L	3.00%	3.48%	4.62%	3.85%	4.71%	5.47%
		(0.11)	(0.15)	(0.15)	(0.05)	(0.11)	(0.19)
0	Strategy H	1.11%	1.08%	1.02%	1.32%	1.47%	1.11%
		(0.09)	(0.09)	(0.09)	(0.09)	(0.10)	(0.09)
	Strategy M	1.29%	1.04%	0.72%	0.71%	0.85%	1.00%
		(0.10)	(0.09)	(0.07)	(0.07)	(0.07)	(0.08)
	Strategy L	0.95%	0.99%	0.95%	1.20%	0.83%	1.32%
		(0.09)	(0.09)	(0.08)	(0.09)	(0.07)	(0.09)
−0.5	Strategy H	0.49%	0.10%	0.12%	0.17%	0.09%	0.05%
		(0.05)	(0.02)	(0.03)	(0.04)	(0.03)	(0.02)
	Strategy M	0.15%	0.17%	0.11%	0.14%	0.08%	0.18%
		(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)
	Strategy L	0.09%	0.06%	0.32%	0.13%	0.00%	0.00%
		(0.02)	(0.02)	(0.05)	(0.03)	0.00	0.00

(b) Scenario B

ρ	V^R	Investor (Type 1 Bank) Case 1	Case 2	Case 3	Investor (Type 2 Bank) Case 1	Case 2	Case 3
0.5	Strategy H	1.95%	3.04%	4.41%	2.63%	4.30%	4.17%
		(0.09)	(0.14)	(0.12)	(0.10)	(0.13)	(0.16)
	Strategy M	2.40%	3.30%	4.40%	3.36%	4.47%	4.91%
		(0.10)	(0.13)	(0.11)	(0.11)	(0.13)	(0.18)
	Strategy L	2.55%	3.16%	4.60%	3.53%	5.66%	5.67%
		(0.10)	(0.14)	(0.10)	(0.12)	(0.15)	(0.19)
0	Strategy H	1.27%	0.67%	1.01%	1.21%	0.86%	1.06%
		(0.10)	(0.07)	(0.09)	(0.09)	(0.08)	(0.08)
	Strategy M	1.45%	1.16%	0.95%	0.87%	1.49%	1.10%
		(0.10)	(0.09)	(0.09)	(0.08)	(0.10)	(0.09)
	Strategy L	1.09%	0.99%	0.11%	1.00%	0.66%	0.67%
		(0.09)	(0.09)	(0.03)	(0.08)	(0.07)	(0.07)
−0.5	Strategy H	0.60%	0.16%	0.06%	0.29%	0.09%	0.05%
		(0.07)	(0.03)	(0.02)	(0.05)	(0.03)	(0.02)
	Strategy M	0.22%	0.14%	0.06%	0.03%	0.25%	0.08%
		(0.04)	(0.03)	(0.02)	(0.02)	(0.04)	(0.02)
	Strategy L	0.14%	0.11%	0.00%	0.10%	0.08%	0.09%
		(0.03)	(0.03)	0.00	(0.03)	(0.03)	(0.02)

Chapter 4

Conclusion

In our study, we contribute to build a model for CoCo issuing banks, taking into account some important features of CoCos in real markets. First, a trigger mechanism is supposed to be a combination of an accounting trigger and a regulatory trigger. An accounting trigger is designed to happen periodically, while a regulatory trigger is expected to take place anytime throughout CoCo's life, subject to regulator's discretion. Two different kinds of regulatory triggers are included in our model. One is directly associated with the bank-asset level and expressed in a first-passage-time model. The other is modeled by a stochastic intensity model, implying that factors other than the asset value can also be the basis of the regulatory decision.

A CoCo investor is also included in our model to enable analysis on how CoCos affect the investor. We suppose that the CoCo investor has correlation with the bank and is also supervised by a financial regulator, which is often the case in practice.

We define regulator's problem as "mitigating systemic risks," which reflects both bank's and investor's default risks. We carry out simulation to investigate how, and to what extent, default probabilities are influenced by regulator's intention with regard to when to trigger the CoCo and how to design relevant CoCo regulations. From the result of numerical examples, we are able to make some observations with regard to effective regulatory policy, which is summarized in the following section.

4.1 Effective Regulatory Policy

Although the numerical experiment is conducted only under the certain parameter sets, we are able to derive some interesting consequences that are worth considering when dealing with a regulatory-trigger CoCo.

Implication 1: Impact of the trigger on the asset-value process should be taken into consideration.

The results of the numerical examples imply that an increase in the asset volatility has an adverse effect to the bank default probabilities. Accordingly, when the volatility hike is expected to follow the trigger, regulators should not set the regulatory threshold at excessively high levels to avoid premature volatility hike. On the other hand, the result implicates that "easy-going," i.e., too late trigger, may not be an effective choice to recover the bank in a timely manner as well. That means, there may be some effective levels with regard to the regulatory-trigger threshold that can effectively mitigate bank-default risks.

In addition, regulators should encourage banks to be aware of the possible impact of the trigger. When a bank does not consider the impact of CoCo on its asset-value process, the bank may issue "too much" CoCos that may result in unintended consequences – higher default probabilities.

Implication 2: Equity-conversion is a preferable loss-absorption mechanism in certain cases.

As for banks, an equity-conversion type CoCo is preferable to a write-down type CoCo as it can attain higher firm value and lower default risks at the same time. Hence, regulators should encourage banks to issue a conversion type CoCo rather than a write-down type CoCo. However, it should be noted that there may not be enough investors who are willing to invest in equity-like instruments. For instance, insurance companies may not be able to make investments in equity-conversion type CoCos because they are often subject to exposure limits against risky stocks.

Implication 3: Financial condition of the investor should be closely monitored when correlation with the issuer is positive.

Correlation between the bank and the investor is an important factor that should be taken into consideration. If the correlation is positive, investors may not be ready to absorb losses required to recover banks, which may result in severe systemic risks. Thus, it is necessary for regulators to assess the financial situation of the investor in addition to that of banks. To enable this assessment, regulators should always be accessible to relevant information with regard to the investor, which is not difficult if banks and investors are supervised by the same authority. However, in some jurisdictions such as the U.S., different authorities are responsible for bank supervision and investor supervision separately. Thus, in such cases, frequent communication between these authorities is important to eliminate systemic risks.

Implication 4: Issuance limit is not always effective to mitigate systemic risks.

Regulators may lay out a regulation with respect to CoCo-issuance limit if the CoCo may induce some undesirable consequences, for instance, the volatility hike. Although an issuance limit seems to be workable to mitigate bank-default risks, however, it is not an effective tool to mitigate investor-default risks when the positive correlation is assumed and the amount of CoCo issuance is associated with the trigger threshold. Instead, one possible action for regulators to eliminate investor-default risks is to impose CoCo-exposure limits, i.e., set a restriction on CoCo investments, to prevent them from unendurable losses ex-ante.

4.2 Future Work

Although the study contributes to highlight some important features of CoCos that impact regulatory policy, additional studies should be done to further deepen our analysis.

Extend the Model

First, we need to extend the current model to make it even more realistic, for example, adding a jump process.

In addition, as shown in Table 1.1, CoCos often allow earlier redemptions; thus, it may be interesting to add a callable feature to our current model. Optimal strategies on callable bonds are proposed in studies such as Brennan and Schwartz (1977) and Ingersoll (1977). However, note that we are not able to follow these studies directly because calls of CoCos are determined based on the regulator's intention, rather than the issuer's.

Besides, we need to add a mechanism to determine the extent of the volatility hike. We find out that the possible volatility hike affects the bank-default risks, however, our current model does not provide an internal mechanism to determine its extent. One possible mechanism is to suppose that the degree of the volatility hike is determined by the bank-asset value at the trigger moment. For instance, if trigger takes place when the bank assets face severe reduction, then the bank experiences a considerably high volatility increase.

Furthermore, we may be able to extend the model to incorporate larger number of entities by studying other related fields such as network theory. It may help us to understand more on how CoCos induce and reduce systemic risks in the financial system.

Pursue More Accurate Solution

We can do another piece of work to improve our study, which relates to pursuing more accurate solution.

We point out that it is not unreasonable to assume that an optimal capital structure of a bank is determined under the framework of a continuous-time accounting trigger, given that conditions with respect to regulatory triggers are not available to banks. For this reason, in the numerical example, we have not calculated the firm value with the model setup which includes regulatory triggers. However, we have demonstrated how we can obtain the value by constructing the integral equation. If we are capable of solving the equation without too much computational costs, we may be able to advance the regulator's problem, for example, we can add another constraint to avoid regulators selecting a strategy that excessively impairs bank's firm value.

Furthermore, we want to conduct more comparative statics to deepen our understanding, for example, comparative statics with regard to exposure limits on investors and drift of the asset-value process.

Appendix A

First Passage Time

In Equations (2.20), (2.28) and (2.29), we have shown that some analytic calculations are available for a stopping time which is a first passage time. In this appendix, we provide a proof for the equations.

First, we provide a following proposition as a preparation of the proof.

Proposition A.1. (First Passage Time) Consider the process $X(t) := \beta t + z^{\mathbb{Q}}(t)$, where $z^{\mathbb{Q}}(t)$ is a Brownian motion under the measure $\mathbb{Q}$, and define τ_m as

$$\tau_m = \inf\{t \geq 0; X(t) \leq m\}, \tag{A.1}$$

for $m > 0$. For some value θ, it follows that

$$\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{-\theta\tau_m}\right] = \mathrm{e}^{-m(-\beta+\sqrt{\beta^2+2\theta})}. \tag{A.2}$$

The proof of the above proposition is provided in Shreve (2003) which can be summarized as follows.

Define $m(t)$ as

$$m(t) := \mathrm{e}^{\xi X(t)-\theta t} = \mathrm{e}^{\xi z^{\mathbb{Q}}(t)-\frac{1}{2}\xi^2 t}, \tag{A.3}$$

where $\theta = \xi\beta - \frac{1}{2}\xi^2$. Therefore, $m(t)$ is an exponential martingale under the measure $\mathbb{Q}$. Given that τ_m is a stopping time, from the optimal stopping theorem, it follows that $\widetilde{m}(t) := m(t \wedge \tau_m)$ is also a martingale. Hence,

$$\begin{aligned} 1 = \widetilde{m}(0) &= \mathbb{E}^{\mathbb{Q}}[\widetilde{m}(t)] \\ &= \mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi X(t\wedge\tau_m)-\theta(t\wedge\tau_m)}\right] \\ &= \mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi m-\theta\tau_m} 1_{\{\tau_m \leq t\}}\right] + \mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi X(t)-\theta t} 1_{\{\tau_m > t\}}\right]. \end{aligned} \tag{A.4}$$

With respect to the first term, given that the sequence $\{a_n\}_{n\in\mathbb{N}} := \mathrm{e}^{\xi m-\theta\tau_m}1_{\{\tau_m\le t_n\}}$, $t_n \le t_{n+1}$ is non-negative and $a_n \le a_{n+1}$ for $\forall n$, we can apply the monotone convergence theorem; thus we have

$$\begin{aligned}\lim_{n\to\infty}\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi m-\theta\tau_m}1_{\{\tau_m\le t_n\}}\right] &= \mathbb{E}^{\mathbb{Q}}\left[\lim_{n\to\infty}\mathrm{e}^{\xi m-\theta\tau_m}1_{\{\tau_m\le t_n\}}\right]\\ &= \mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi m-\theta\tau_m}1_{\{\tau_m\le\infty\}}\right].\end{aligned} \tag{A.5}$$

As for the second term in (A.4), given that

$$0 \le \mathrm{e}^{\xi X(t)-\theta t}1_{\{\tau_m>t\}} \le \mathrm{e}^{\xi X(t)-\theta t} \le \mathrm{e}^{\xi m}, \tag{A.6}$$

we confirm that the sequence $\{b_n\}_{n\in\mathbb{N}} := \mathrm{e}^{\xi X(t)-\theta t_n}1_{\{\tau_m>t_n\}}$ is non-negative and bounded above. Thus, from the dominated convergence theorem, we have

$$\begin{aligned}\lim_{n\to\infty}\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi X(t)-\theta t_n}1_{\{\tau_m>t_n\}}\right] &= \mathbb{E}^{\mathbb{Q}}\left[\lim_{n\to\infty}\mathrm{e}^{\xi X(t)-\theta t_n}1_{\{\tau_m>t_n\}}\right]\\ &\le \mathbb{E}^{\mathbb{Q}}\left[\lim_{n\to\infty}\mathrm{e}^{\xi X(t)-\theta t_n}\right] = 0\end{aligned} \tag{A.7}$$

Consider the limit of equation (A.4) and collect above, we have

$$\begin{aligned}1 &= \lim_{n\to\infty}\mathbb{E}^{\mathbb{Q}}[\widetilde{m}(t)] = \lim_{n\to\infty}\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi m-\theta\tau_m}1_{\{\tau_m\le t\}}\right] + \lim_{n\to\infty}\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi X(t)-\theta t}1_{\{\tau_m>t\}}\right]\\ &= \mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{\xi m-\theta\tau_m}1_{\{\tau_m\le\infty\}}\right],\end{aligned} \tag{A.8}$$

which is equivalent to

$$\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{-\theta\tau_m}1_{\{\tau_m\le\infty\}}\right] = \mathrm{e}^{-\xi m} = \mathrm{e}^{-m(-\beta+\sqrt{\beta^2+2\theta})}. \tag{A.9}$$

In the case of $\tau=\infty$, $\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{-r\infty}1_{\{\tau=\infty\}}\right] = 0$. Hence, we obtain (A.2).

Now we prove Equation (2.20) by using the proposition. To make the following calculation simple, define $U(t) := \log V(t)$, so that $U(t)$ is a Brownian motion with a drift and we have

$$U(t) = y + \left(r - \frac{\sigma^2}{2}\right)t + \sigma z^{\mathbb{Q}}(t), \tag{A.10}$$

where $y := U(0) = \log x$. Let ℓ_D be the default barrier corresponding to $U(t)$, i.e., $\ell_D = \log V_D$. It follows that

$$V(t) \le V_D \iff U(t) \le \ell_D \iff \eta t + z^{\mathbb{Q}}(t) \le \frac{1}{\sigma}(\ell_D - y)$$

where $\eta = (r - \frac{\sigma^2}{2})/\sigma$. Given $\eta = \beta$, $U(t)$ and $X(t)$ become equivalent and thus τ_D defined in (2.5) and τ_m in (A.1) seem the same stopping time. However, note that

$\frac{1}{\sigma}(\ell_D - y) < 0$ while (A.1) assumes $m > 0$; thus we need to consider the flipped process with regard to $X(t)$ (i.e., $-X(t)$) to apply (A.2).[1] Thus, substituting $m = -\frac{1}{\sigma}(\ell_D - y)$ and then applying (A.2), we get

$$\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{-\theta\tau_D}\right] = \mathrm{e}^{\frac{1}{\sigma}(\ell_D - y)(-\beta+\sqrt{\beta^2+2\theta})}. \tag{A.11}$$

Substituting $\theta = r$ and recalling that $\beta = \eta = (r - \frac{\sigma^2}{2})/\sigma$, we have

$$\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right] = \mathrm{e}^{\frac{2r}{\sigma^2}(\ell_D - y)}. \tag{A.12}$$

Finally, substituting $\ell_D = \log V_D$ and $y = \log x$, we obtain

$$\mathbb{E}^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_D}\right] = \left(\frac{x}{V_D}\right)^{-\frac{2r}{\sigma^2}}. \tag{A.13}$$

We can prove Equations (2.28) and (2.29) similarly.

[1]We can prove that (A.2) is also applicable to the flipped process $-X(t)$ given the symmetric feature of the Brownian motion.

Appendix B

Calculation of Firm Value

Calculation of the firm value become complicated when we consider a combination of different triggers; we need to solve an integral equation (2.30) to obtain the firm value. Before going into details of how we can solve the equation, first we build a similar integral equation for a simpler case, i.e., the case of discrete-time accounting trigger only. In the simple case, we are able to compute the firm value without simulation.

Hereinafter, a density function of a random valuable X conditional on A is denoted as $f_A^{\{X\}}(z)$. For example, $f_T^{\{U,M\}}(z,m)$ is the joint density of $U(t)$ and $M(t)$ conditional on $t = T$.

B.1 Discrete-Time Accounting Trigger

In the combination case, $\tau_T < \tau_D$ is assured because $V(t)$ must pass the regulatory threshold V_R to reach V_D. However, if we omit the regulatory trigger and consider the discrete-time accounting trigger only, the probability of $\tau_D < \tau_T = \tau_A$ becomes non-zero quantity. Note that the default barrier in this case is $\widehat{V}_D := (\bar{D}+\bar{C})/(1-\chi_D)$ for $t < \tau_T$, which is not equivalent to V_D of any bank type. The threshold V_D is only applicable for $\tau_T \leq t$.

Hence, the CoCo issuing bank experiences one of the following three cases if we consider one period of time until the next evaluation date T:

1. Neither the trigger nor the default happens.
2. No default, but the accounting trigger happens.
3. Default before or at T.

Let us denote the firm value of this simple case by $\hat{v}(x)$ to differentiate it from the original combination case. An integral equation with respect to $\hat{v}(x)$ become

$$\begin{aligned}\hat{v}(x) &= \mathrm{e}^{-rT}\mathbb{E}_x^{\mathbb{Q}}\Big[\hat{v}(V(T))1_{\{\tau_A>T\}}1_{\{\tau_D>T\}}\Big] \\ &\quad + \mathrm{e}^{-rT}\mathbb{E}_x^{\mathbb{Q}}\Big[\hat{v}_T(V(T))1_{\{\tau_A=T\}}1_{\{\tau_D>T\}}\Big] + \mathbb{E}_x^{\mathbb{Q}}\Big[\mathrm{e}^{-r\tau_D}\hat{v}(\widehat{V}_D)1_{\{\tau_D\le T\}}\Big], \qquad \text{(B.1)}\end{aligned}$$

where $\hat{v}_T(V(t))$ is the firm value after the trigger which is analytically known. From the discussion in Chapter 2, we have

$$\begin{aligned}B(\widehat{V}_D) &= \alpha\widehat{V}_D \\ I(\widehat{V}_D) &= \bar{D} - (1-\alpha)\widehat{V}_D \\ F(\widehat{V}_D) &= 0 \qquad \text{(B.2)}\end{aligned}$$

for any type of bank. It follows that

$$\hat{v}(\widehat{V}_D) = \widehat{V}_D - B(\widehat{V}_D) + F(\widehat{V}_D) + I(\widehat{V}_D) = \bar{D}. \qquad \text{(B.3)}$$

Thus $\hat{v}(\widehat{V}_D)$ is a constant value.

(B.1) is equivalent to

$$\begin{aligned}\hat{v}(x) &= \mathrm{e}^{-rT}\mathbb{E}_x^{\mathbb{Q}}\Big[\hat{v}(V(T))1_{\{V(T)>V_A\}}1_{\{L(T)>\widehat{V}_D\}}\Big] \\ &\quad + \mathrm{e}^{-rT}\mathbb{E}_x^{\mathbb{Q}}\Big[\hat{v}_T(V(T))1_{\{\widehat{V}_D\le V(T)\le V_A\}}1_{\{L(T)>\widehat{V}_D\}}\Big] + \bar{D}\mathbb{E}_x^{\mathbb{Q}}\Big[\mathrm{e}^{-r\tau_D}1_{\{\tau_D\le T\}}\Big], \qquad \text{(B.4)}\end{aligned}$$

where $L(t) := \min_{0\le s\le t} V(s)$. Note that the event $\tau_D > T$ is equivalent to the event $L(T) > \widehat{V}_D$.

Consider the process $U(t)$ as defined in (A.10) and let ℓ_A and $\hat{\ell}_D$ be the relevant barriers corresponding to $U(t)$, i.e., $\ell_A = \log V_A$ and $\hat{\ell}_D = \log \widehat{V}_D$. Denote the firm value calculated from the process $U(t)$ as $\hat{u}(y)$ and $\hat{u}_T(y)$, i.e., $\hat{v}(x) = \hat{u}(y)$ and $\hat{v}_T(x) = \hat{u}_T(y)$. We have

$$\begin{aligned}\hat{u}(y) &= \mathrm{e}^{-rT}\mathbb{E}_y^{\mathbb{Q}}\Big[\hat{u}(U(T))1_{\{U(T)>\ell_A\}}1_{\{M(T)>\hat{\ell}_D\}}\Big] \\ &\quad + \mathrm{e}^{-rT}\mathbb{E}_y^{\mathbb{Q}}\Big[\hat{u}_T(U(T))1_{\{\hat{\ell}_D\le U(T)\le\ell_A\}}1_{\{M(T)>\hat{\ell}_D\}}\Big] + \bar{D}\mathbb{E}_y^{\mathbb{Q}}\Big[\mathrm{e}^{-r\tau_D}1_{\{\tau_D\le T\}}\Big] \qquad \text{(B.5)}\end{aligned}$$

where $M(t) = \min_{0\le s\le t} U(s)$ and $\mathbb{E}_y^{\mathbb{Q}}[\cdot] := \mathbb{E}^{Q}[\cdot \mid U(0) = y]$. Given relevant density functions, the above equation can be expressed as

$$\begin{aligned}\hat{u}(y) &= \mathrm{e}^{-rT}\int_{\hat{\ell}_D}^{\infty}\int_m^{\infty}\hat{u}(z)f_{y,T}^{\{U,M\}}(z,m)\mathrm{d}z\mathrm{d}m \\ &\quad + \mathrm{e}^{-rT}\int_{\hat{\ell}_D}^{\ell_A}\int_m^{\ell_A}\hat{u}_T(z)f_{y,T}^{\{U,M\}}(z,m)\mathrm{d}z\mathrm{d}m + \bar{D}\int_0^T \mathrm{e}^{-r\tau_D}f_y^{\{\tau_D\}}(t)\mathrm{d}t \\ &=: \mathrm{e}^{-rT}\int_{\hat{\ell}_D}^{\infty}\int_m^{\infty}\hat{u}(y)f_{y,T}^{\{U,M\}}(z,m)\mathrm{d}z\mathrm{d}m + \hat{h}(y), \qquad \text{(B.6)}\end{aligned}$$

which can be solved numerically because density functions $f_{y,T}^{\{U,M\}}(z,m)$ and $f_y^{\{\tau_D\}}(t)$ have closed-form expressions, i.e.,

$$f_{y,T}^{\{U,M\}}(z,m) = \frac{2(z+y+(r-\sigma^2/2)T-2m)}{\sigma^3 t\sqrt{2\pi T}} \exp\left\{-\frac{(z+y+(r-\sigma^2/2)T-2m)^2}{2\sigma^2 T}\right\}, \tag{B.7}$$

for $m \le y,\ m \le z$ (otherwise 0), and

$$f_y^{\{\tau_D\}}(t) = \frac{(y-(r-\sigma^2/2)t-\hat{\ell}_D)}{2\sigma t\sqrt{2\pi t}} \exp\left\{-\frac{(y-(r-\sigma^2/2)t-\hat{\ell}_D)^2}{2\sigma^2 t}\right\}. \tag{B.8}$$

(B.6) can be solved recursively. Set $\hat{u}_0(y) = \hat{h}(y)$, then define

$$\hat{u}_n(y) = \mathrm{e}^{-rT}\int_{\hat{\ell}_D}^{\infty}\int_m^{\infty} \hat{u}_{n-1}(z) f_{y,T}^{\{U,M\}}(z,m)\mathrm{d}z\mathrm{d}m + \hat{h}(y). \tag{B.9}$$

Note that $\hat{u}_n(y)$ is monotonically increasing in n for all y and it converges to $\hat{u}(y)$ as $n \to \infty$.

B.1.1 Proof of (B.7)

Following Shreve (2003), from the reflection principle, we have

$$\mathbb{Q}\{M(T) \le m, U(T) \ge z\} = \mathbb{Q}\{U(T) \le 2m - z\}. \tag{B.10}$$

for $m \le y,\ m \le z$. Since $U(t)$ is a Brownian motion, the RHS become

$$\begin{aligned}\mathbb{Q}\{U(T) \le 2m-z\} &= \mathbb{Q}\left\{y + \left(r - \frac{\sigma^2}{2}\right)T + \sigma z^{\mathbb{Q}}(T) \le 2m - z\right\} \\ &= \Phi\left(\frac{1}{\sigma\sqrt{T}}(2m - z - y - \left(r - \frac{\sigma^2}{2}\right)T\right),\end{aligned} \tag{B.11}$$

where $\Phi(\cdot)$ is a cumulative density function of a standard Brownian motion. Hence, (B.10) is equivalent to

$$\int_{-\infty}^{m}\int_{z}^{\infty} f_{y,T}^{\{U,M\}}(u,w)\mathrm{d}u\mathrm{d}w = \int_{-\infty}^{\frac{1}{\sigma\sqrt{T}}(2m-z-y-(r-\frac{\sigma^2}{2})T} \mathrm{e}^{-\frac{w^2}{2}}\mathrm{d}w. \tag{B.12}$$

Differentiate the above by m and then by z, we obtain (B.7).

B.1.2 Proof of (B.8)

Recall that $\tau_D = \inf\{t \geq 0; U(t) \leq \hat{\ell}_D\}$, we have

$$\begin{aligned} f_y^{\{\tau_D\}}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbb{Q}\{U(t) \leq \hat{\ell}_D\} = \frac{\mathrm{d}}{\mathrm{d}t}\mathbb{Q}\left\{y + \left(r - \frac{\sigma^2}{2}\right)t + \sigma z^{\mathbb{Q}}(t) \leq \hat{\ell}_D\right\} \\ &= \frac{\mathrm{d}}{\mathrm{d}t}\int_{-\infty}^{\frac{1}{\sigma\sqrt{T}}(\hat{\ell}_D - y - (r - \frac{\sigma^2}{2})T} \mathrm{e}^{-\frac{z^2}{2}}\mathrm{d}z, \end{aligned}$$

which results in equation (B.8).

B.2 Regulatory Trigger Added

Next let's consider the case of the combination trigger, i.e., $\tau_T = \min\{\tau_A, \tau_{R_1}, \tau_{R_2}\}$. Note that the event $\tau_{R_2} > T$ is equivalent to the event $L(t) > V_R$. Rewrite the equation (2.30), we have

$$\begin{aligned} v(x) = \mathrm{e}^{-rT}\mathbb{E}_x^{\mathbb{Q}}&\Big[v(V(T))1_{\{V(T)>V_A\}}1_{\{L(T)>V_R\}}1_{\{\tau_{R_1}>T\}}\Big] \\ &+ \mathrm{e}^{-rT}\mathbb{E}_x^{\mathbb{Q}}\Big[v_T(V(T))1_{\{V_D \leq V(T) < V_A\}}1_{\{L(T)>V_R\}}1_{\{\tau_{R_1}>T\}}\Big] \\ &\quad + \mathbb{E}_x^{\mathbb{Q}}\Big[\mathrm{e}^{-r\tau_R}v_T(V(\tau_R))1_{\{\tau_R<T\}}\Big]. \end{aligned} \tag{B.13}$$

Rewrite again with respect to the logarithmic process $U(t)$ and its minimum $M(t)$, we have

$$\begin{aligned} u(y) = \mathrm{e}^{-rT}\mathbb{E}_y^{\mathbb{Q}}&\Big[u(U(T))1_{\{U(T)>\ell_A\}}1_{\{M(T)>\ell_R\}}1_{\{\tau_{R_1}>T\}}\Big] \\ &+ \mathrm{e}^{-rT}\mathbb{E}_y^{\mathbb{Q}}\Big[u_T(U(T))1_{\{\ell_D \leq U(T) < \ell_A\}}1_{\{M(T)>\ell_R\}}1_{\{\tau_{R_1}>T\}}\Big] \\ &\quad + \mathbb{E}_y^{\mathbb{Q}}\Big[\mathrm{e}^{-r\tau_R}u_T(U(\tau_R))1_{\{\tau_R\leq T\}}\Big] \end{aligned} \tag{B.14}$$

where $\ell_R = \log V_R$ and $v(x) = u(y)$.

The first term on the RHS of the above equation is expressed as

$$
\begin{aligned}
&\mathrm{e}^{-rT}\mathbb{E}_y^{\mathbb{Q}}\left[u(U(T))1_{\{U(T)>\ell_A\}}1_{\{M(T)>\ell_R\}}1_{\{\tau_{R_1}>T\}}\right] \\
&\quad = \mathrm{e}^{-rT}\int_{\ell_R}^{\infty}\int_{\ell_A}^{\infty}\int_{T}^{\infty} u(z) f_{y,T}^{\{U,M,\tau^{R_1}\}}(z,m,t)\mathrm{d}t\mathrm{d}z\mathrm{d}m \\
&\quad = \mathrm{e}^{-rT}\int_{\ell_R}^{\infty}\int_{\ell_A}^{\infty} u(y) f_{y,T}^{\{U,M\}}(z,m)\mathrm{d}z\mathrm{d}m \\
&\qquad\qquad \times \mathbb{Q}\{\tau_{R_1} > T \mid U(0)=y,\ U(T)=z,\ M(T)>\ell_R\} \\
&\quad = \mathrm{e}^{-rT}\int_{\ell_R}^{\infty}\int_{\ell_A}^{\infty} u(y) f_{y,T}^{\{U,M\}}(z,m)\mathrm{d}z\mathrm{d}m \\
&\qquad\qquad \times \mathbb{E}\left[\mathrm{e}^{\int_0^T h(V(s))\mathrm{d}s} \mid U(0)=y,\ U(T)=z,\ M(T)>\ell_R\right], \qquad \text{(B.15)}
\end{aligned}
$$

where the last equation follows from (2.9).

The second term on the RHS of (B.14) can be expressed similarly, i.e.,

$$
\begin{aligned}
&\mathrm{e}^{-rT}\mathbb{E}_y^{\mathbb{Q}}\left[u_T(U(T))1_{\{\ell_D\le U(T)<\ell_A\}}1_{\{M(T)>\ell_R\}}1_{\{\tau_{R_1}>T\}}\right] \\
&\quad = \mathrm{e}^{-rT}\int_{\ell_R}^{\ell_A}\int_{m}^{\ell_A}\int_{T}^{\infty} u_T(z) f_{y,T}^{\{U,M,\tau^{R_1}\}}(z,m,t)\mathrm{d}t\mathrm{d}z\mathrm{d}m \\
&\quad = \mathrm{e}^{-rT}\int_{\ell_R}^{\infty}\int_{m}^{\ell_A} u_T(z) f_{y,T}^{\{U,M\}}(z,m)\mathrm{d}z\mathrm{d}m \\
&\qquad\qquad \times \mathbb{E}\left[\mathrm{e}^{\int_0^T h(V(s))\mathrm{d}s} \mid U(0)=y,\ U(T)=z,\ M(T)>\ell_R\right]. \qquad \text{(B.16)}
\end{aligned}
$$

Although the density function $f_{y,T}^{\{U,M\}}(z,m)$ is analytically known as shown in (B.7), we need to run some simulation to evaluate these terms since the expectation part is path dependent. Given that the initial and the terminal values are fixed, we may use techniques such as a Brownian bridge to reduce computational costs.

As for the third term on the RHS of (B.14), note that $U(\tau_{R_2}) = \ell_R$, hence

$$
\begin{aligned}
&\mathbb{E}_y^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_R}u_T(U(\tau_R))1_{\{\tau_R\le T\}}\right] \\
&\quad = \mathbb{E}_y^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_{R_1}}u_T(U(\tau_{R_1}))1_{\{\tau_{R_1}\le T\}}1_{\{M(\tau_{R_1})>\ell_R\}}\right] + u_T(\ell_R)\mathbb{E}_y^{\mathbb{Q}}\left[\mathrm{e}^{-r\tau_{R_2}}1_{\{\tau_{R_2}\le T\}}1_{\{\tau_{R_2}<\tau_{R_1}\}}\right] \\
&\quad = \int_{\ell_R}^{\infty}\int_{m}^{\infty}\int_{0}^{T} \mathrm{e}^{-rt} f_y^{\{\tau_{R_1},U(\tau_{R_1}),M(\tau_{R_1})\}}(t,z,m)u_T(z)\mathrm{d}t\mathrm{d}z\mathrm{d}m \\
&\qquad + u_T(\ell_R)\int_0^T\int_{t_2}^{T} \mathrm{e}^{-rt_2} f_y^{\{\tau_{R_1},\tau_{R_2}\}}(t_1,t_2)\mathrm{d}t_1\mathrm{d}t_2. \qquad \text{(B.17)}
\end{aligned}
$$

Joint probability density functions $f_y^{\{\tau_{R_1},U(\tau_{R_1}),M(\tau_{R_1})\}}(t,z,m)$ and $f_y^{\{\tau_{R_1},\tau_{R_2}\}}(t_1,t_2)$ need to be evaluated by simulation, which is not straightforward as both involve assessment of the rare-event probabilities.

As an alternative method, if we could approximate $v(x)$ by using $\hat{v}(x)$, it would be easier to obtain the firm value because we know that $\hat{v}(x)$ can be calculated numerically. Determine some time horizon $\hat{T}$ to run simulation, and then we obtain $v_{sim}(x)$ and $\hat{v}_{sim}(x)$ by the simulation as approximations of $v(x)$ and $\hat{v}(x)$. Unfortunately, these may not be good approximations given that $\hat{T}$ is finite, while our model assumes that the life of a bank is perpetual unless it goes bankrupt. However, we can find a function $\hat{g}(x)$ that satisfies

$$\hat{g}(x) = v_{sim}(x) - \hat{v}_{sim}(x), \tag{B.18}$$

and then we can assume that

$$v(x) \approx \hat{v}(x) + \hat{g}(x). \tag{B.19}$$

In this way, we can obtain the approximate value of $v(x)$.

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